Fault-tolerant dynamically-decoupled hyper-Ramsey spectroscopy of optical clock transitions

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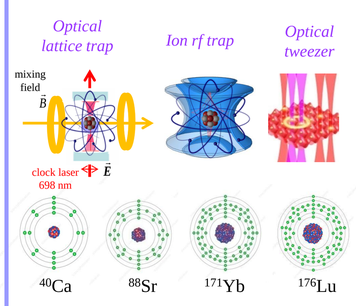
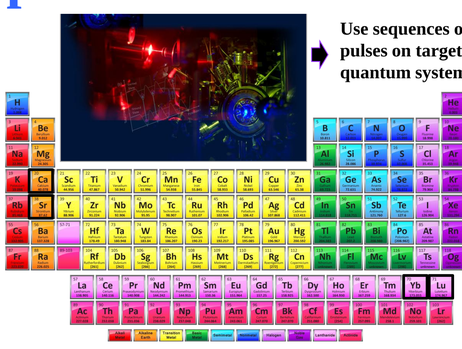
NANYANG TECHNOLOGICAL UNIVERSITY · NUS National University of Singapore · SCIENCES SORBONNE UNIVERSITÉ · MajuLab · UNIVERSITÉ CÔTE D'AZUR · CGT Centre for Quantum Technologies · LPL Laboratoire de physique des lasers · SOFIA UNIVERSITY ST. KLIMENT OHRIDSKI · MONARIS · cnrs

❑ I. SU(2) hyper-clock combinatorics :

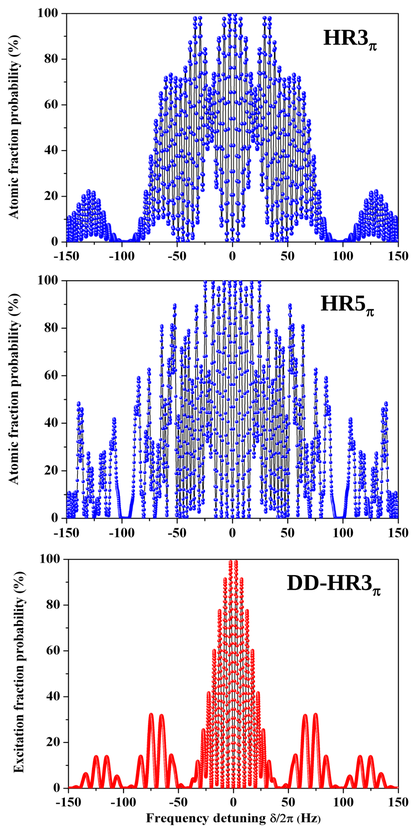


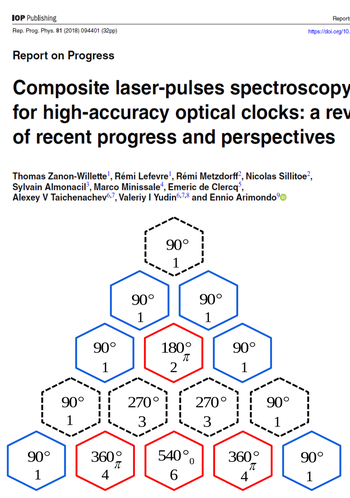


➧ Use sequences of laser pulses on targeted quantum systems

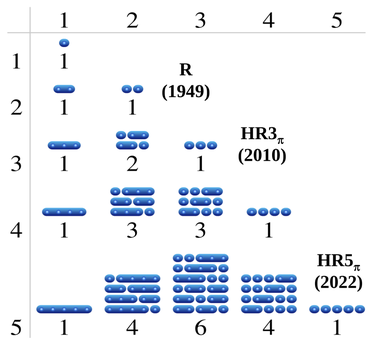
➧ Select composite pulses to dress atoms with structured-light

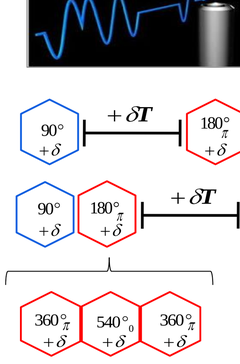
➧ Shield quantum interferences against external perturbations

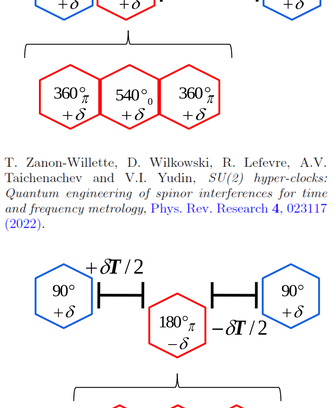




Clock interrogation protocols (Pascal's triangle)



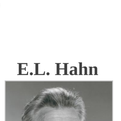




T. Zanon-Willette, D. Wilkowski, R. Lefevre, A.V. Taichenachev and V.I. Yudin, *SU(2) hyper-clocks: Quantum engineering of spinor interferences for time and frequency metrology*, Phys. Rev. Research **4**, 023117 (2022).

N.F. Ramsey

M.H. Levitt

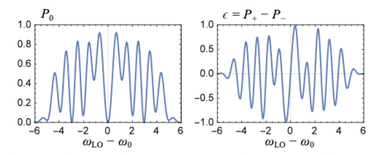


E.L. Hahn

A.J. Shaka and A. Pines, *Symmetric Phase-Alternating Composite Pulses*, J. Magn. Reson. **71**, 495 (1987).
M.H. Levitt, *Symmetry in the design of NMR multiple-pulse sequences*, J. Chem. Phys. **128**, 052205 (2008).

N.V. Vitanov, T.F. Gloger, P. Kaufmann, D. Kaufmann, T. Collath, M. Tanveer Baig, M. Johanning and C. Wunderlich, *Fault-tolerant Hahn-Ramsey interferometry with pulse sequences of alternating detuning*, Phys. Rev. A **91**, 033406 (2015).

Protocols	Composite pulses θ_l $(\varphi_l^+,\varphi_l^-)$
R	$\mathbf{90}_{(+\frac{\pi}{2},-\frac{\pi}{2})} \dashv \delta T \vdash \mathbf{90}_{(0,0)}$ (†) $\mathbf{90}_{(0,0)} \dashv \delta T \vdash \mathbf{90}_{(-\frac{\pi}{2},+\frac{\pi}{2})}$
HR	$\mathbf{90}_{(+\frac{\pi}{2},-\frac{\pi}{2})} \dashv \delta T \vdash \mathbf{270}_{(0,0)}$ (†) $\mathbf{270}_{(0,0)} \dashv \delta T \vdash \mathbf{90}_{(-\frac{\pi}{2},+\frac{\pi}{2})}$
HR-π	$\mathbf{90}_{(+\frac{\pi}{2},-\frac{\pi}{2})} \dashv \delta T \vdash \mathbf{180}_{(\pi,\pi)}\mathbf{90}_{(0,0)}$ (†) $\mathbf{90}_{(0,0)}\mathbf{180}_{(\pi,\pi)} \dashv \delta T \vdash \mathbf{90}_{(-\frac{\pi}{2},+\frac{\pi}{2})}$
MHR	$\mathbf{90}_{(+\frac{\pi}{2},0)} \dashv \delta T \vdash \mathbf{180}_{(\pi,\pi)}\mathbf{90}_{(0,-\frac{\pi}{2})}$ (†) $\mathbf{90}_{(-\frac{\pi}{2},0)}\mathbf{180}_{(\pi,\pi)} \dashv \delta T \vdash \mathbf{90}_{(0,+\frac{\pi}{2})}$
GHR($\frac{\pi}{4}$)	$\mathbf{90}_{(0,0)} \dashv \delta T \vdash \mathbf{180}_{(+\frac{\pi}{4},-\frac{\pi}{4})}\mathbf{90}_{(0,0)}$ (†) $\mathbf{90}_{(0,0)}\mathbf{180}_{(-\frac{\pi}{4},+\frac{\pi}{4})} \dashv \delta T \vdash \mathbf{90}_{(0,0)}$
GHR($\frac{3\pi}{4}$)	$\mathbf{90}_{(0,0)} \dashv \delta T \vdash \mathbf{180}_{(+3\frac{\pi}{4},-3\frac{\pi}{4})}\mathbf{90}_{(0,0)}$ (†) $\mathbf{90}_{(0,0)}\mathbf{180}_{(-3\frac{\pi}{4},+3\frac{\pi}{4})} \dashv \delta T \vdash \mathbf{90}_{(0,0)}$



Generating a dispersive error signal by difference of two phase-shifted transition probabilities:

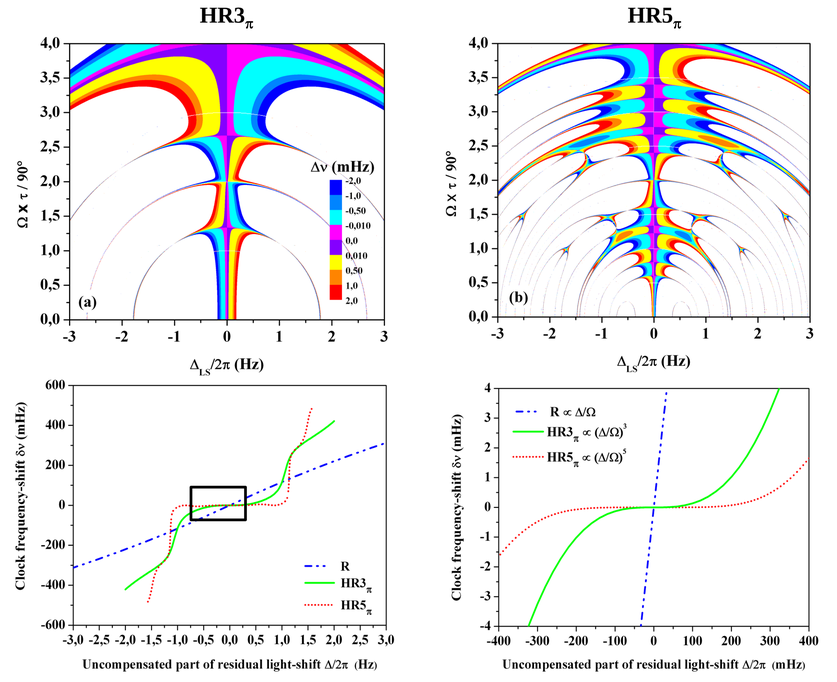
$$\Delta E = \mathrm{P}_{|g\rangle\mapsto|e\rangle}(\varphi_1^+) - \mathrm{P}_{|g\rangle\mapsto|e\rangle}(\varphi_1^-)$$

$$P_{|g\rangle\mapsto|e\rangle} = A + B\cos(\delta T + \Phi)$$

$$\pm\varphi \;\Rightarrow\; \Delta E = \rho \times \sin(\delta T + \Phi)$$

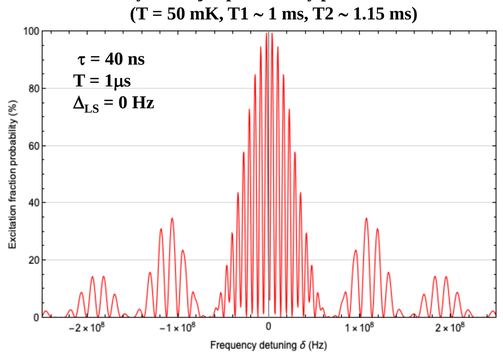
Elimination of spurious phase-shifts

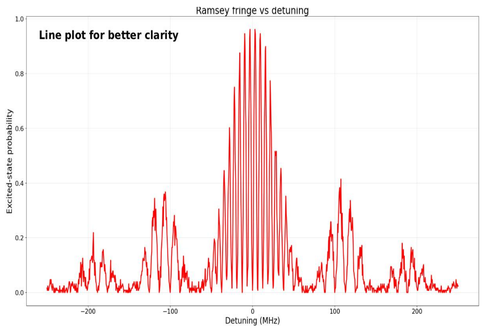
❑ II. Phase-shift tomography :

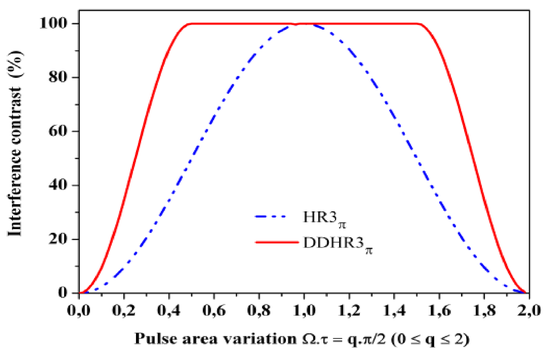


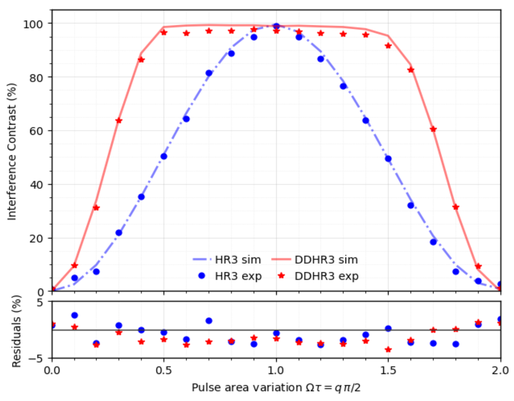
❑ III. 2025 experimental DD-HR results on IQM computer :

work by B. Ilikj supervized by prof . N.V. Vitanov
(T = 50 mK, T1 ~ 1 ms, T2 ~ 1.15 ms)









The propagator matrix of the l-th pulse $\left[\tilde{\vartheta}_l\right]_\pm$ (l= 1, 2, 3) driving SU(2) spinor dynamics reads [44]

$$\tilde{\vartheta}_l^{\pm} \equiv \begin{pmatrix} \cos\tilde{\vartheta}_l e^{i\phi_l} & -ie^{-i\varphi_l}\sin\tilde{\vartheta}_l \\ -ie^{i\varphi_l}\sin\tilde{\vartheta}_l & \cos\tilde{\vartheta}_l e^{-i\phi_l} \end{pmatrix}, \qquad (1)$$

with a pulse phase φ_l related to the Rabi frequency Ω_l and where $\pm$ means we can apply a positive or a negative effective probe detuning $\tilde{\delta}_l \to \pm\tilde{\delta}_l + \varepsilon_P$ where $\tilde{\delta}_l = \delta - \Delta_{LS}$. Cayley-Klein phase angles are introduced as:

$$\tilde{\vartheta}_l = \arcsin\left[\frac{\Omega_l}{\omega_l}\sin\tilde{\theta}_l\right], \quad \phi_l = \arctan\left[\frac{\tilde{\delta}_l}{\omega_l}\tan\tilde{\theta}_l\right]. \qquad (2)$$

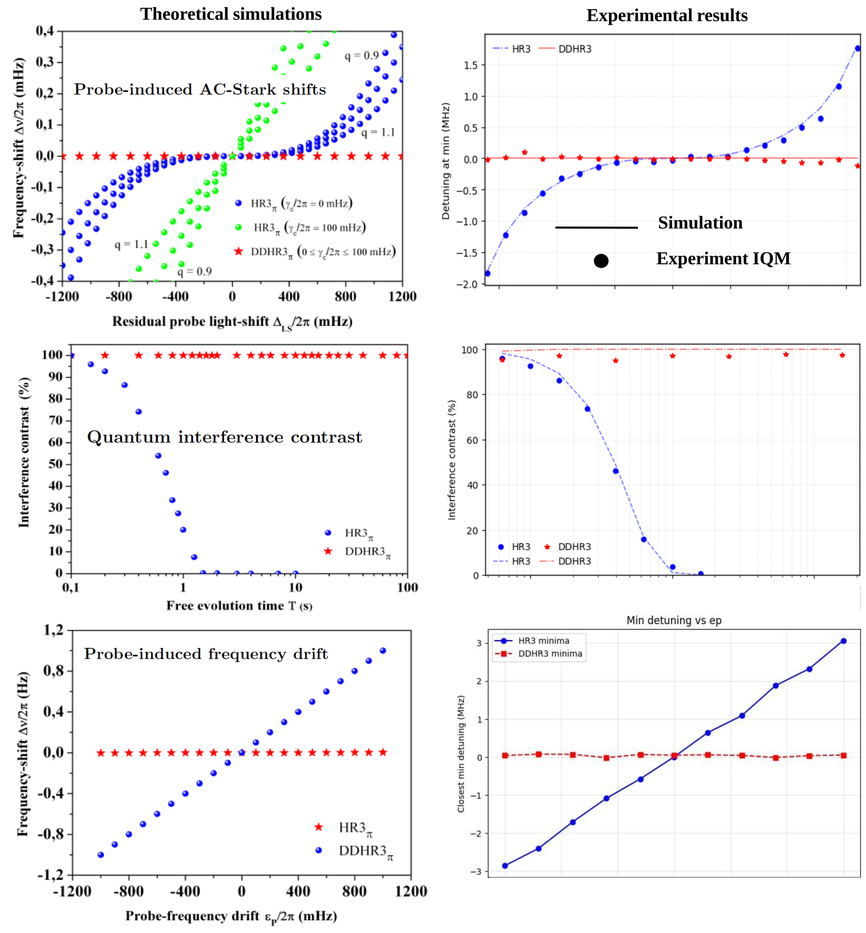
The pulse area is $\tilde{\theta}_l = \omega_l\tau_l/2$ with a generalized Rabi frequency denoted as $\omega_l = \sqrt{\tilde{\delta}_l^2 + \Omega_l^2}$. The free evolution propagator $[\pm]$ while switching laser fields is

$$[\pm] \equiv \begin{pmatrix} e^{i(\pm\delta+\varepsilon_P)T/2} & 0 \\ 0 & e^{-i(\pm\delta+\varepsilon_P)T/2} \end{pmatrix}. \qquad (3)$$

Transition probabilities associated to our protocols are computed by multiplying propagator matrices leading to:

$$\mathrm{HR3}_\pi \equiv \left|\langle e\left|90_0^{\circ+}180_\pi^{\circ+}[+]90_0^{\circ+}\right|g\rangle\right|^2, \qquad (4)$$

$$\mathrm{DDHR3}_\pi \equiv \left|\langle e\left|90_0^{\circ+}[-]180_\pi^{\circ-}[+]90_0^{\circ+}\right|g\rangle\right|^2. \qquad (5)$$



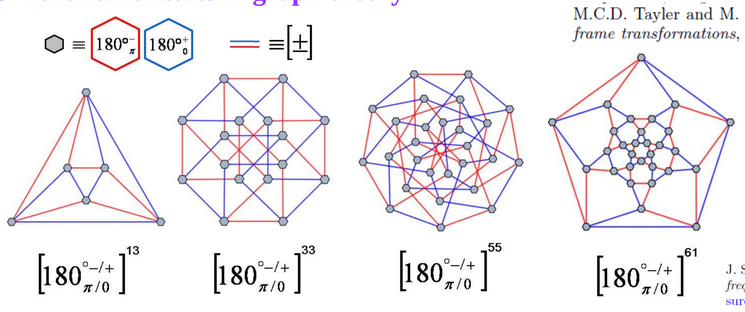
❑ IV. Towards fault-tolerant dynamical-decoupling SU(2) hyper-clocks :

Dynamical-decoupling sequences of **multiple refocusing rotary-echoe pulses** to shied optical clock transitions against decoherence.

→ **Eulerian circuits in graph theory**

L. Viola, *Advances in decoherence control*, Journal of Modern Optics **51**, 2357 (2004).

M.C.D. Tayler and M. Sabba, *Cyclicity of interaction-frame transformations*, Phys. Rev. A (2025).



J. Schat, *Using the Thue-Morse sequence to cancel low-frequency fluctuations*, IEEE Instrumentation and Measurement Technology Conf. IMTC, 1 (2007).

A natural choice is to start with the generation of Walsh-Hadamard WH_{2^k} building-blocks by a recursive application following:

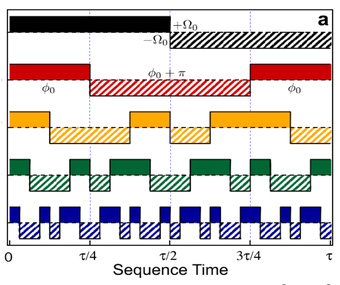
$$WH_{2^k} = \begin{bmatrix} WH_{2^{k-1}} & WH_{2^{k-1}} \\ WH_{2^{k-1}} & -WH_{2^{k-1}} \end{bmatrix}_H = WH_2 \otimes WH_{2^{k-1}}$$

where we have the initilization stage (k= 1) WH_2 as:

$$\Rightarrow \begin{bmatrix} + & + \\ + & - \end{bmatrix}_H$$

WAHAHA-HR protocols

$$\begin{bmatrix} WH_4 & WH_4 \\ WH_4 & -WH_4 \end{bmatrix}_S = \Rightarrow \begin{bmatrix} + & + & + & + & + & + & + & + \\ + & + & + & + & - & - & - & - \\ + & + & - & - & - & - & + & + \\ + & + & - & - & + & + & - & - \\ + & - & - & + & + & - & - & + \\ + & - & - & + & - & + & + & - \\ + & - & + & - & - & + & - & + \\ + & - & + & - & + & - & + & - \end{bmatrix}_S \Rightarrow$$

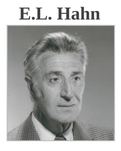


Filter design by Walsh synthesis

J.L. Walsh

J. Hadamard

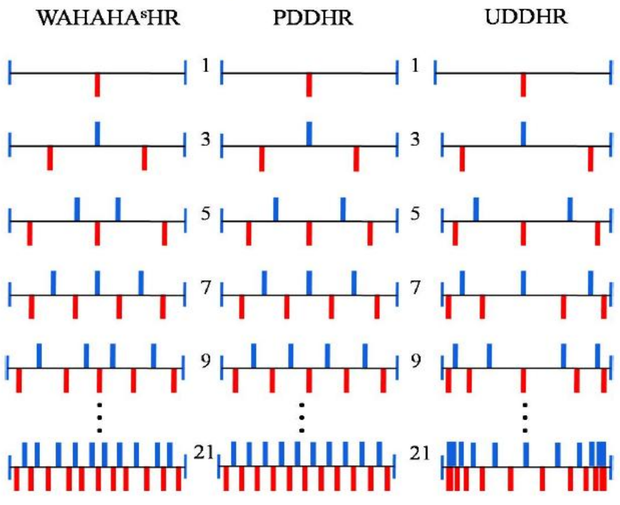
E.L. Hahn



H. Ball and M.J. Biercuk, *Walsh synthetized noise filters for quantum logic*, EPJ Quantum Technology **2**, 11 (2015).

Periodic and CPMG DD-HR protocols $(1 \le n \le 2N-1)$

$$180^{\circ-}_{\pi} \to \left[[(-1)^n]\,180^{\circ,(-1)^n}_{n\pi}\,[(-1)^{n-1}]\right]^{2N-1}$$

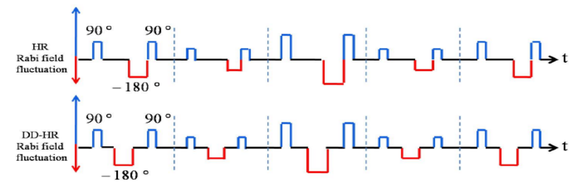


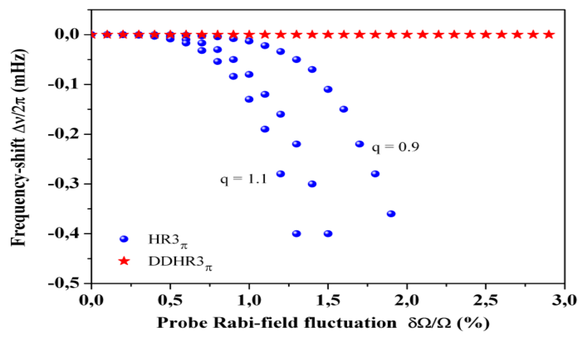
Uhrig DD-HR protocols

refocusing pulses within *magic* time intervals

$$T \to T\left(\cos\left[(n-1)\frac{\pi}{2N}\right] - \cos\left[n\frac{\pi}{2N}\right]\right) \quad (\text{for } 1 \le n \le 2N)$$

K. Beloy, *Hyper-Ramsey spectroscopy with probe-laser-intensity fluctuations*, Phys. Rev. A **97**, 031406(R) (2018).





Conclusion of this work

Hyper-Ramsey protocols effectively reduce AC-Stark shifts in probing ultra-narrow optical clock transitions but they remain sensitive to laser intensity noise, decoherence, frequency drifts, and low-frequency perturbations. We address these limitations by incorporating dynamical decoupling, using sequences of rotary Hahn-echo pulses that toggle the probe frequency detuning and phase between opposite signs. Implementing time-optimized Eulerian cycling circuits of multiple refocusing pulses, we generate high-contrast hyper-Ramsey interferences that are completely free from AC-Stark shifts and robust against environmental noise and laser probe parameters imperfections. Fault-tolerant dynamically-decoupled SU(2) hyper-clocks are a significant step toward universal, noise-resilient quantum sensors, enabling fault-tolerant metrology for searches about new physics beyond the Standard Model.