Geneva, 29th October 2025

Tracking & timing performance of SNSPDs under 160 GeV SPS pion beams

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Warsow University of Technology: Georgy Kornakov, Blazej Zylinski

•Introductions (10+ years in R&D)….

- **Process Tunning via SiMS**
- **Gallium Process Developpement**
- **Dopant Radiation Studies**

SiMS measurements & Simulation, Varied bias rail geometry structures characterization and TCAD simulation	24th RD50 Workshop @ Bucarest
Doping profile Simulations and measurements	25th RD50 Workshop @ CERN
Study of irradiated NinN production and LGAD doping profiles	26th RD50 Workshop @ Santander
Test beam and clean room studies of ATLAS PPS modules with alternative bias rail geometries	26th RD50 Workshop @ Santander
LGAD and irradiated doping profiles	27th RD50 Workshop @ CERN
Neutron Irradiated doping profile evaluation	28th RD50 Workshop @ Torino

- **LGAD Timing studies**
- **Carbon process development**
- **First documented observation of SEB**

Timing performance and gain analysis of heavily irradiated LGAD diodes	30th RD50 Workshop @ Krakow
Radiation hardness of 6" SoI CNM LGADs	35th RD50 Workshop @ Krakow
Acceptor removal and gain Reduction in proton and neutron irradiated LGADs	36th RD50 Workshop @ CERN
Efficiency estimation on irradiated LGAD with respect to sensor stability	37th RD50 Workshop @ Zagreb
Comprehensive MIP particle measurement and analyses system	37th RD50 Workshop @ Zagreb

- **3D detectors for timing**
- **Bandwidth induced bias**
- **Radiation up to 1e17**

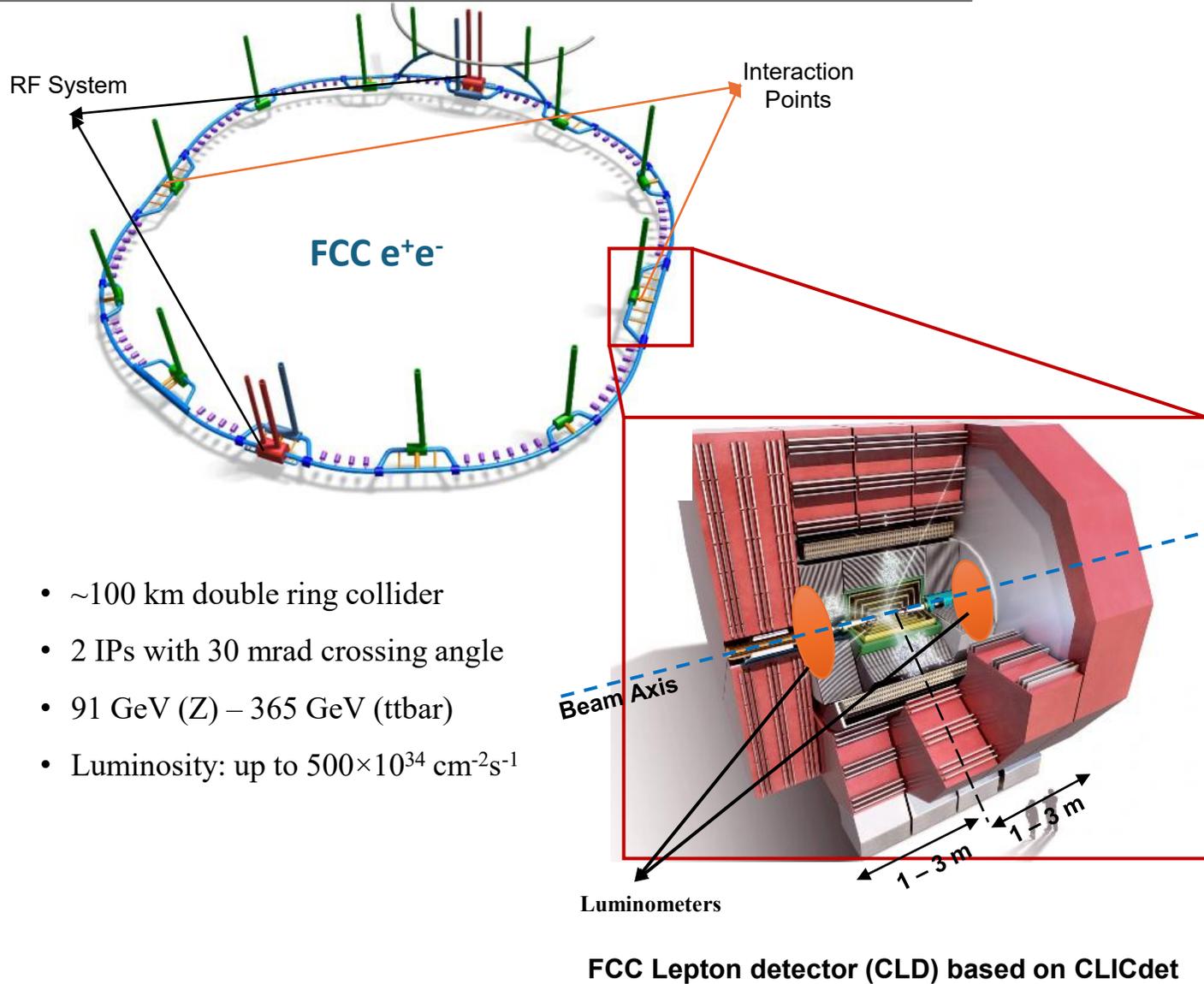|	
Comparative doping profile evaluation of Carbonated LGADs	39th RD50 Workshop@ Valencia
Time resolution of single cell 3D devices on SPS pion beams	40th RD50 Workshop @ CERN
Time resolution and field uniformity study of single cell 3D pixel structures neutron and proton irradiated	43th RD50 **W**orkshop @ CERN

•Introduction and Scope

RF System

FCC e⁺e⁻

Interaction Points

- ~100 km double ring collider
- 2 IPs with 30 mrad crossing angle
- 91 GeV (Z) – 365 GeV (ttbar)
- Luminosity: up to 500×10^{34} cm^{-2}s^{-1}

Beam Axis

Luminometers

1 – 3 m   1 – 3 m   1 – 3 m

FCC Lepton detector (CLD) based on CLICdet

SN-SPDs based Luminometer

✓ Sub-per mill resolution through small angle Bhabha scattering
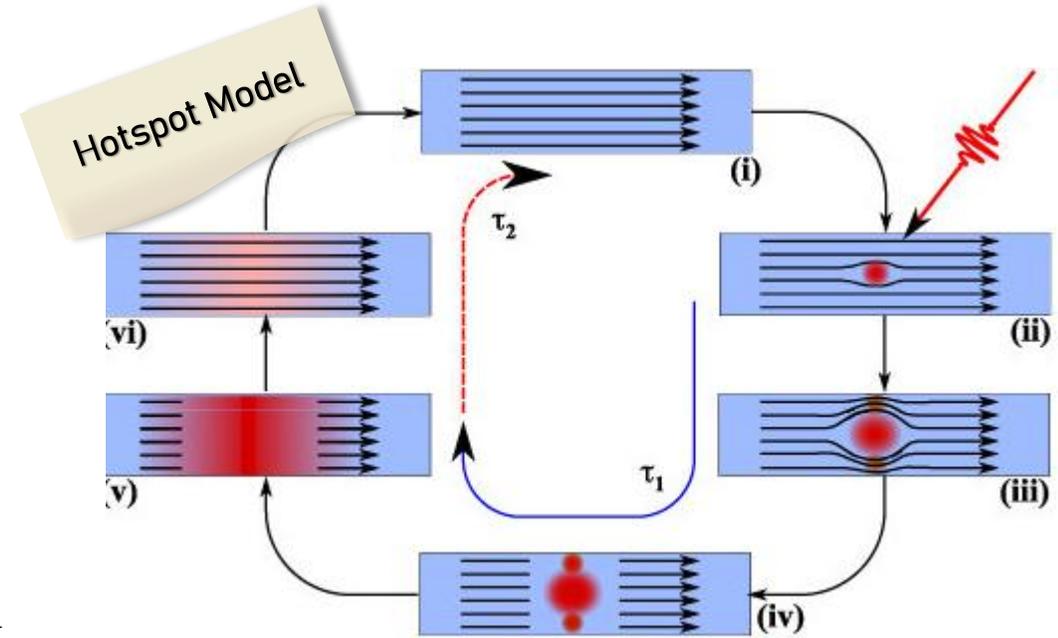
$$\sigma = \frac{16\pi\alpha^2}{s}\left(\frac{1}{\theta_{\min}^2} - \frac{1}{\theta_{\max}^2}\right)$$

✓ Goal of 10^{-4} precision on absolute luminosity
✓ Readout at Bunch-crossing rates with < 20ns shaping time
✓ 1.5 MGray dose at Z pole
✓ Pile-up ~ 1E-3 / event
✓ σ(Z) ~ 40nb, σ(Bhabha)~14 nb

Current project:

- **Test pilot of MgB_2, $MoSi$, $La_{2-x}Sr_xCuO_2$, $NbTiN$ as possible materials**

- **Differentiate between poly- / mono crystalline and amorphous material response**

- **Test different widths of nano-wire to characterise high-rate capability and recovery time**

•Superconductive Nanowire SPD

➢ ultrathin, ~100-nm-wide superconducting strip on Si /SiN /SiC /sapphire/ diamond / substrate

➢ Current biased at a few K, operated just below it's transition point

➢ Single photon triggers brief resistive state, high efficiency at telecom bands, picosecond-scale timing, and ultra-low dark counts

➢ Absorbed photon locally breaks Cooper pairs forming nonequilibrium quasiparticle

➢ Current crowds around hotspot, exceeding the despairing current, resistive belt

Hotspot Model

Cryostat
SNSPD
Bias-tee
Amplifier (G)
Read-Out Chain

L_K $R_n(t)$

Z_L

I_{bias}

$t_{Rise}=L_K/(Z_L+R_n(t))$
$t_{Decay}=L_K/Z_L$
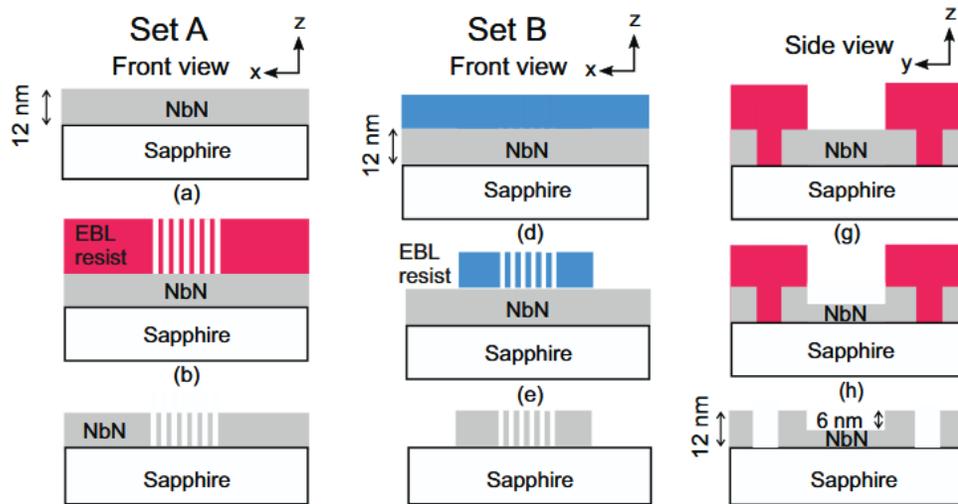
} **Timing Characteristics**

•Superconductive Nanowire SPD

Film Growth
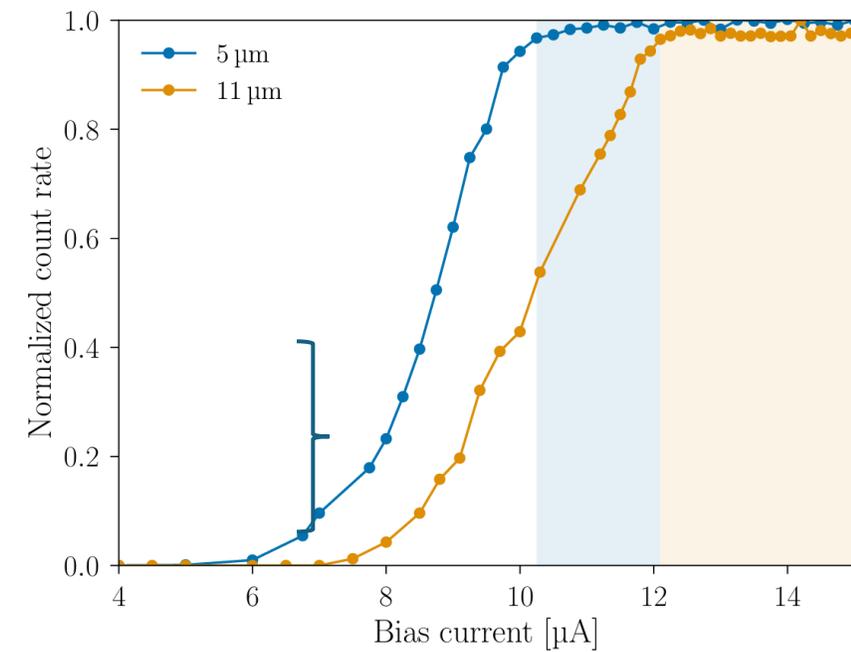- Deposition with sputtering tools
- Growth by chemical vapor deposition
- Atomic layer deposition
- Molecular beam epitaxy
- Laser ablation (NbN)

Patterning
- EBL-Electron beam lithography
- PL-Photolithography
- CE-Chemical etching
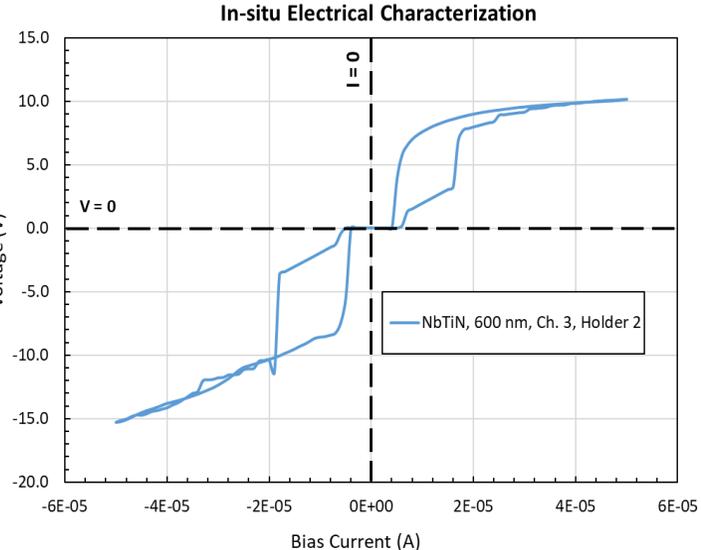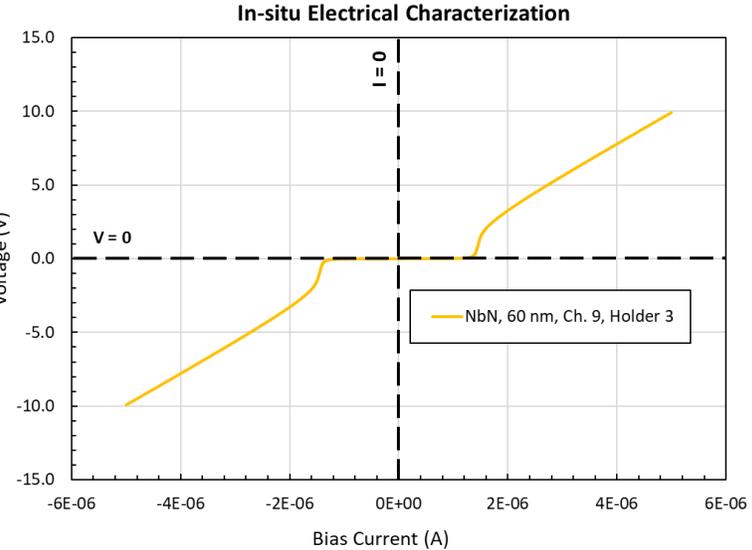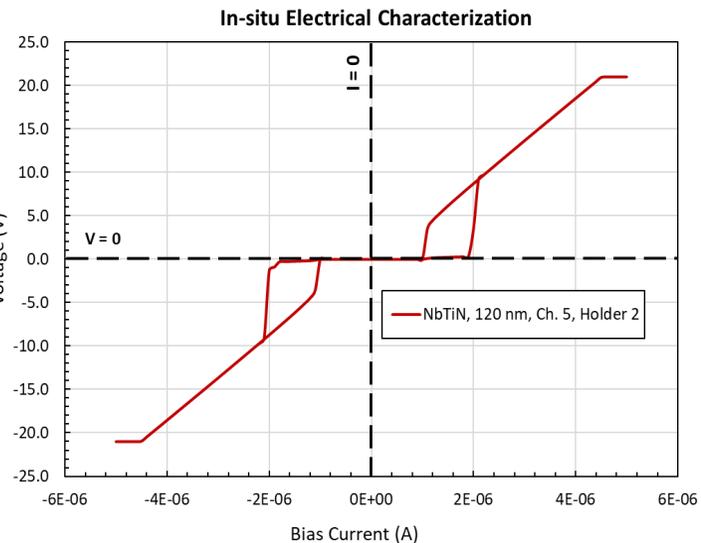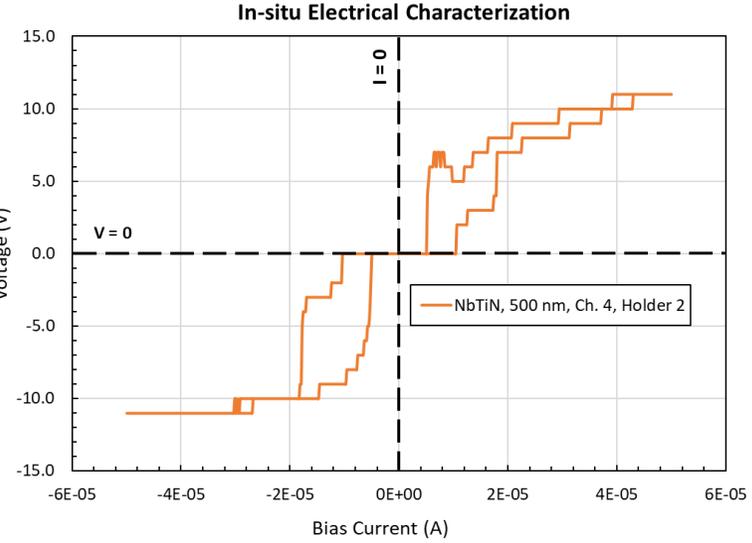- IBE-Ion beam etching
- FIB-Focused ion beam

(poly-) ccrystalline

Materials	NbN	NbTiN	WSi	TaN	MoGe	MoSi	NbSi	MgB$_2$	YBCO
Bulk T$_c$ (K)	16	17	5	5	7.4	7.5	3.1	39	93
T$_c$ (K)	8.6	9.6	3.7	8.3	4.4	4.2	2	39	85.5
Thickness (nm)	(3nm)	(4.5nm)	(4.5nm)	(5nm)	(7.5nm)	(4nm)	(10nm)	(350nm)	(3 unit cell)
Band gap 2Δo (meV)	4.9	5.17	1.52	1.52	2.2	2.28	0.94	11.86	28.29
J$_C$ (MA/cm^2) Meas. Temp. (K)	2-4 (4.2K)	8 (2.9 K)	0.8 (250 mK)	4 (4.2 K)	1.2 (250mK)	1.1– 2.5 (1.7 K)	0.14 (300mK)	1.1 (3 K)	14.4 (78 K)

•Electrical Characterization

In-situ Electrical Characterization — NbTiN, 500 nm, Ch. 4, Holder 2

In-situ Electrical Characterization — NbTiN, 120 nm, Ch. 5, Holder 2

In-situ Electrical Characterization — NbN, 60 nm, Ch. 9, Holder 3

In-situ Electrical Characterization — NbTiN, 600 nm, Ch. 3, Holder 2

In-situ Electrical Characterization — NbTiN 1 μm, Ch.13, Holder 2

$$\delta V = IR_0 \times \left[\frac{1}{R_0}\left(\frac{dR}{dT} \right) \right] \times \delta T$$

Voltage Variation — **Bias Current** — **Material Resistance** — **Temp. dependent resistance change** — **Temperature variation**

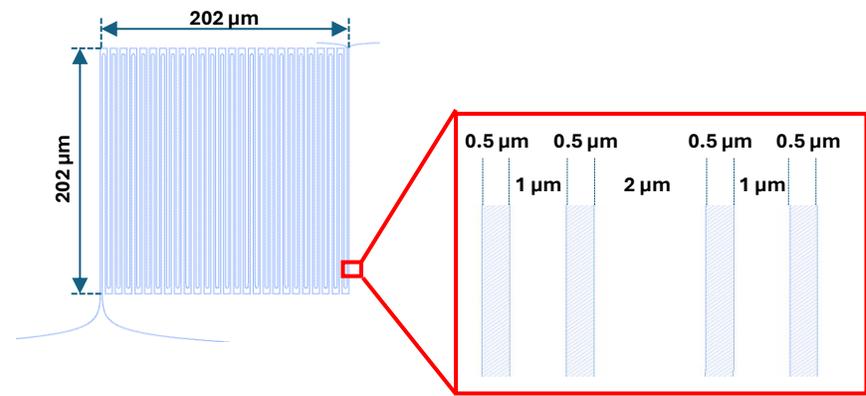**Positive sweep:** Slow I_{bias} ramp-up until sharp voltage jump (I_c)

Retrace: Decrease I_{bias} until voltage drop to 0 (I_h)

Negative sweep: Negative I_{bias} ramp to voltage transition ($-I_c$)

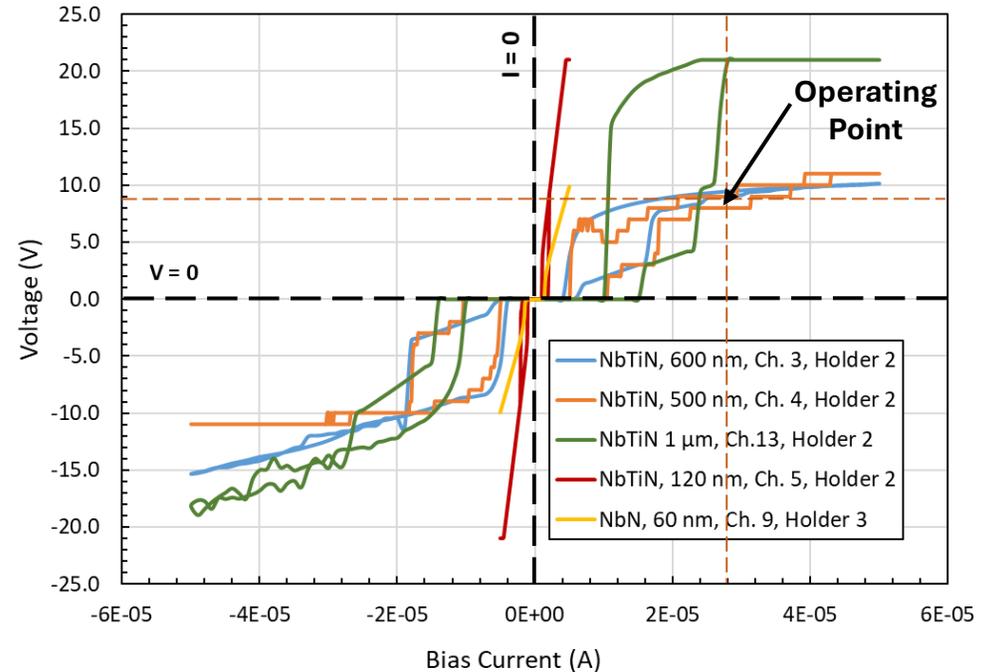**Retrace to 0:** Decrease negative I_{bias} until voltage drop to 0 ($-I_h$)

•Samples and Materials

DUT Plane	Material	Wire Width	R (300 K)	Cryostat Channel	Super-conductive	Osc. Chan.	Particle Induced Signals
Holder no. 1 Last in beam direction	NbTiN	100 nm	123 kΩ	1	No	X	X
		100 nm	1.8 MΩ	2	No		
		100 nm	590 kΩ	15	No	X	
		100 nm	153 kΩ	16	No	X	X
Holder no. 2 Middle	NbTiN	600 nm	2.61 MΩ	3	Yes	Ch2, Osc1	No
		500 nm	1.58 MΩ	4	Yes	Ch3, Osc1	Yes
		120 nm	3.27 MΩ	5	Yes	Ch4, Osc1	No
		400 nm	1.01 MΩ	12	No	X	X
		1 µm	1.25 MΩ	13	Yes	Ch3, Osc2	No
		100 nm	1.9 MΩ	14	No	X	X
Holder no. 3 First in beam direction	NbN	110 nm	2.31 MΩ	6	No	X	X
		90 nm	770 kΩ	7	No	X	X
		120 nm	1.44 MΩ	8	No	X	X
		60 nm	1.71 MΩ	9	Yes	Ch2, Osc2	No
		80 nm	2.74 MΩ	10	No	X	X
		100 nm	2.8 MΩ	11	No	X	X

No temperature monitoring!!

➤ Double loop geometry with 51 meandering branches

➤ 7 nm active layer thickness deposited on 400 µm Poly-Si substrate atop the native SiO_2 (2 nm)

In-situ Electrical Characterization

➤ The 500 nm wire width NbTiN sample operated at 28 - 30 µA @ 9.3 V (5th metastable resistive state after I_c, phase slips, vortices, inhomogeneities)

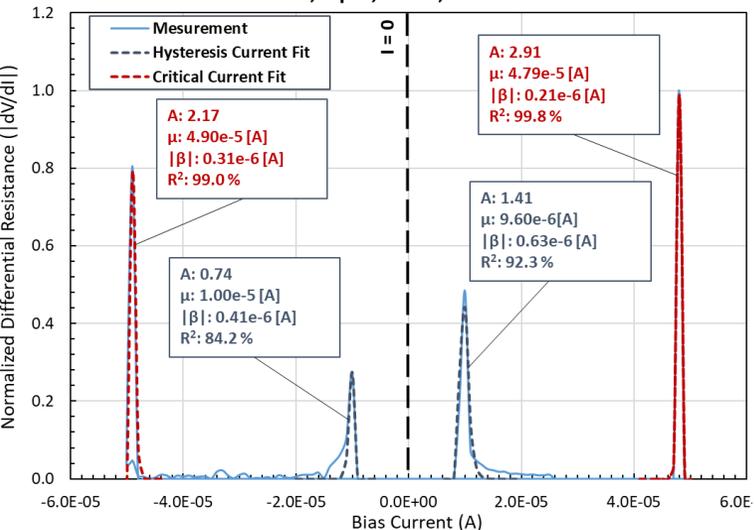➤ 600 nm wire width NbTiN sample operated at 14.0 µA @ 10 V (no particle induced signals, holder 2)
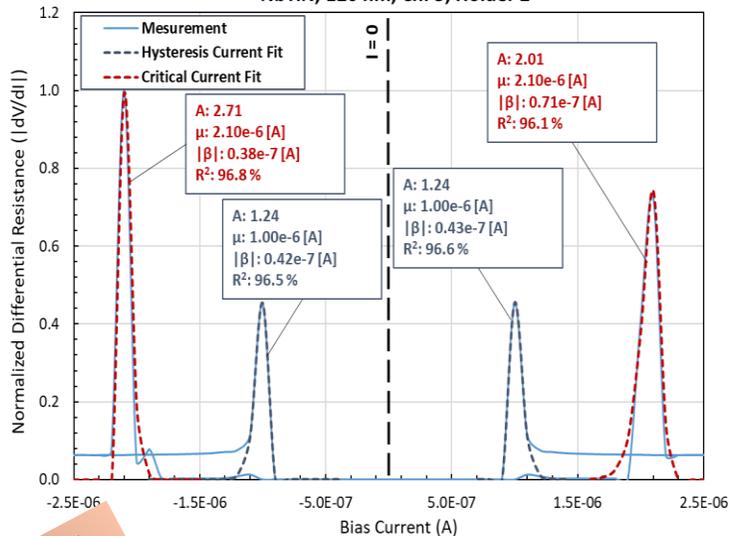
•Samples and Materials

➢ Differential resistivity peaks at Critical / Hysteresis current

➢ Asymmetric distribution driven by stochastic effects and quantum phase slips

➢ Mirrored Gumbel PDFs, covariant matrix uncertainties

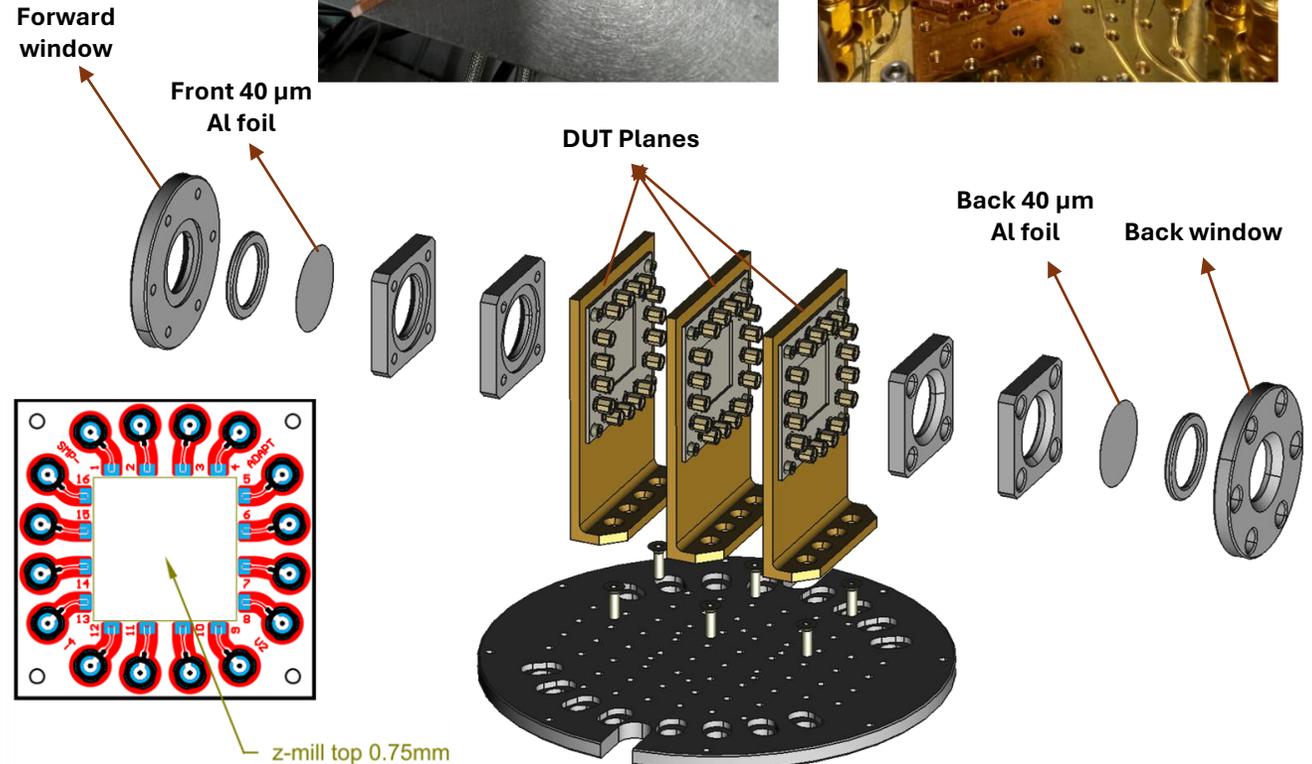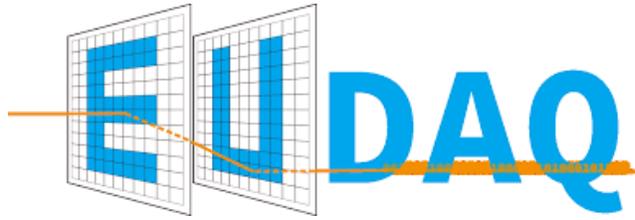| Sample	Critical Current [A]	Hysteresis Current [A]
600 nm, NbTiN	$(1.81 \pm 0.10) \times 10^{-5}$	$(4.04 \pm 0.49) \times 10^{-6}$
500 nm, NbTiN	$(1.06 \pm 0.01) \times 10^{-5}$	$(5.01 \pm 0.10) \times 10^{-6}$
120 nm, NbTiN	$(2.1 \pm 0.03) \times 10^{-6}$	$(1.00 \pm 0.03) \times 10^{-6}$
60 nm, NbN	$(1.50 \pm 0.03) \times 10^{-6}$	$(1.40 \pm 0.06) \times 10^{-6}$
1 µm, NbTiN	$(4.83 \pm 0.05) \times 10^{-5}$	$(9.87 \pm 0.52) \times 10^{-6}$

•DUT Holder & Assembly

Material	Density (kg/m³)	Glass Transition (°C)	Melting Point (°C)	Electrical Conductivity (Ω*m)⁻¹	Volume Resistivity (Ω · cm)	Thermal conductivity W/(m·K)	Thermal expansion coefficient K⁻¹	Dielectric Strength (kV/mm)
Aluminum	2700	N / A	660.3	$\sim 3.5 \times 10^7$	7.2×10^{-5}	151–202	16–18×10^{-6}	N / A
Stainless Steel A2	8000	N / A	$1400 - 1450$	$\sim 1.4 \times 10^6$	3.99×10^{-8}	14–17	23.6×10^{-6}	N / A
Nitril	1200	-29	250 - 300	$\sim 1 \times 10^{-8}$	$> 10^{15}$	0.25	170×10^{-6}	20 -30
AlN	3255	N / A	2200	$\sim 10^{-12} - 10^{-11}$	10^{14}	> 170	$\sim 4.5 \times 10^{-6}$	15
Al₂O₃	3950	N / A	2.072	$\sim 10^{-14} - 10^{-13}$	10^{15}	25 - 35	~ 7.5–8.5×10^{-6}	13 - 30
Silicon	2329	N / A	1412	0.005	2×10^4	150	2.6×10^{-6}	500 - 800
Capton	1420	360 - 410	500 decomposition	$\sim 10^{-16} - 10^{-15}$	10^{17}	0.12	20×10^{-6}	150 - 300

➢ FR4 PCB with 16 SMP high bandwidth connectors and a 2.0 x 2.0 central window

➢ Cu Sample holder with 2.0 x 2.0 cm center opening for beam passthrough and multiple scattering mitigation

➢ AlN sample support with 12 mm overlap with coper support on all sides and 750 um central thickness

➢ 40 um thick Al windows at entre/exit of the cryostat and no intermediate barriers for a total thickens of 2 x 40 um Al.

Forward window
Front 40 µm Al foil
DUT Planes
Back 40 µm Al foil
Back window
z-mill top 0.75mm

•EUDEQ Telescopes @ CERN

- 6 Mimosa26 COMS planes featuring 4 μm resolution
- EUDAQ 1/2 and AIDA TLU 1/2 as options
- Can be booked as an option in PS/SPS beam request
- 35 weeks access time across all telescopes

ACONITE

- ✓ Permanently installed in SPS/H6A
- ✓ Equipped with cold box allowing stable -60°C using two ethanol cooled deep temperature chillers
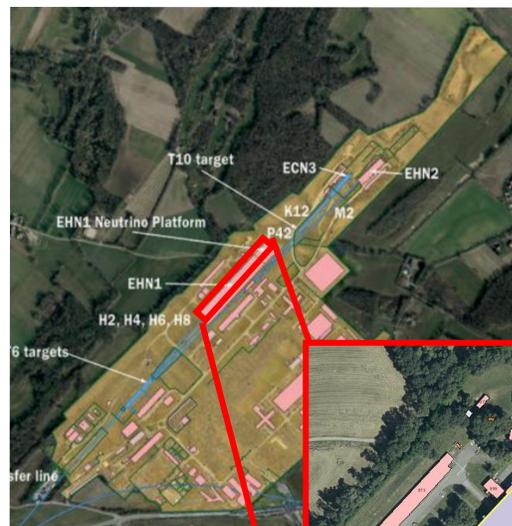- ✓ Currently maintained and used by ATLAS Pixels

AIDA

- ✓ Permanently installed in SPS/H6B
- ✓ Upgraded with 500 kg micrometric stage and -70°C cold box
- ✓ 2-stage ethanol cooling system with 5 kC cater chiller

AZALEA
- ✓ Mobile telescope
- ✓ Primarily used at PS but also transported to SPS

General Information: H6B

Schedule Runs SPS H6 1.1.1 :: Status 2025-04-22 08:19 (UTC)

Building 887

CERN Prévessin

Area: SPS – North Area (NA)
Beam line: H6B
Access Door: PPE156
Counting Room: 887/1-Q54

AIDA Telescope in H6B Current setup

- 6 MIMOSA 26 planes
- One 2-axis ILE20 stage, up to 500 kg support

The Setup

- AIDA Telescope
- Closed Cycle He Cryostat
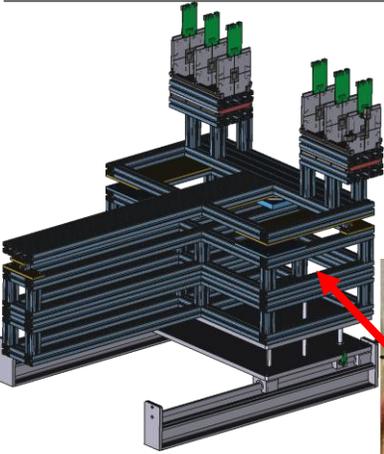- 2 x LGAD timing planes on UCSC single channel board
- Pixelated alignment & ROI plane

•Setup @ SPS

•Setup @ SPS

New 500 Kg Stage Installation (part of AIDA WP2 project)

Modified telescope frame, 1.4 m back extension

Instrumentation Rack

Turbo-Molecular vacuum pump (9 ×10⁻⁹ bar)

3-phase, He₂ , water cooled Cryo-Compressor

Dedicated Trigger & synchronization Board

•AIDA Project infrastructure

Problem: Low temperature environment needed for leakage current mitigation at high fluences $>10^{15}$ n_{eq}/cm^2 and charge carrier mobility for impact ionization-based technologies (LGADs)

Create a common framework within AIDA INNOVA WP2 to implement necessary infrastructure
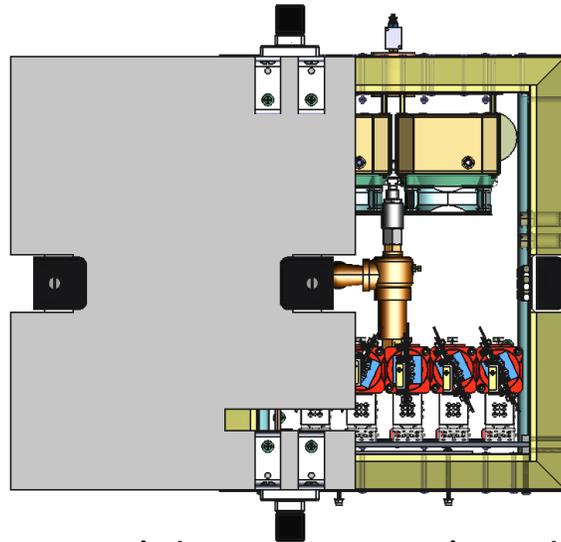
2-stage deep Temperature Chiller

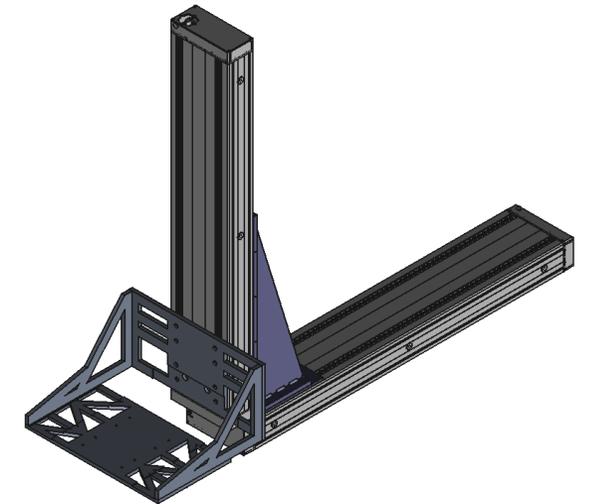Hitema ENR 0.10,
9.3 kW water chiller

Hubert P815W,
1.2 kW, -80 °C

Controllable multi-plane cold box

Low material, permanent installation, removable DUT assembly

High-capacity precision stage, ILE

34.2 kN dynamic carrying load, 5 mm pitch, and 1 m travel range
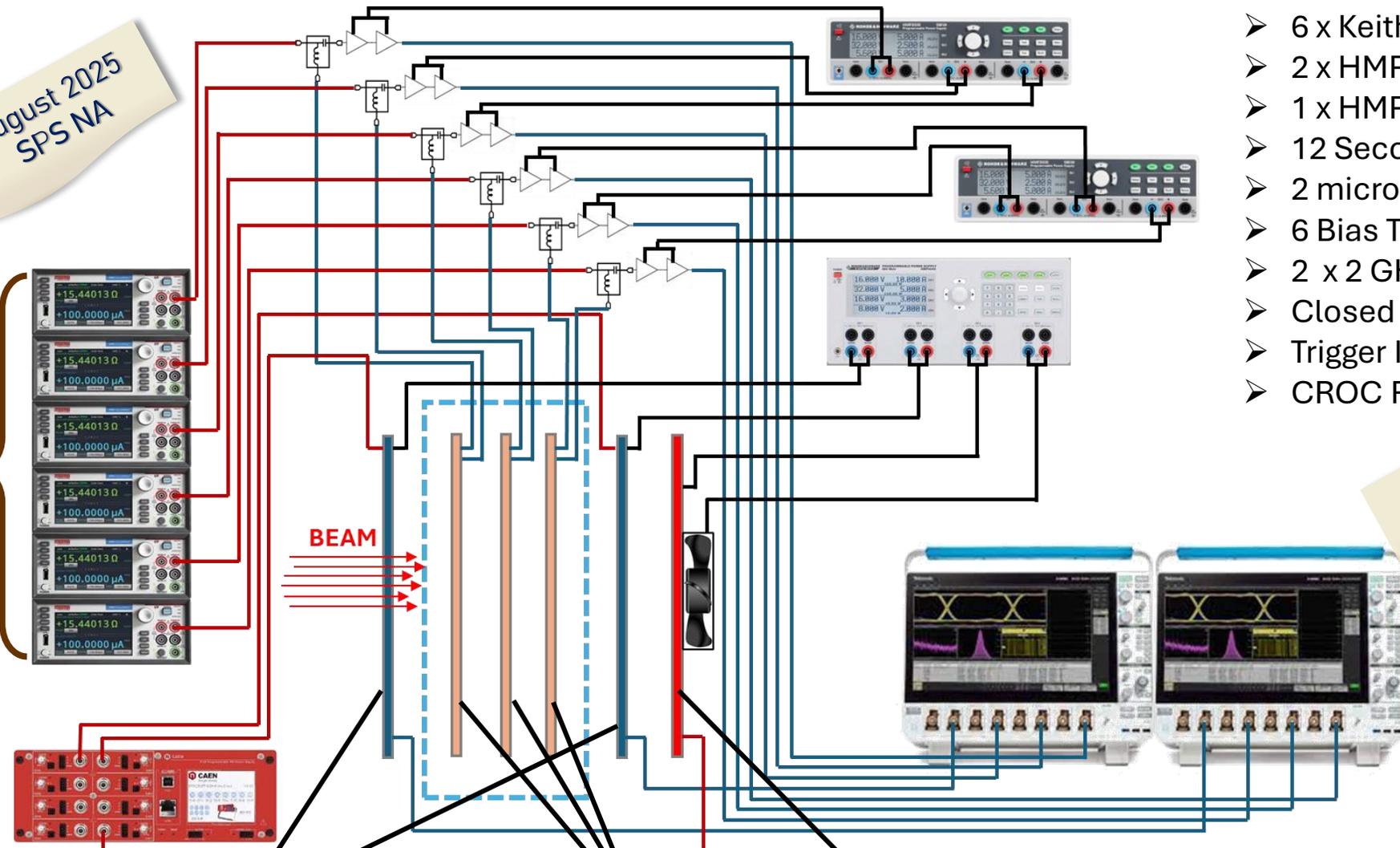
•Instrumentation & Readout

August 2025
SPS NA

Equipment

- ➢ 2 x 6 GHz Oscilloscopes (40 Gs/s)
- ➢ 6 x Keithley 2450 SMUs (10 pA resolution)
- ➢ 2 x HMP2030 LV PSUs
- ➢ 1 x HMP404 LV PSU
- ➢ 12 Second stage amplifiers (3 GHz)
- ➢ 2 micro-positioning stages
- ➢ 6 Bias Tees
- ➢ 2 x 2 GHz Bandwidth 2nd stage amplifiers
- ➢ Closed Cycle He CRYOSTAT
- ➢ Trigger Interface Board V2.0
- ➢ CROC RoI pixelated plane + FC7 Readout

Signal Chain

Current SMUs for SNSPD biasing

HV PSU for Si planes

BEAM

LGAD Planes for timing

SNSPD planes

Pixelated RoI plane

Amplifier: ZX60-3018G-S+
- Bandwidth: 3GHz
- Output power: 12.8dBm
- Max current: 45mA
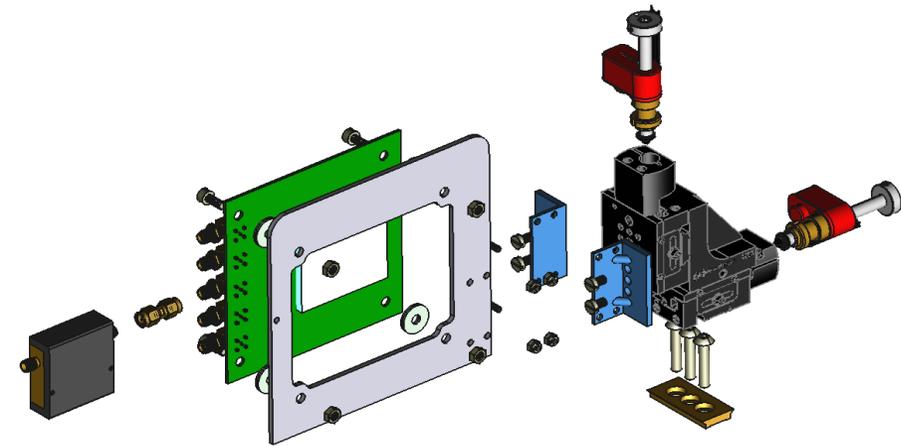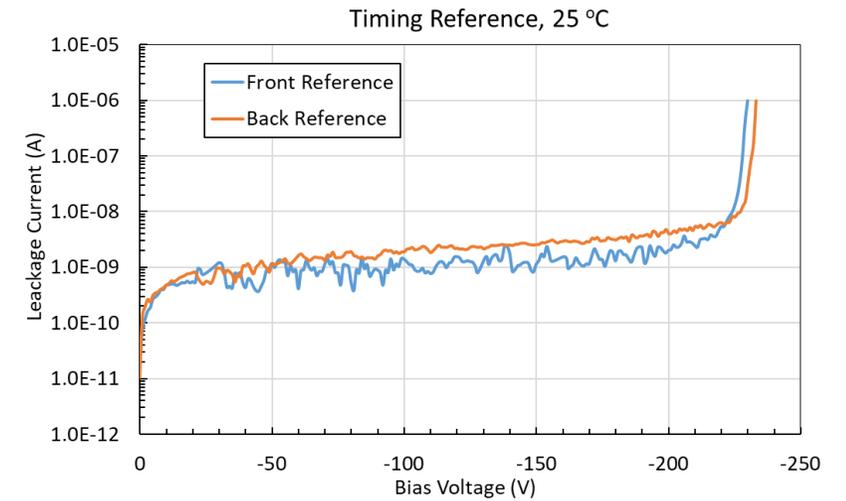- Max voltage: 12.5V

Bias Tee: ZX85-12G-S+
- Max voltage: 25V
- Max current: 400mA
- Insertion Losses: 0.6 dB
- Bandwidth: 0.2 MHz – 12 GHz

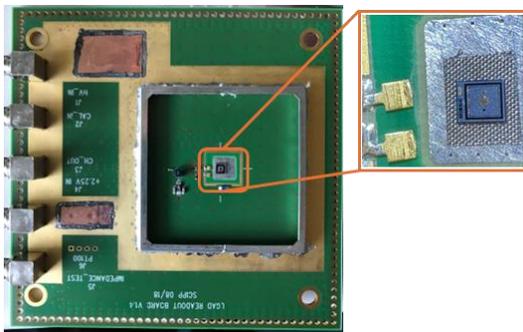# •Reference Timing Planes

Sensors

- 2 x 1 mm2, 50 um active thickness HPK LGADs (HPK-LGAD-L20-P3 front & HPK-LGAD-L20-P4 back)
- Breakdown ~ 230 V, depletion ~25 V
- Operated at fixed 150 V bias @ 25 C throughout the campaign
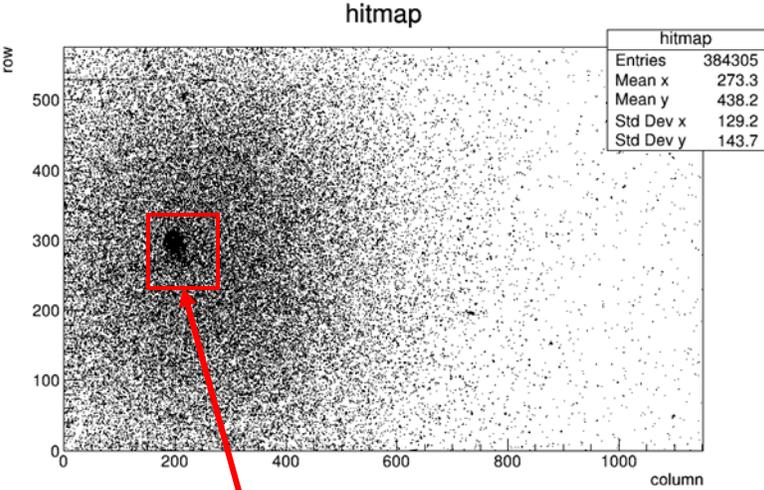- One plane per oscilloscope for ps-level synchronization

Timing Reference, 25 °C

- 30 nm precision 2-axis (Y-Z) movement through piezo-electric stages
- Precision alignment within a 2 x 2 mm maximum range
- Geometric overlap between reference planes and DUTs for three object coincidence timing system
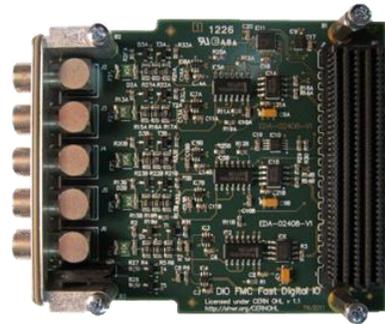
1st Stage Amplifier

- High frequency SiGe (~12 GHz) common emitter first stage charge amplifier (470Ω trans-impedance)
- Fully enclosed faraday cage surrounding sensor
- RMS sensor + amplifier noise < 1.8 mV, gain of 10
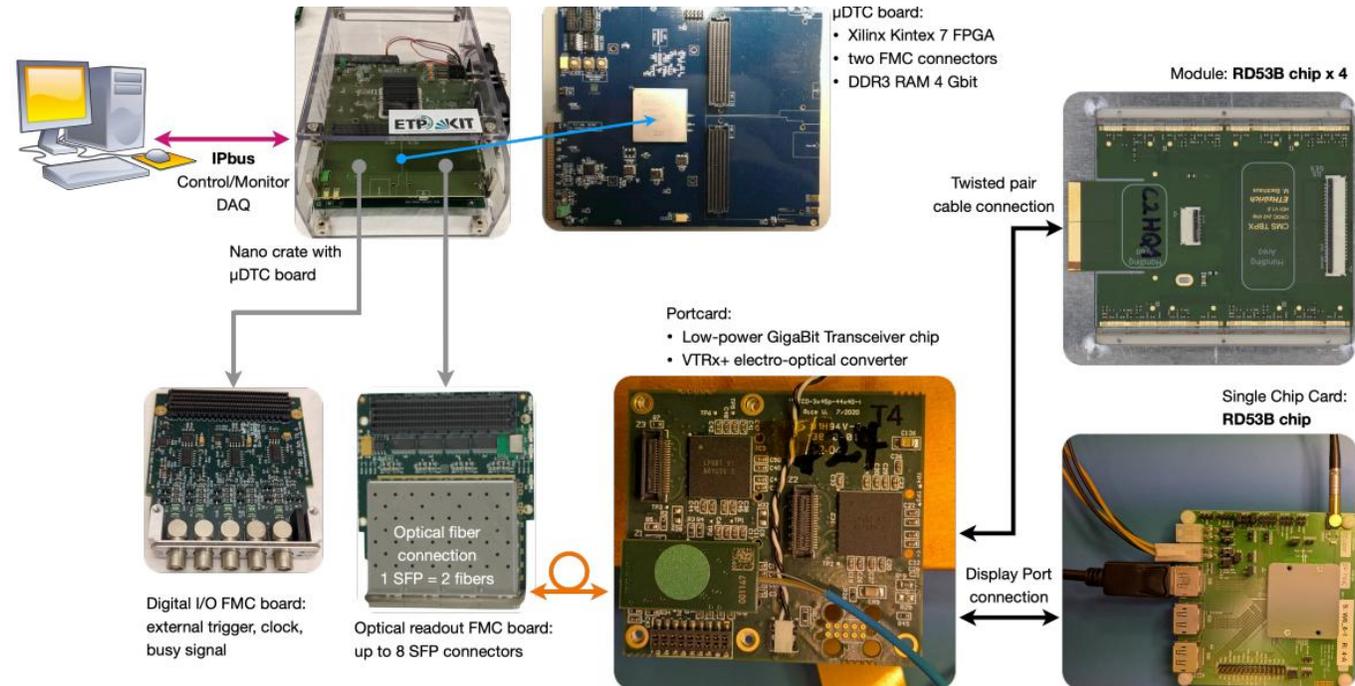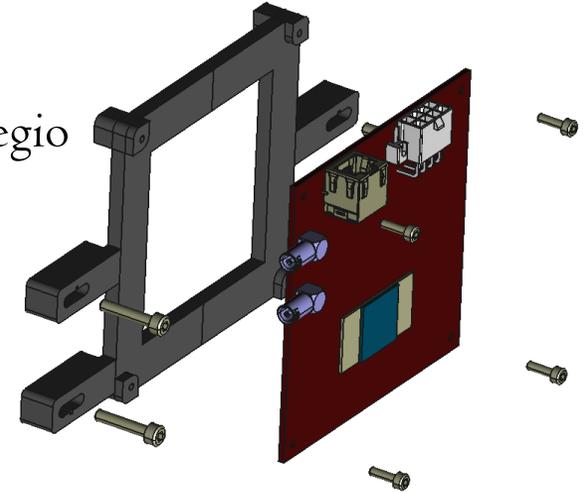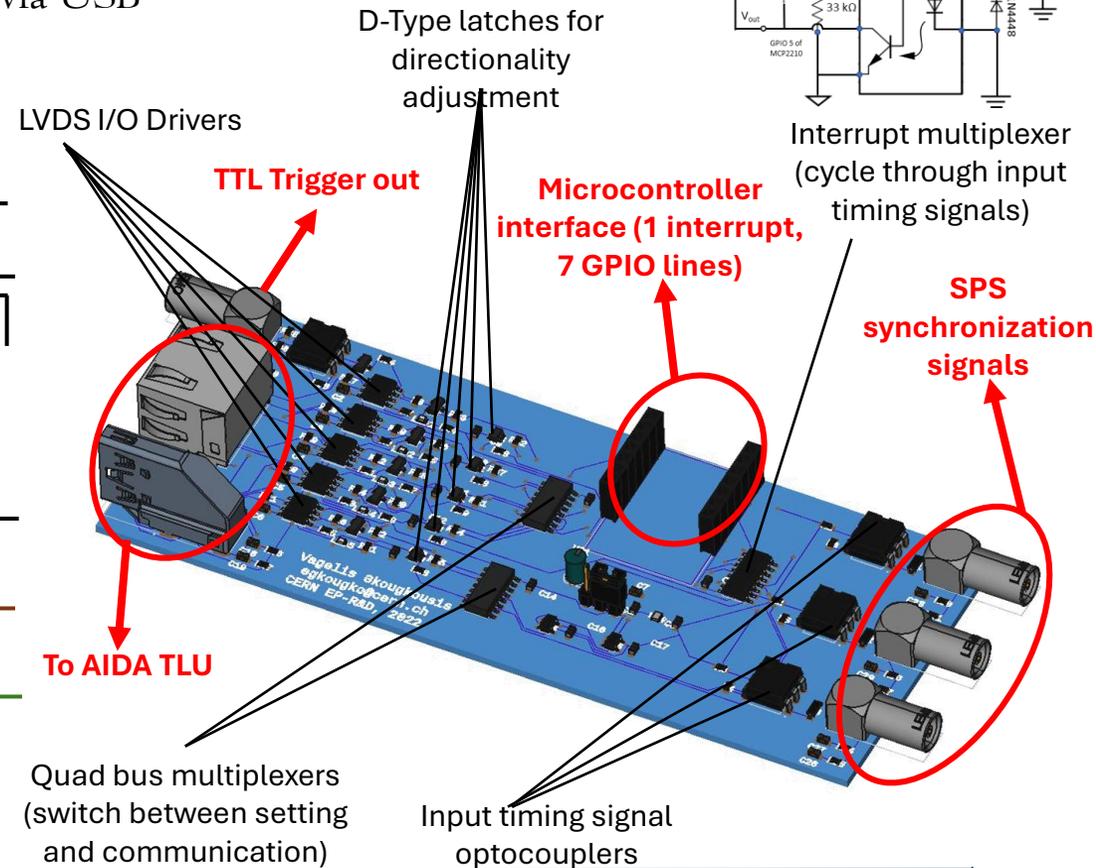
•Triggering Scheme & ROI

hitmap

hitmap	
Entries	384305
Mean x	273.3
Mean y	438.2
Std Dev x	129.2
Std Dev y	143.7

200 x 200 µm ROI
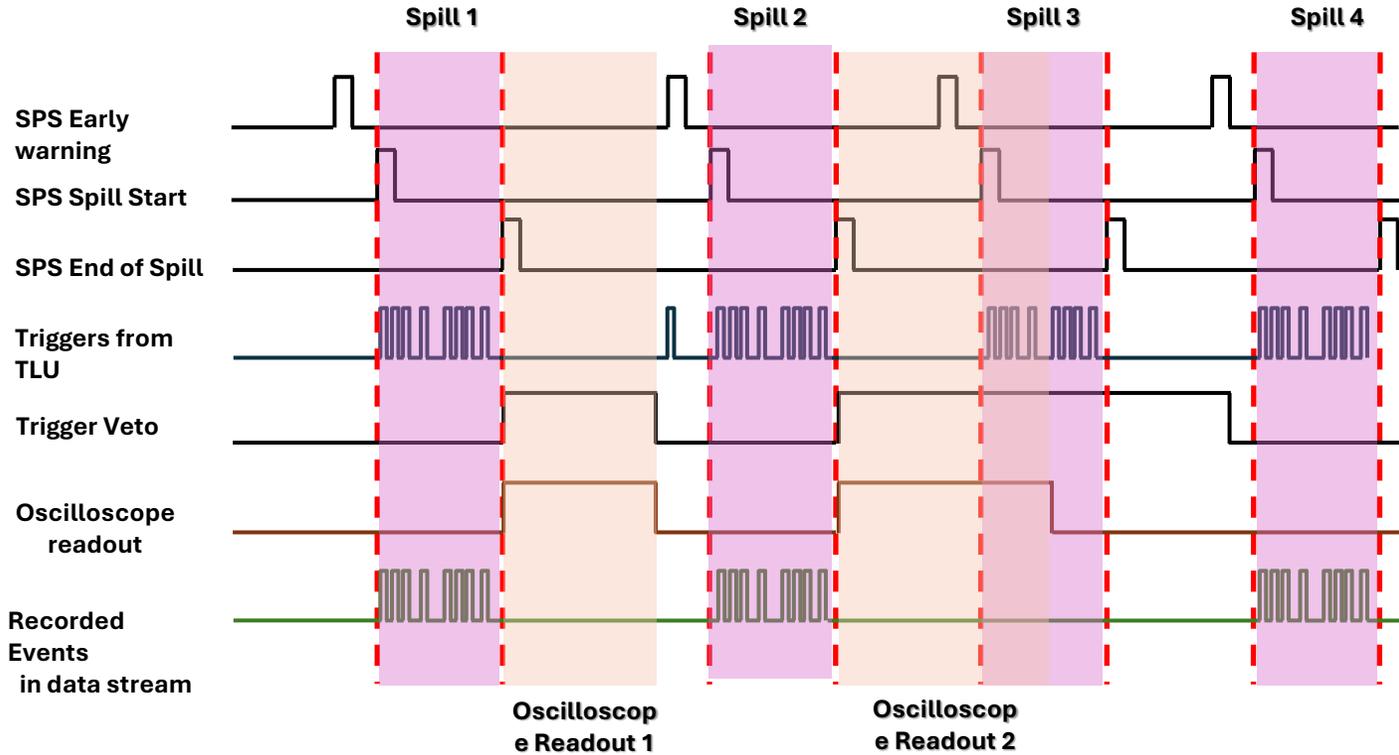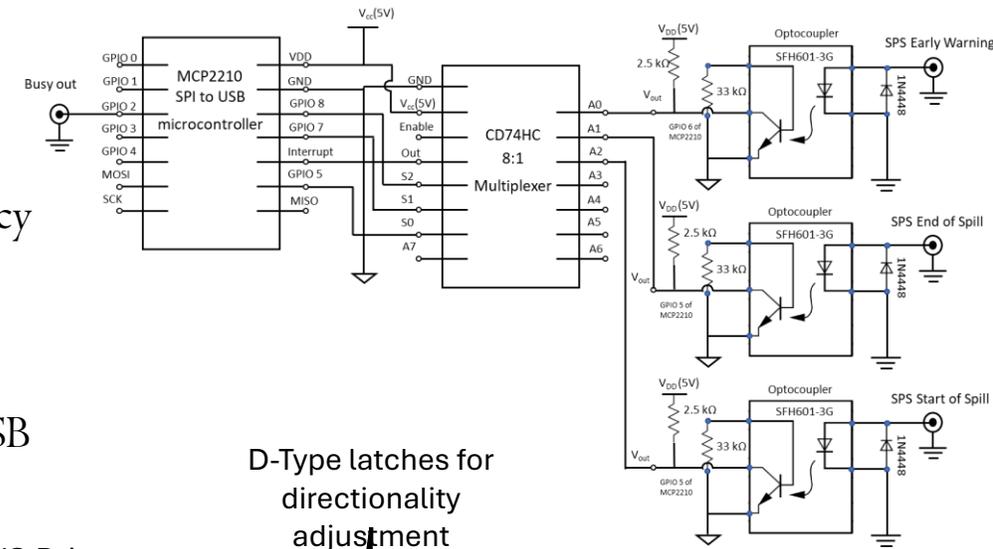
➤ Extremely small sensors, use of an ROI for triggering

➤ Trigger as coincidence of front scintillator and ROI Regio

➤ CROC + FC7 in combination with FDIO 5

➤ 16 - pixel mask in the CROC (4 x 4 region)

➤ Phase-2 Acquisition and Control Framework

➤ Have a look at Mauro's talk for details here
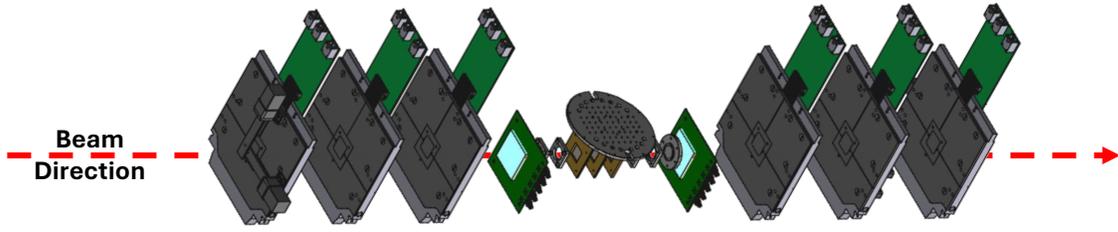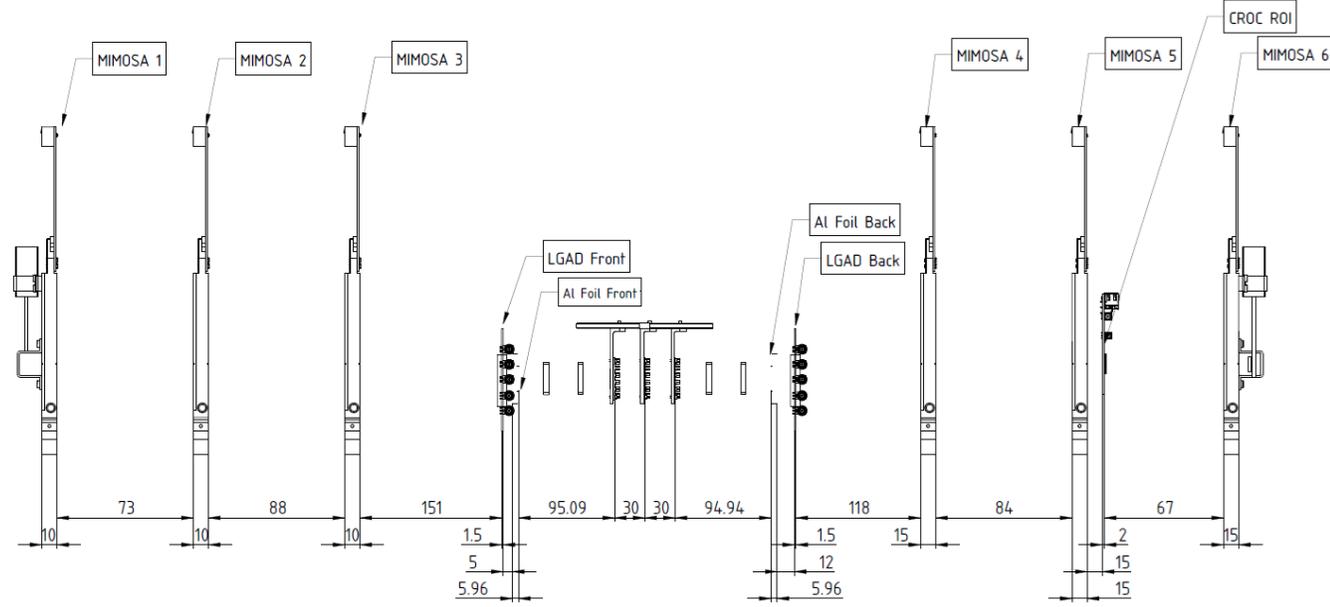

- FDIO 5 FMC Carrier

•Data Acquisition Scheme

➢ Oscilloscope in fast readout mode with binary format

➢ Event readout only between SPS-spills or when event buffer full to increase efficiency

➢ TLU Synchronization by vetoing data taking during read-out

➢ RJ-45 or HDMI for EUDET TLU communication (EUDET 2 compatible)

➢ **Versatile design**, I/Os **Reconfigurable** and microcontroller **Reprogrammable** via USB
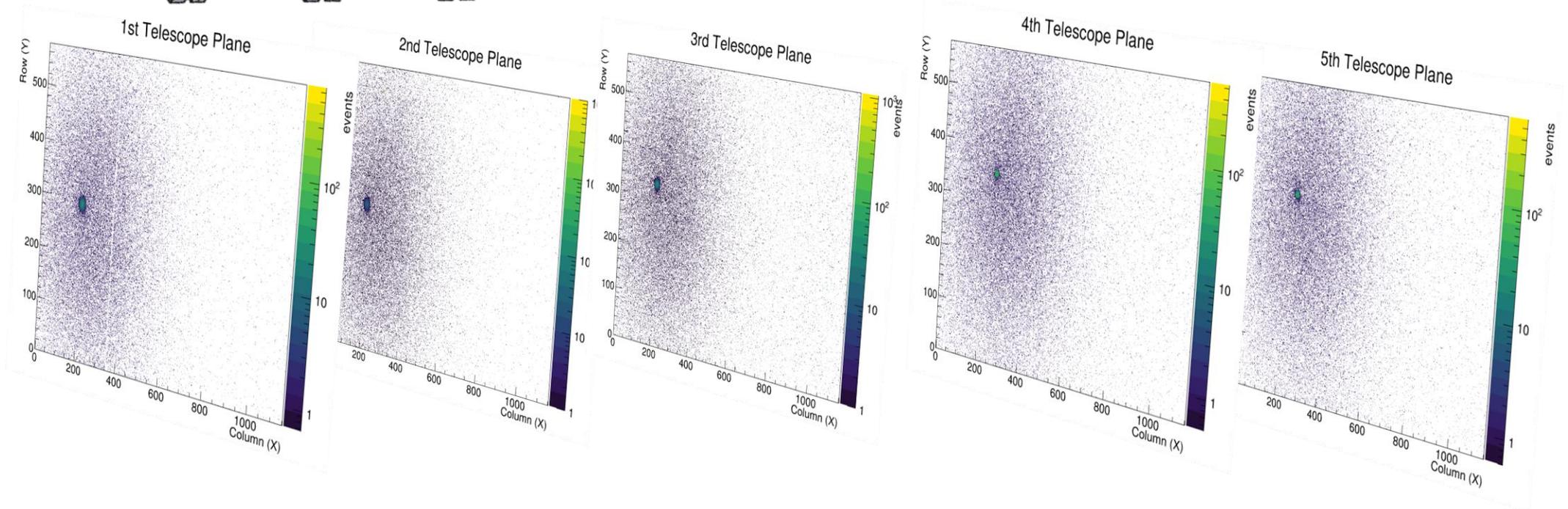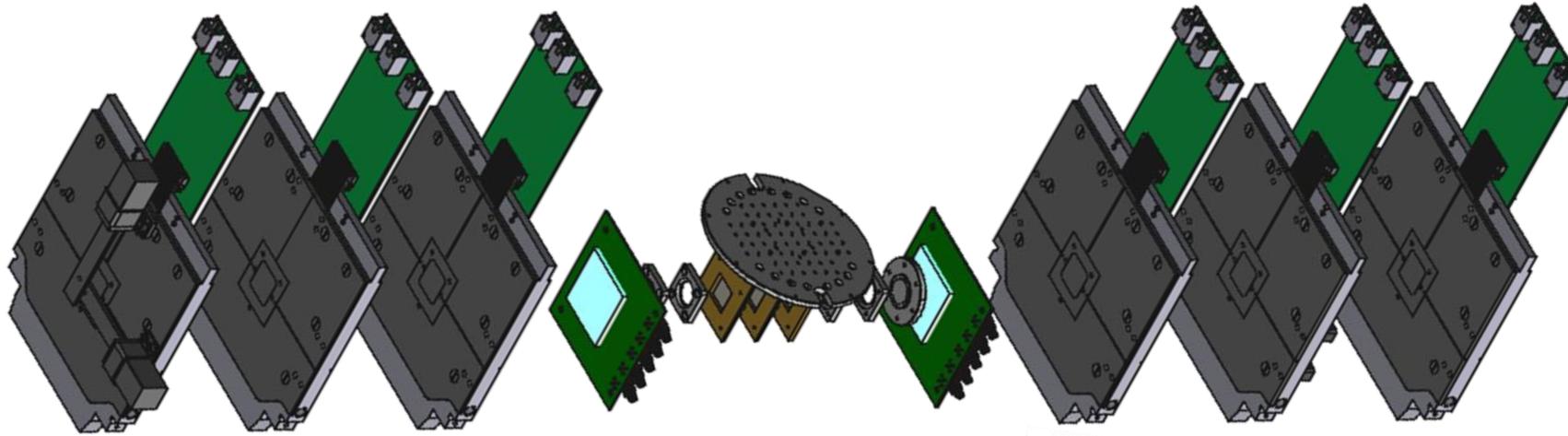
•Data & Track Reconstruction

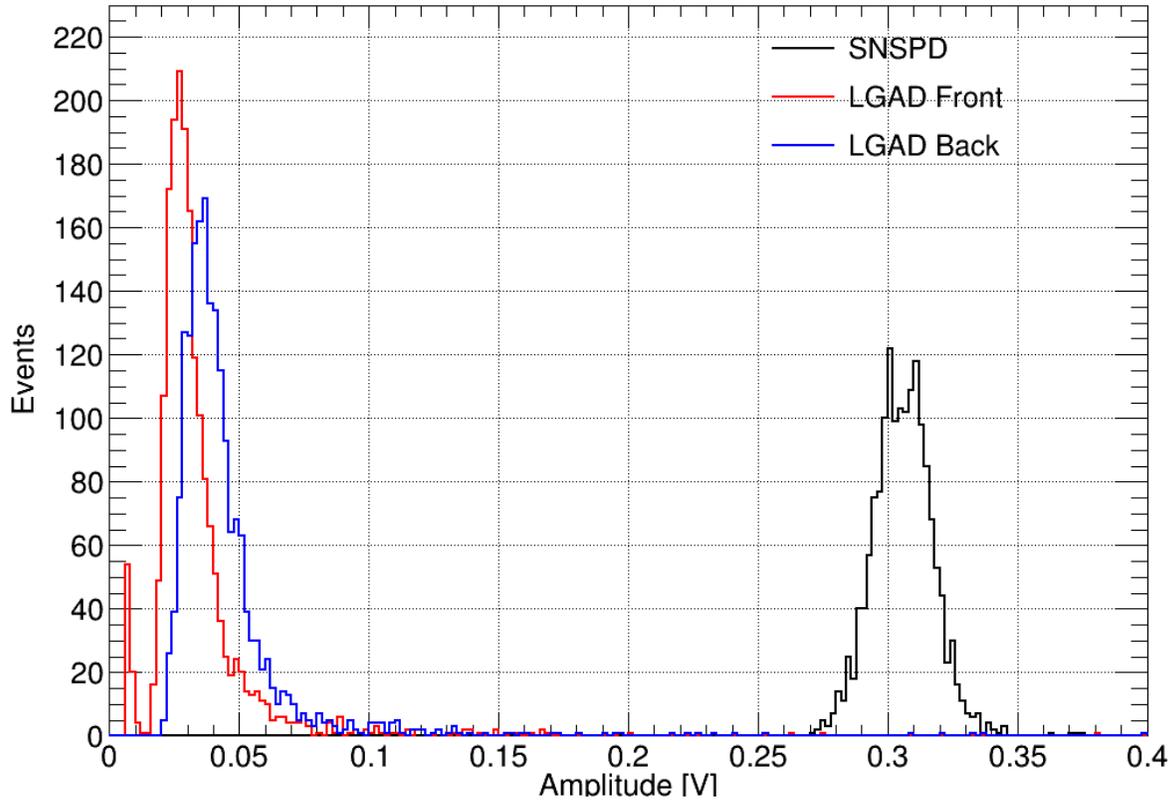| Run Configuration	No. of Events	I_{bisas} (SNSPD)	Rate (Hz)	LGAD Bias	Date
Timing Run Coincidence Trig.	2019	28 μA	0.000054	-150 V	10 / 8, 23:52
	2933	30 μA	0.000439	-150 V	10 / 8, 12:30
Tracing & Efficiency Run, RoI Trigger	6061	30 μA	0.011477	-150 V	12 / 8, 1:10
	6178	30 μA	0.011431	-150 V	12 / 8, 1:20
	7261	30 μA	0.011578	-150 V	12 / 8, 1:31
	7111	30 μA	0.010707	-150 V	12 / 8, 1:45
	568	30 μA	0.00529	-150 V	12 / 8, 9:51
	1942	30 μA	0.00343	-150 V	12 / 8, 10:26
	2560	30 μA	0.00343	-150 V	12 / 8, 10:38
	1928	30 μA	0.00343	-150 V	12 / 8, 10:50
Tracing & Efficiency Run, RoI Trigger SPS Sync.	51138	30 μA	0.004267	-150 V	12 / 8, 13:40
	59167	30 μA	0.00343	-150 V	12 / 8, 18:11
	186905	30 μA	0.009263	-150 V	12 / 8, 23:07
	110635	30 μA	0.007672	-150 V	13 / 8, 4:48
Total	**446406**		**(0.5 M events in 1.5 days)**		

➢ 5 Functioning MIMOSA26 CMOS planes (18.4 μm² pixel size, 115.2 μs readout time, 80 Mhz clock in rolling shutter mode)

➢ 1 x 150 μm active thickness CMS phase 2 Upgrade planar pixel module, 50 x 50 μm pixel, CROC readout, 2 x 50 μm active tickets LGADs
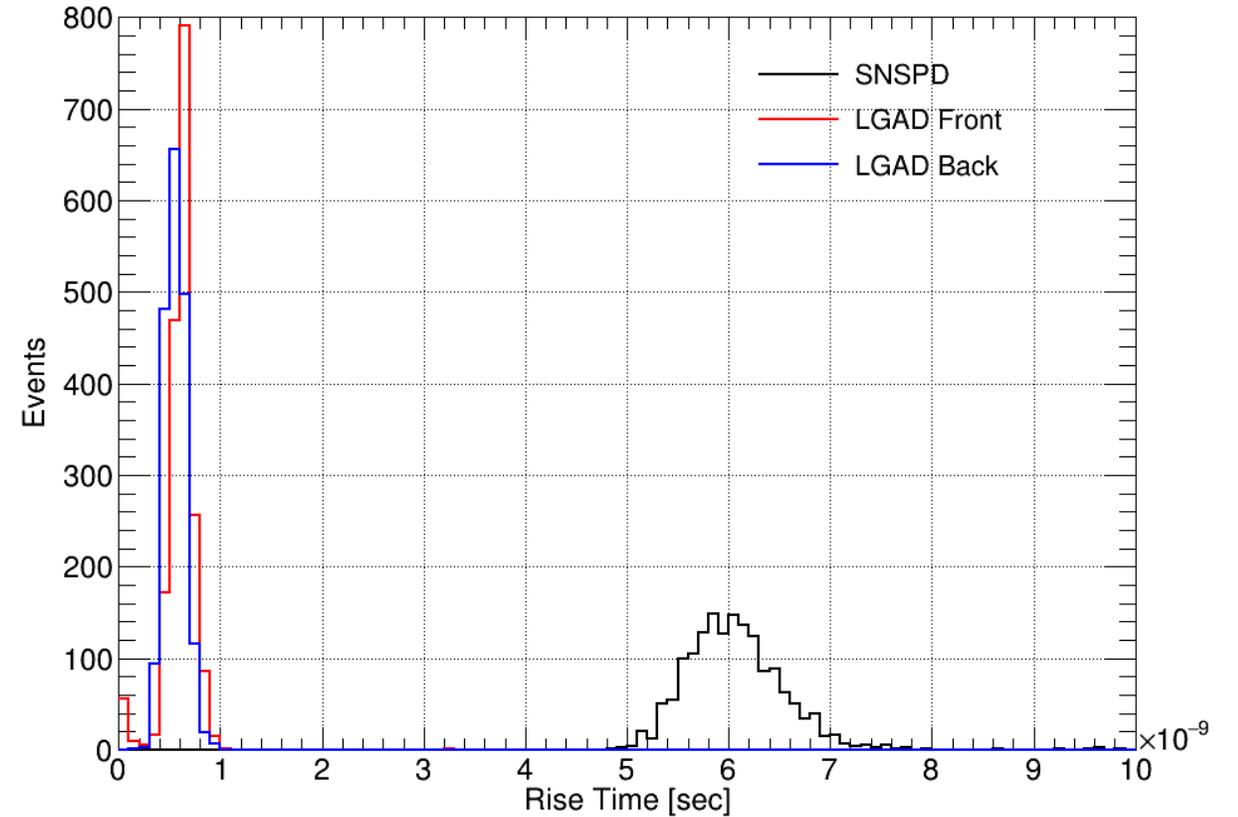
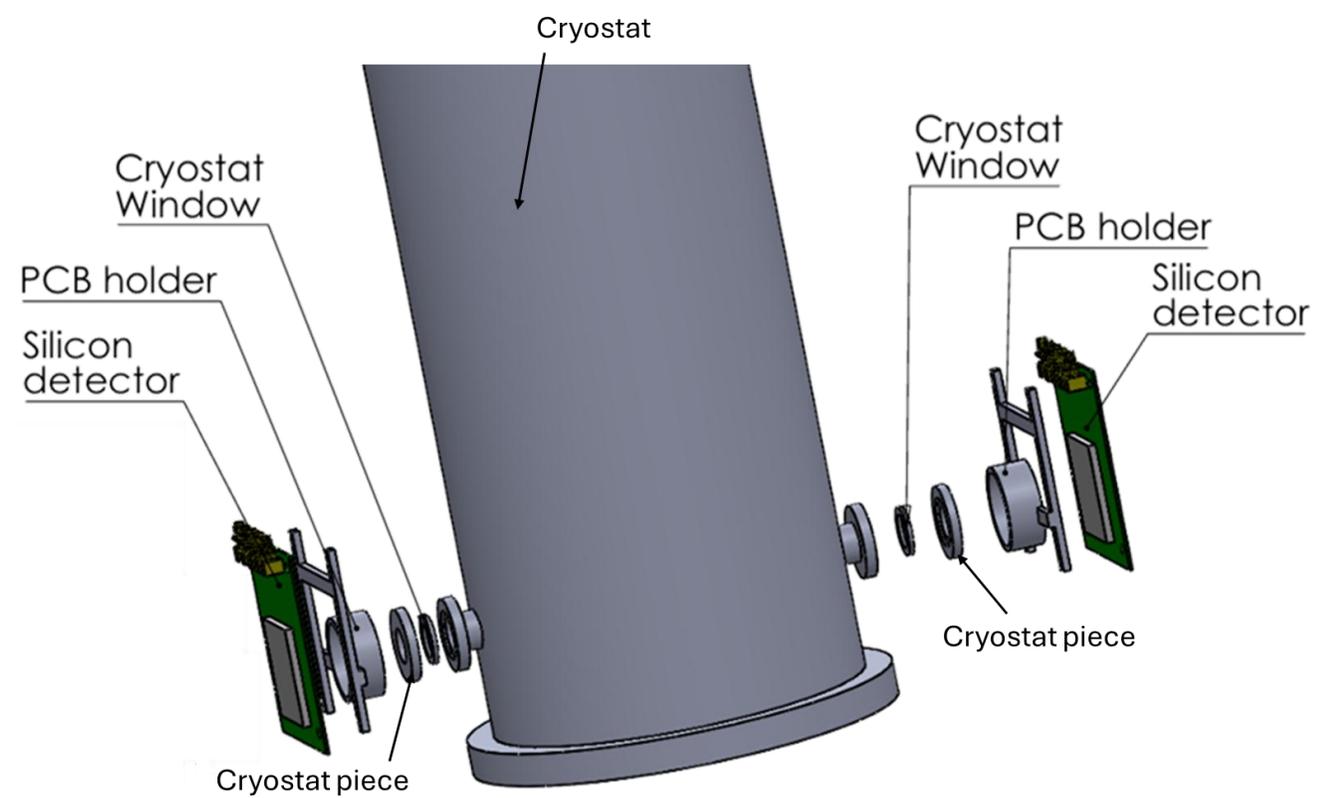➢ Expected resolution @ 160 GeV with 5 planes ~ 5 μm

•Waveform analysis

Amplitude distributions
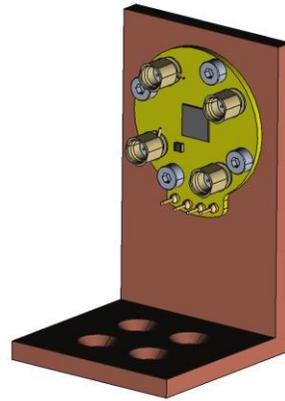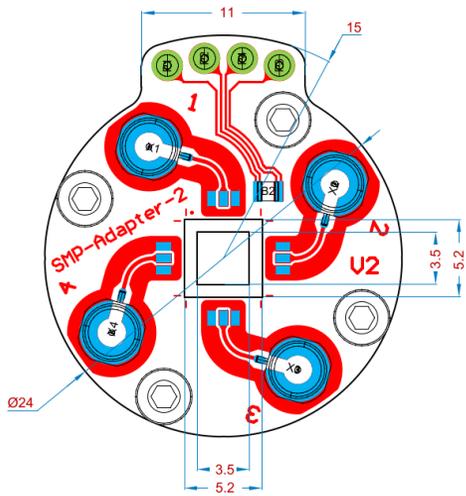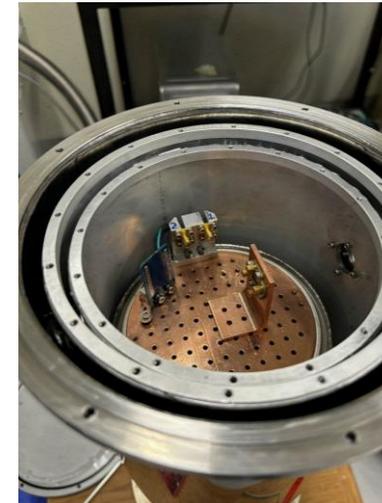

Rise Time distributions

➢ Significantly higher amplitudes with respect to LGADs, ~ 300 mV (SNSPD Bias 28 μA)

➢ Gaussian distribution (no visible Landau tail as at the Vavilov / Landau Straggling limit)

➢ Rise time in the order of 6 nsec with respect to the 800 psec for the LGADs
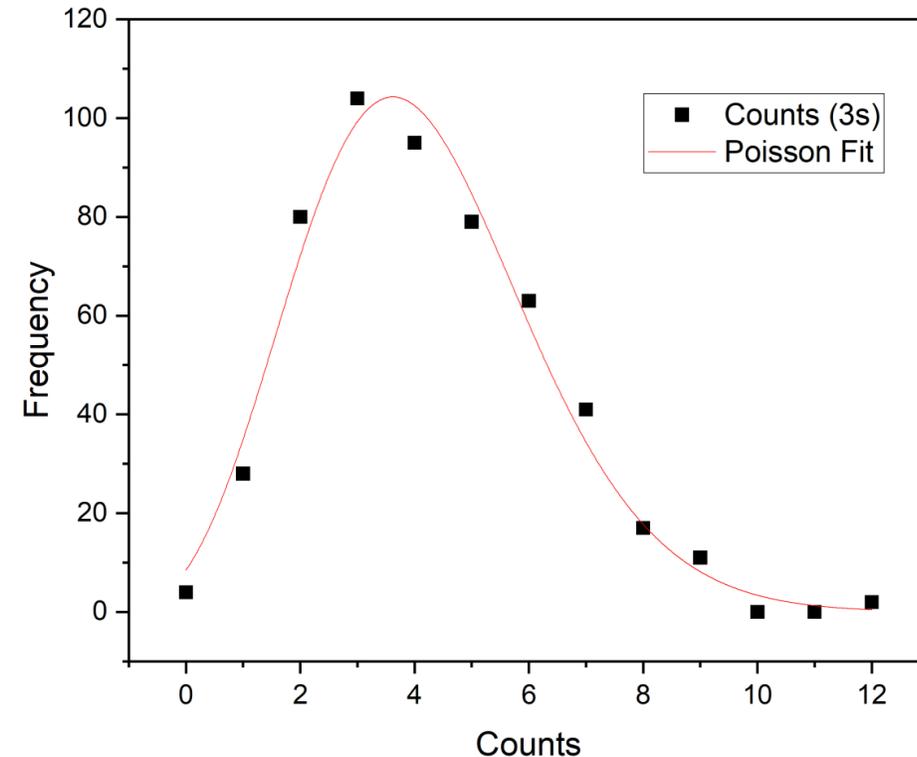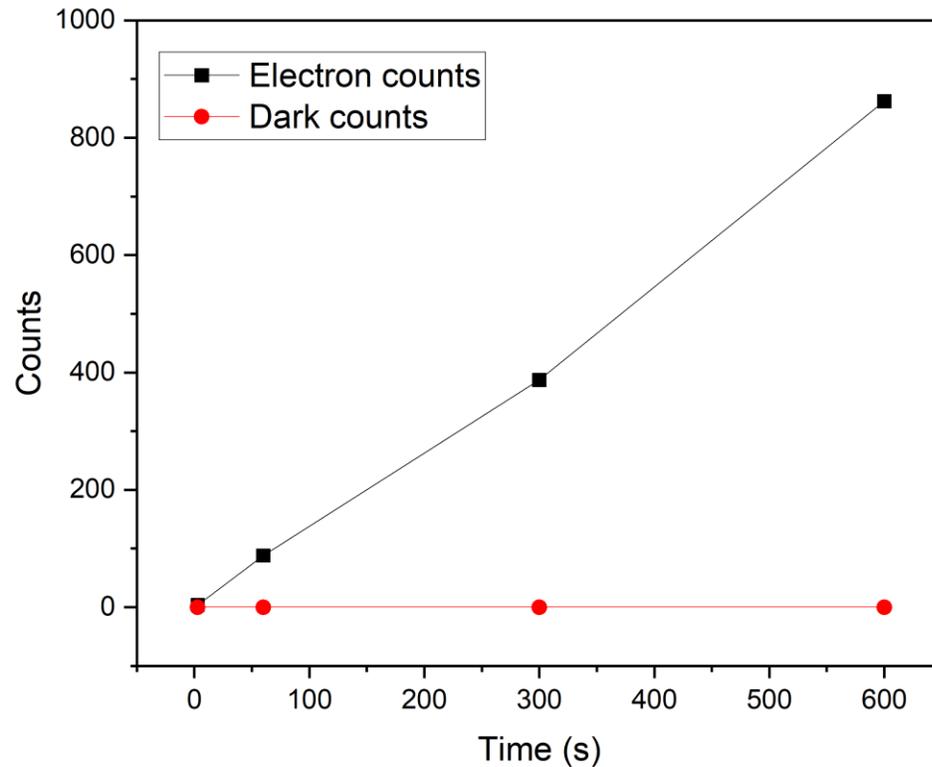
•β-particle measurements

- AlN PCB with high thermal conductivity but low X_0
- Coper Sample holder with 0.5 x 0.5 cm opening behind the sample for beam passthrough and ,multiple scattering mitigation
- Aluminized Millar windows at both sides of the cryostat and intermediate barriers with a total thickens of 2 x 40 um Al.
- Open cycle cryostat with no active temperature control
- 2 GHz oscilloscope at 5 Gs/s with 28 Mbq ($\pm$ 15%) ^{90}Sr beta source
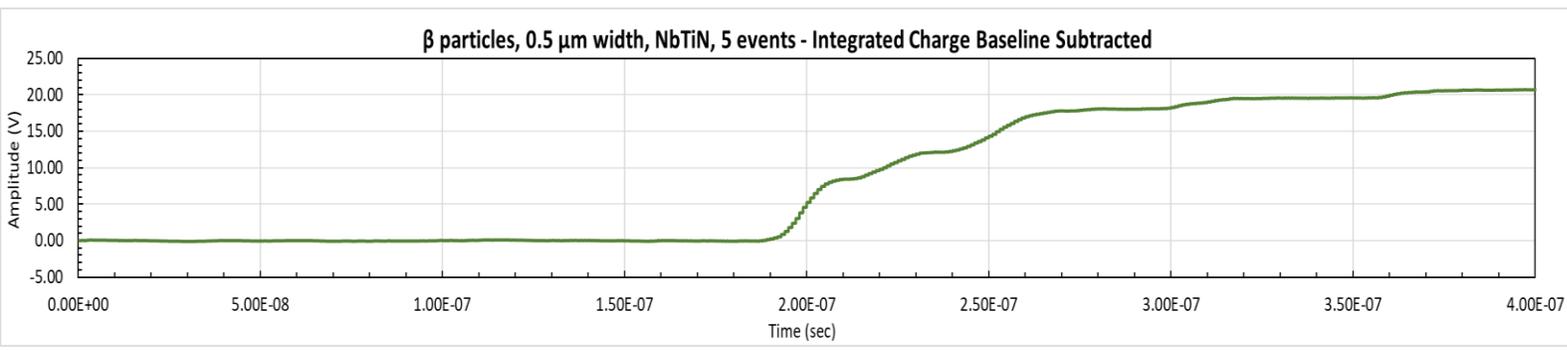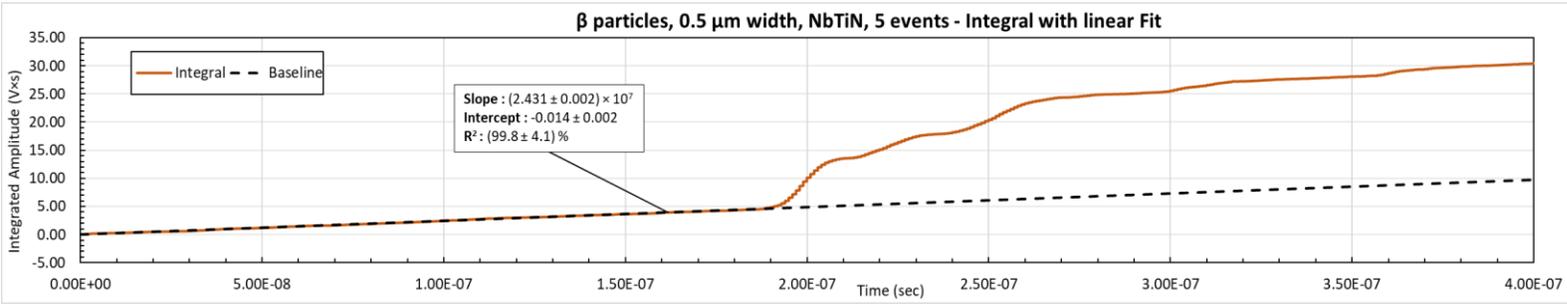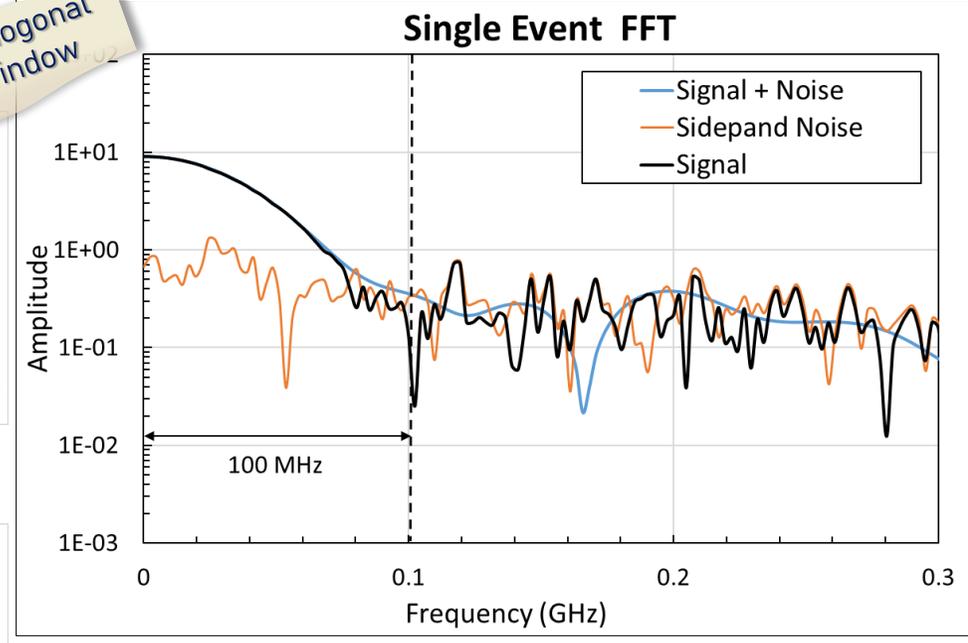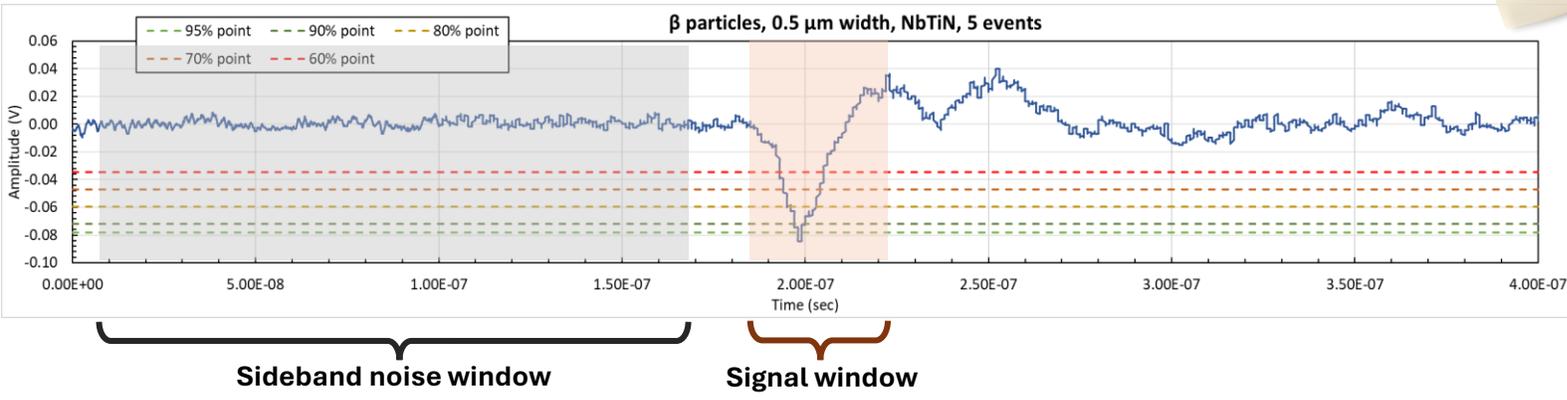- 3 GHz MiniCircuits voltage amplifiers in a 2-stage configuration

β-particle measurements

- Measured rates in the lab to verify functionality
- Poissonian distribution of frequency of counts
- Mylar windows with cooper support holder, similar response to test bema

•β-particle measurements

Orthogonal window

β particles, 0.5 µm width, NbTiN, 5 events

- - - 95% point   - - - 90% point   - - - 80% point
- - - 70% point   - - - 60% point

Sideband noise window   **Signal window**

β particles, 0.5 µm width, NbTiN, 5 events - Integral with linear Fit

— Integral   - - - Baseline

Slope : $(2.431 \pm 0.002) \times 10^7$
Intercept : -0.014 ± 0.002
R^2 : $(99.8 \pm 4.1)\,\%$

β particles, 0.5 µm width, NbTiN, 5 events - Integrated Charge Baseline Subtracted

Single Event FFT

— Signal + Noise
— Sidepand Noise
— Signal

100 MHz

$$A_{\text{sig}}(f) = \sqrt{\max\left(A_{\text{meas}}(f)^2 - A_{\text{noise}}(f)^2, 0\right)}$$

Signal Caractéristiques

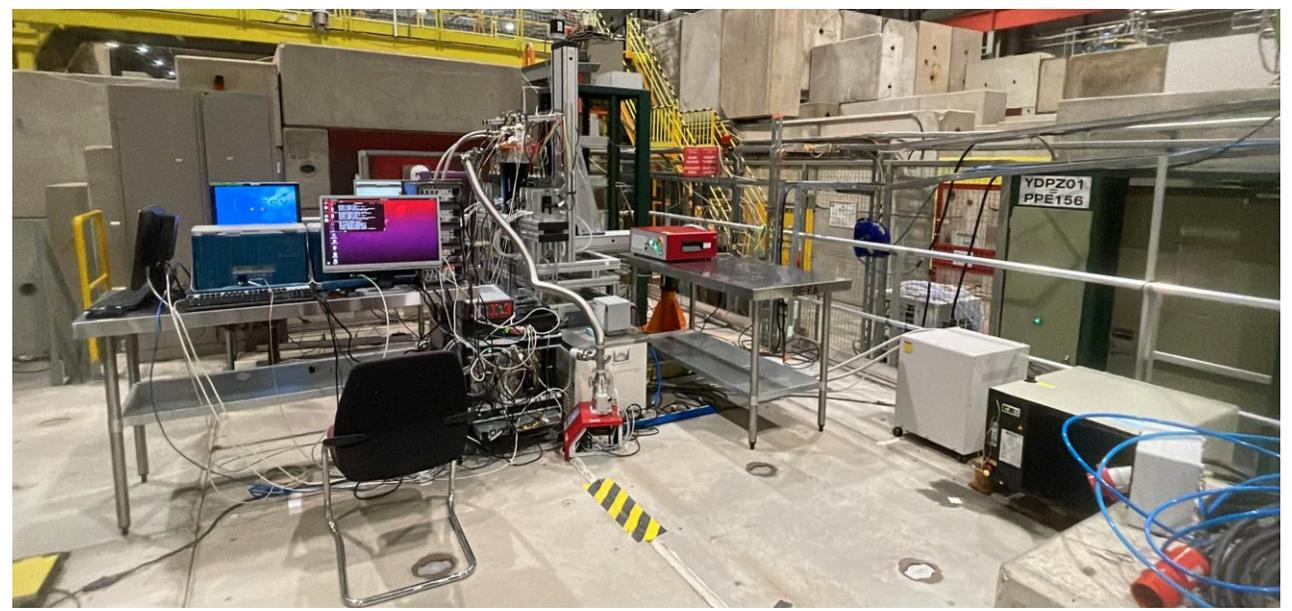|	
Min Voltage	- 40 mV
Max Voltage	084.3 mV
Baseline	3.6 mV
RMS noise	3.9 mV
Rise Time	9.7 nsec
Signal Duration	36 nsec
Integrated Charge (22.4^2 gain @ 100 MHz)	2.88×10^{-11} Q

50 Ω Chain

•Conclusions & Outlook

➤ First Test of NbTiN and Nb N samplers in test beam proved operating principal

➤ Temperature issues resolved in the second three-week period (19 September – 1 October)

➤ Tracing analysis on-going, first results encouraging but demonstrate slow signals

➤ Simulations under way with GEANT4 to understand substrate implication

➤ Future target higher temperature materials (MgB_2)

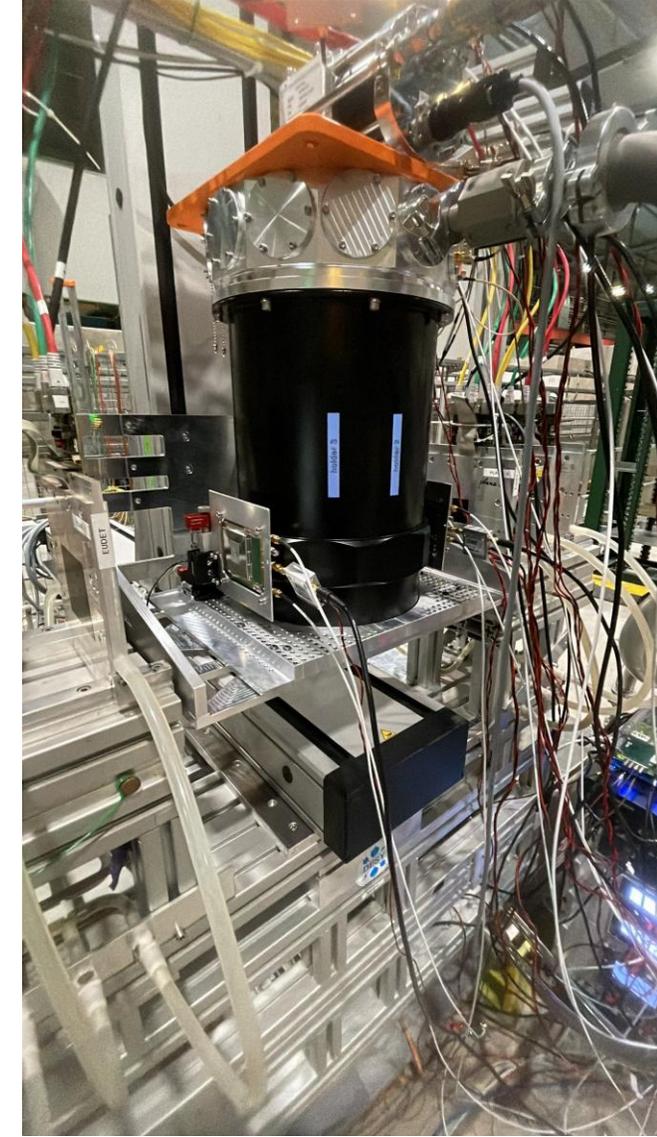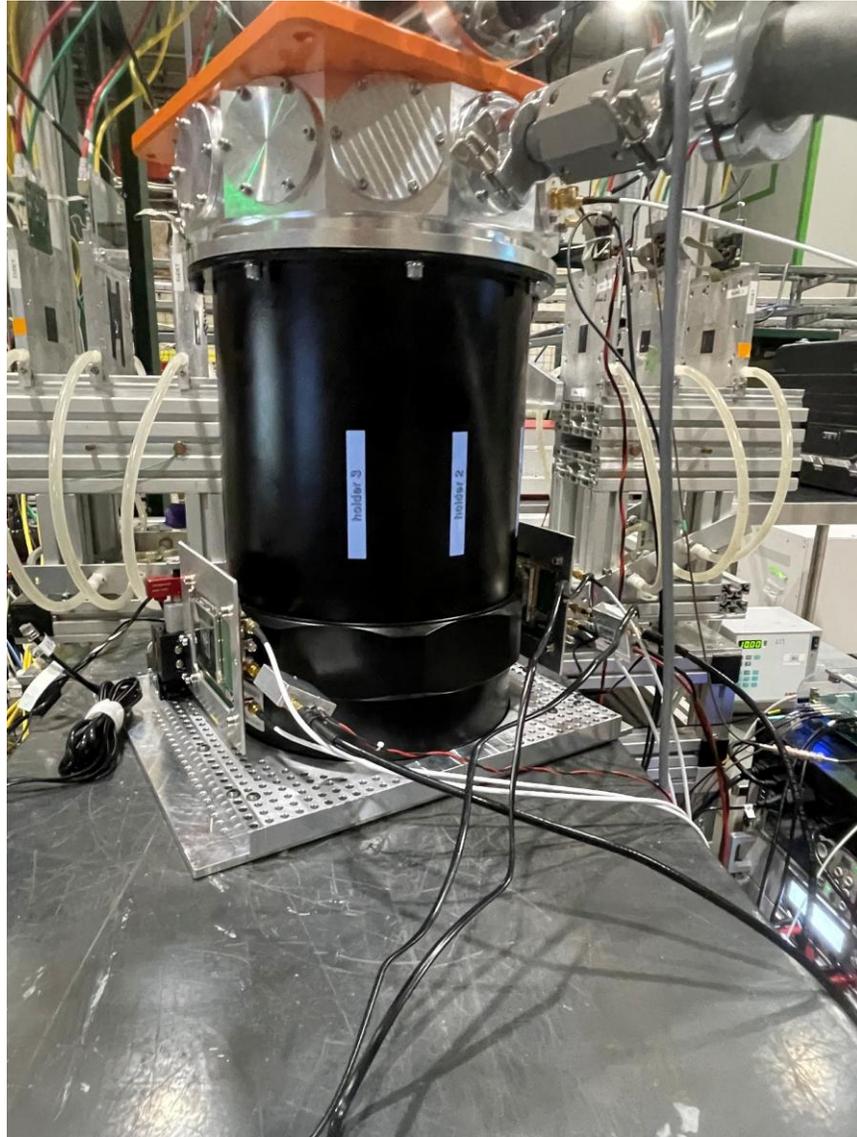➤ Design optimization for faster signals and higher fill factors
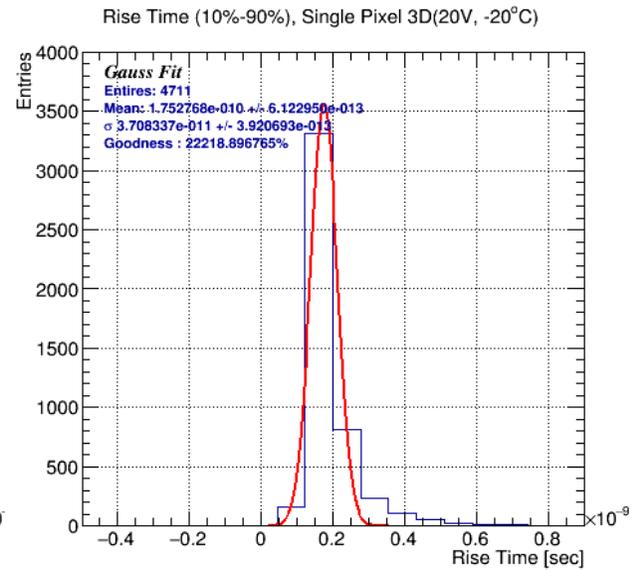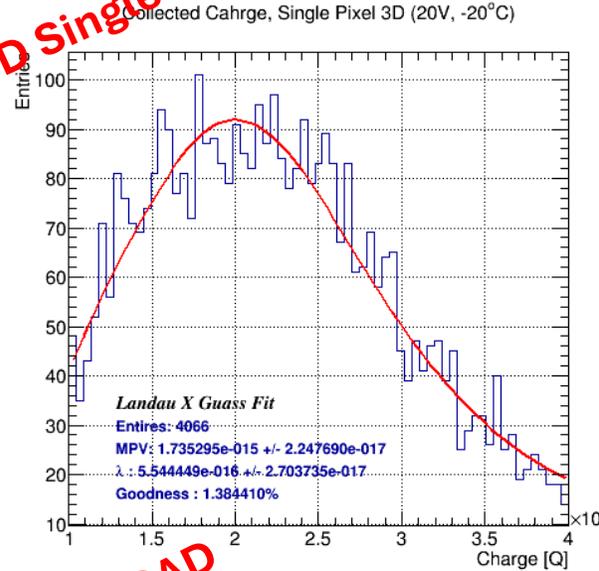
•Backup

Timming planes Assembly

Timming planes Assembly

•Timing Reference

3D Single Cell

HPK LGAD

$$(\sigma_{\text{Dut}})_{CFD_{ij}} = \sqrt{(\sigma^2_{\text{Tot}})_{CFD_{ij}} - (\sigma^2_{\text{Ref}})_{CFD_i}}$$

CFD Map, LGAD - Single Pixel 3D (-20°C, 20V)

2D optimization plot – 0.5% binning

$$Time\ Resolution: \quad \sigma^2_{tot} = \sigma^2_{timewalk} + \sigma^2_{jitter} + \sigma^2_{conversion} + \sigma^2_{Clock}$$

$$\sigma^2_{Dist.} + \sigma^2_{Landau} \qquad \left(\frac{t_{rise}}{S/N}\right)^2 \qquad \left(\frac{TDC_{bin}}{\sqrt{12}}\right)^2 \qquad Fixed\ Term \\ \sim 5\text{-}7\ psec$$

•ROI trigger – CROC

FMC DIO 5ch TTL a
CERN Open Hardware Project

Product Name	Company	Contact
fmc dio 5chttla	iNCAA Computers	sales@incaacomputers.com
FMC DIO	Sundance Technology	sales@sundance.com
FMC DIO 5CH TTL A	Creotech	kontakt@creotech.pl

Plane	Distances (mm)	
	Interplane	Relative
MIMOSA 1	0	0
MIOMSA 2	83	83
MIMOSA 3	98	181
LGAD Front	157.5	338.5
Holder 3	106.05	444.55
Holder 2	30	474.55
Holder 1	30	504.55
LGAD Back	112.36	616.91
MIMOSA 4	125.5	742.41
MIMOSA 5	99	841.41
CROC	24.5	865.91

$$\frac{dy}{dx}(I) = A \times \exp\left(-\frac{I-\mu}{\beta} - e^{-(I-\mu)/\beta}\right)$$

$$\frac{dy}{dx}(I) = A \times \exp\left(+\frac{I-\mu}{\beta} - e^{(I-\mu)/\beta}\right)$$

β : asymmetric skew parameter
    measures spread of switching events
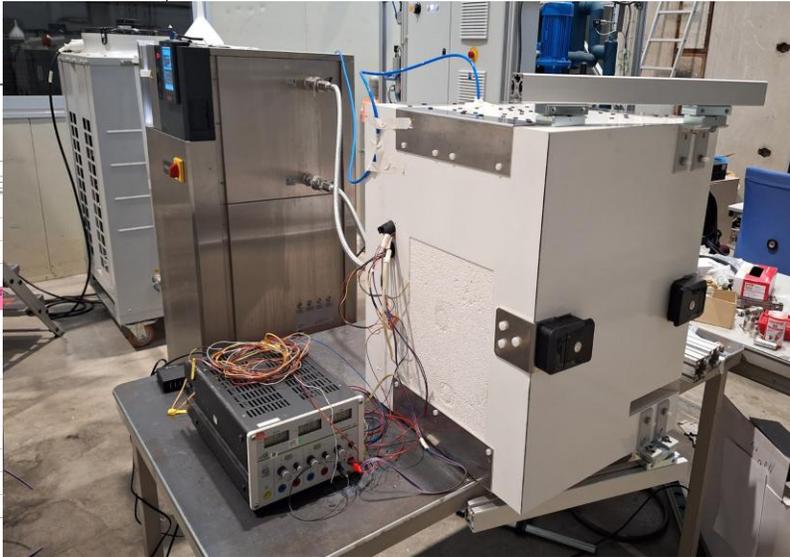μ : x- axis location parameter
    most probable switching current
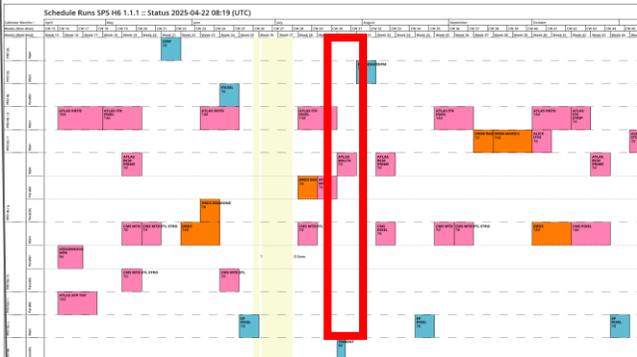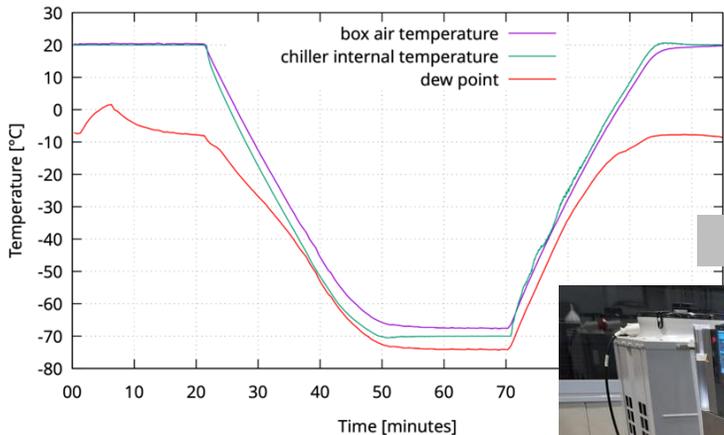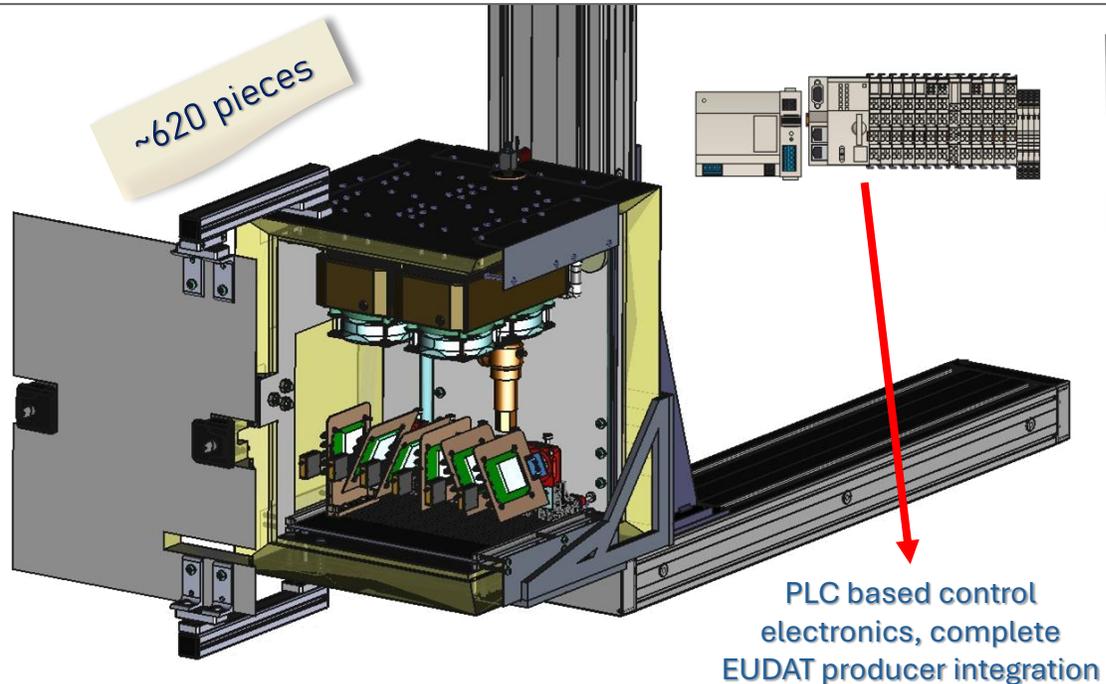A : amplitude scaling parameter

https://docs.google.com/spreadsheets/d/1vLE-evXXKtkaPMSj6GC_cAlBCw8hYmCW9g4qliC3hTk/edit?usp=sharing

•Infrastructure upgrade @H6B

Team: Vagelis Gkougkousis
Andre Rummler
Dominik Dannheim

More info at Vagelis AIDA Innova [prestation](#)

~620 pieces

PLC based control electronics, complete EUDAT producer integration

- - 80 ^{o}C with ethanol cooling
- 20% ^{0}X for pions using aluminum clad XPS fly-ash reinforced-core wall, 8 % in beam region
- Carbon fiber hydraulic pass-through, conductive
- 5 plane X - Y DUT translation stages
- Independent DUT rotation up to 35°

Complete drawings and step files:
https://github.com/VGkougkousis/VBox/tree/main

✓ Chiller / stages / box delivered end of 2024, acceptance test complete

✓ Setup currently in Building 180, Transfer to H6B end July

✓ Installation foreseen for the week of August 3rd, fist user CMS Pixel

✓ Services to be finalized (water supply / power infrastructure / safety)

✓ Universal software integration with **ALL** available LV / HJV units to a single framework (Unified infrastructure producer)

Cryostat window minimum thickness

Minimum window thickness equation:

$$S_{max} = \frac{KD^2P}{4T} = \frac{F_a}{Sf} \qquad T = D\sqrt{\frac{SfKP}{4F_a}}$$

- K : 0.75 (empirical factor accounting for the clamping method)
- D : Diameter
- P : Pressure difference
- Fa : Rupture modulus
- Sf : Safety factor taken as 4
- T : Window thickness
- S_{max} : Maximum force

Material	Aluminium Nitrite	Aluminium Oxide	Aluminium	Plexiglass	Silicon
Density (g/cm³)	3.255	3.98	2.7	1.18	2.38
Rupture Modulus (MPa)	367	800	280	72	200
Minimum Thickness (mm)	0.26	0.3	0.18	0.58	0.35

To chose optimal window material, need to analyse the individual stopping power of each material, and the post window beta spectrum
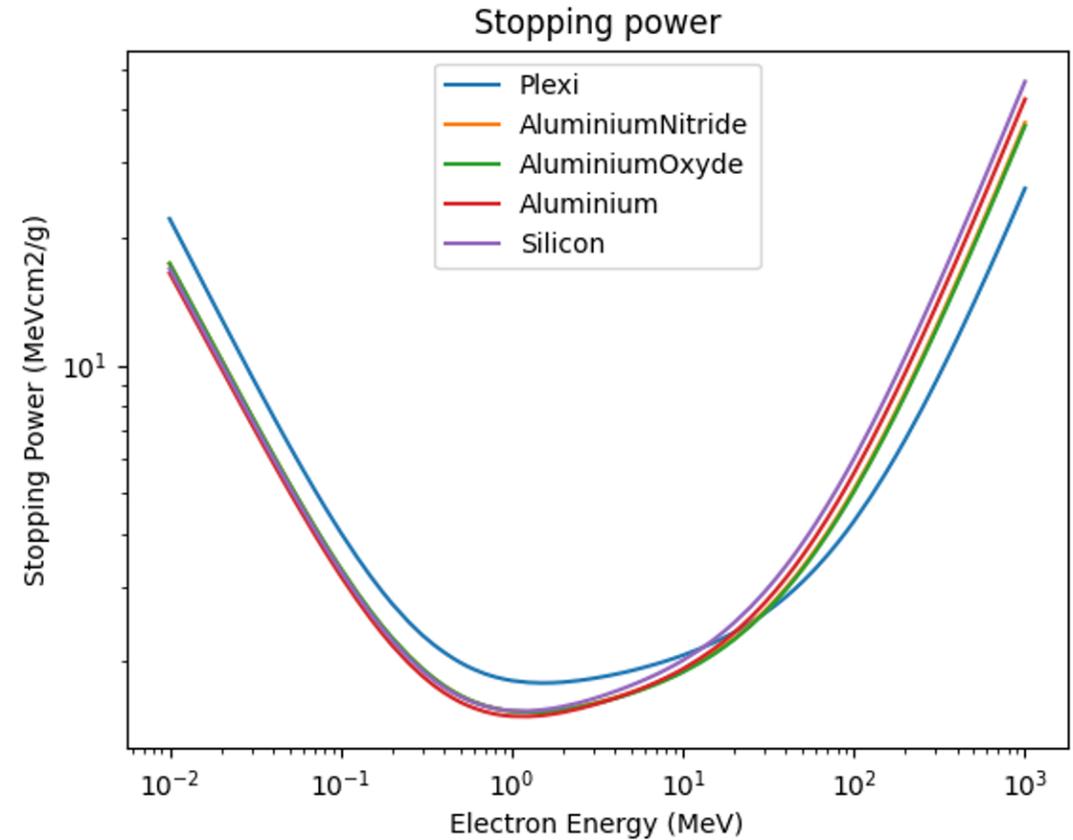
Stopping power analysis

Bethe Bloch equation:

$$-\frac{dE}{dx} = Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln\left(\frac{2m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} \right) - \beta^2 - \frac{\delta}{2} \right]$$

- T_{max}: Maximum kinetic energy
- K: Constant
- A: Atomic mass of medium
- Z: Atomic number of medium
- m_e: Electron mass
- $\beta = \frac{v}{c}$
- δ: Density effect correction
- z: Electron charge
- I: Average excitation energy

Considering the Stopping power for the selected materials is relatively close, we needed an analysis of the Beta Spectrum to make an educated choice

The data was taken from the NIST ESTAR database