# neBEM
# Recent Developments and Applications

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# Outline

- *Numerical adventures*: Taming stubborn system of equations
- *Applications as a component of the simulation framework:* RPC, Micromegas, GEM, MHSP
- *Future plans:*

# Numerical Adventures

# BEM Solvers

- Numerical implementation of boundary integral equations (BIE) based on Green's function by discretization of boundary.
- Boundary elements endowed with distribution of sources, doublets, dipoles, vortices (singularities).
- Useful in fluid dynamics, fracture mechanics, acoustics, optics, gravitation, electromagnetics, quantum mechanics …

**Electrostatics BIE**

$$\Phi(\vec{r}) = \int_S G(\vec{r}, \vec{r}\,')\,\rho(\vec{r}\,')\,ds'$$

Potential at r

Charge density at r'

Green's function

$$G(\vec{r}, \vec{r}\,') = \frac{1}{4\pi\varepsilon\,|\vec{r} - \vec{r}\,'|}$$

ε - permittivity of medium

discretization

$$[A]\{\rho\} = \{\Phi\}$$

Influence Coefficient Matrix

$$\{\rho\} = [A]^{-1}\{\Phi\}$$

Accuracy depends critically on the estimation of [A], in turn, the integration of G, which involves singularities when $r \rightarrow r'$.

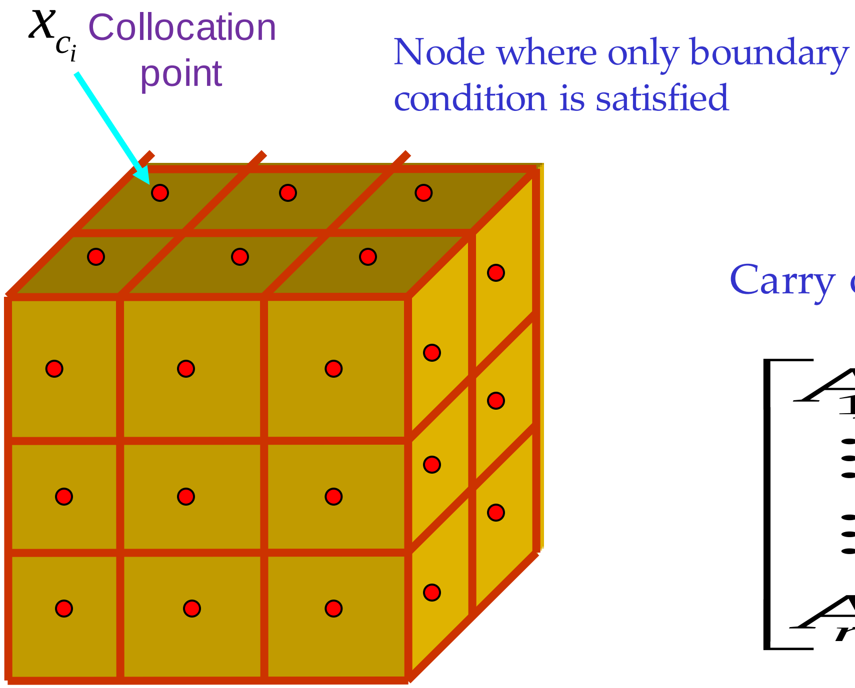
Most BEM solvers fail here.

# nearly exact BEM

**Centroid Collocation**

**Basis Function Approach**

$$\Psi(\vec{x}_{c_i}) = \sum_{j=1}^{n} \alpha_j \underbrace{\int_{panel\ j} \frac{1}{\left\| \vec{x}_{c_i} - \vec{x}' \right\|} \, dS'}_{A_{i,j}}$$




Carry out the integrations!

$$A_{i,j} = \int_S G(\vec{x}_{c_i}, \vec{x}') \, \varphi_j(\vec{x}') \, dS'$$

$$\begin{bmatrix} A_{1,1} & \cdots & A_{1,n} \\ \vdots & \ddots & \vdots \\ A_{n,1} & \cdots & A_{n,n} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{bmatrix} = \begin{bmatrix} \Psi(\vec{x}_{c_1}) \\ \vdots \\ \Psi(\vec{x}_{c_n}) \end{bmatrix}$$

No singularities, no special treatments, no additional formulations

# System of algebraic equations

*Important definitions*

The Singular Values of the rectangular / square matrix A with real elements is defined as the square root of the eigenvalues of $A^TA$.

The Condition Number is the ratio of the largest to the smallest singular value.

A matrix is Ill-Conditioned Matrix if the condition number is too large. How large the condition number can be, before the matrix is ill-conditioned, is determined by the machine precision. Well-conditioned matrices have condition numbers close to 1.
A matrix is Singular if the condition number is infinite. The determinant of a singular matrix is zero.

The Rank of a matrix, is the dimension of the range of the matrix. This corresponds to the number of non-singular values for the matrix, i.e. the number of linear independent rows of the matrix.

# Why need we be concerned?

*Encounters of strange kind*

A square matrix A is ill-conditioned if it is invertible but can become non-invertible (singular) if some of its entries are changed ever so slightly.

Solving linear systems whose coefficient matrices are ill-conditioned is tricky also because even a small change in the right-hand side vector can lead to radically different answers.

In presence of round-off errors, ill-conditioned matrices are very hard to handle.

They occur when degenerate boundaries, degenerate scales related to special geometries, mixed boundary conditions, large difference in dielectric permittivity are present.

Besides unit circle, the degenerate scale problem under different contours with specific geometry may be encountered, and the accurate degenerate scale is dependent on the discretization boundary density in BEM. Mathematically speaking, the singularity pattern distributed along a ring boundary resulting in a zero field introduces a degenerate scale.

# Good old days

## *LU decomposition*

$$\begin{bmatrix} \alpha_{11} & 0 & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & 0 \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} \end{bmatrix} \cdot \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ 0 & \beta_{22} & \beta_{23} & \beta_{24} \\ 0 & 0 & \beta_{33} & \beta_{34} \\ 0 & 0 & 0 & \beta_{44} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

$$\alpha_{ii} = 1$$

The LU decomposition is a method reducing a square matrix to a product of two triangular matrices (lower triangular and upper triangular). It does not require a positive definite matrix, but there is no guarantee that it is equivalent to solving a system of linear equations by the Gauss elimination. Its advantages include easy implementation, speed and disadvantages include numerical instability without pivoting.

The LU decomposition is most commonly used in the solution of systems of simultaneous linear equations. The method is at least twice as fast as other decomposition algorithms. Unfortunately, the method has less desirable numerical problems.

For ill-conditioned problems, or when high accuracy is needed, other decomposition would be preferred.

# Enters Singular Value Decomposition

SVD of $A \in \mathbf{R}^{m \times n}$ with $\mathbf{Rank}(A) = r$:

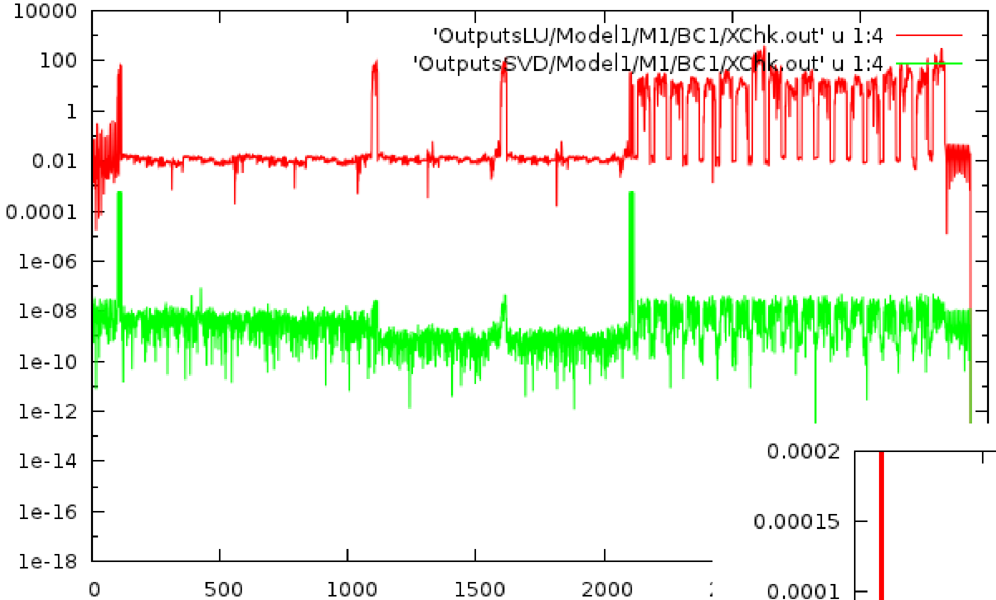
$$A = U_1 \Sigma_1 V_1^T = \begin{bmatrix} u_1 & \cdots & u_r \end{bmatrix} \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_r \end{bmatrix} \begin{bmatrix} v_1^T \\ \vdots \\ v_r^T \end{bmatrix}$$

Using two orthonormal matrices, SVD can diagonalize any matrix and the results of SVD can tell a lot about (numerical) properties of the matrix. SVD offers a numerically stable way to solve a system of linear equations. The diagonal entries of Σ are the singular values of the least-squares matrix A. For N charges there will be N singular values. The condition number of A is defined by the ratio of the largest to the smallest singular value .

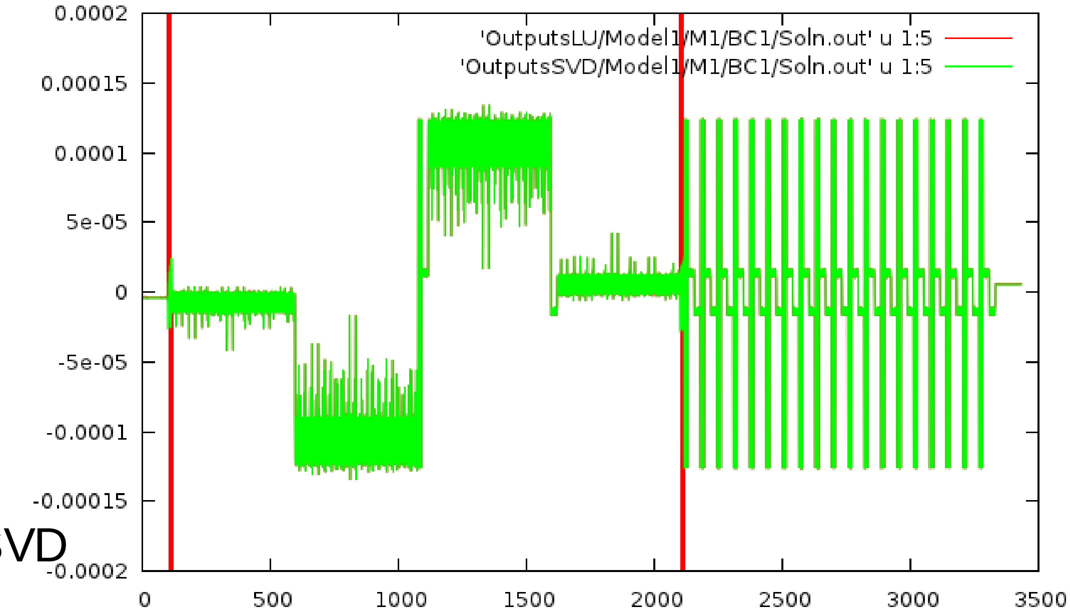
Thus, one can compute the eigenvalues of directly from the original matrix. Second, the number of nonzero singular values equals the rank of a matrix. Consequently, SVD can be used to find an effective rank of a matrix, to check a near singularity and to compute the condition number of a matrix. That is, it allows to assess conditioning and sensitivity to errors of a given system of equations.

# Observation of significant improvements

*Error estimates*



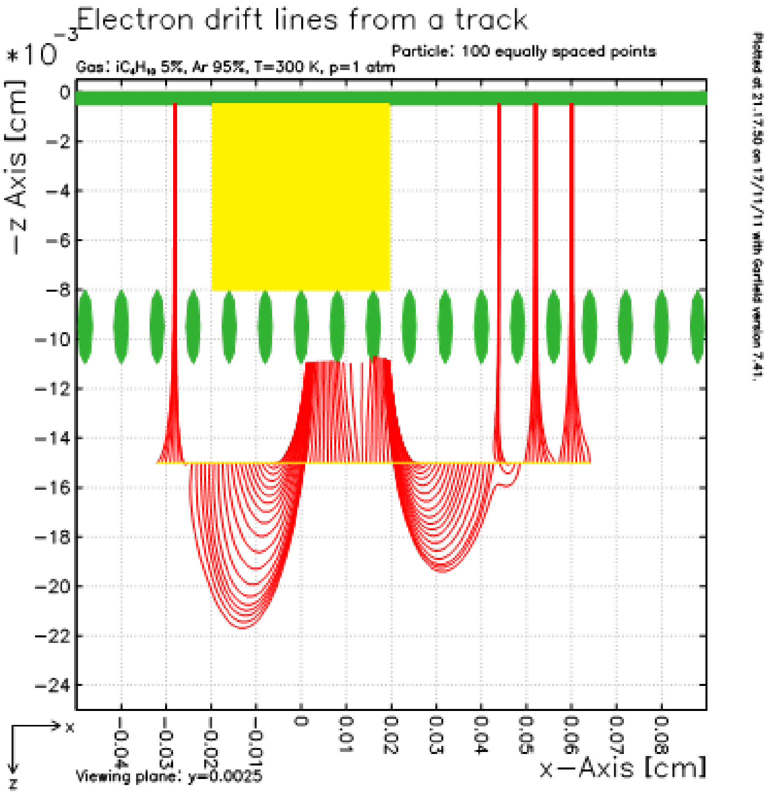

Errors using LU and SVD (note the log scale!)

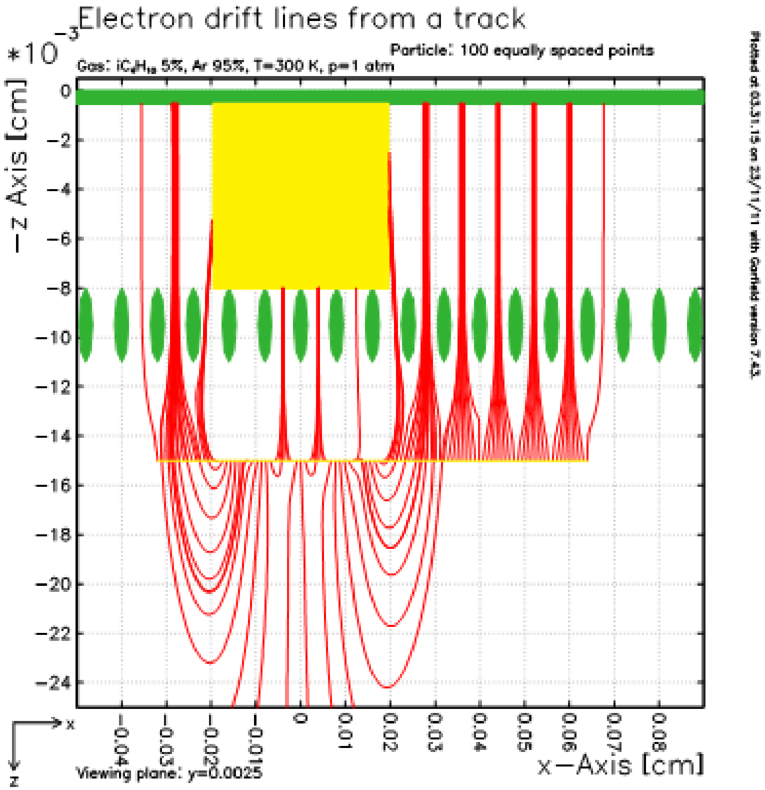



Charge densities using LU and SVD

# Observation of significant differences

*Effect on detector physics*




Drift lines in a bulk micromegas (LU)

Drift lines in a bulk micromegas (SVD)

Recent applications

# neBEM Strengths

- Can handle large length scale variation, including thin structures
- Can provide potential and field at arbitrary locations
- Can handle intricate 3D geometries
- Can handle multiple-material complex systems
- Has been made reasonably robust and fast

- Integrated to the fortran version of Garfield
- Using its toolkit nature, neBEM can be easily interfaced

# New simulation framework

*Garfield+Magboltz+Heed+neBEM*

*neBEM* (nearly exact Boundary Element Method) as a toolkit

*A new formulation based on green's function that allows the use of exact close-form analytic expressions while solving 3D problems governed by Poisson's equation. It is very precise even in critical near-field regions and microscopic length scale.*

It is easy to use, interface and integrate neBEM

Stand-alone
A driver routine
An interface routine
Post-processing, if necessary

Garfield
Garfield prompt
Garfield script

Interface to Magboltz, Heed

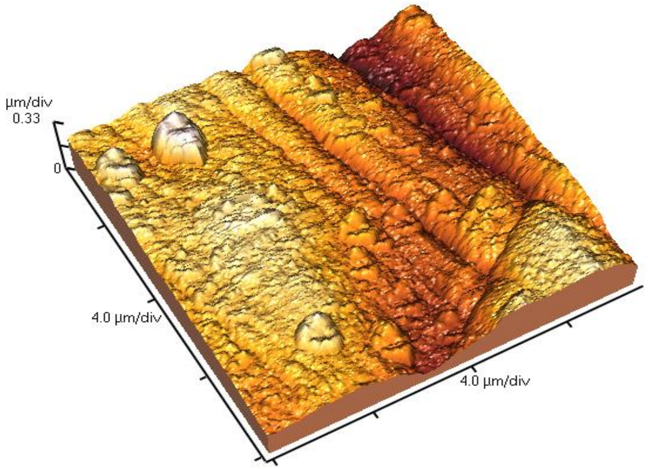
Charge density at all the interfaces

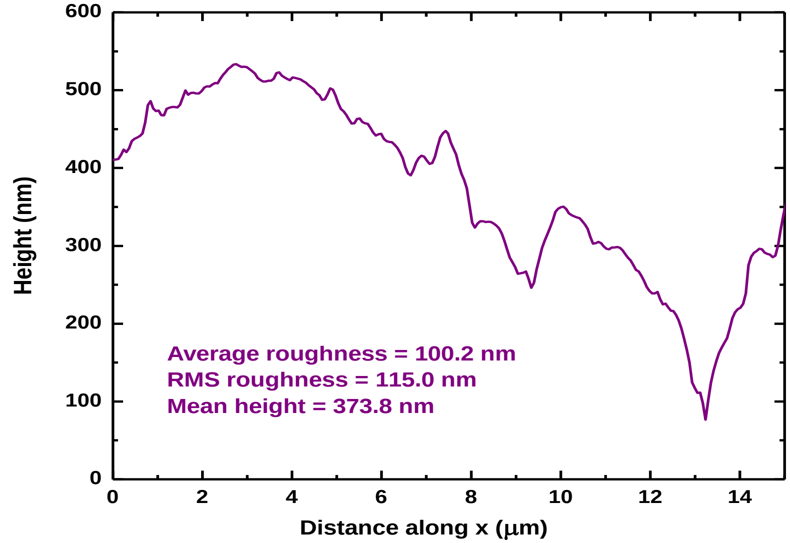
Potential at any arbitrary point

Field at any arbitrary point

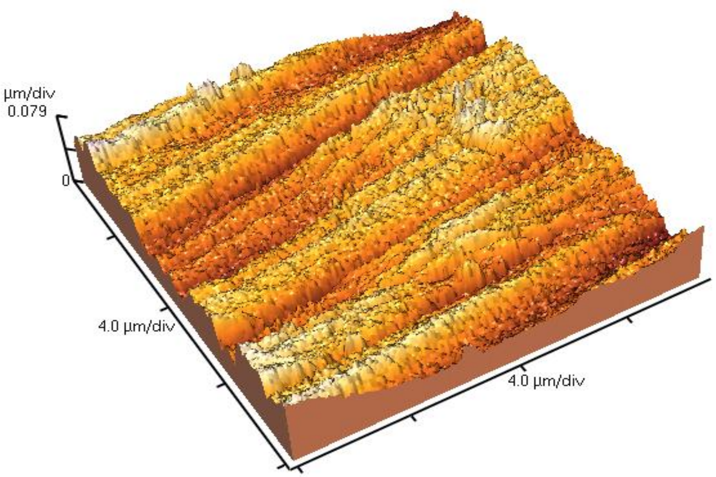
Capacitance, forces on device components properties can be obtained by post-processing

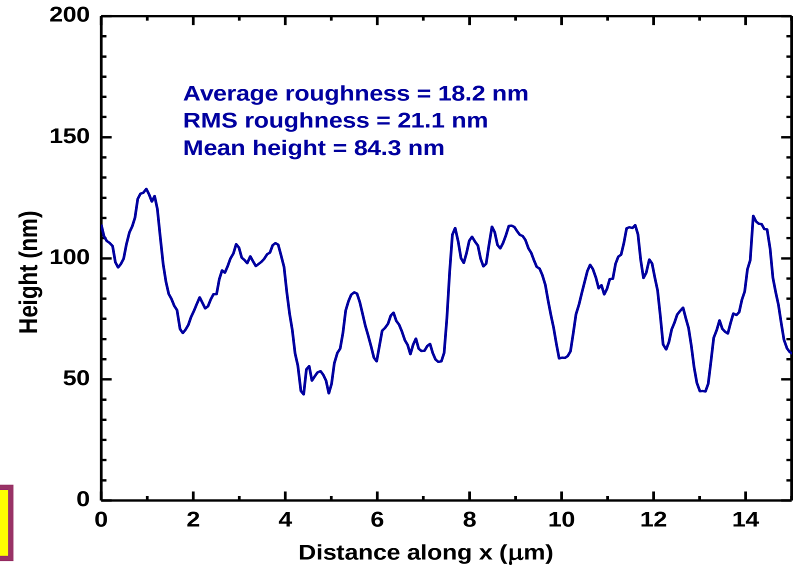
# Roughness studies of RPCs




**2 mm thick**

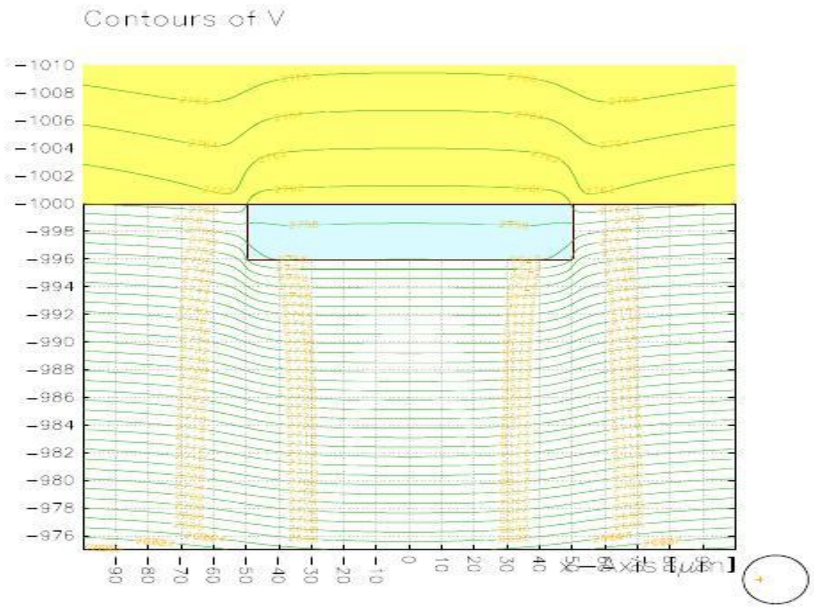


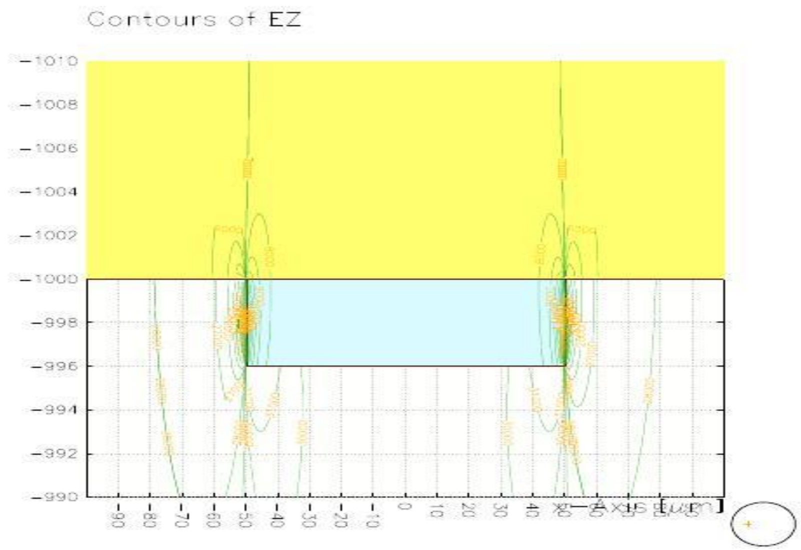



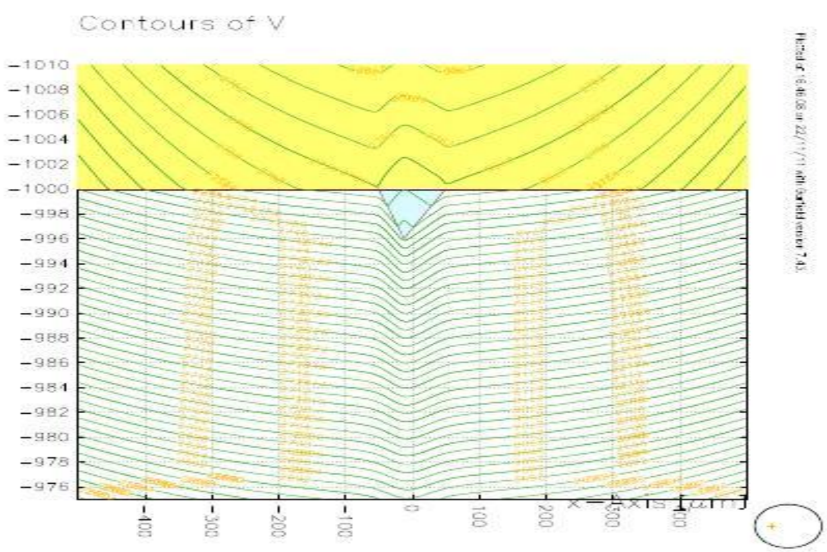

**1.6 mm thick (Smoother surface)**

**AFM Images and analysis of uncoated P-120 bakelite surfaces**

# Roughness studies on RPCs



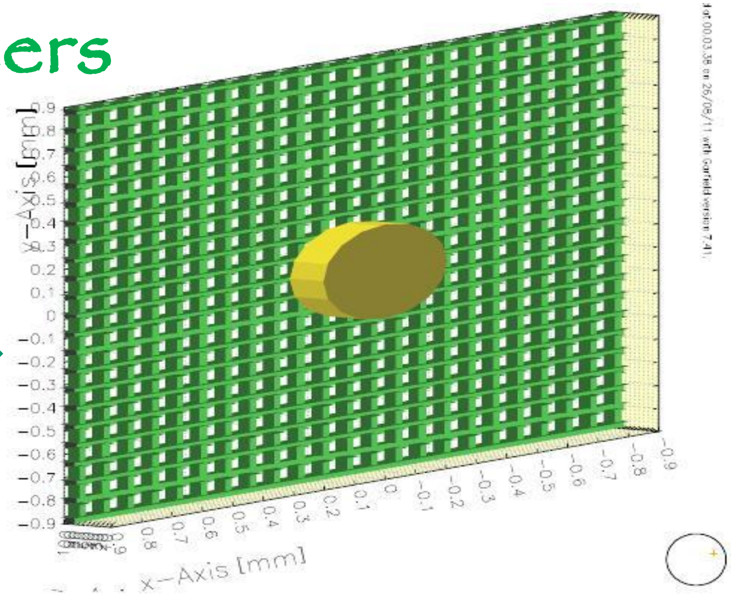




- Regularly shaped 4 micron high dielectric pillar/pyramid on bakelite surface
- Base of the rectangular pillar and the pyramid shaped one is 100 micron square
- Gas gap is 2 mm

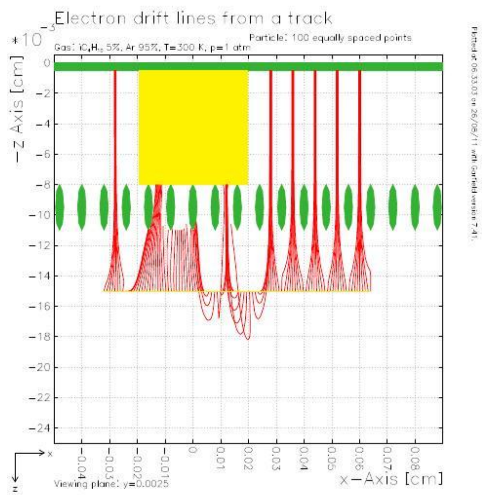
# Micromegas
# Effect of spacers

- *Regularly spaced insulating pillars between grid and anode plane, guarantee the uniformity of gap*
- *Pillars radius larger than mesh cell size, particles are not detected in this region*

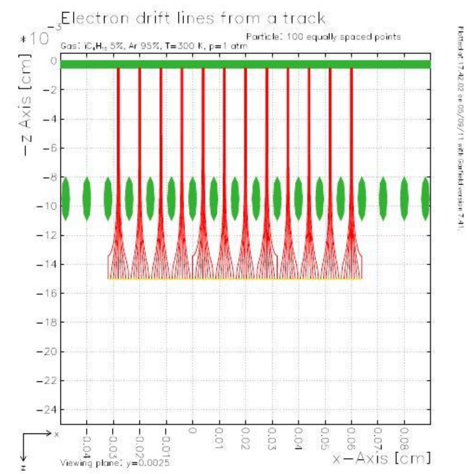
***Mesh with one spacer : mesh pitch 80 $\mu m$, spacer diameter 400 $\mu m$***




- ✓ **Electron drift lines get distorted in spacer region, do not reach anode plane**
- ✓ **Total charge collected by anode affected**
- ✓ **Effect on spatial resolution, energy resolution etc.**
- ✓ **Only preliminary results, need further investigation**




**With Spacer (LU)**

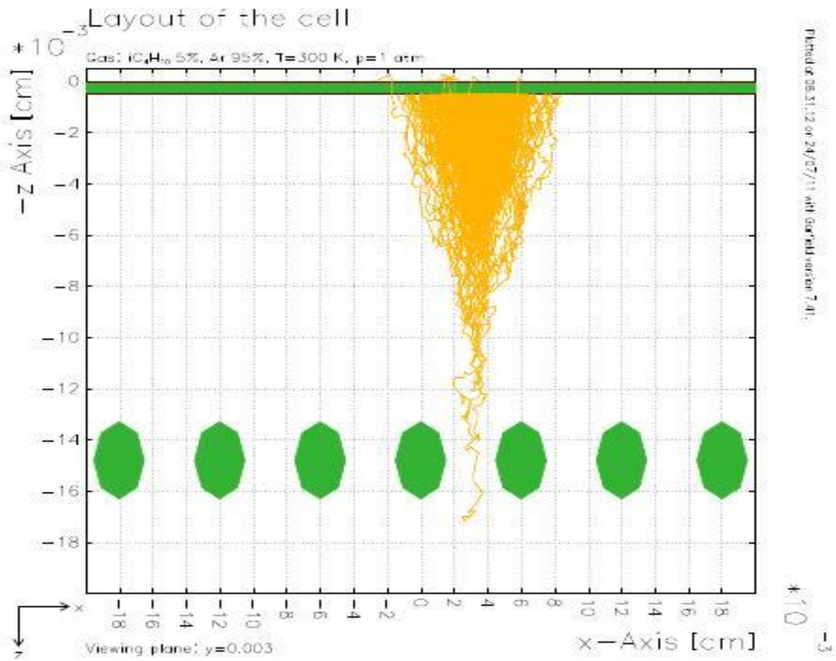



**Without Spacer**

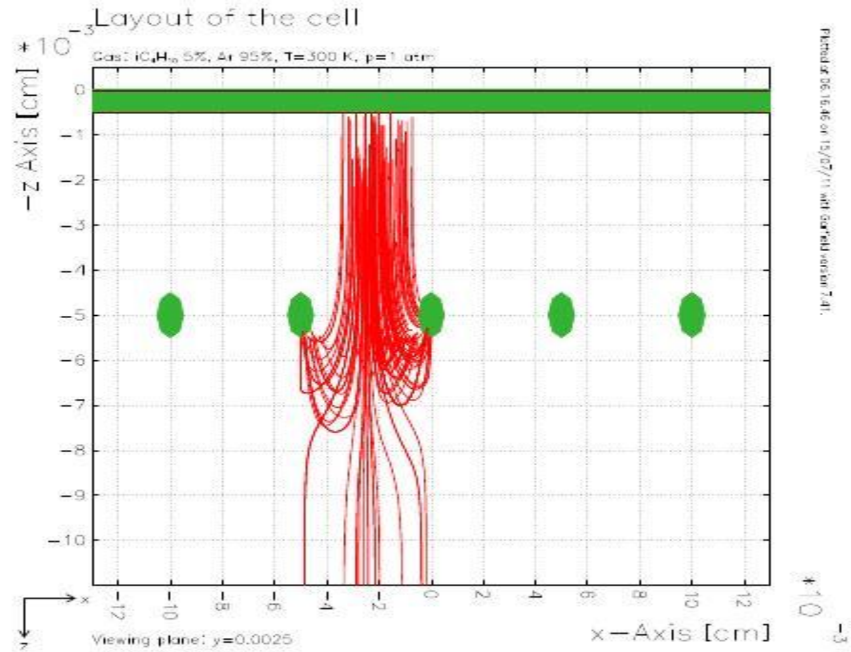
***Electron Drift Lines (2D Picture)***

# Micromegas
# Ion back-flow

- ***Secondary ions from amplification region drift to drift region***
- ***Distortion of electric field***
- ***Micromegas micromesh stops a large fraction of these ions***

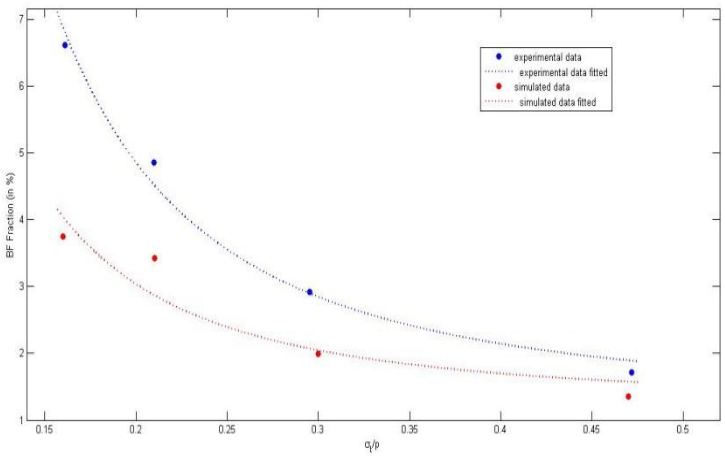



**Avalanche of Electrons (2D picture)**

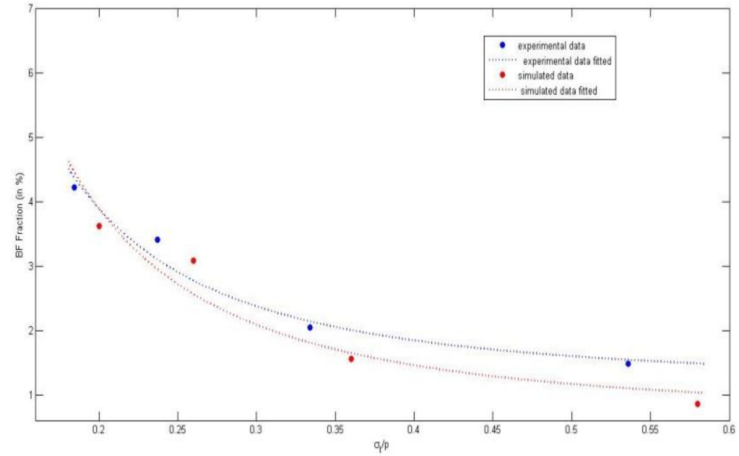



**Drift of Secondary Ions (2D picture)**

## *Variation of Backflow fraction at a field ratio of 100 for various values of the ratio $\sigma_t/p$*

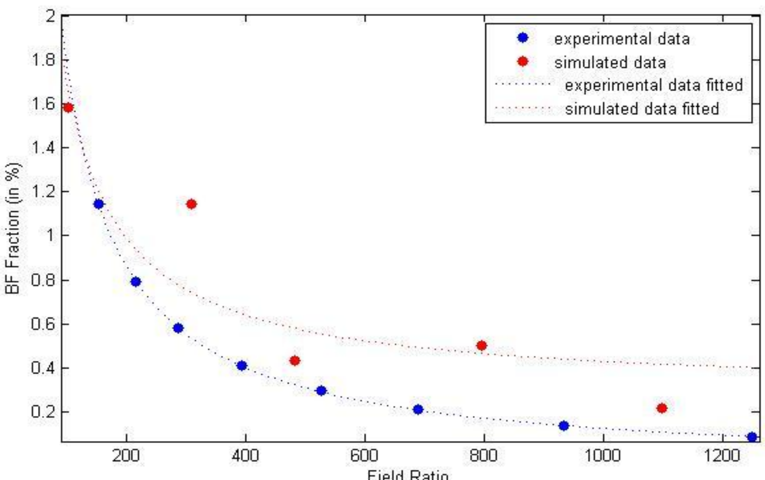



**(Amplification gap – 45 μm)**

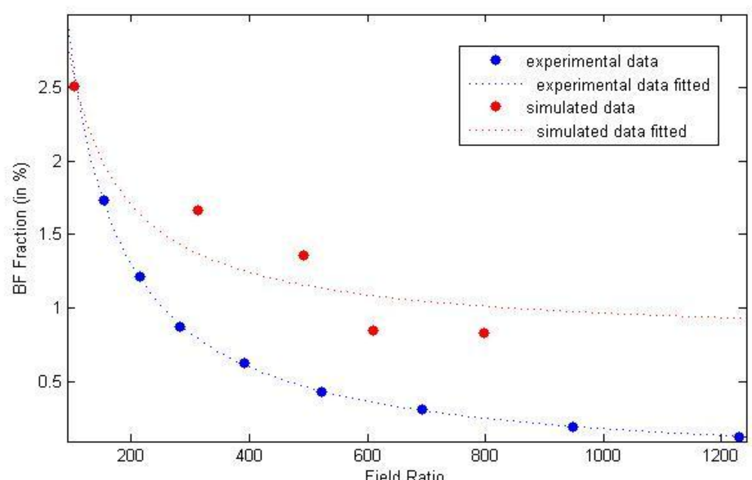



**(Amplification gap – 58 μm)**

## *Variation of Backflow fraction with field ratio for 70 μm amplification gap*

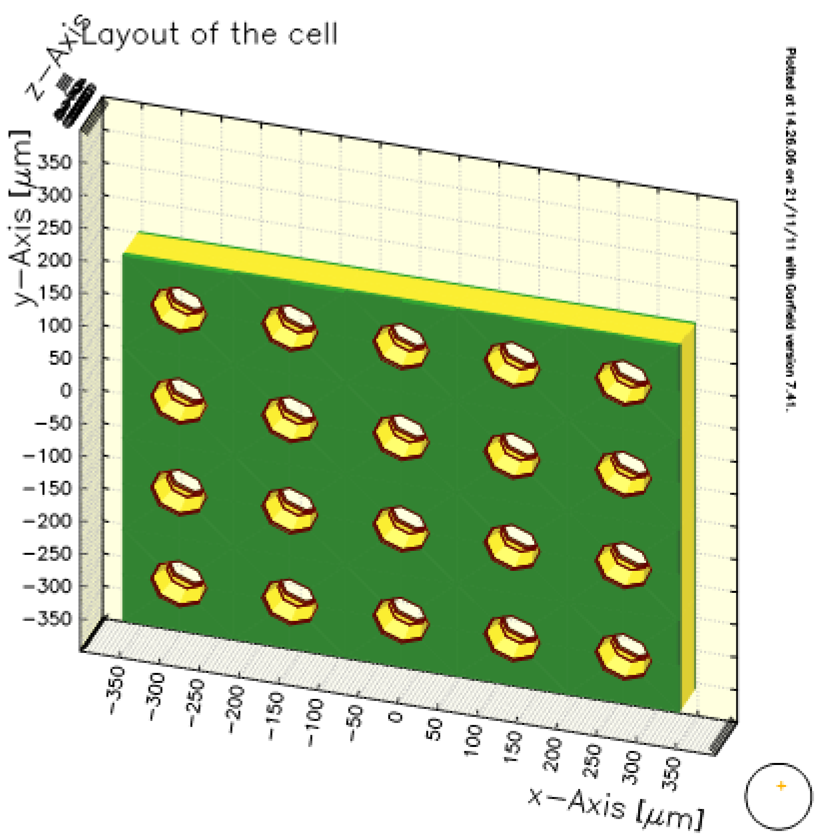


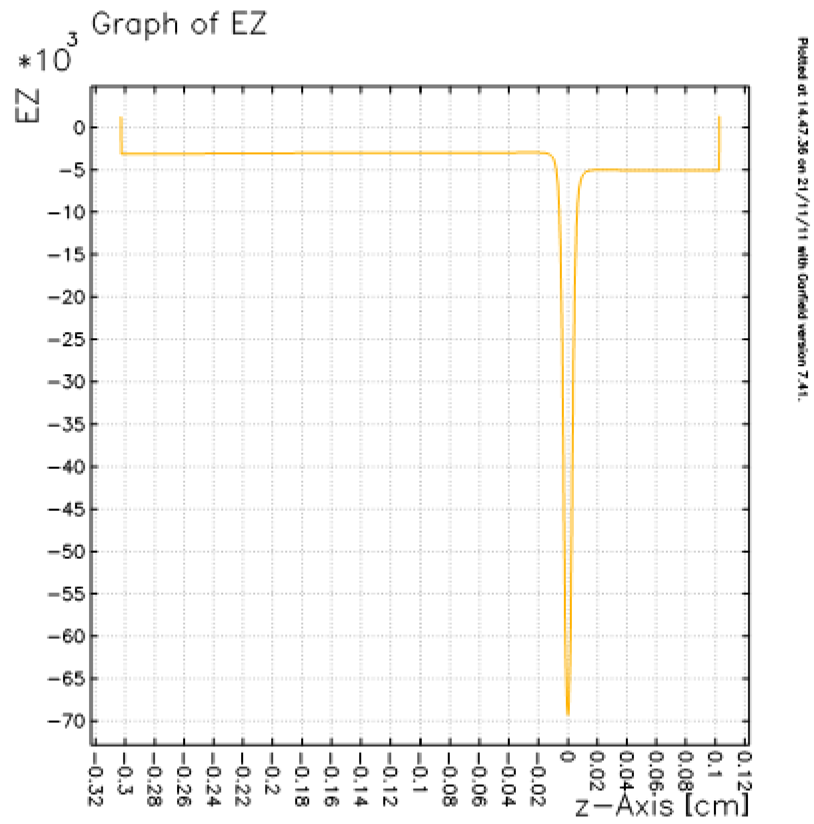

**(Pitch – 32 μm)**



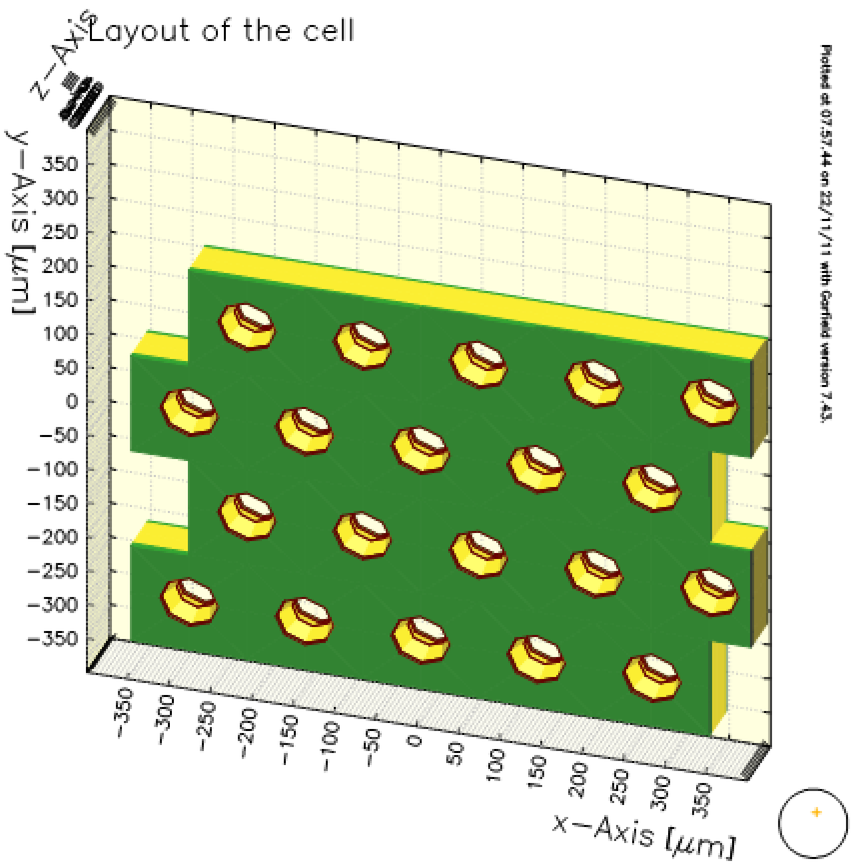

**(Pitch – 45 μm)**

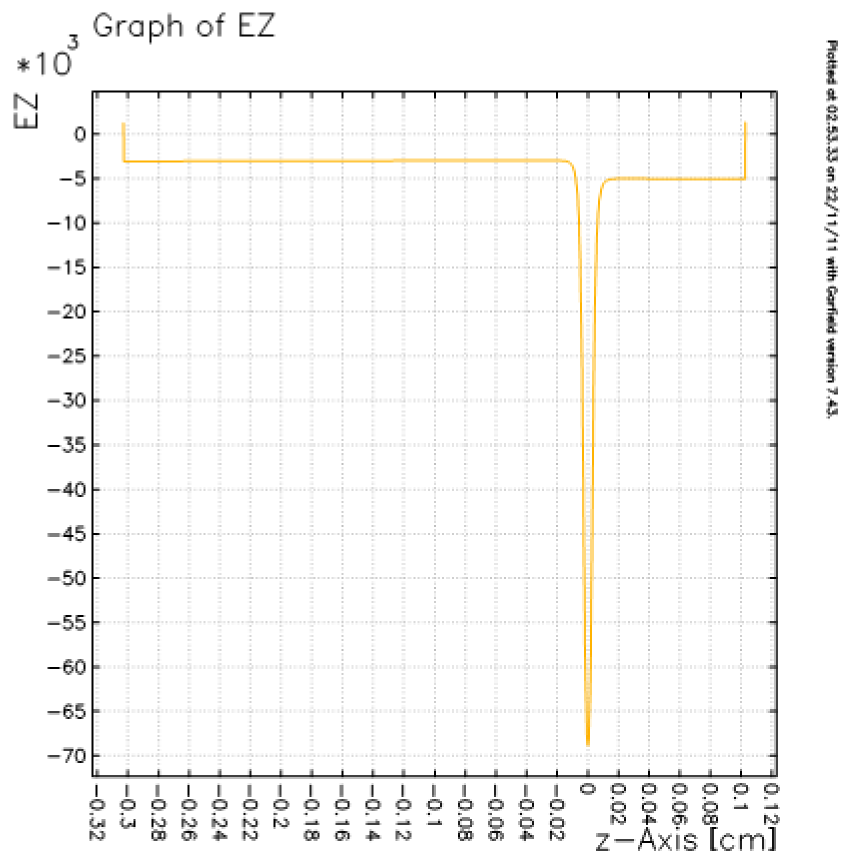
# Non-staggered GEM



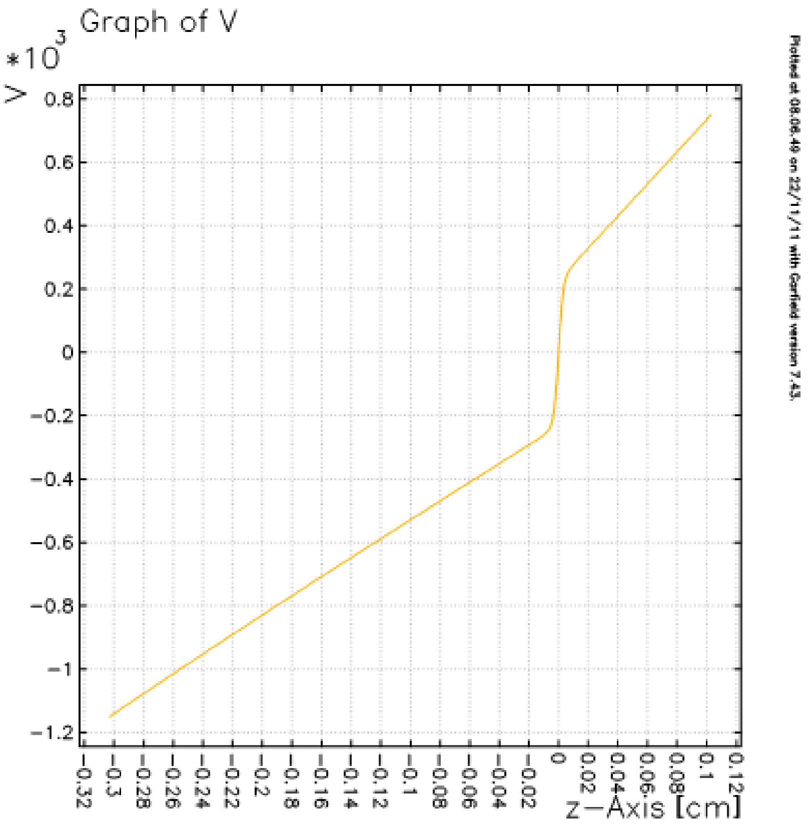



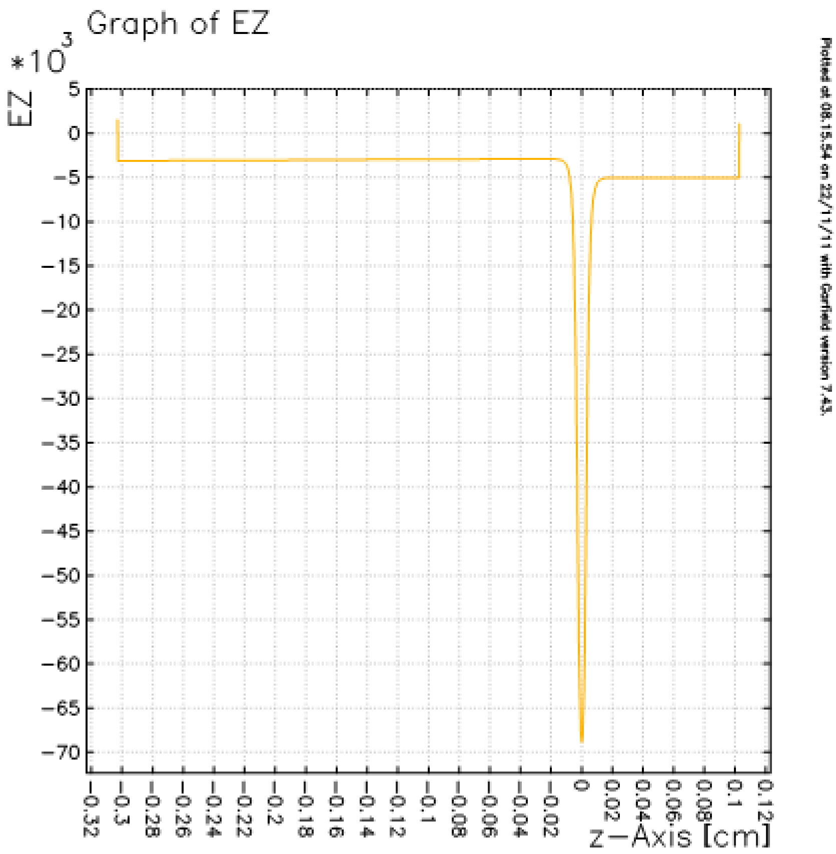
# Staggered GEM with LU



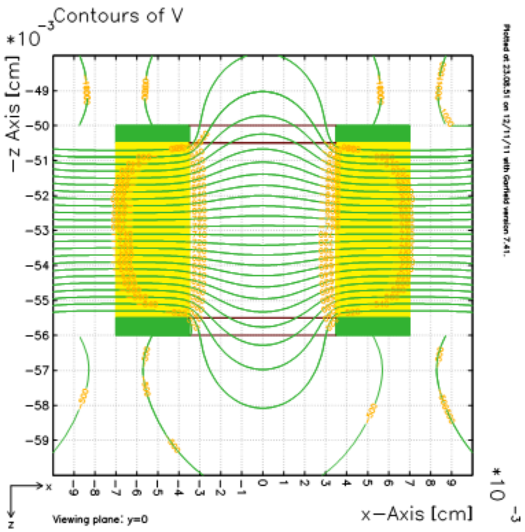



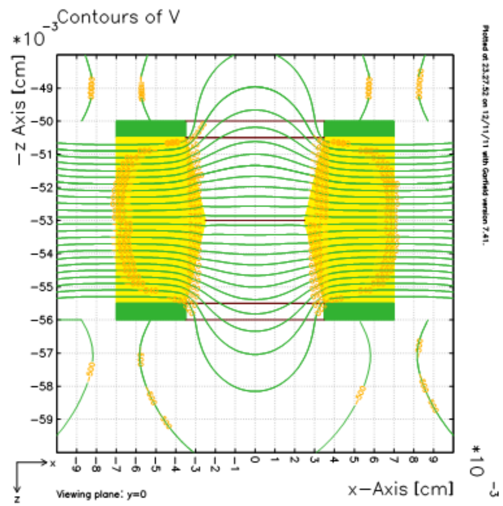
# Staggered GEM with SVD

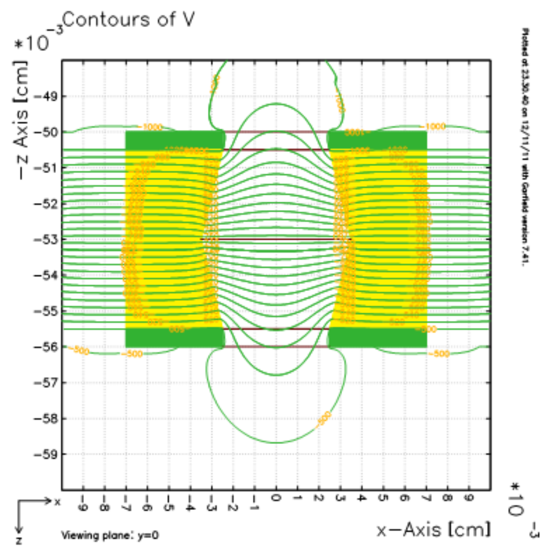


# Potential contours in several GEMs



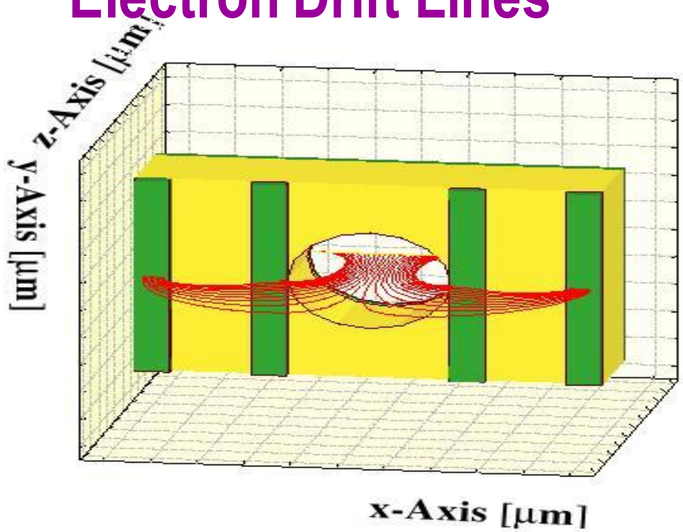



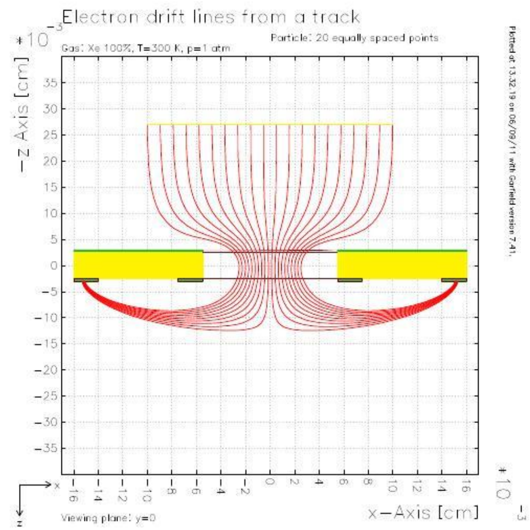
# MHSP

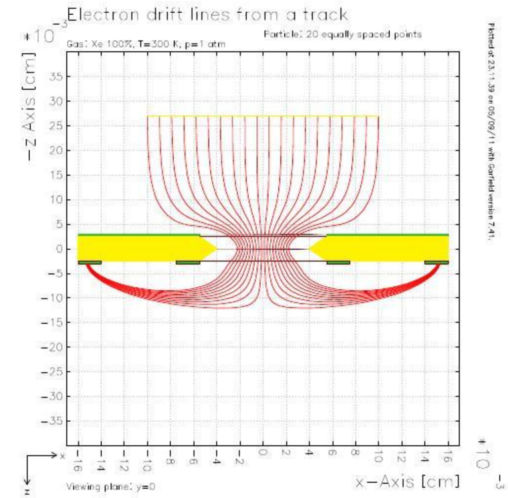
*efficiency, gain*

## Electron Drift Lines



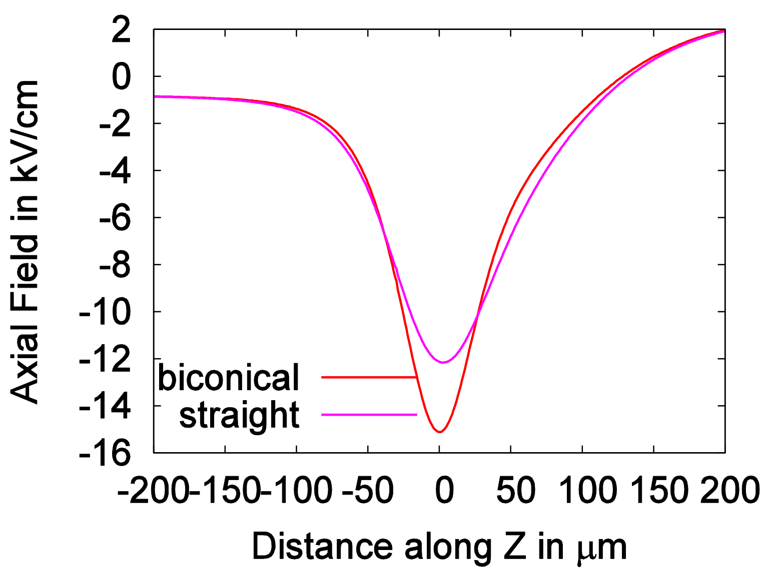

**3D Picture**




**Straight Hole** **Bi-conical Hole**

**2D Picture**

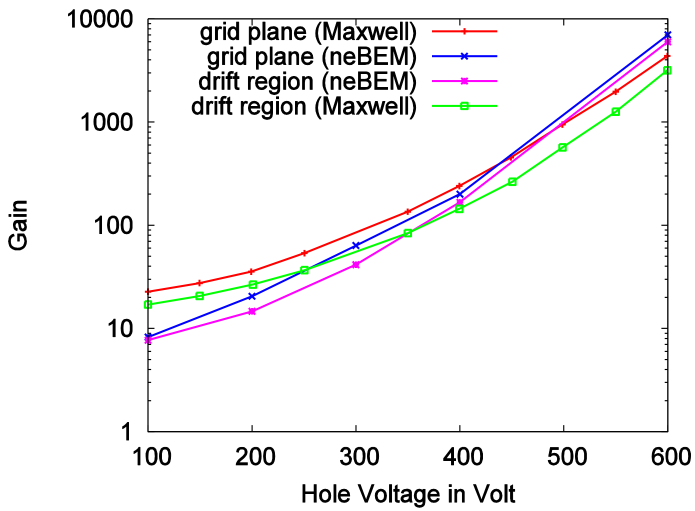


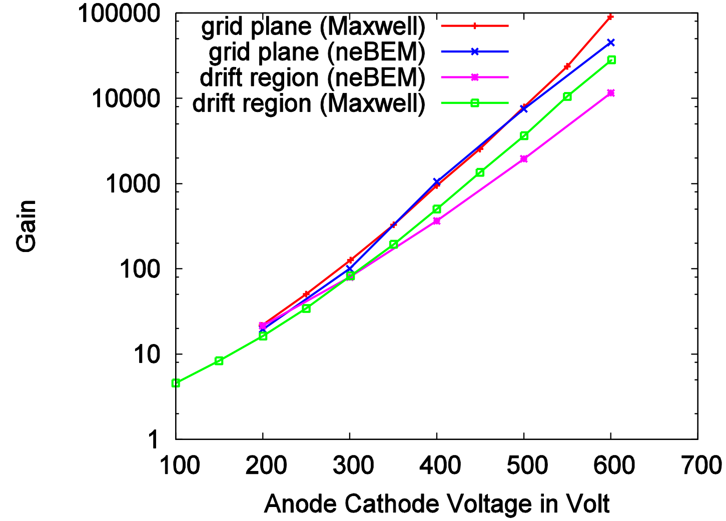

**Gas : Pure Xenon Temp.: 300 K, Pressure : 1 Atm**

**Axial Electric Field**

# Comparison with existing simulated results (2D calculations using Maxwell)

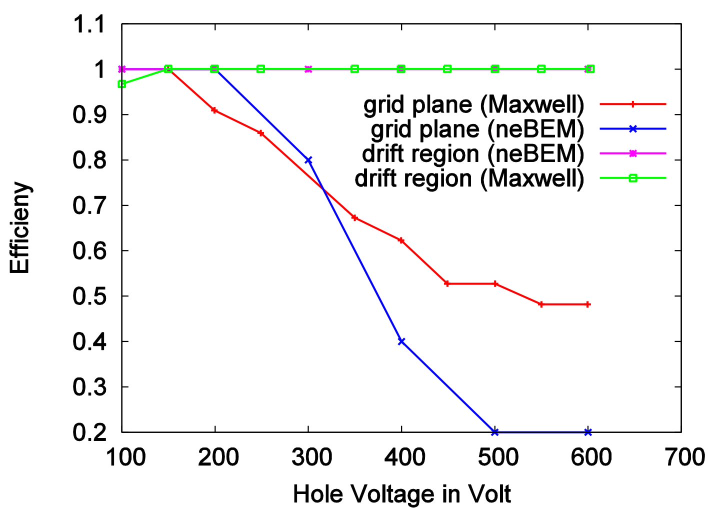
## Variation of Total Gain

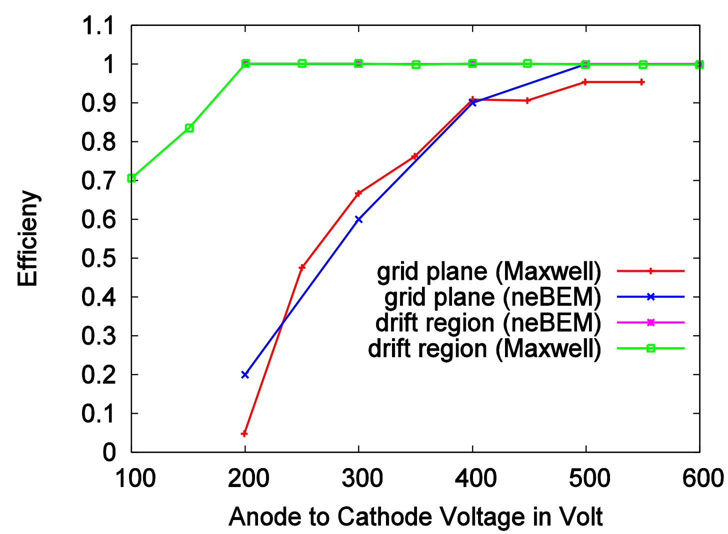



**With Hole Voltage**

**With Anode to Cathode Voltage**




## Variation of Electron Collection Efficiency of Anode

# Plans for the near-future

# Interface to Garfield++

- In the fortran version, neBEM essentially uses the Garfield geometry modeler to set up its model.

- An improved version of the geometry modeler is being developed for Garfield++. As soon as it is there, neBEM will be happily interfaced to the new Garfield++.

# Improved geometry modeler, user interface

- Simple enhancements such as mirror repetitions
- Interface to ROOT and Geant 4 geometries; user interfaces based on such and similar programs will immensely help users
- A geometry modeler tuned for neBEM (likely to be based on CGAL or a similar library)

Thank you!