Applications of String Field Theory

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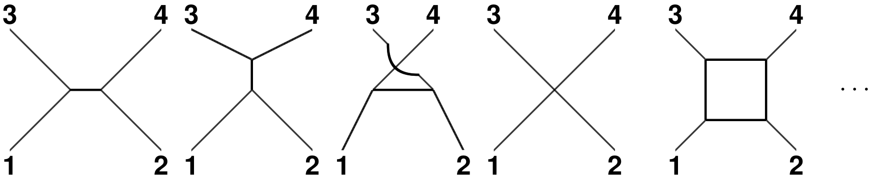
Our set-up: String theory in asymptotically flat space-time

Goal: Compute S-matrix

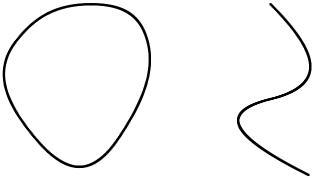
In a weakly coupled quantum field theory S-matrix is computed using Feynman diagrams

e.g. in a scalar field theory with cubic and quartic interactions, the four-point amplitude has contributions from the diagrams



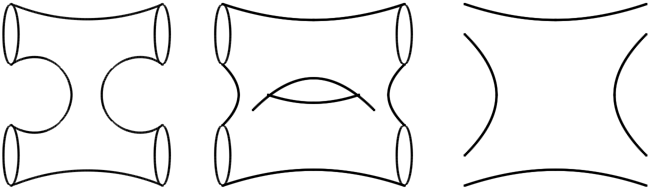
String theory is an attempt to formulate a theory of quantum gravity and all other forces

– based on the idea that the elementary constituents of matter are one dimensional objects

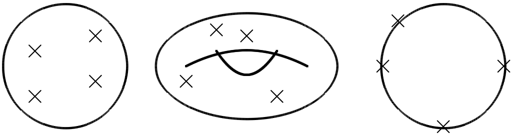


Observables of the theory are again S-matrices

– computed using string diagrams by thickening the lines of Feynman diagrams into tubes or strips



For external states of fixed mass, we can close the 'gaps' associated with external strings and replace them by 'vertex operators'



$\times$: punctures where the vertex operators are inserted

World-sheet theory gives expression for the S-matrix to any order in perturbation theory

n-point amplitude:

$$\int_{M_n} I_n$$

M_n: moduli space of Riemann surfaces with n punctures

I_n: correlation function of n vertex operators inserted at the n punctures of the Riemann surface + universal ghost and PCO insertions*

$\int_{M_n}$ includes sum over Riemann surfaces of different topologies

– sphere, torus, disk etc

*PCO: Picture changing operator

Example: Veneziano amplitude

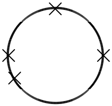
– four point function of open string tachyons in bosonic theory

$(p_1, p_2) \to (p_3, p_4)$

$$\int_0^1 dx\, x^{-2-s}(1-x)^{-2-t}$$

$s = -(p_1 + p_2)^2 = (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2, \quad t = -(p_1 - p_3)^2$

x parametrizes the moduli space of disk with four punctures at the boundary



$$z_{UHP} = i\,\frac{1 - z_{disk}}{1 + z_{disk}}$$

Note: The integral diverges for $s \geq -1$ and / or $t \geq -1$

Usual procedure: Define this by analytic continuation

String field theory (SFT) is a quantum theory of infinite number of fields, designed to formally reproduce the amplitudes given by the world-sheet theory

As in a quantum field theory, the amplitudes in SFT are given by sum over Feynman diagrams

i-th Feynman diagram gives $\int_{R_n^{(i)}} I_n$, where $R_n^{(i)}$ is a subregion $R_n^{(i)}$ of M_n

Sum over all Feynman diagrams gives the full amplitude

$$\cup_i R_n^{(i)} = M_n$$

The decomposition of M_n provided by SFT is not unique

SFT comes with intrinsic set of parameters

Different choice of parameters correspond to different ways of dividing up the moduli spaces

These different SFT's can be shown to be related by field redefinition

– all physical results are independent of which version of SFT we use

Why do we need string field theory?

Schwinger parameter representation of the SFT propagator

$$(k^2 + m^2)^{-1} = 2\pi \int_0^\infty ds\, e^{-2\pi s(k^2+m^2)}$$

s for different propagators become the world-sheet modular parameters

Divergences come from $s \to \infty$ limit from states with $k^2 + m^2 \leq 0$

For $k^2 + m^2 < 0$ states we simply use the correct representation $(k^2 + m^2)^{-1}$

– equivalent to analytic continuation we saw earlier

For $k^2 + m^2 = 0$ states we have genuine divergence

We have to remove their contribution from the propagator and treat them separately using insights from QFT

Some examples of cases where we need string field theory:

1. **Mass and wave-function renormalization**

2. **Massless tadpoles**

3. **Solitons and instantons**

Review: A.S., Zwiebach, arXiv:2405.19421

Mass and wave-function renormalization

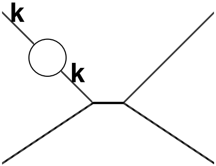
$\int_{M_n} I_n$ gives the SFT amplitudes by naively extending the tree level rules to loop amplitudes

1. Use the quadratic terms in the classical action to determine the masses of particles

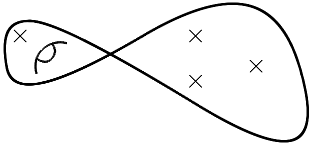
2. Multiply the Green's function by $\prod_i (k_i^2 + m_i^2)$ and set $k_i^2 + m_i^2 = 0$

This works for tree amplitudes, but at loop level we need to resum self-energy insertion on external lines and renormalize masses and wave-functions following LSZ

SFT can do this but world-sheet theory cannot



SFT



world-sheet at s=∞

The internal propagator of momentum k is on-shell as long as the external state is on-shell

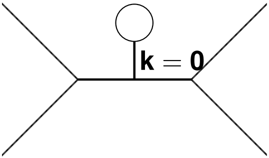
$$(k^2+m^2)^{-1} = 2\pi\int_0^\infty e^{-2\pi s(k^2+m^2)} ds = 2\pi\int_0^\infty ds \quad \text{for} \quad k^2+m^2=0$$

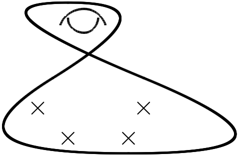
$\Rightarrow$ in the world-sheet description we get $2\pi\int_0^\infty ds$

In SFT we use the standard LSZ procedure to deal with this

Massless tadpoles

String theory can have massless fields whose one point function becomes non-zero at loop level





SFT world-sheet at s=∞

On the world-sheet we again get $\int_0^\infty ds$

In SFT we treat this as a linear contribution to the effective potential

$$V_{eff}(\phi) = c\, g_s\, \phi + V(\phi)$$

Find solution to $c\, g_s + V'(\phi) = 0$ and expand the SFT action around the new solution to compute S-matrix

Even when mass renormalization and tadpole problems are absent, we need SFT to deal with wave-function renormalization

World-sheet results take the form 0 $\times\ \infty \rightarrow$ ambiguous

These ambiguities can be absorbed into Witten

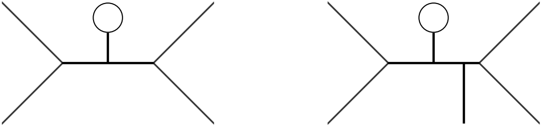
1. Wave-function renormalization of external states

2. Shift of massless background scalars $\phi \rightarrow \phi + c$

Problem: One can do this for one amplitude, but not for every amplitude independently

Example: Once a four point amplitude has been computed in a given background, it is meaningful to ask what the five point amplitude will be in the same background

– not possible to address in the world-sheet theory



In string field theory, all amplitudes are computed at the same background values of the scalar fields)

D-branes

Besides fundamental strings, string theory also has other extended objects known as D-branes

A Dp-brane is a p-dimensional extended object on which the open strings can end

External states in a scattering are open and closed string

Open strings are constrained to move along the brane

– carry p+1 dimensional momenta

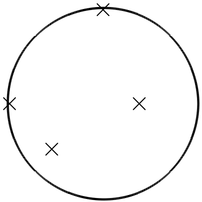
Closed strings live in the bulk

– carry D dimensional momenta (D=10 or 26)

S-matrices are computed as integrals over moduli spaces of punctured Riemann surfaces with boundary

Open string vertex operators are inserted on the boundary

Closed string vertex operators are inserted in the bulk



The same amplitudes can also be reproduced by Feynman diagrams of string field theory of open and closed strings

This procedure works for p>1 but runs into problem for low p

There are massless open string fields

– e.g. translation along transverse directions (collective modes)

In loop amplitudes this could lead to IR divergences

$\int d^{p+1}k/k^2$ has divergence from k=0 for p=-1, 0, 1

D-instanton, D0-brane, D1-brane

In the world-sheet approach, these show up as divergences in the integral over moduli space of Riemann surfaces

$$\int d^{p+1}k/k^2 = 2\pi \int_0^\infty ds \int d^{p+1}k\, e^{-2\pi s k^2} \sim \int_0^\infty ds\, s^{-(p+1)/2}$$

– diverges for p=-1,0,1

– has no remedy in the world-sheet theory which only knows how to do naive perturbation theory

In quantum field theory we know how to address this (al least for p=-1,0)

1. We separate out the massless collective modes and do path integral over the other modes first

– can be done using Feynman diagrams

2. Treat integration over the collective modes exactly without using perturbation theory

For p $= -1$ this involves ordinary integration over a finite set of variables

For p=0 this involves ordinary quantum mechanics of a finite set of degrees of freedom

Since SFT is a QFT the same procedure can be followed in SFT

This resolves some long standing problems in string theory

For example, D-instantons and D0-branes were discovered in the 90's

However computations involving these ran into divergences beyond the leading order

– resolved only recently with the help of string field theory

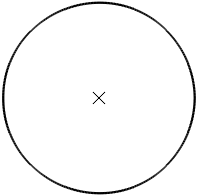
D0-brane string scattering

We shall illustrate this by analyzing the first subleading correction to the D0-D0-tachyon amplitude in bosonic string theory in D+1=26

Stefanski, A.S.

(not physical due to the presence of closed and open string tachyon but useful for illustration)

In the world-sheet approach, the leading contribution is given by the disk one point function with D0-brane boundary condition on the boundary

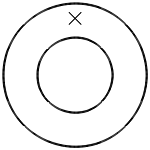


Result

$$2\pi\,\delta(k^0)\,\frac{1}{2}\,g_s\,M$$

M: mass of the D0-brane

The first subleading correction is given by the one point amplitude of a closed string tachyon on the annulus, with D0-brane boundary condition on both boundaries



World-sheet result:

$$\int_0^\infty ds \int_0^{1/4} dx\, F(x,s)$$

$$F(x,s) = 2\pi\delta(k^0)\,\frac{g_s \eta'_c}{\sqrt{2}\pi}\, s^{-1/2}\, \eta(is)^{-24} \left[\frac{\vartheta_1(2x|is)}{\vartheta'_1(0|is)}\right]^{-2}, \qquad \eta'_c \equiv \frac{1}{2\pi}$$

$$\int_0^\infty ds \int_0^{1/4} dx\, F(x,s)$$

$$F(x,s) = 2\pi\delta(k^0)\, \frac{g_s \eta_c'}{\sqrt{2}\pi}\, s^{-1/2}\, \eta(is)^{-24} \left[\frac{\vartheta_1(2x|is)}{\vartheta_1'(0|is)}\right]^{-2}, \qquad \eta_c' \equiv \frac{1}{2\pi}$$

Examine this for large s

$$F(x,s) = 2\pi\delta(k^0)\, \frac{g_s \eta_c'}{\sqrt{2}\pi}\, \frac{\pi^2}{\sin^2(2\pi x)}\, s^{-1/2}\, \left[e^{2\pi s} + 20 + 4\cos(4\pi x) + \cdots\right]$$

The leading divergences as $s \to \infty$ and at x=0 is due to the open string tachyon on the D0-brane

– will be absent in superstring theory and can be 'managed' in bosonic string theory

But the subleading terms for large s:

$$\int dx ds\, s^{-1/2}\, 2\pi\delta(k^0)\, \frac{g_s \eta_c'}{\sqrt{2}\pi}\, \frac{\pi^2}{\sin^2(2\pi x)}\, [4\cos(4\pi x) + 20]$$

can be traced to massless open string modes propagating in the loop and will be present even in superstring theory

Kinematics:

Usually three point functions vanish due to kinematic constraints on momenta

– includes also the three graviton amplitude

Remedy: Allow external momenta to be complex.

For D0-D0-tachyon amplitude we do the same

Incoming D0-brane: $(M, \vec{k}_1)$, $\vec{k}_1^2 = 0$

Incoming tachyon: $(0, \vec{k})$, $\vec{k}^2 = 4$, $(\vec{k} + \vec{k}_1)^2 = 0$

Outgoing D0-brane: $(M, \vec{k} + \vec{k}_1)$

Note: We have anticipated full momentum conservation

Strategy:

1. Use SFT to compute the effective action of the closed string tachyon Σ and the massless open string fields χ^i to desired order after integrating out all other fields

– sum over Feynman diagrams after removing the contribution from massless open string fields in the internal propagator

2. Use symmetry principles to bring the action into a ‘standard form’ after appropriate field redefinition

Note: The field redefinition can lead to non-trivial jacobian, contributing additional term to the effective action

Effective field theory description:

A non-relativistic particle of mass M (D0-brane) interacting with a scalar field Σ of mass m (closed string state) in D+1 space-time dimensions

$\vec{\chi}(t)$: **The position of the particle at time t**

Action:

$$
\begin{aligned}
- \quad & \frac{1}{2}\int dt\, d^D x\ \left[\eta^{\mu\nu}\,\partial_\mu \Sigma(t,\vec{x})\,\partial_\nu \Sigma(t,\vec{x}) + m^2 \Sigma(t,\vec{x})^2\right] \\
+ \quad & \frac{M}{2}\int dt\,\partial_t \vec{\chi}(t).\partial_t \vec{\chi}(t) + \int dt\,\mathcal{F}(\vec{\nabla})\,\Sigma(t,\vec{\chi}(t)) + \cdots
\end{aligned}
$$

$$
\mathcal{F}(\vec{\nabla})\,\Sigma(t,\vec{\chi}(t)) \equiv \mathcal{F}(\vec{\nabla})\,\Sigma(t,\vec{x})|_{\vec{x}=\vec{\chi}}
$$

Translation symmetry:

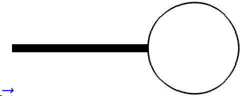
$$
\Sigma(t,\vec{x}) \to \Sigma(t,\vec{x}+\vec{a}), \qquad \vec{\chi}(t) \to \vec{\chi}(t) - \vec{a}
$$

First we shall try to compute one point function of Σ using Feynman diagrams.

Expand the action in powers of $\vec{\chi}$:

$$-\frac{1}{2}\int dt\, d^D x\, \left[\eta^{\mu\nu}\,\partial_\mu\Sigma(t,\vec{x})\,\partial_\nu\Sigma(t,\vec{x}) + m^2\Sigma(t,\vec{x})^2\right] + \frac{M}{2}\int dt\,\partial_t\vec{\chi}(t).\partial_t\vec{\chi}(t)$$

$$+\int dt\,\left(\mathcal{F}(\vec{\nabla})\,\Sigma(t,\vec{0}) + \chi^i(t)\,\mathcal{F}(\vec{\nabla})\partial_i\Sigma(t,\vec{0}) + \frac{1}{2}\chi^i(t)\chi^j(t)\,\mathcal{F}(\vec{\nabla})\partial_i\partial_j\Sigma(t,\vec{0})\right) + \cdots$$



Thick line: Σ, Thin line: $\vec{\chi}$

$$\mathcal{F}(i\vec{k})\,k_i\,k_j\,\delta_{ij}\,\frac{1}{M}\int\frac{d\omega}{2\pi}\,\frac{1}{\omega^2+i\epsilon} = \infty$$

$$\int\frac{d\omega}{\omega^2+i\epsilon} = -i\int\frac{d\omega_E}{\omega_E^2} = -2\pi i\int_0^\infty ds\int d\omega_E\, e^{-2\pi s\omega_E^2} \sim \int_0^\infty ds\, s^{-1/2}$$

Remedy: Treat $\vec{\chi}$ as collective mode and not via perturbative expansion

External states must be momentum eigenstates, obtained via quantizing $\vec{\chi}$ as a free particle quantum mechanics.

Easiest approach:

1. Treat $\vec{\chi}$ using Hamiltonian formalism treating Σ as background field

2. Calculate the relevant matrix element of the interaction term $\int \mathbf{dt}\, \mathcal{F}(\vec{\nabla})\, \Sigma(\mathbf{t}, \vec{\chi}(\mathbf{t}))$ between incoming and outgoing D0-brane states

3. This matrix element gives the D0-D0-Tachyon amplitude

Normalize the momentum eigenstates of the D0-brane as:

$$\langle \vec{k}_{out} | \vec{k}_{in} \rangle = (2\pi)^D \delta^{(D)}(\vec{k}_{in} - \vec{k}_{out})$$

and introduce the Fourier transform $\widetilde{\Sigma}$ of Σ via

$$\Sigma(t, \vec{x}) = \int \frac{d\omega}{2\pi} \int \frac{d^D k}{(2\pi)^D} \, e^{-i\omega t + i\vec{k}.\vec{x}} \, \widetilde{\Sigma}(\omega, \vec{k}) \, .$$

This gives the linear term in the effective action of Σ to be

$$\int dt \, \langle \vec{k}_{out} | \mathcal{F}(\vec{\nabla}) \, \Sigma(t, \vec{\chi}(t)) | \vec{k}_{in} \rangle = \int dt \int \frac{d\omega}{2\pi} \frac{d^D k}{(2\pi)^D} \, \mathcal{F}(i\vec{k}) \, \widetilde{\Sigma}(\omega, \vec{k}) \, \langle \vec{k}_{out} | e^{-i\omega t + i\vec{k}.\vec{\chi}(t)} | \vec{k}_{in} \rangle$$

$$= \int dt \int \frac{d\omega}{2\pi} \int \frac{d^D k}{(2\pi)^D} \, e^{i(\omega_{out} - \omega_{in} - \omega)t} \, \mathcal{F}(i\vec{k}) \, \widetilde{\Sigma}(\omega, \vec{k}) \langle \vec{k}_{out} | e^{i\vec{k}.\vec{\chi}(t=0)} | \vec{k}_{in} \rangle$$

$$= \int \frac{d\omega}{2\pi} \int \frac{d^D k}{(2\pi)^D} \, \mathcal{F}(i\vec{k}) \, \widetilde{\Sigma}(\omega, \vec{k}) \, 2\pi \, \delta(\omega_{out} - \omega_{in} - \omega) \, (2\pi)^D \delta^{(D)}(\vec{k} + \vec{k}_{in} - \vec{k}_{out})$$

$$\omega_{in} = \vec{k}_{in}^2 / (2M), \qquad \omega_{out} = \vec{k}_{out}^2 / (2M)$$

The first delta function comes from the t integral and the second delta function comes from the matrix element.

$$\int \frac{d\omega}{2\pi} \int \frac{d^D k}{(2\pi)^D} \mathcal{F}(i\vec{k})\, \widetilde{\Sigma}(\omega, \vec{k})\, 2\pi\, \delta(\omega_{out} - \omega_{in} - \omega)\, (2\pi)^D \delta^{(D)}(\vec{k} + \vec{k}_{in} - \vec{k}_{out})$$

The relevant amplitude for external Σ of energy ω and momentum $\vec{k}$ is then given by,

$$\mathcal{F}(i\vec{k})\, 2\pi\, \delta(\omega - \omega_{out} + \omega_{in})\, (2\pi)^D \delta^{(D)}(\vec{k} + \vec{k}_{in} - \vec{k}_{out})$$

Main task: Compute $\mathcal{F}(i\vec{k})$

Leading contribution from the disk:

$$\mathcal{F}_{disk} = \frac{1}{2} g_s M$$

The challenge is to compute $\mathcal{F}_{annulus}$

1. Use SFT to compute the annulus one point function $\mathcal{F}_{\text{annulus}}$ of Σ after removing massless mode contribution from internal propagators

2. Compute terms in the effective action with one Σ and many χ^i's

3. Find an appropriate field redefinition of $\vec{\chi}$ that brings the coupling to the form

$$\frac{M}{2}\int dt\,\partial_t\vec{\chi}(t).\partial_t\vec{\chi}(t) + \int dt\,\mathcal{F}(\vec{\nabla})\,\Sigma(t,\vec{\chi}(t)) + \cdots$$

The jacobian could give additional contribution to $\mathcal{F}(i\vec{k})$

Result:

$$\mathcal{F}_{\text{annulus}} = (1.15899 - 0.438710\,i)g_s$$

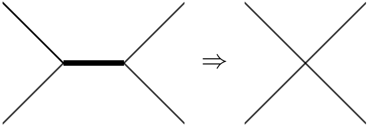
In the rest of these lectures I shall describe how to get this

To begin with we have a theory of open and closed strings

– a typical Feynman diagram will have open and closed string propagators

We shall integrate out the closed strings

– any Feynman diagram with a closed string internal propagator will be considered as part of the interaction vertex, e.g.



– increases the size of the regions of M_n associated with open string contact interactions

Closed string fields will appear as external legs as if they are background fields

World-sheet theory has $X^\mu,\ b,\ c,\ \bar{b},\ \bar{c}$

Suppose $\{\phi_{r,k}(z)\}$ be a basis of (off-shell) open string vertex operators on Dp-brane

r: discrete label,   k: p+1 dimensional momentum

Using state operator correspondence this provides a basis of open strings states

$$|\phi_{r,k}\rangle = \phi_{r,k}(0)|0\rangle$$

Examples of $\phi_{r,k}$:

Tachyon: $c\, e^{ik.X_\parallel}$,   translation modes: $c\,\partial X_\perp^i\, e^{ik.X_\parallel}$

vectors: $c\,\partial X_\parallel^s e^{ik.X_\parallel}$

Refs: A.S., B. Zwiebach, 2405.19421, A.S., B. Stefanski, 2509.02716

A generic open string state has the form:

$$|\psi_O\rangle = \sum_r \int d^{p+1}k\, \psi_r(k)|\phi_{r,k}\rangle$$

$\psi_r(k)$: arbitrary coefficient functions of momenta

In string field theory, $\{\psi_r(k)\}$ become the fields in Fourier space

Kinetic term:

$$\frac{1}{2}\langle\psi_O|Q_B|\psi_O\rangle$$

Q_B: BRST operator of the world-sheet theory, $Q_B^2 = 0$

Gauge invariance: $\delta|\psi_O\rangle = Q_B|\lambda\rangle$ for any state $|\lambda\rangle$

We shall denote by $\psi_O(z) = \sum_r \int d^{p+1}k\, \psi_r(k)\phi_{r,k}(z)$ the vertex operator for $|\psi_O\rangle$

In $b_0|\psi_O\rangle = 0$ gauge the propagator $\propto L_0^{-1} = 2\pi \int_0^\infty ds e^{-2\pi s L_0}$

Next we shall introduce interaction terms

If z=f(w) is a conformal transformation and V is a vertex operator, then we define

$f \circ V(w)$ is the conformal transform of V(w) by f

e.g. for a primary vertex operator V of dimension h,

$$f \circ V(w) = (f'(w))^h V(f(w))$$

w is known as the local coordinate at the puncture where V is inserted

Closed string fields ψ_C will be taken to be the on-shell tachyon state $c\bar{c}e^{ik.X}$ with $k^2 = -4$.

$$f \circ \psi_C(0) = \psi_C(f(0)) = c\bar{c}e^{ik.X}(f(0))$$

Interaction terms for n open string fields and m closed string fields have the form

$$\int_{R_{n,m}} \langle f_1 \circ \psi_O(0) \cdots f_n \circ \psi_O(0) g_1 \circ \psi_C(0) \cdots g_m \circ \psi_C(0) \times b-\text{ghosts} \rangle$$

$R_{n,m}$: subspace of $M_{n,m}$, – moduli space of Riemann surfaces with n open string and m closed string punctures

Different versions of SFT have different $R_{n,m}$ and f_i, g_j

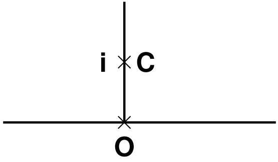
– related by field redefinition

For g_j, only $g_j(0)$ is needed since ψ_C is conformally invariant

Example: Open-closed interaction term on disk / UHP

– has 0-dimensional moduli space

Place the closed string vertex operator at i and the open string vertex operator at 0



Since ψ_O is off-shell we need the local coordinate at O

Choose w = λ z for some real constant λ

Then z=f(w) = w/λ, $\quad f \circ \psi_O(0) = \lambda^{-h_O} \psi_O(0)$

The expression for the interaction vertex is

$$\langle \psi_C(i)\, f\circ\psi_O(0)\rangle = \lambda^{-h_O}\langle \psi_C(i)\,\psi_O(0)\rangle$$

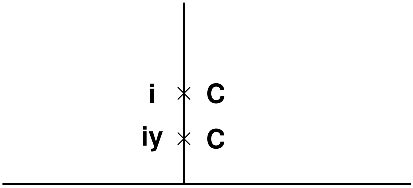
Note: Large λ suppresses propagation of states of positive dimension

– simplifies the computations considerably

Final result is independent of λ since SFT for different λ are related by field redefinition

Next example: C-C interaction vertex

The CC amplitude has one dimensional moduli space



$0 < y < 1$

This gets contribution from two Feynman diagrams:

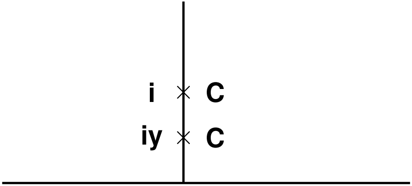
First task is to figure out the region of moduli space of CC amplitude covered by diagram (a)

Rule: Take two copies of CO interaction and dentify the local coordinates w, w′ of the open string punctures via

$$w\,w' = -q, \qquad q \equiv e^{-2\pi s}, \quad s : \text{Schwinger parameter}$$

$$\lambda z\,\lambda z' = -q, \qquad 0 \le q \le 1$$

Closed string punctures are at z=i, z'=i $\to z = iq/\lambda^2$



y=q/λ^2 takes value in the range $0 \le y \le 1/\lambda^2$

Diagram (b) containing the CC interaction vertex must cover the rest of the region $\lambda^{-2} < y < 1$

General procedure:

Use lower order interaction vertices to build Feynman diagrams

At the end, whatever region is not covered by Feynman diagrams, is covered by introducing a new interaction term

For each new interaction term, we need to choose local coordinates at the punctures

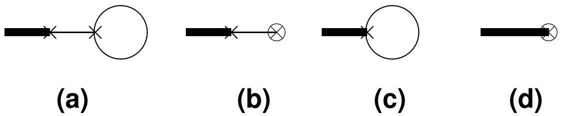
– new parameters of SFT

In the D0-D0-tachyon scattering problem we use two parameters: λ from CO, α from OOO and a function f(β) from COO

SFT's for different α, λ and f(β) are related by field redefinition and give the same answer

Now consider the annulus one point function of closed strings

$\mathcal{F}_{\text{annulus}}$ gets contribution from four Feynman diagrams



$\times$: interaction term from disk amplitude

$\otimes$: interaction term from annulus amplitude

We work with large α and λ so that massive mode propagation is suppressed

Collective modes χ^i are removed from the propagators, yielding a finite result for each diagram

Only for diagram (d) we use the world-sheet expression

Since it has no propagators there are no divergences

For other regions we use the corresponding Feynman diagrams

Massive propagator contribution is suppressed in the limit of large λ and α

χ^i propagators are removed since they have to be treated separately

Tachyon propagator gives $(k^2 + m^2 - i\epsilon)^{-1}$ with $m^2 = -1$

We get a finite result

Field redefinition:

According to the postulated form of the effective action, the coupling of closed string tachyon Σ to open string massless mode χ^i is given by

$$\int dt \left(\mathcal{F}(\vec{\nabla})\,\Sigma(t,\vec{0}) + \chi^i(t)\,\mathcal{F}(\vec{\nabla})\partial_i\Sigma(t,\vec{0}) + \frac{1}{2}\chi^i(t)\chi^j(t)\,\mathcal{F}(\vec{\nabla})\partial_i\partial_j\Sigma(t,\vec{0}) \right) + \cdots$$

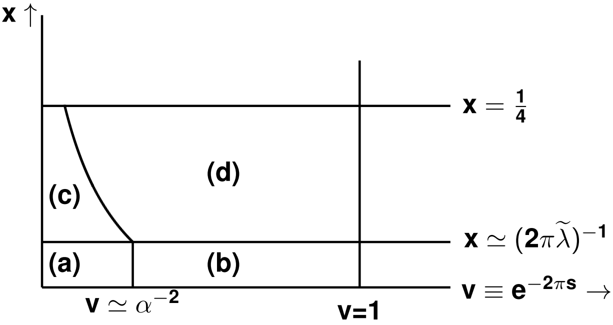
Explicit SFT calculation of C-O and C-O-O interaction term gives different result

Remedy: Need a redefinition of the fields χ^i:

$$\chi^i \to \chi^i + C^i_j\chi^j\Sigma + \cdots$$

Jacobian $\propto (1 + C^i_i\Sigma) \sim \exp[C^i_i\Sigma]$ gives additional contribution to one point function of Σ

In the x-s plane the different diagrams cover different region:



α a new SFT parameter that appears in O-O-O interaction

$$\widetilde{\lambda} = \lambda\,\alpha$$

The curve separating regions (c) and (d) is given by

$$v = \frac{1}{\alpha^2 \widetilde{\lambda}^2 \sin^2(2\pi x)} \left(1 + \frac{1}{4\widetilde{\lambda}^2}\right)^{-2} \left[1 - 2\left\{\cot^2(2\pi x) - \widetilde{\lambda}^2 f(\tan \pi x)^2\right\} \alpha^{-2} \widetilde{\lambda}^{-2} \left(1 + \frac{1}{4\widetilde{\lambda}^2}\right)^{-2}\right]$$

$f(\beta)$: A new function that appears in the expression for the C-O-O interaction term

Result:

$$\mathcal{F}_{\text{annulus}} = \mathbf{g_s}\eta'_{\mathbf{c}} \lim_{\substack{\alpha,\widetilde{\lambda}\to\infty \\ \alpha/\widetilde{\lambda}=\text{fixed}}} \left[\mathcal{F}_{(\mathbf{a})} + \mathcal{F}_{(\mathbf{b})} + \mathcal{F}_{(\mathbf{c})} + \mathcal{F}_{(\mathbf{d})} + \mathcal{F}_{(\mathbf{e})} + \mathcal{F}_{(\mathbf{f})} + \mathcal{F}_{(\mathbf{g})} + \mathcal{F}_{\text{jac}} + \mathcal{F}_{\text{cor}}\right]$$

$$\eta'_{\mathbf{c}} = \frac{\mathbf{1}}{\mathbf{2}\pi}$$

The result should be independent of $\alpha, \widetilde{\lambda}$ but we need to take this limit since we simplify the analysis by dropping terms with inverse powers of $\alpha, \widetilde{\lambda}$

$$\mathcal{F}_{(a)} = -\frac{i}{2} \times \int_{-i\infty}^{i\infty} \frac{d\omega'}{2\pi} \, \widetilde{\lambda} \, \alpha^{2+2\omega'^2} \times (\omega'^2 + 1 + i\epsilon)^{-1}$$

$$\mathcal{F}_{(b)} = -\frac{1}{2\sqrt{2}} \left(1 + \alpha^{-2}\right) \widetilde{\lambda} \int_{s_c}^{\frac{1}{2\pi}\ln(\alpha^2 - 1/2)} ds \, s^{-1/2} \, \eta(is)^{-24}$$

$$\mathcal{F}_{(c)} = i \int_{-i\infty}^{i\infty} \frac{d\omega'}{2\pi} \left\{ \alpha^2 \widetilde{\lambda}^2 \left(1 + \frac{1}{4\widetilde{\lambda}^2}\right)^2 \right\}^{1+\omega'^2} \int_{1/(2\widetilde{\lambda})}^{1} \frac{d\beta}{(1+\beta^2)^{1+2\omega'^2}} (2\beta)^{2\omega'^2} \frac{1}{\omega'^2 + 1 + i\epsilon}$$

The lower limit 0 of t integration has divergences due to closed string tachyon.

– can be avoided using Witten's iϵ prescription (or closed SFT)

$$s_c = (s_0 + i\infty)^{-1}, \qquad s_0\text{: arbitrary number}$$

$$\mathcal{F}_{(d)} = \frac{1}{\sqrt{2}\pi}\int_A^{1/4} dx \int_{s_c}^{B(x)} ds\; s^{-1/2}\,\eta(is)^{-24}\left[\frac{\vartheta_1(2x|is)}{\vartheta_1'(0|is)}\right]^{-2},$$

$$A \equiv (2\pi\widetilde{\lambda})^{-1}(1-\alpha^{-2})$$

$$e^{2\pi B(x)} \equiv \alpha^2\widetilde{\lambda}^2\sin^2(2\pi x)\left(1+\frac{1}{4\widetilde{\lambda}^2}\right)^2 + 2\sin^2(2\pi x)\left\{\cot^2(2\pi x)-\widetilde{\lambda}^2 f(\tan\pi x)^2\right\}$$

$$\mathcal{F}_{(e)} = \frac{1}{8}\,\widetilde{\lambda}\,\frac{1}{\sqrt{2\pi}}(\ln\alpha)^{-1/2}$$

$$\mathcal{F}_{(f)} = \frac{1}{\sqrt{2\pi}}\,\widetilde{\lambda}^2\int_{1/(2\widetilde{\lambda})}^{1} d\beta\, f(\beta)^2\,\frac{1}{1+\beta^2}\,\frac{1}{\sqrt{\ln\alpha+\ln\frac{4\widetilde{\lambda}^2+1}{4\widetilde{\lambda}}+\ln\frac{2\beta}{1+\beta^2}}}$$

$$\mathcal{F}_{(g)} = -\frac{1}{4}\,\widetilde{\lambda}\,\frac{1}{\sqrt{2\pi}}(\ln\alpha)^{-1/2}$$

f: a function encoding another set of SFT parameters

$$\mathcal{F}_{\rm jac} = \frac{1}{\sqrt{2\pi}} \left[-\ 2 \int_{1/(2\widetilde{\lambda})}^{1} d\beta \left(\ln\alpha + \ln\frac{4\widetilde{\lambda}^2+1}{4\widetilde{\lambda}} + \ln\frac{2\beta}{1+\beta^2} \right)^{-1/2} \right.$$

$$\left\{ \frac{25}{8\beta} - \frac{25}{8}\beta + \vec{k}^2 \tan^{-1}\beta \right\} \left(\frac{1}{\beta} - \frac{2\beta}{1+\beta^2} \right)$$

$$\left. + \pi\,\vec{k}^2 \left(\ln\alpha + \ln\frac{4\widetilde{\lambda}^2+1}{4\widetilde{\lambda}} \right)^{1/2} \right], \qquad \vec{k}^2 = 4$$

$$\mathcal{F}_{\rm cor} = \left[\frac{1}{4\sqrt{2\pi}} \widetilde{\lambda}^2 \alpha^{-2} \int_{1/(2\widetilde{\lambda})}^{1} \frac{d\beta}{1+\beta^2} \left\{ \ln\alpha + \ln\widetilde{\lambda} + \ln\frac{2\beta}{1+\beta^2} \right\}^{-3/2} f(\beta)^4 \right.$$

$$\left. + \frac{i}{8\widetilde{\lambda}} \int_{-i\infty}^{i\infty} \frac{d\omega'}{2\pi} \alpha^{2+2\omega'^2} \frac{1}{\omega'^2+1+i\epsilon} + \frac{1}{6\sqrt{2}} \widetilde{\lambda}^{-1} \int_{s_c}^{\frac{1}{2\pi}\ln\alpha^2} ds\, s^{-1/2} e^{2\pi s} \right]$$

The formula for $\mathcal{F}_{\text{annulus}}$ can be shown to be independent of $\alpha, \widetilde{\lambda}$, f up to terms that scale as inverse power of $\alpha, \widetilde{\lambda}$

In principle we can fix the missing terms uniquely by requiring that the full answer is independent of $\alpha, \widetilde{\lambda}$ and f.

Alternatively we can numerically evaluate the integrals by taking $\alpha, \widetilde{\lambda}$ large and get a number

$$\mathcal{F}_{\text{annulus}} = (1.15899 - 0.438710\, i)g_s$$

The imaginary part can be traced to closed string tachyons

Other applications

Shifting background

In a QFT we can expand the action around any classical solution and quantize the theory

The same thing can be done for SFT

For NSNS background this corresponds to deforming the world-sheet CFT

For (nearly) marginal deformation this can be done in the world-sheet description using conformal perturbation theory.

Even for this problem SFT is more systematic since it does not encounter the UV divergence of the world-sheet from collision of operators

$$\int d^2z_1 d^2z_2 V(z_1, \bar{z}_1) V(z_2, \bar{z}_2)$$

For RR background SFT is a must in the RNS formulation

Tachyon condensation

Open string field theory can be used to find exact solutions describing the minimum of the open string tachyon potential

– can be generalized to find other solutions

e.g. any D-brane can be regarded as a solution in the theory of any other D-brane

Wilsonian effective action

Even though SFT has infinite number of fields, it is possible to integrate out all the heavy fields and write an effective action of light fields

– this has full information about string theory

– a UV finite, gauge invariant theory of light fields only

Furthermore, by adjusting the SFT parameters (stubs) it is possible to ensure that we not only integrate out the massive fields but also the high momentum modes of light fields

The effective momentum cut-off can be adjusted by adjusting the SFT parameters