Thrust in SCET at Next-to-leading Power

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1. Motivation and Introduction to Thrust

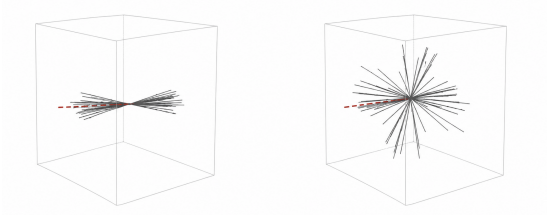
- Definition of the Thrust observable:

  [M. D. Schwartz, Quantum Field Theory and the Standard Model]

$$T \equiv \max_{\vec{n}} \frac{\sum_j \left|\vec{p}_j \cdot \vec{n}\right|}{\sum_j \left|\vec{p}_j\right|}, \quad \frac{1}{2} \leq T \leq 1$$

$$\tau \equiv (1 - T) \sim \lambda^2$$

- Examples of collider events with different Thrust values: *[T. Becher, 1803.04310]*

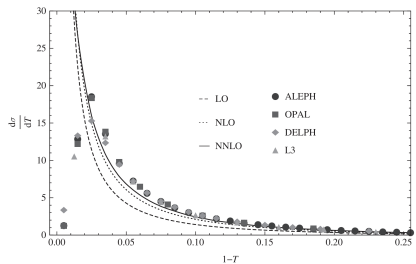


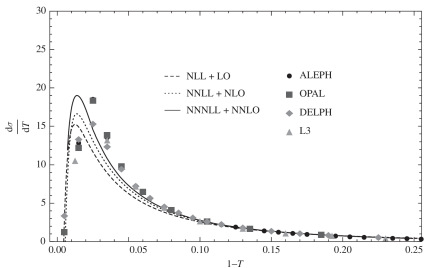
(a) $\tau \approx 0.002$ **(b)** $\tau \approx 0.35$

1. Motivation and Introduction to Thrust

- Fixed order computation of Thrust vs. Resummation of Thrust using SCET:

  [M. D. Schwartz, Quantum Field Theory and the Standard Model]





- Schematic presentation: *[R. A. Davison, B. R. Webber, 0809.3326]*

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \frac{1}{\tau}\left[\,\bar{\alpha}_s\left(c_{12}L + c_{11}\right) + \bar{\alpha}_s^2\left(c_{24}L^3 + c_{23}L^2 + c_{22}L + c_{21}\right) + \cdots\right] + \mathcal{O}(\tau^0)\,,\quad L = \log(\tau)$$

2. Introduction to SCET in the Context of Thrust

- Short introduction of SCET: We choose two light-like vectors (n and $\bar{n}$) for the decomposition of a momentum p:

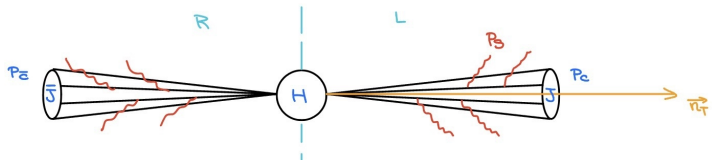
$$p^\mu = (n\cdot p)\frac{\bar{n}^\mu}{2} + (\bar{n}\cdot p)\frac{n^\mu}{2} + p_\perp^\mu = (n\cdot p,\ \bar{n}\cdot p,\ p_\perp^\mu)$$

- Scaling of hard, collinear, anti-collinear, soft region:

$$p_h^\mu \sim (1,1,1)\,Q\,,\ p_c^\mu \sim (\lambda^2,1,\lambda)\,Q\,,\ p_{\bar{c}}^\mu \sim (1,\lambda^2,\lambda)Q\,,\ p_s^\mu \sim (\lambda^2,\lambda^2,\lambda^2)Q$$

- Splitting of field in SCET: e.g. $\phi(x) \to \phi_c(x) + \phi_{\bar{c}}(x) + \phi_s(x)$
- In the case of Thrust, the reference vectors would be:

$$n^\mu = (1, \vec{n_T})\ ,\ \bar{n}^\mu = (1, -\vec{n_T})$$



[T. Becher, 1803.04310]

2. Introduction to SCET in the Context of Thrust

- Hard contributions into Wilson coefficients:

$$\boxed{\text{Full theory}} \xrightarrow{\text{matching}} \boxed{C(Q,\mu) \times J_{\text{soft+coll.}}}$$

- SCET Lagrangian:

$$\mathcal{L}_{\text{SCET}} = \underbrace{\mathcal{L}^{(0)}}_{\mathcal{O}(\lambda^0)} + \underbrace{\mathcal{L}^{(1)}}_{\mathcal{O}(\lambda^1)} + \underbrace{\mathcal{L}^{(2)}}_{\mathcal{O}(\lambda^2)} + \cdots$$

$$\mathcal{L}^{(0)} = \underbrace{\mathcal{L}_s^{(0)}}_{\text{soft}} + \underbrace{\mathcal{L}_c^{(0)} + \mathcal{L}_{\bar{c}}^{(0)}}_{\text{(anti-)collinear}} + \underbrace{\mathcal{L}_{s-c}^{(0)} + \mathcal{L}_{s-\bar{c}}^{(0)}}_{\text{(anti-)collinear–soft}}$$

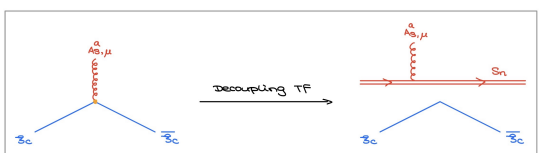
2. Introduction to SCET in the Context of Thrust

- Example: the collinear-soft interaction term:

$$\mathcal{L}^{(0)}_{s-c} = \bar{\xi}_c \frac{\not{n}}{2} in\cdot D\, \xi_c,$$
$$in\cdot D = in\cdot\partial + g\, n\cdot A_c(x) + g\, n\cdot A_s(x_-)$$

- Decoupling transformation (field-redefinition):

$$\xi_c(x) \to S_n(x_-)\,\tilde{\xi}_c(x),$$
$$A^\mu_c(x) \to S_n(x_-)\,\tilde{A}^\mu_c(x)\,S^\dagger_n(x_-),$$
$$S_n(x) = \mathrm{P}\exp\left[ig\int_{-\infty}^{0} \mathrm{d}s\; n\cdot A_s(x+sn)\right]$$



- After transformation:

$$\mathcal{L}^{(0)}_{s-c} \to \bar{\tilde{\xi}}_c \frac{\not{n}}{2} in\cdot \tilde{D}_c\, \tilde{\xi}_c$$

[C. W. Bauer, D. Pirjol, I. W. Stewart, 0109045 (2001)]

3. Factorization of Thrust using SCET

- Leading power (LP) scattering process:

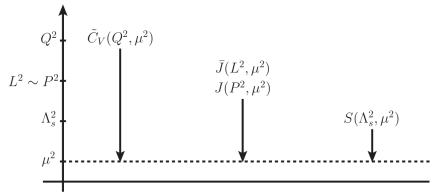
$$e^+e^- \to \gamma^* \to q\bar{q} \quad (+ \text{ soft gluon emissions})$$

- Factorization formula for thrust (2-jet):

[T. Becher, M. D. Schwartz, JHEP 07 (2008) 034, Eq. (1.4), 0803.0342]

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma_2}{\mathrm{d}\tau} = H\left(Q^2,\mu\right)\int \mathrm{d}p_L^2\,\mathrm{d}p_R^2\,\mathrm{d}k\, J\left(p_L^2,\mu\right) J\left(p_R^2,\mu\right) S_T\left(k,\mu\right)\delta\left(\tau - \frac{p_L^2+p_R^2}{Q^2} - \frac{k}{Q}\right) + \mathcal{O}(\tau)$$

- Resummation using RGEs for each function:



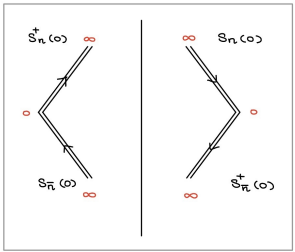
[T. Becher, arXiv:1803.04310]

4. Leading power (LP) soft function

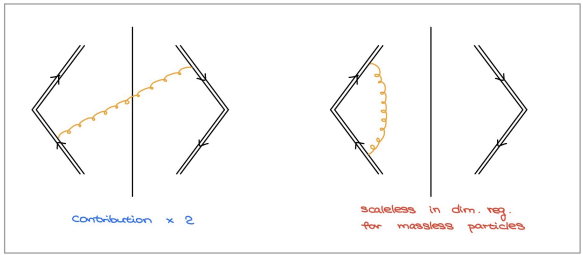
- Expression of leading power soft function: *[T. Becher, 1803.04310]*

$$S(k) = \frac{1}{N_c} \sum\!\!\!\!\!\!\int \langle 0| \left[S_{\bar{n}}^{\dagger}(0)\, S_n(0) \right]_{ab} |X_s\rangle \langle X_s| \left[S_n^{\dagger}(0)\, S_{\bar{n}}(0) \right]_{ba} |0\rangle \, \delta\left(k - n\cdot p_{X_s}^{R} - \bar{n}\cdot p_{X_s}^{L} \right)$$

- Corresponding LP Thrust soft function diagram: (presentation of LO and NLO)



LO at $\mathcal{O}(\alpha_s^0)$



NLO at $\mathcal{O}(\alpha_s^1)$

5. Motivation for next-to-leading power (NLP)

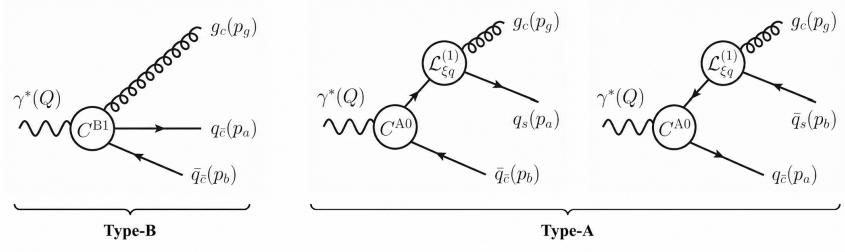
- The leading-power (LP) resummation of thrust has been well studied:
  1. N^3LL resummation of LP Thrust using SCET:
     [T. Becher, M. D. Schwartz, 0803.0342]
  2. N^3LL resummation + N^4LL effects using conventional QCD resummation in Laplace space:
     [U. G. Aglietti et al., Phys. Rev. Lett. 134 (2025) 251904]
- The correction from NLP might be more significant than higher resummation accuracy of logs at LP.

6. Factorization of Thrust at Next-to-leading Power

- Next-to-leading power (NLP) scattering process: (starting with $\mathcal{O}(\alpha_s)$)

$$e^+e^- \to \gamma^* \to [q\bar{q}]_{\bar{c}} + [g]_c \quad (+ \text{ soft gluon emissions})$$

- Two configurations of 2-jet "off-diagonal" gluon-thrust amplitude:



[M. Beneke et al., 2205.04479]

6. Factorization of Thrust at Next-to-leading Power

- Type A factorization formula of differential cross section:

$$\frac{1}{\sigma_0}\frac{\widetilde{\mathrm{d}\sigma}}{\mathrm{d}s_R\,\mathrm{d}s_L}\Big|_{\text{A-type}} = \frac{2C_F}{Q}\,f(\epsilon)\,\left|C^{\text{A0}}\left(Q^2\right)\right|^2\,\widetilde{\mathcal{J}}_{\bar{c}}^{(\bar{q})}\left(s_R\right)\int \mathrm{d}\omega\,\mathrm{d}\omega'$$
$$\times\left\{\widetilde{\mathcal{J}}_c\left(s_L,\omega,\omega'\right)\widetilde{S}_{\text{NLP}}\left(s_R,s_L,\omega,\omega'\right)+\widetilde{\hat{\mathcal{J}}}_c\left(s_L,\omega,\omega'\right)\widetilde{\hat{S}}_{\text{NLP}}\left(s_R,s_L,\omega,\omega'\right)\right\}$$

[M. Beneke et al., 2205.04479]

- After decoupling transformation, the collinear-soft interactions still remain.

7. Next-to-leading power (NLP) soft function

- Type A NLP soft function:

$$
\begin{aligned}
&S_{\text{NLP}}\left(l_+, l_-, \omega, \omega'\right) \\
&= g_s^2 \int \frac{\mathrm{d}x_-}{2\pi} \frac{\mathrm{d}x'_-}{2\pi} e^{-i(x_-\omega - x'_-\omega')} \langle 0| \overline{\mathrm{T}} \left\{ \left[S_{\bar{n}}^\dagger(0)\, S_n(0) \right]_{cb'} \left[S_n^\dagger\, q_s \right]_{\alpha' a'} \left(x'_-\right) \right\} \\
&\qquad \times \mathcal{P}_s\left(l_+, l_-\right) \mathrm{T} \left\{ \left[\bar{q}_s\, S_n \right]_{\alpha a} \left(x_-\right) \left[S_n^\dagger(0)\, S_{\bar{n}}(0) \right]_{bc} \right\} |0\rangle
\end{aligned}
$$

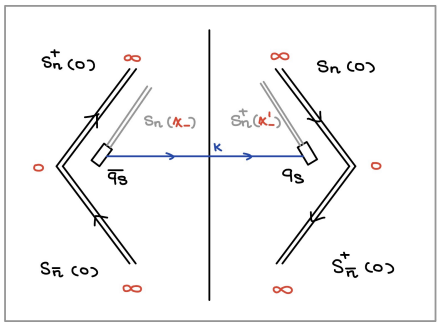
- Measurement function:

$$
\mathcal{P}_s\left(l_+, l_-\right) = \sum_{X_s} \int d\mathrm{PS}_{X_S} \delta\left(l_+ - \bar{n}\cdot p_{X_s}^R\right) \delta\left(l_- - n\cdot p_{X_s}^L\right) |X_s\rangle \langle X_s|
$$

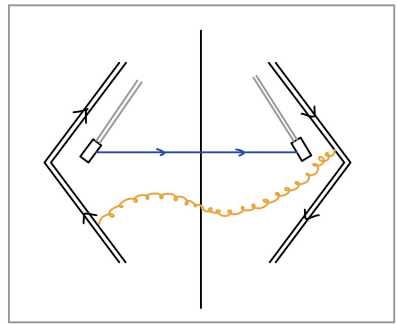
[M. Beneke et al., 2205.04479]

7. Next-to-leading power (NLP) soft function

- Soft function of Thrust at NLP starts with $\mathcal{O}(\alpha_s)$:



- In progress: computation of soft function contributing to $\mathcal{O}(\alpha_s^2)$

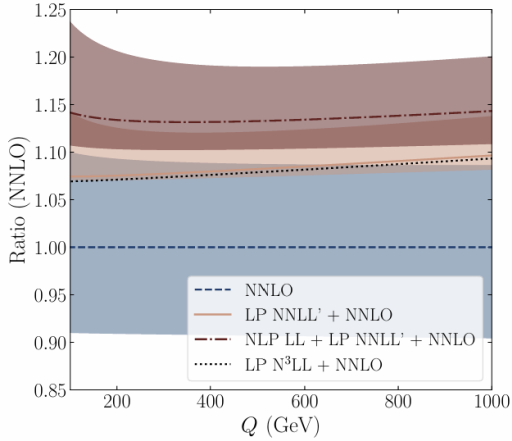


8. Conclusion

- **Small-τ thrust:** Fixed-order perturbation theory contains large logarithms. Resummation is needed.
- **SCET factorization:** SCET provides a systematic factorization of the thrust distribution into hard, (anti-) collinear and soft contributions.
- **Leading vs. next-to-leading power:** At LP, soft and collinear sectors decouple, while at NLP, additional soft-collinear interactions remain.
- **NLP soft function:** We studied its structure and we are calculating its contribution at $\mathcal{O}(\alpha_s^2)$ as a step towards NLP resummation.

Thank you for your attention.

Backup Slide: Motivation for next-to-leading power

- The correction from NLP might be more significant than higher resummation accuracy of logs at LP:

  example borrowed from Higgs production: [M. van Beekveld et al., JHEP 05 (2021) 114, arXiv:2101.07270]