Exploring universality classes of nonequilibrium statistical physics

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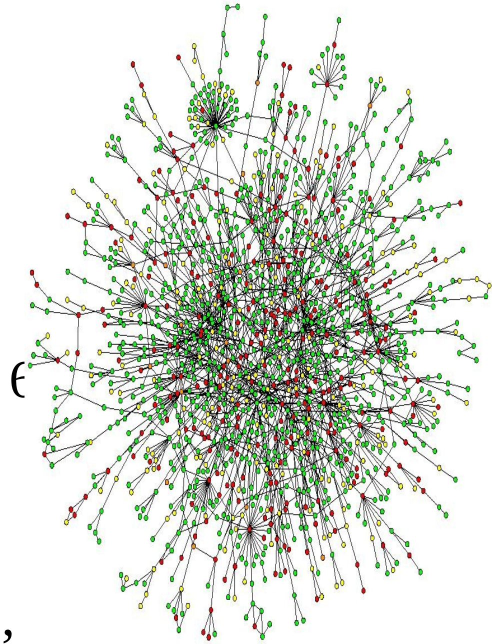
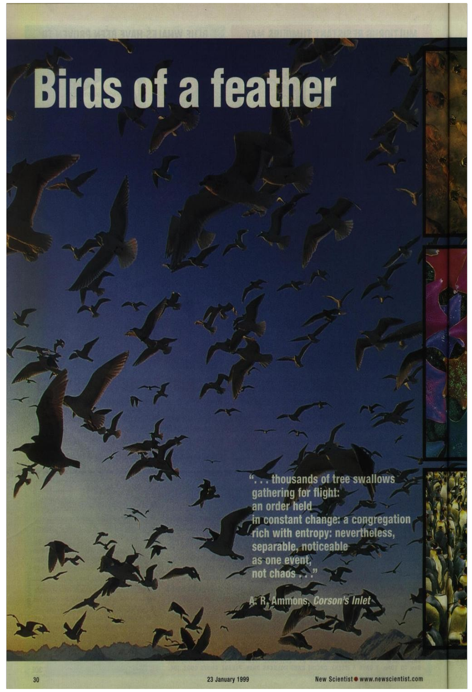
- **The aim**: Understanding the basic universality classes of nonequlibrium systems. Universal scaling behavior is an attractive feature in statistical physics because a wide range of models can be classified purely in terms of their collective behavior due to a **diverging correlation length**. Scaling phenomenon has been observed in many branches of physics, chemistry, biology, economy ... etc., most frequently by **critical phase transitions** and **surface growth.** This is a basic science topic, the results can be used in applied sciences:

- Nonequilibrium **critical phase transitions** may appear in models of
*Spatiotemporal intermittency***:** Phys. Rev. E 72 (2005) 016202,
Population dynamics : E. V. Albano, J. Phys. A 27 (1994) L881,
Epidemics spreading : T. Ligget, Interacting particle systems 1985,
Catalysis : Da-yin Hua, Phys. Rev E 70 (2004) 066101,
Itinerant electron systems : D. E. Feldman, Phys. Rev. Lett 95 (2005) 177201,
Cooperative transport : S. Havlin and D. ben-Avraham, Adv. Phys. 36 (1987) 6
Enzyme biology : H. Berry, Phys. Rev. E 67 (2003) 031907,
Biological control systems : K. Kiyono, et al., PRL95 (2005) 058101.
Plasma physics : C. A. Knapek et al. Phys. Rev. Lett 98 (2007) 015001.
Stock-prize fluctuations and markets: K. Kiyono, et al., PRL96 (2006) 068701,
Meteorology and Climatology: O. Petres and D. Neelin, Nature Phys. 2 (2006) 393.



Universal scaling behavior in nature

- The concept of self-organized critical (**SOC**) phenomena has been introduced some time ago to explain the frequent occurrences of **scaling laws** experienced in nature. The term SOC usually refers to a mechanism of slow energy accumulation and fast energy redistribution, driving a system toward a **critical state**. The prototype of SOC systems is the sand-pile model in which particles are randomly dropped onto a two dimensional lattice and the sand is redistributed by fast avalanches. Therefore in SOC models instead of tuning the parameters an inherent mechanism is responsible for driving it to criticality. SOC mechanism has been proposed to model earthquakes the evolution of biological systems, solar flare occurrence, fluctuations in confined plasma, snow avalanches and rain fall.

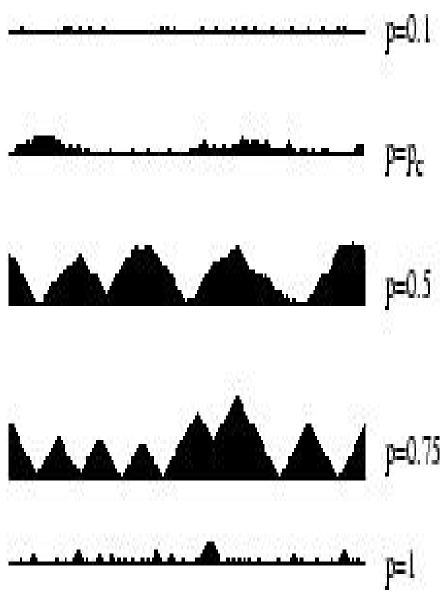


- **Diverging correlation length and scaling may also occur away from the phase transition point.** For example in quantum matter near absolute zero temperature thermal equilibration can be obstructed in case of topological ordered ground states, where only the slow dynamical relaxation of defects pairs -- via annihilation-diffusion -- can occur (see example : *C. Chamon, Phys. Rev. Lett. 94 040402 (2005)*).

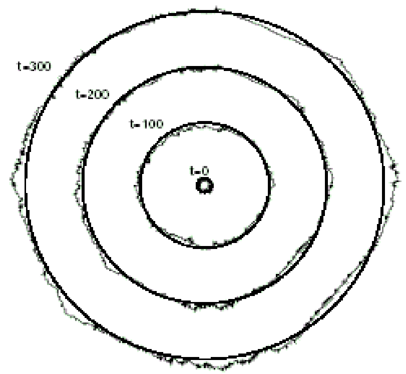
- A + A -> A reaction (~ A + A -> 0)

Universal scaling behavior at interfaces

- Rough **surfaces** and **interfaces** are ubiquitous in nature and from technological point of view the control of their roughness is becoming critical for applications in fields such as microelectronics, *image formation, surface coating or thin film growth* S. Chow, Mesoscopic Physics of Complex Materials Texts in Contemporary Physics, Springer 2000).
  Related topics are the depinning transition of elastic systems in disordered media Colton et al., Phys. Rev. Lett 97 (2006) 057001) and the wetting transition transitions.
  In application to *parallel and distributed computations*, the important consequence of the derived scaling is the existence of the upper bound for the desynchronization in a conservative update algorithm for parallel discrete-event simulations (A. Kolakowska et al. Phys. Rev. E 70 051602 (2004)).



  Understanding the fundamental laws driving the tumor development is one of the biggest challenges of contemporary science. Internal *dynamics of a tumor* reveals itself in a number of phenomena, one of the most obvious ones being the *growth*
  (see example : B. Brutovsky at al.: http://arxiv.org/abs/physics/0610134.



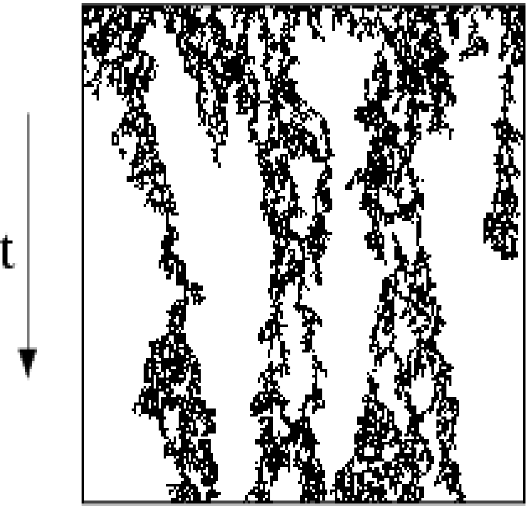
- Mapping between surface-interface and reaction-diffusion models !

Overview of dynamical universality classes

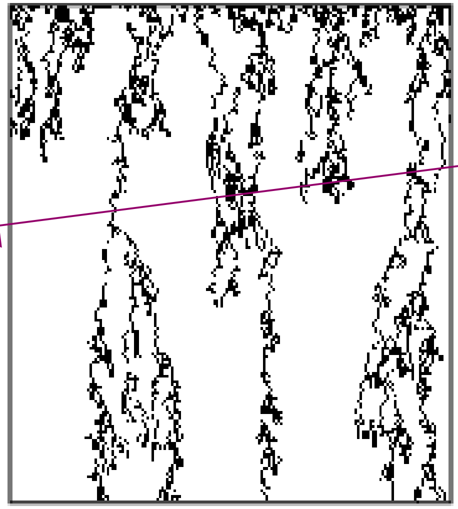
G. Ó.: Rev. Mod. Phys. 76 (2004) 663

- Extension of static equilibrium classes (Ising, Potts, O(N)...) with different dynamics : Glauber, Kawasaki ...
- Mixture of the above dynamics to create out of equilibrium systems
- Genuine nonequilibrium classes appear by phase transitions to absorbing state, example in reaction-diffusion systems :
  a) Directed percolation class : *A->2A, 2A->A, A->0, rapidity reversal*
  b) Dynamical percolation: *As a) with long time memory*
  c) Voter model class: *Diffusion and annihilation at surfaces*
  d) Branching with *kA->0 (k>2)* classes (mostly mean-field like)
  d) Parity conserving class: *A->3A, 2A->0*
- Multi-component reaction-diffusion model classes:
  e) DP with coupled diffusive or non-diffusive random walk
  f) The same as e) with global particle conservation...

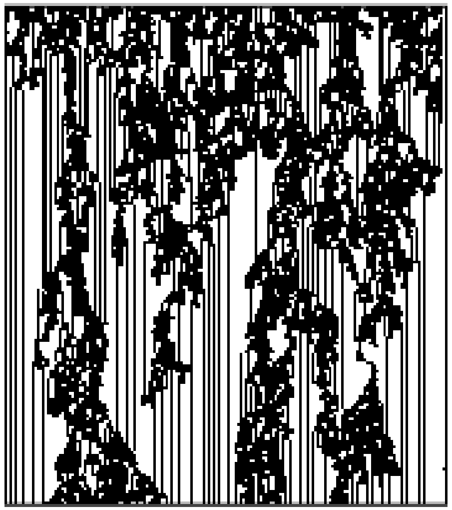
Space-time evolution of basic, universal nonequilibrium spreading models with absorbing states



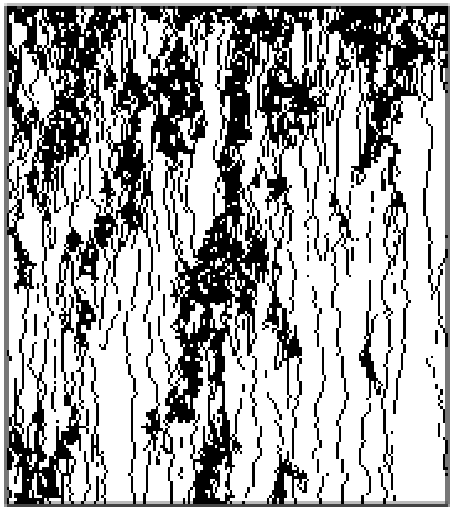
DP



BAW2



PCP



PCPD

- **Unary** production spreading without and with *parity conservation*:

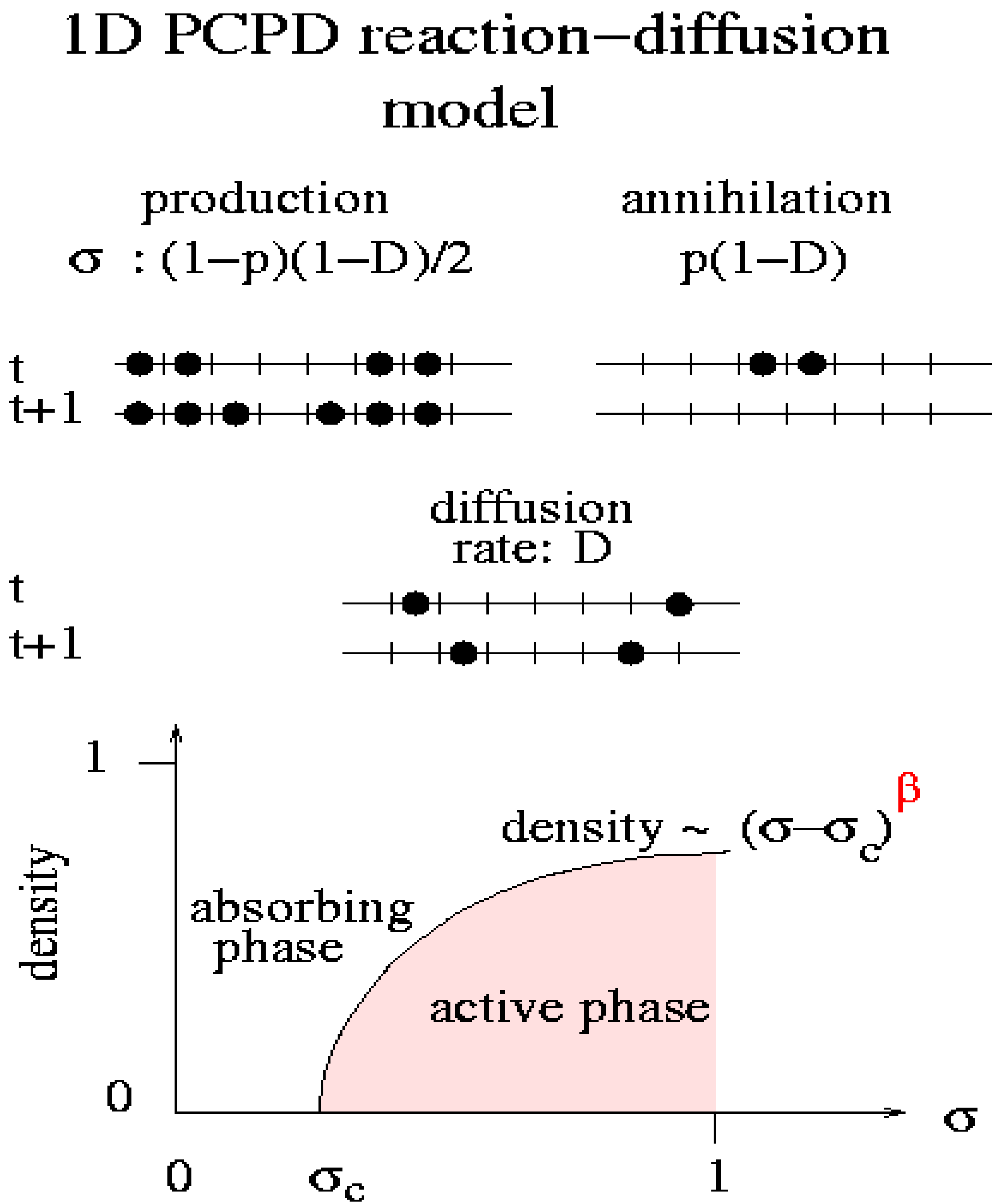
  $\mathbf{A} \to (m+1)A, \;\; 2A \to 0$

- **Binary** production spreading coupled to slave modes without and with *diffusion*:

  $\mathbf{2A} \to (m+2)A, \;\; 2A \to 0$

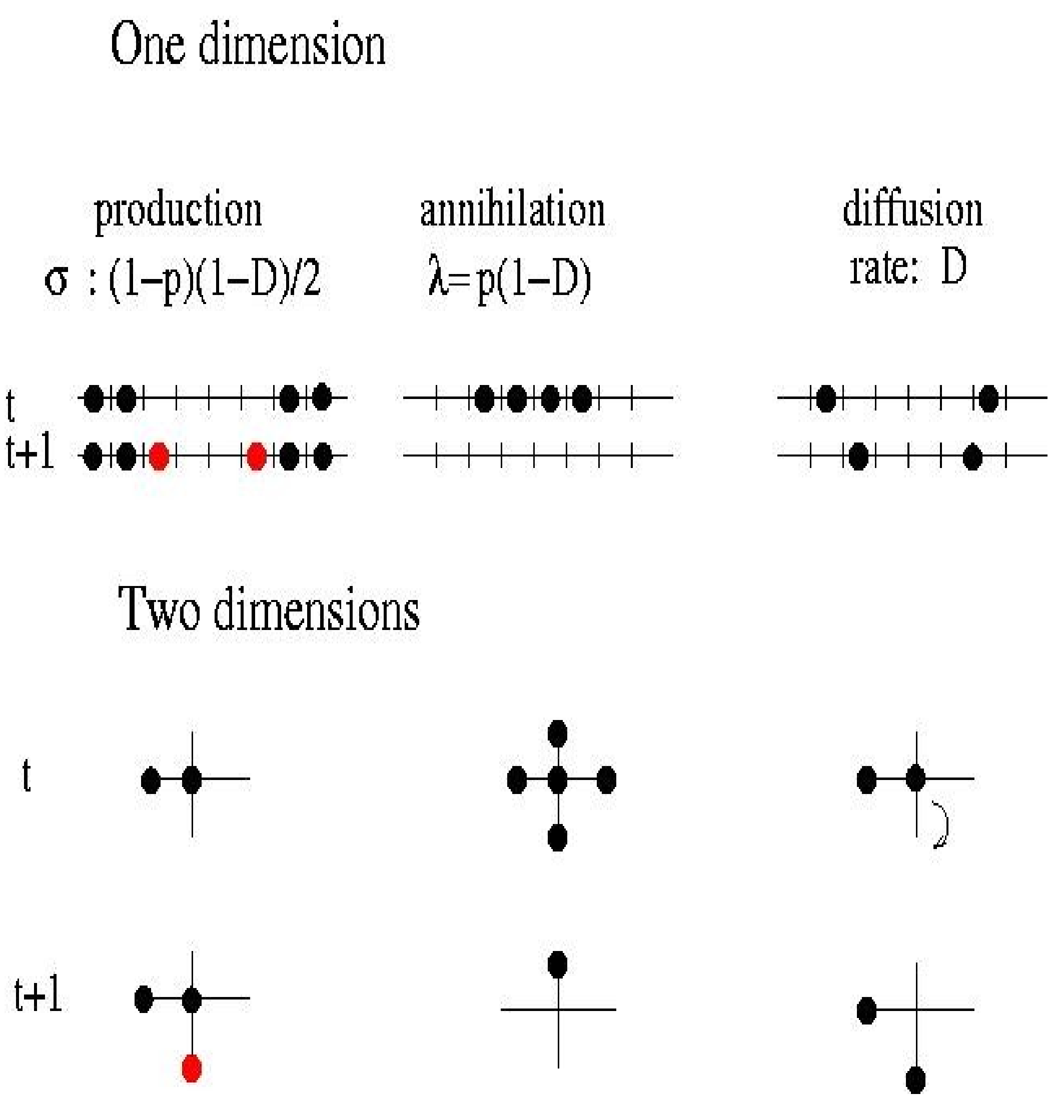
  Reactive and diffusive sectors, changing exponents by varying the diffusion rate: *G.Ódor, Phys. Rev. E 62 (2000) R3027.*

The challenging binary production, diffusive pair contact process (PCPD)



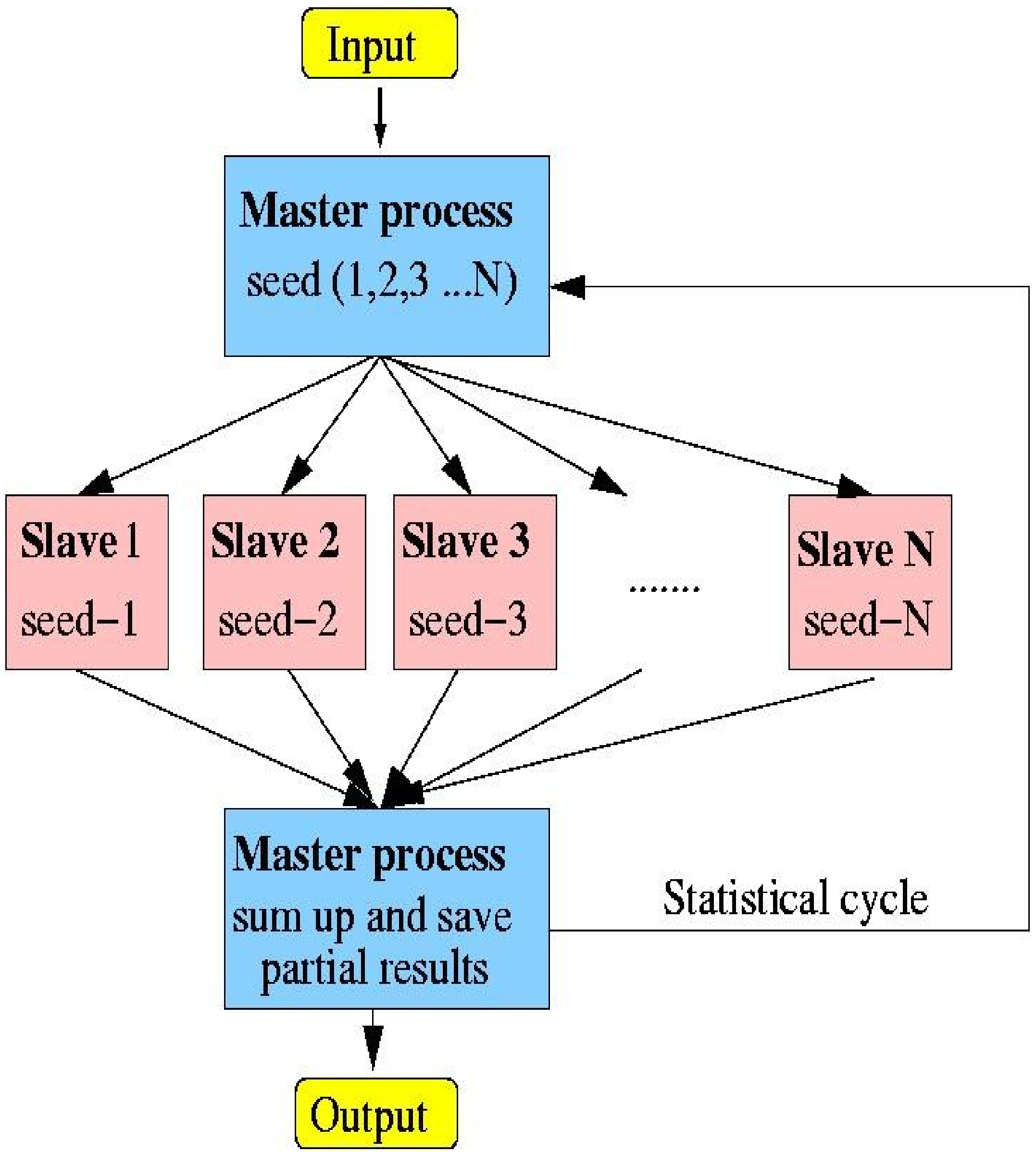
- Two absorbing states **without symmetry,** one of them is diffusive (*Carlon et al.(PRE 2001)*). Numerical methods show new exponents, but are controversial. No extra symmetries or conservation laws found to explain it.
- Bosonic field theories (1997) failed to describe critical behavior. In the bosonic model the active phase particle density diverges.
- Fermionic model shows different critical behavior, but perturbative field theoretical renormalization did not find stable fixed point corresponding to the novel scaling behavior (2004). For a review see: *M. Henkel and H. Hinrichsen, J. Phys. A37, R117-R159 (2004)*

Lattice simulations



- Random sequential update of occupied sites
- Full or random half filled initial conditions
- L ≤ 500 000 in 1d,
  L ≤ 7000 x 7000 in 2d
  periodic boundary conditions
- $t_{max} \leq 5 \times 10^8$ MCS
- Master-slave parallel algotihms on international computing GRIDS (~ 100 - 500 CPU-s)

Parallel algorithm realizations



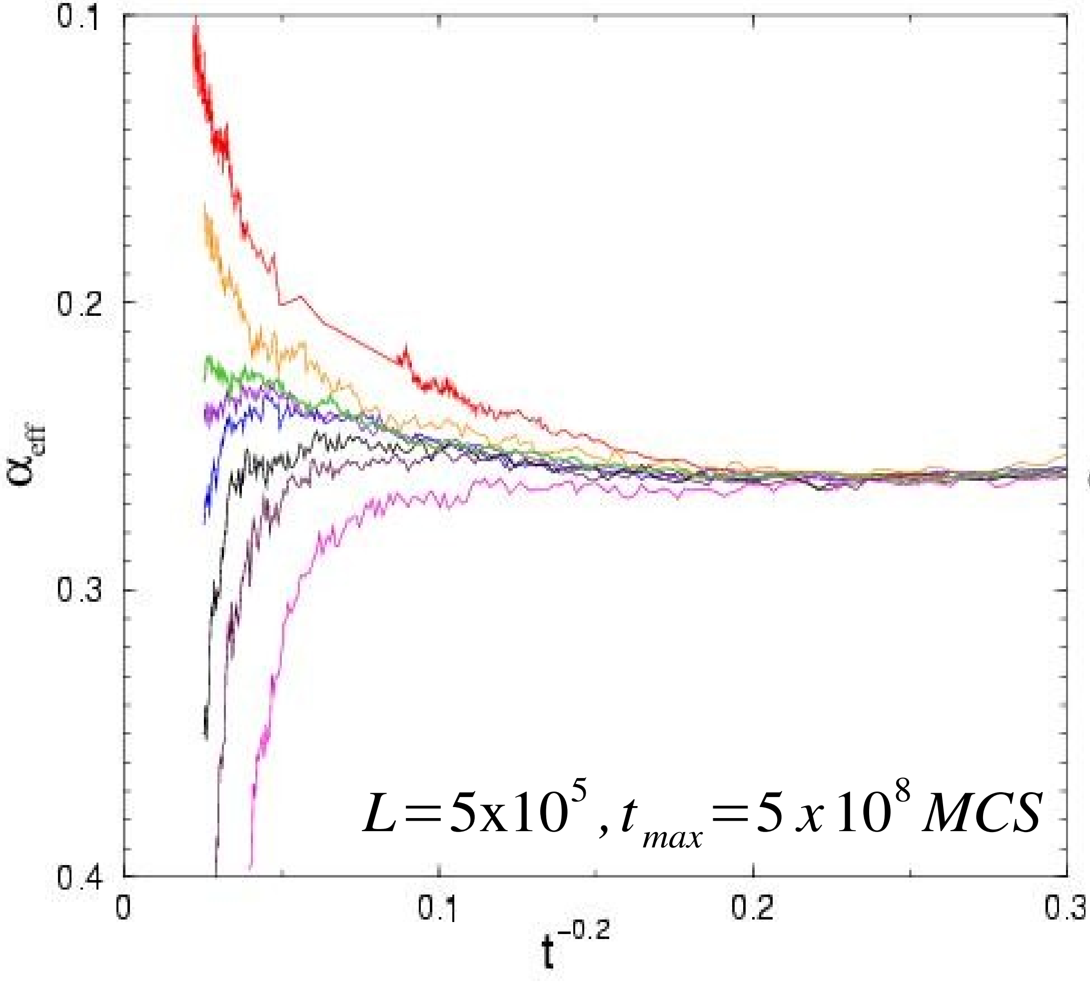
- Master-worker setup, Single Program Multiple Data (SPMD) algorithm.
- The slave processes are completely identical and sequential. Minimal communication losses, easy program development
- Either PVM, PMI or manual parallelization (condor standard universe with check pointing)

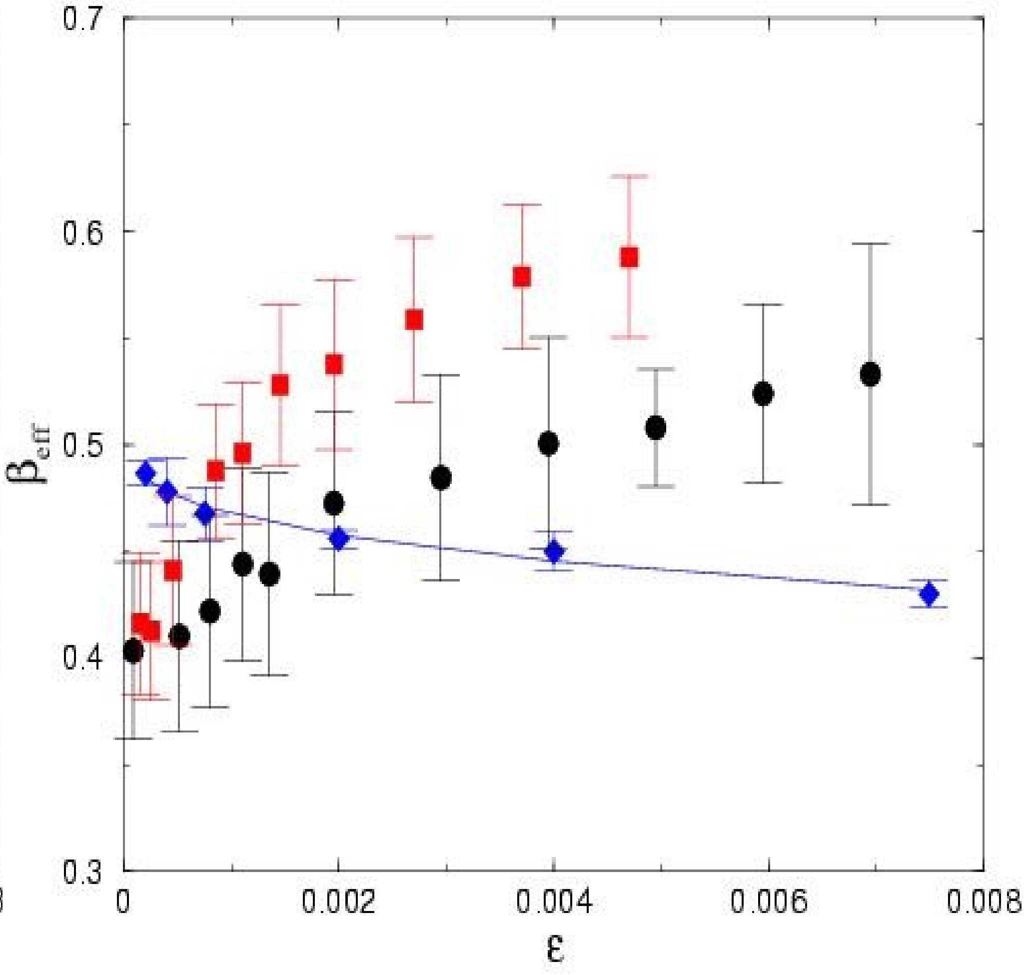
Simulation results for the 2A -> 3A, 4A -> 0 model in 1 dimension

Density decay local exponents with : α = 0.21(2) (~PCPD), at D = 0.5, σ_c = 0.42075.

Steady state local exponents near σ_c=0.42 for D = 0.5, 0.2, with β = 0.40(2) (~PCPD). For D = 0.9: only mean-field with β =1/2.

2A -> 0 process becomes irrelevant !



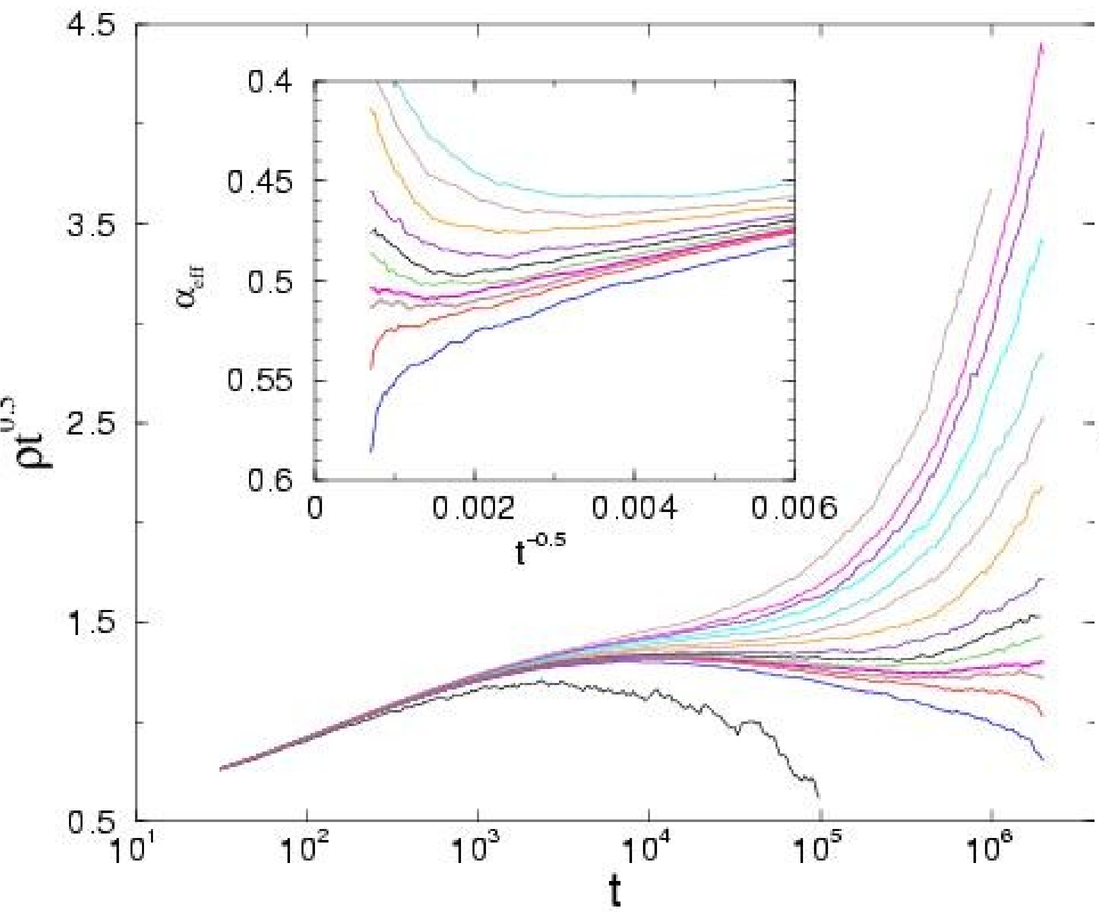


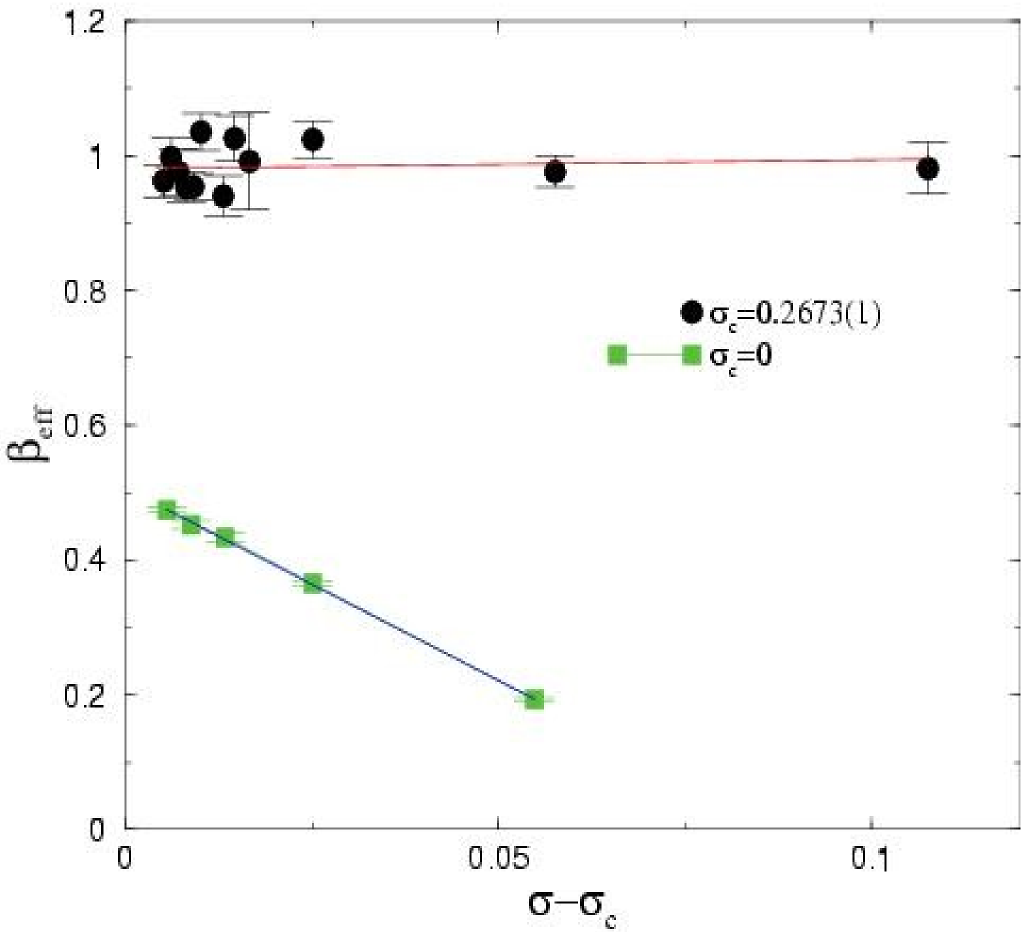
Simulation results for the 2A -> 3A, 4A -> 0 model in 2 dimensions

Density decay local exponents with :
α = 0.50(1) (MF-PCPD) at σ_c =0.26715

$L=7000, t_{max}=2\text{x}\,10^6\,MCS\,,\,D=0.05$

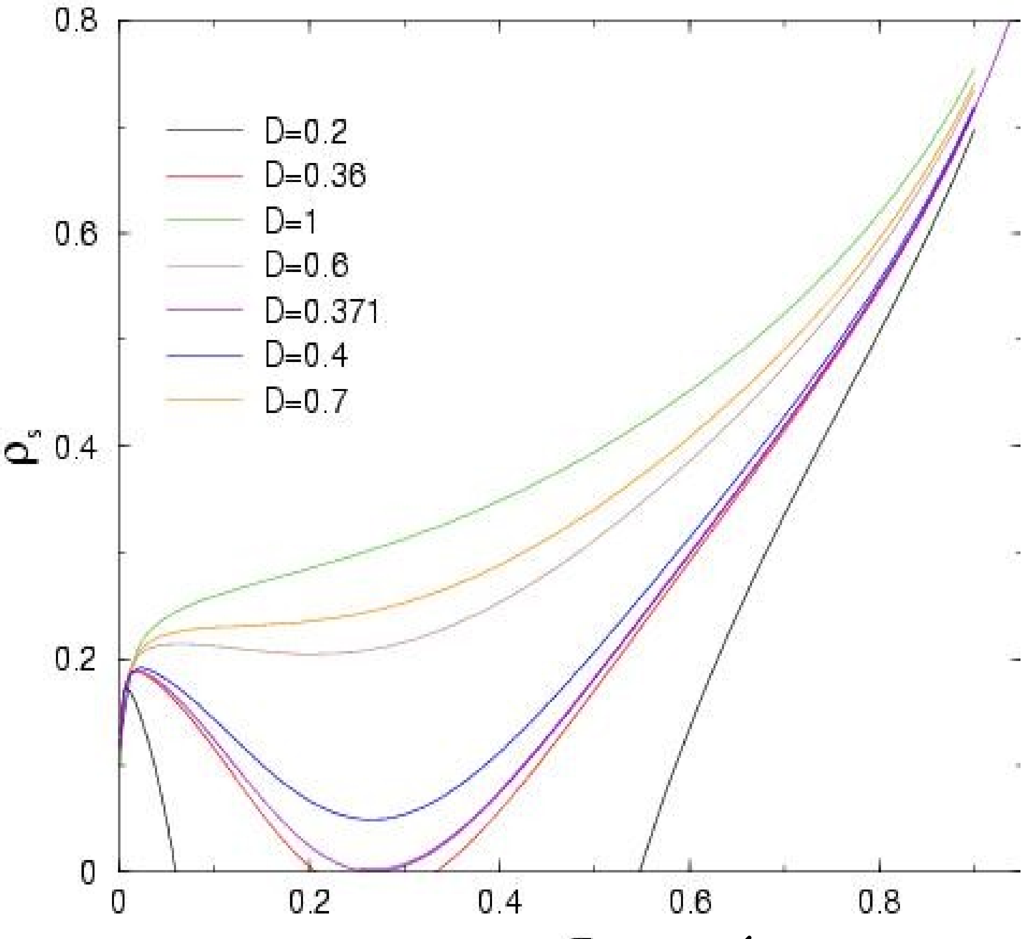
Steady state local exponents at D = 0.05
with β = 0.98(2) (MF-PCPD).
For D=0.9: only mean-field with β=1/2.
2A -> 0 process becomes irrelevant !



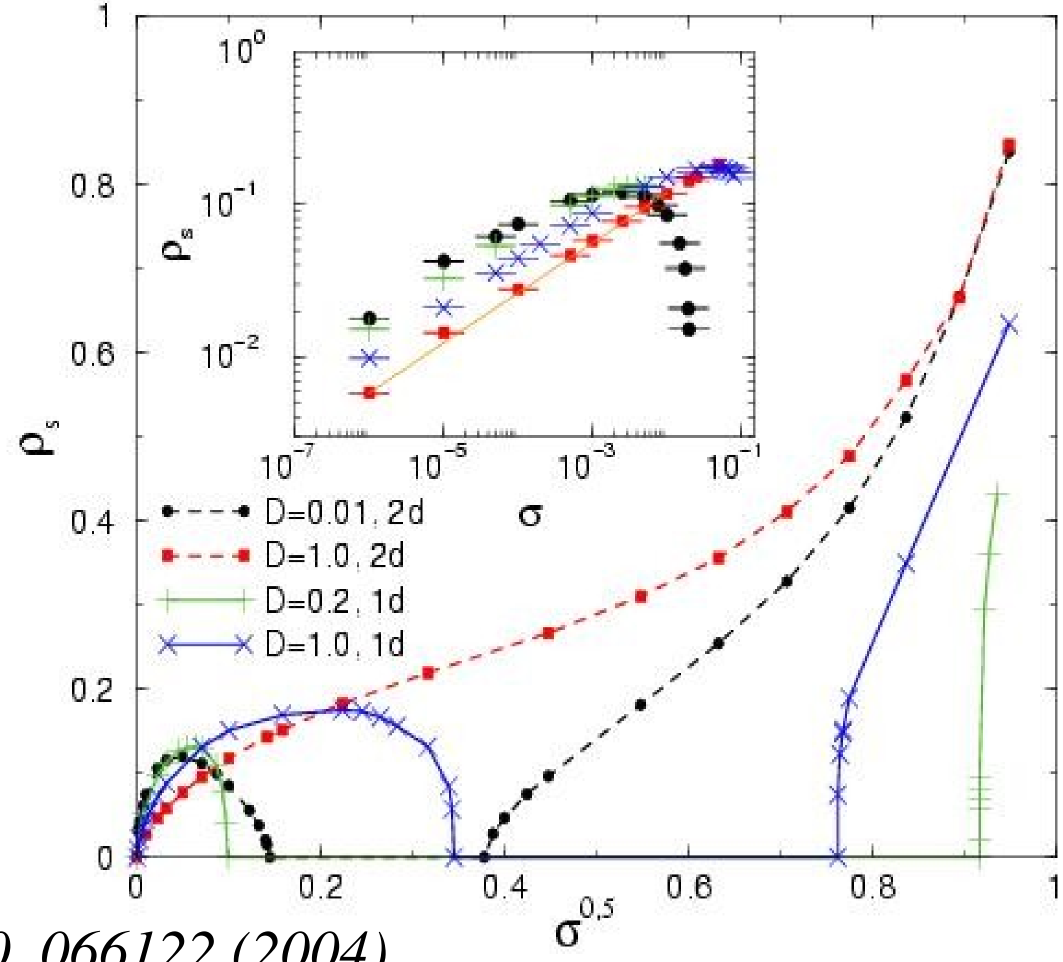


The role of diffusion A->2A, 4A->0: Reentrant phase diagram

N-cluster approximation



Simulations:
2d: L= 4000x4000, 1d: L= 100000



G. Ódor: PRE 70, 066122 (2004)

Further problems studied

The effect of disorder:

Géza Ódor and Nóra Menyhárd:

Critical behavior of an even offspringed branching and annihilating random walk cellular automaton with spatial disorder,

Phys. Rev. E 73 (2006) 036130

Local scale invariance in dynamical systems:

Géza Ódor

Local scale invariance in the parity conserving nonequilibrium kinetic Ising model

J. Stat. Mech. (2006) L11002

Topological constraints in low dimensions:

Géza Ódor :

Universal behavior of one-dimensional multispecies branching and annihilating random walks with exclusion

Phys. Rev. E 63(2001) 056108.

Oscillating states in population dynamics:

Tibor Antal, Michel Droz, Adam Lipowski and Géza Ódor: On the critical behavior of a lattice prey-predator model,

Phys. Rev. E 64(2001) 036118.

List of publications referencing GRID simulations

36.
G. Odor:
Critical behavior of the one-dimensional diffusive pair contact process,
Phys.Rev. E 67 (2003) 016111.

37.
G. Odor:
Critical behavior of binary production reaction-diffusion systems, in
Proc of 7-th Granada Lectures (page 58-75), ed. P.L.Garrido and J. Marro,
AIP Conference Proceedings 661 (2003).

38.
G. Odor:
Phase transition classes in triplet and quadruplet reaction-diffusion models,
Phys. Rev. E 67 (2003) 056114.

39.
G. Odor:
Critical behavior in reaction-diffusion systems exhibiting absorbing phase transitions.
Braz. J. of Phys. 33 (2003) 431.

40.
N. Menyhard and G. Odor,
Multispecies annihilating random walk transition at zero branching rate:
Cluster scaling behavior in a spin model
Phys. Rev. E 68, (2003) 056106.

41.
G. Odor:
Phase transitions of the binary production 2A->3A, 4A->0 model,
Phys. Rev. E 69 (2004) 036112.

List of publications referencing GRID simulations

43.
G. Odor:
Critical behavior of the two-dimensional 2A -> 3A, 4A -> 0 binary system.
Phys. Rev. E 70 (2004) 026119.

44.
G. Odor:
Role of diffusion in branching and annihilation random walk models,
Phys. Rev. E 70 (2004) 066122.

45.
G. Odor and A. Szolnoki,
Cluster mean-field study of the parity-conserving phase transition,
Phys. Rev. E 71 (2005) 066128.

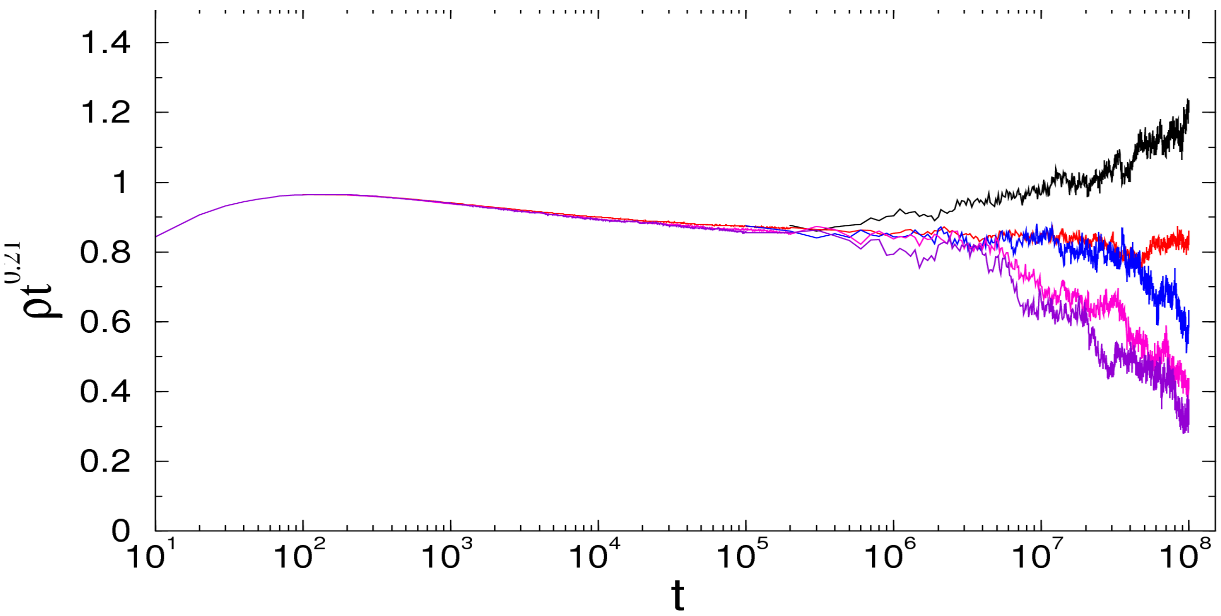
46.
G. Odor and N. Menyhard,
Critical behavior of an even offspringed branching and annihilating random walk
cellular automaton with spatial disorder,
Phys. Rev. E 73, 036130 (2006)

47.
G. Odor,
Phase transition of triplet reaction-diffusion models,
Phys. Rev. E 73, 047103 (2006).

48.
G. Odor,
Local scale invariance in the parity conserving nonequilibrium
kinetic Ising model,
J. Stat. Mech. (2006) L11002

Some open problems

- Field theoretical understanding
- Understanding the role symmetries and conservation laws
- In 1d two classes, non-universal or DP scaling ? Numerical confirmation following density decay



- More complex: nA -> (n+m) A type reaction-diffusion models with n > 2 show similar new universal behavior in low dimensions ...

Outlook/what would be good for a GRID user?

- Reliable GRIDS: no fundamental system changes, reconfigurations, long shutdowns in every month
- Only a few lost jobs per month
- Intelligent queuing system (avoiding long batch waiting times)
- Checkpointing in parallel PVM/MPI applications