Remarks on theory errors and the choice of scales

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- To calculate cross sections, one needs to choose the renormalization scale and the factorization scale.

    * What to choose?

- The freedom to choose provides a way to estimate the error from not having calculated beyond NLO.

    * How does that work?

    * Is there more freedom?

Scale dependence

- The coupling depends on the renormalization scale and the parton distributions depend on the factorization scale.

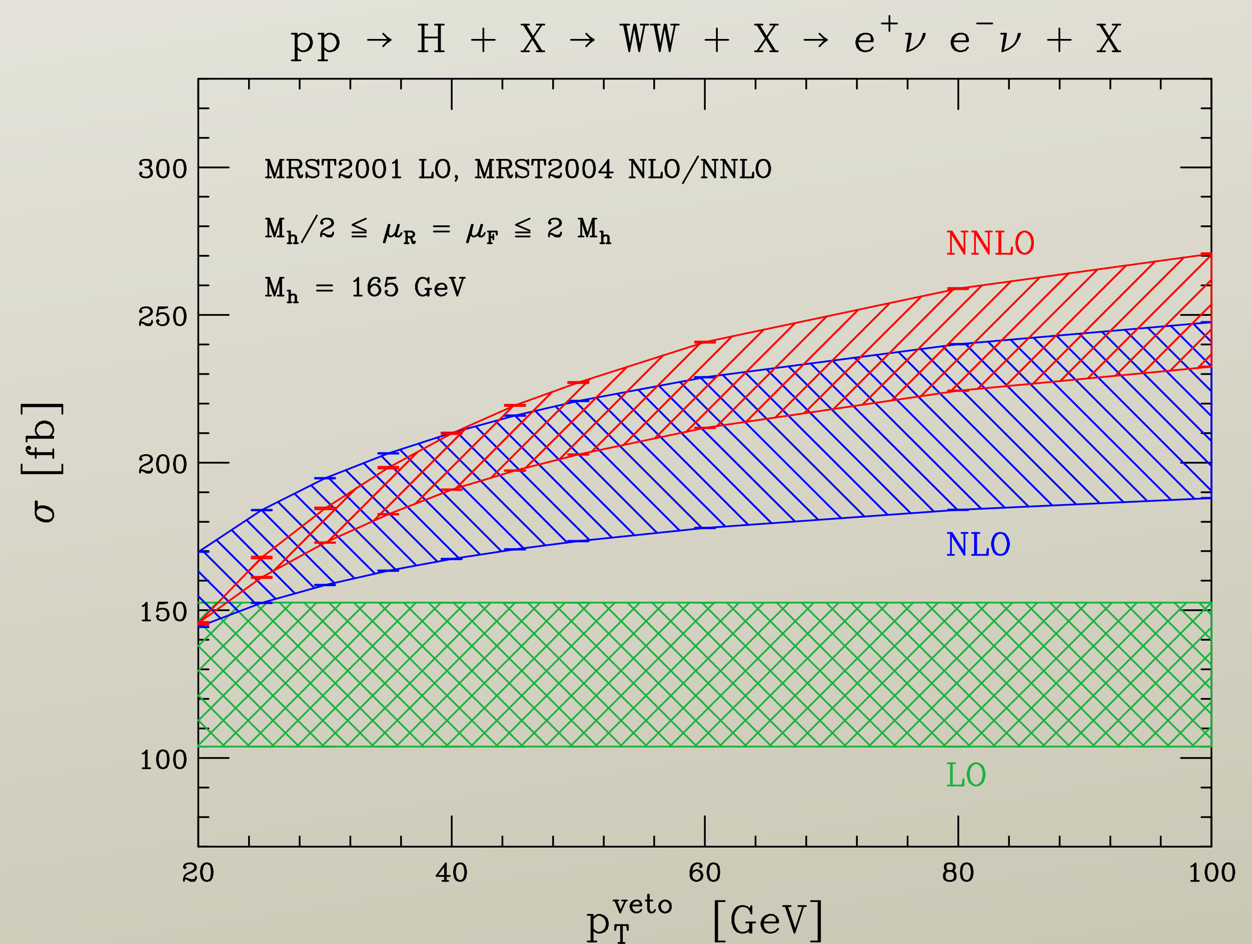
- If we had an NNLO calculation, the dependence on the scales would be cancelled to that order.

- Thus the dependence on the scales shows us how big *some* of the NNLO terms are.

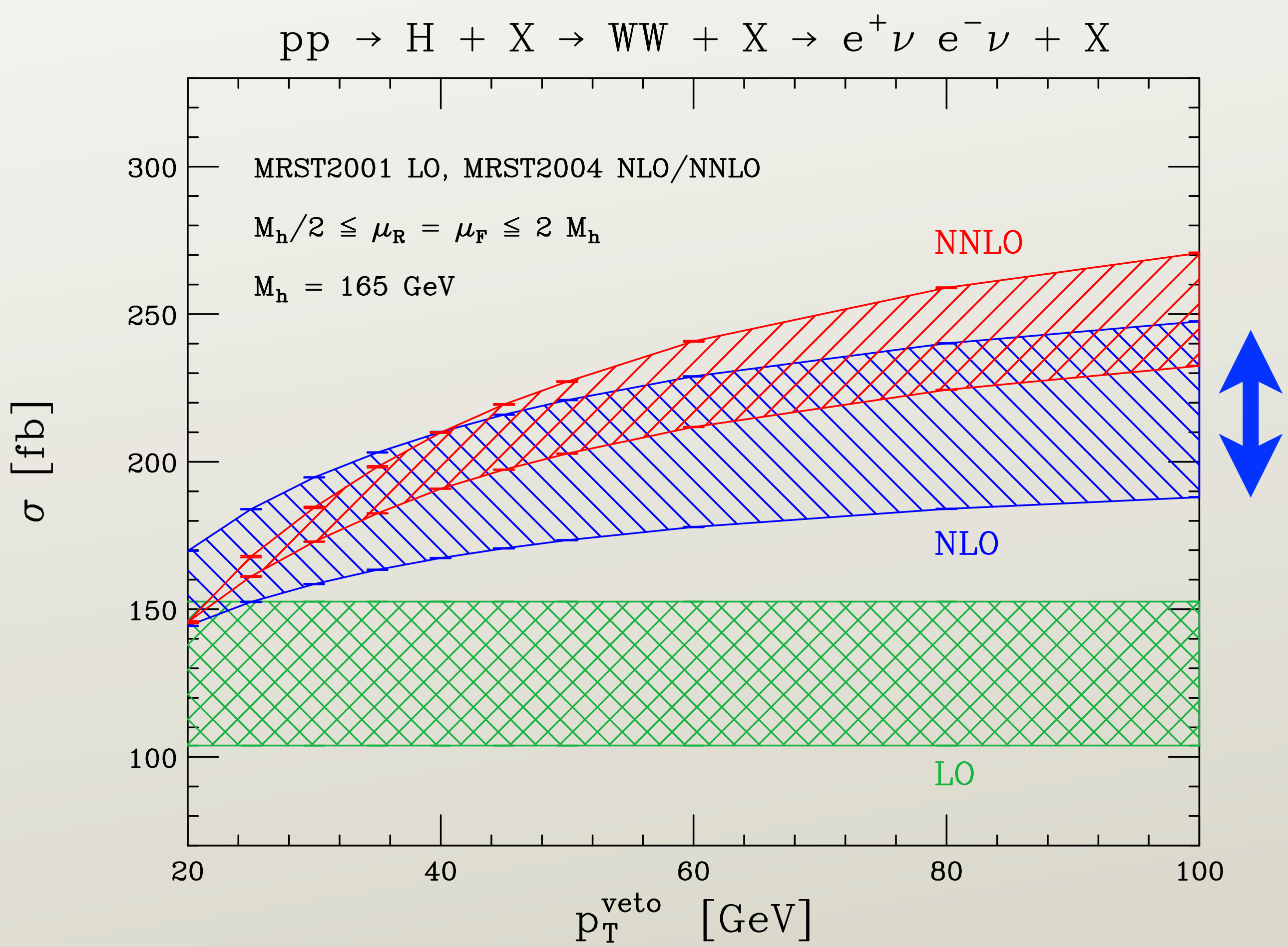
- This supplies an error estimate.

Error estimates

- Perturbative calculations are usually presented with an error estimate.
- For example, Anastasiou, Dissertori, and Stockli, JHEP 0709, 018 (2007):



- Suppose that we have only NLO.



- We hope our NLO error band gives a range where NNLO will fall.

- Varying the scales by a factor 2 is more or less a guess, but it usually works pretty well.

Scale dependence plots

with Fred Olness

Plot one jet inclusive cross section

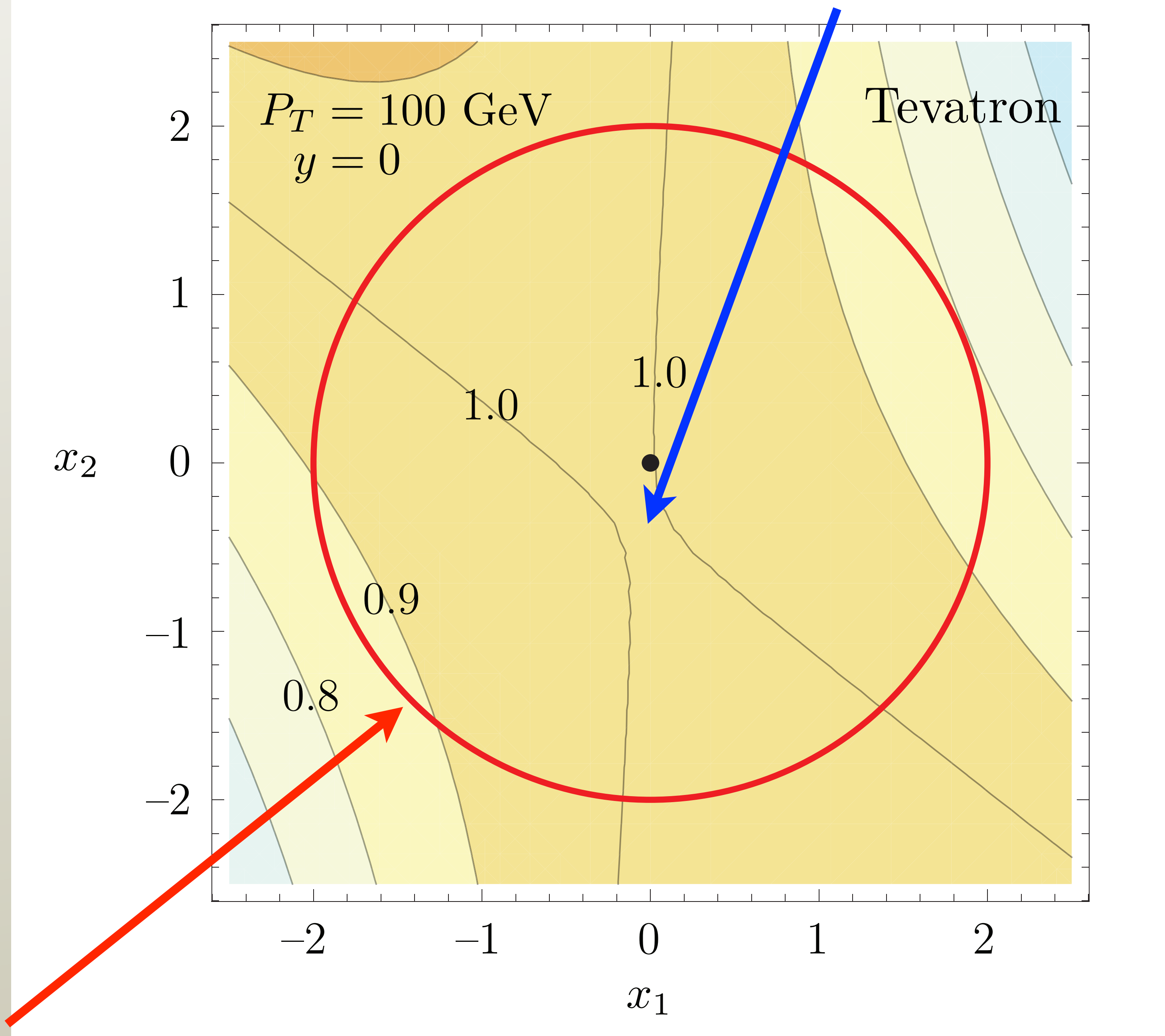
$$\frac{d\sigma}{dP_T\,dy}$$

versus

$$x_1 = \log_2\left(\frac{\mu_R}{P_T/2}\right)$$

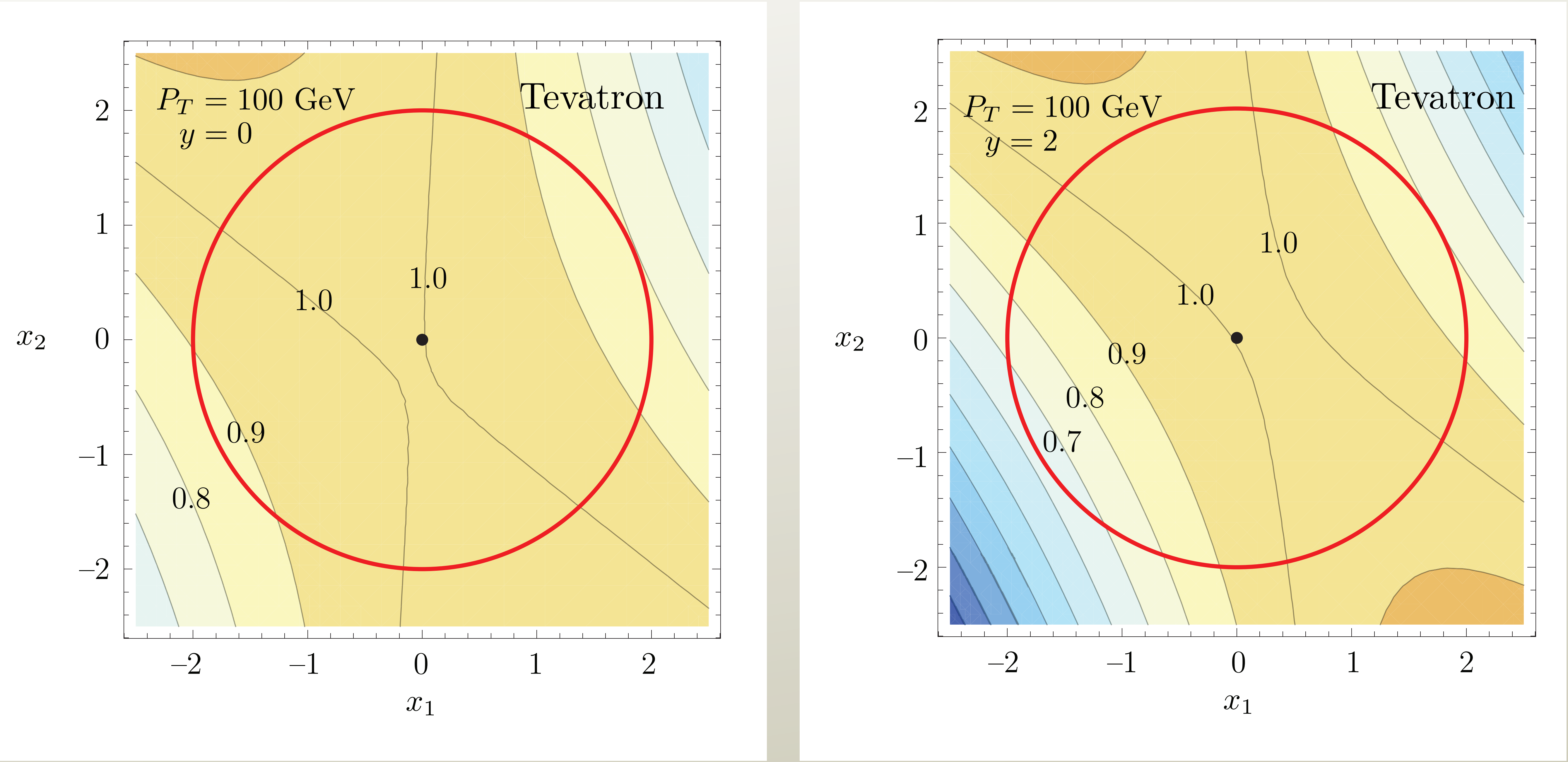
$$x_2 = \log_2\left(\frac{\mu_F}{P_T/2}\right)$$

Choose standard scales near saddle point.



Use rms deviation around circle for error estimate.

Rapidity matters



Is there anything else that you can vary?

You can vary the definition of parton distribution functions

new pdf

$\overline{\mathrm{MS}}$ pdf

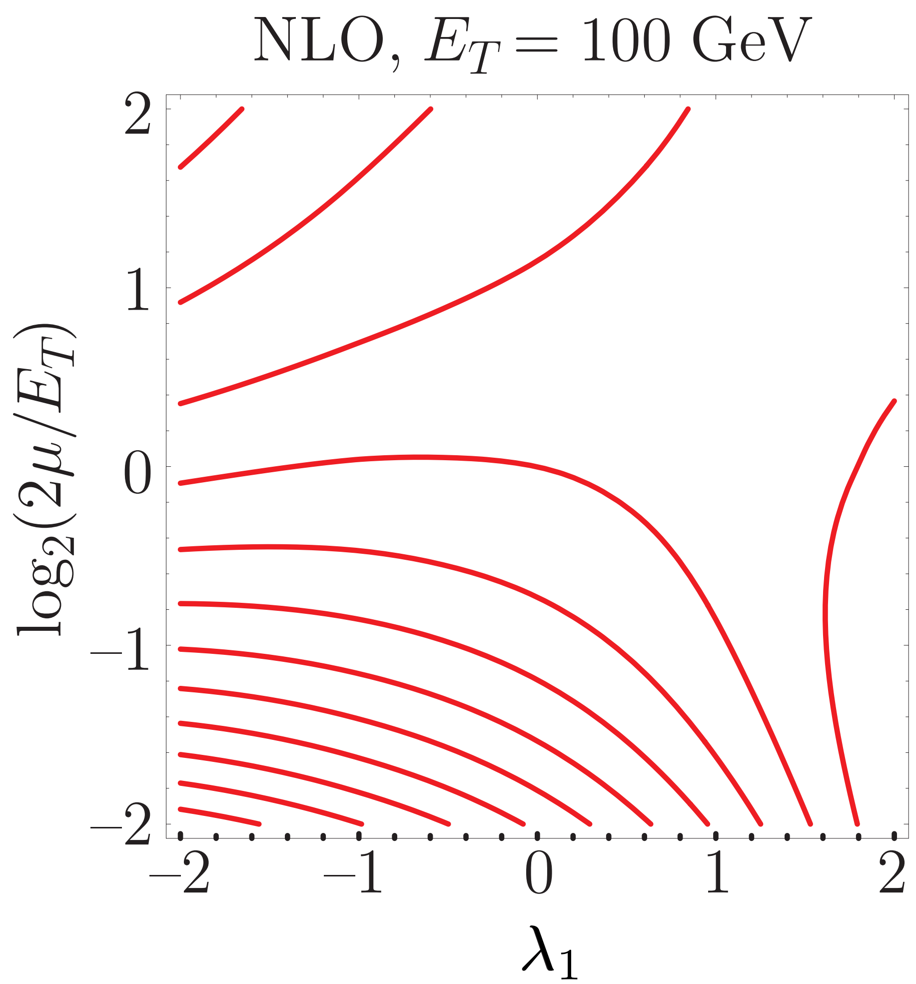
$\overline{\mathrm{MS}}$ pdf

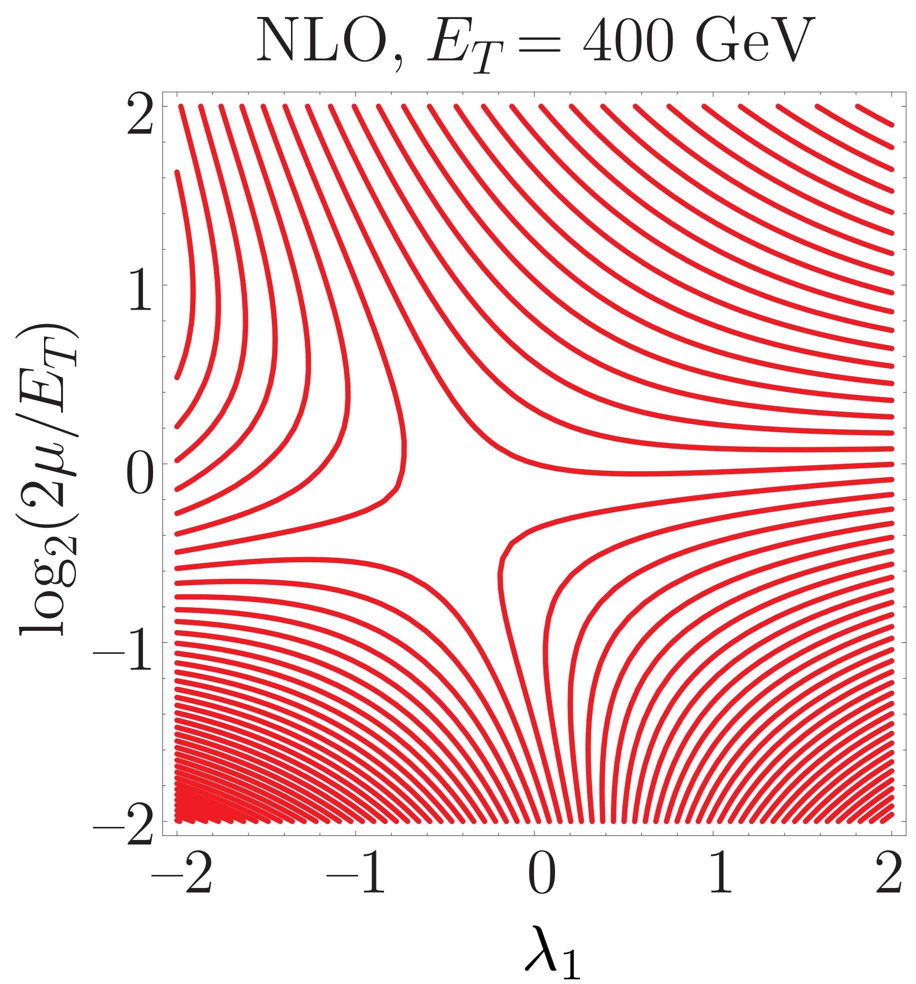
$$
\begin{aligned}
\tilde{f}_{a/A}(x,\mu) = & f_{a/A}(x,\mu) \\
& + \frac{\alpha_s(\mu)}{2\pi} \sum_{J=0}^{N} \lambda_J \int_x^1 \frac{d\xi}{\xi} \sum_b K_{ab}^{(J)}(x/\xi)\, f_{b/A}(\xi,\mu)\,.
\end{aligned}
$$

Example: $\lambda_1 = 1$ can be DIS pdfs.

Parvez Anandam explored this.

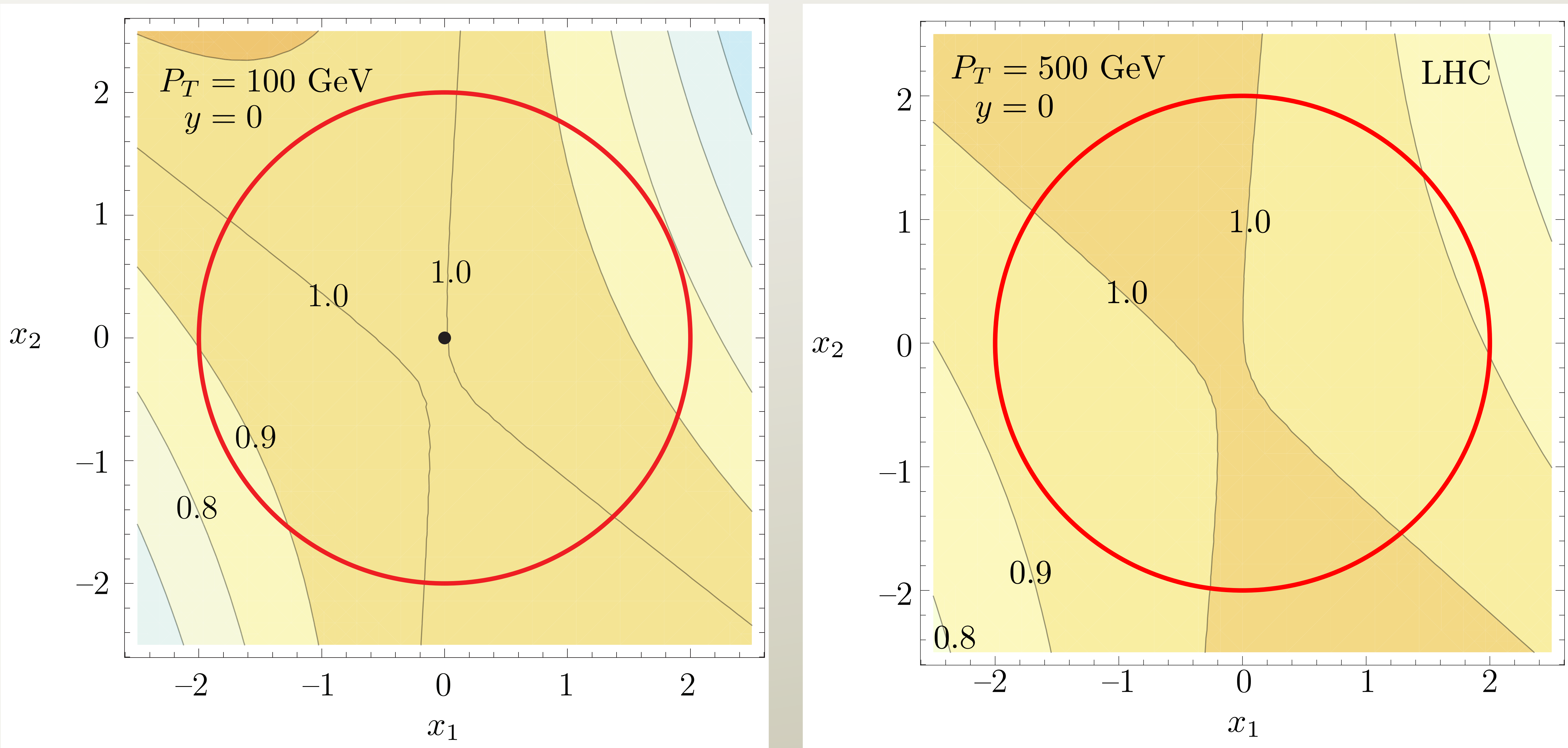
Vary λ_1 and μ_F.





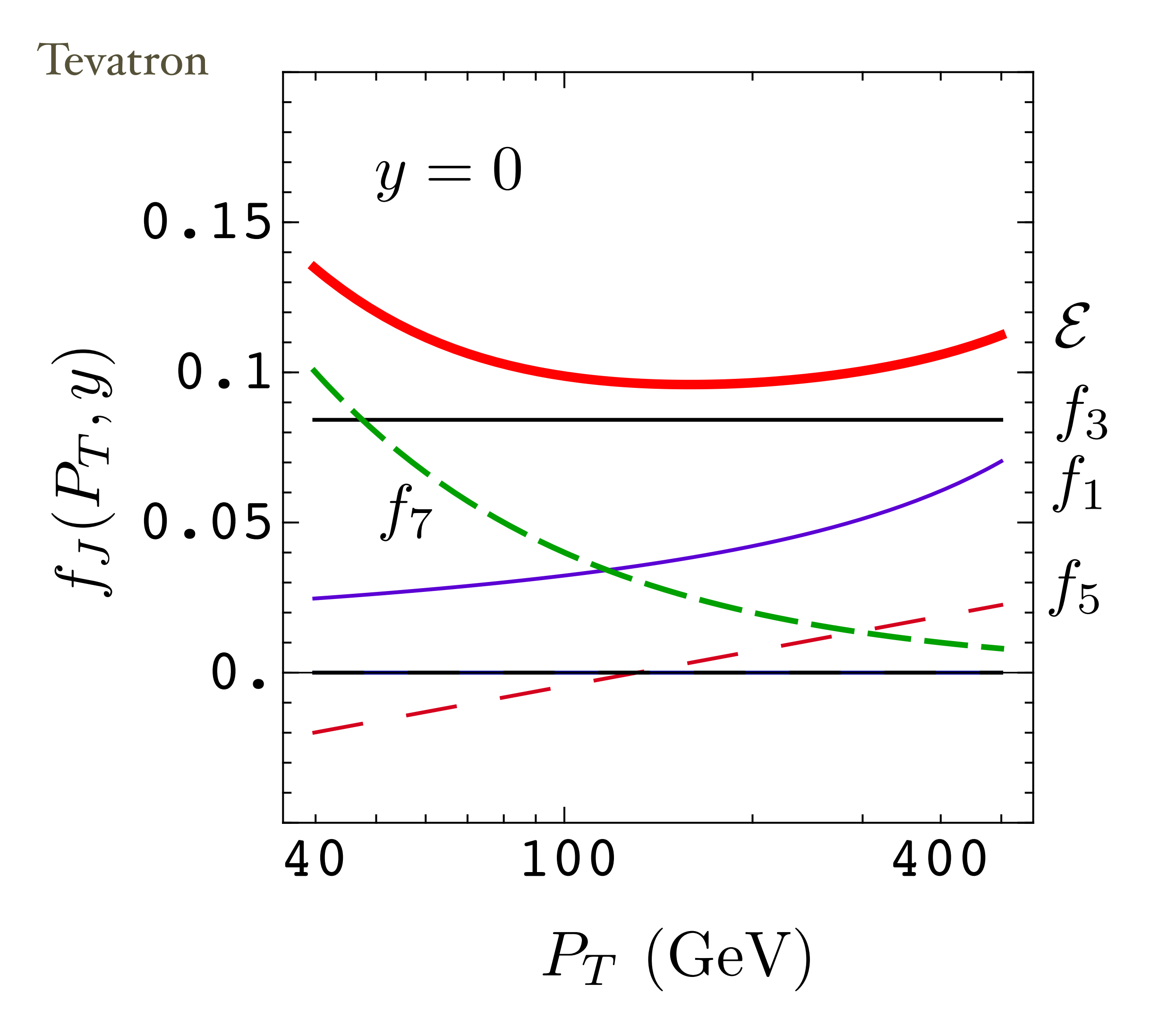
These are 1% contour lines.

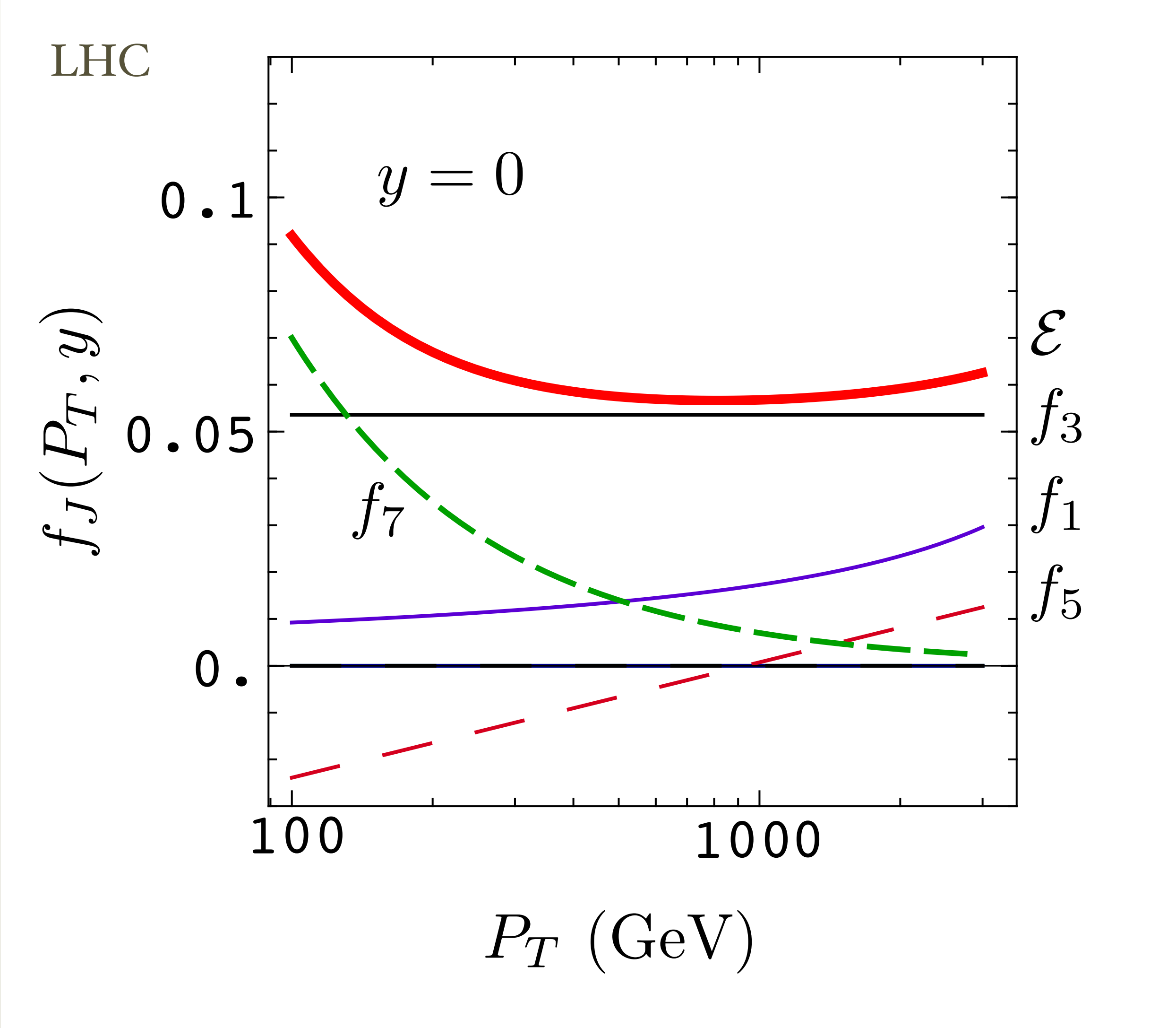
Scale dependence less at LHC



Error estimates

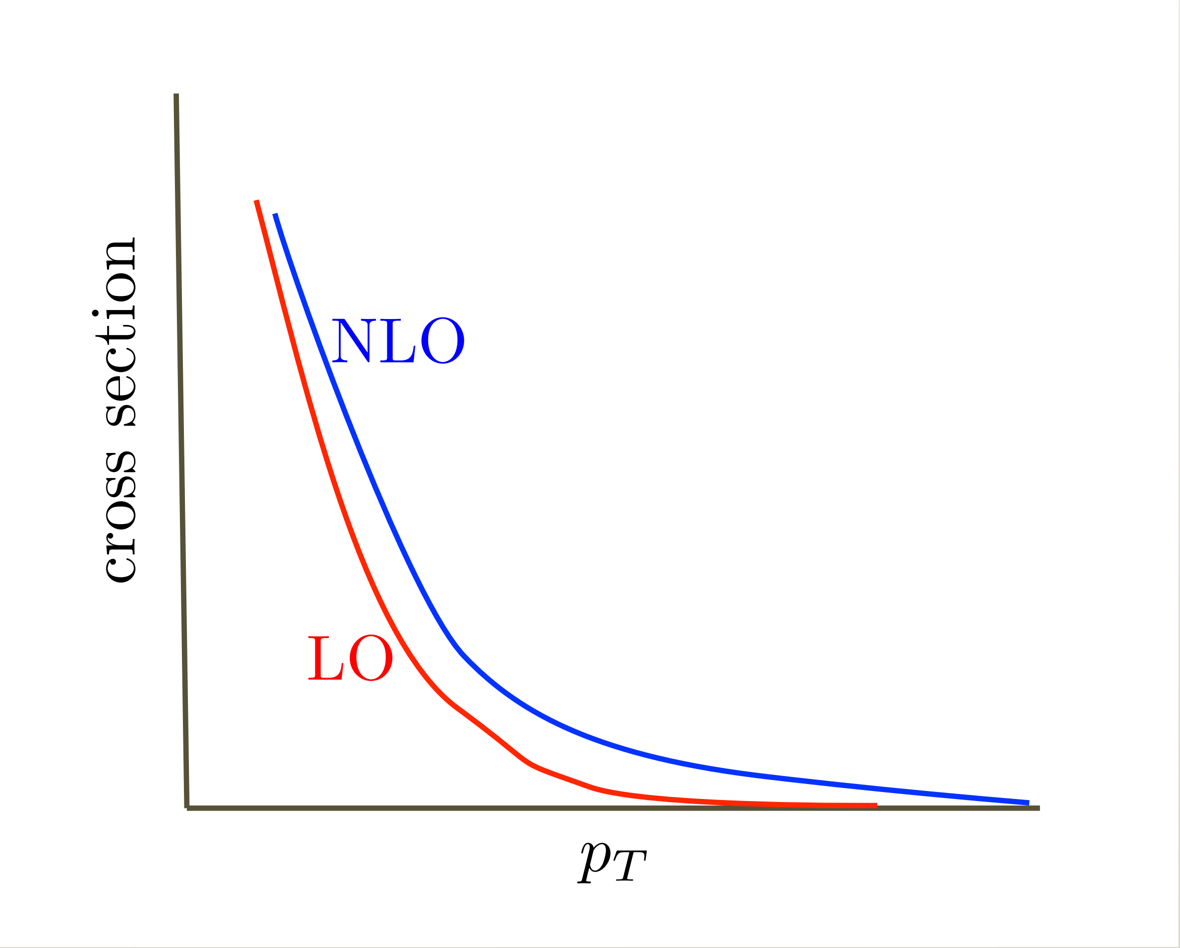
with Fred Olness





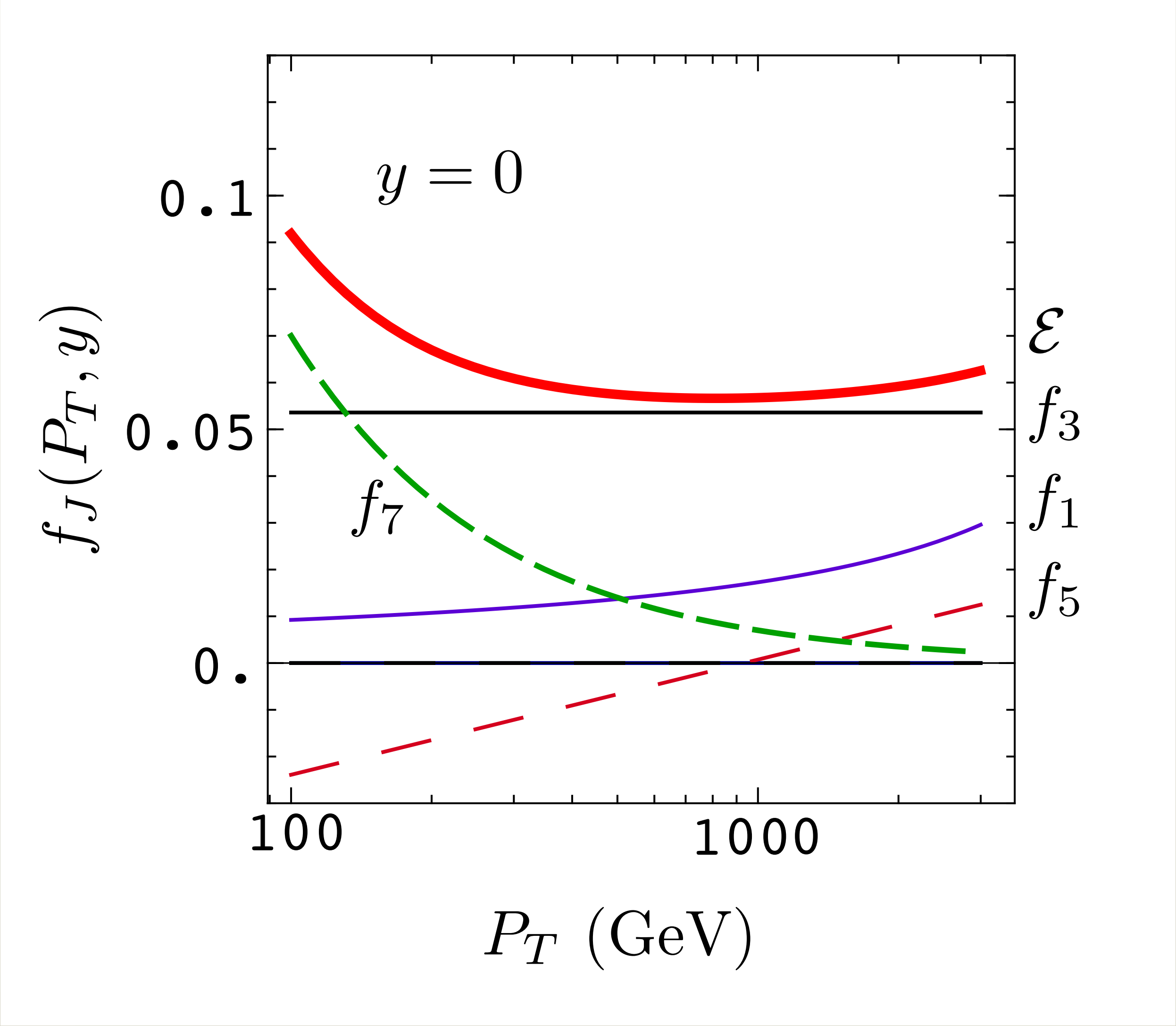
Estimated errors are smaller at LHC

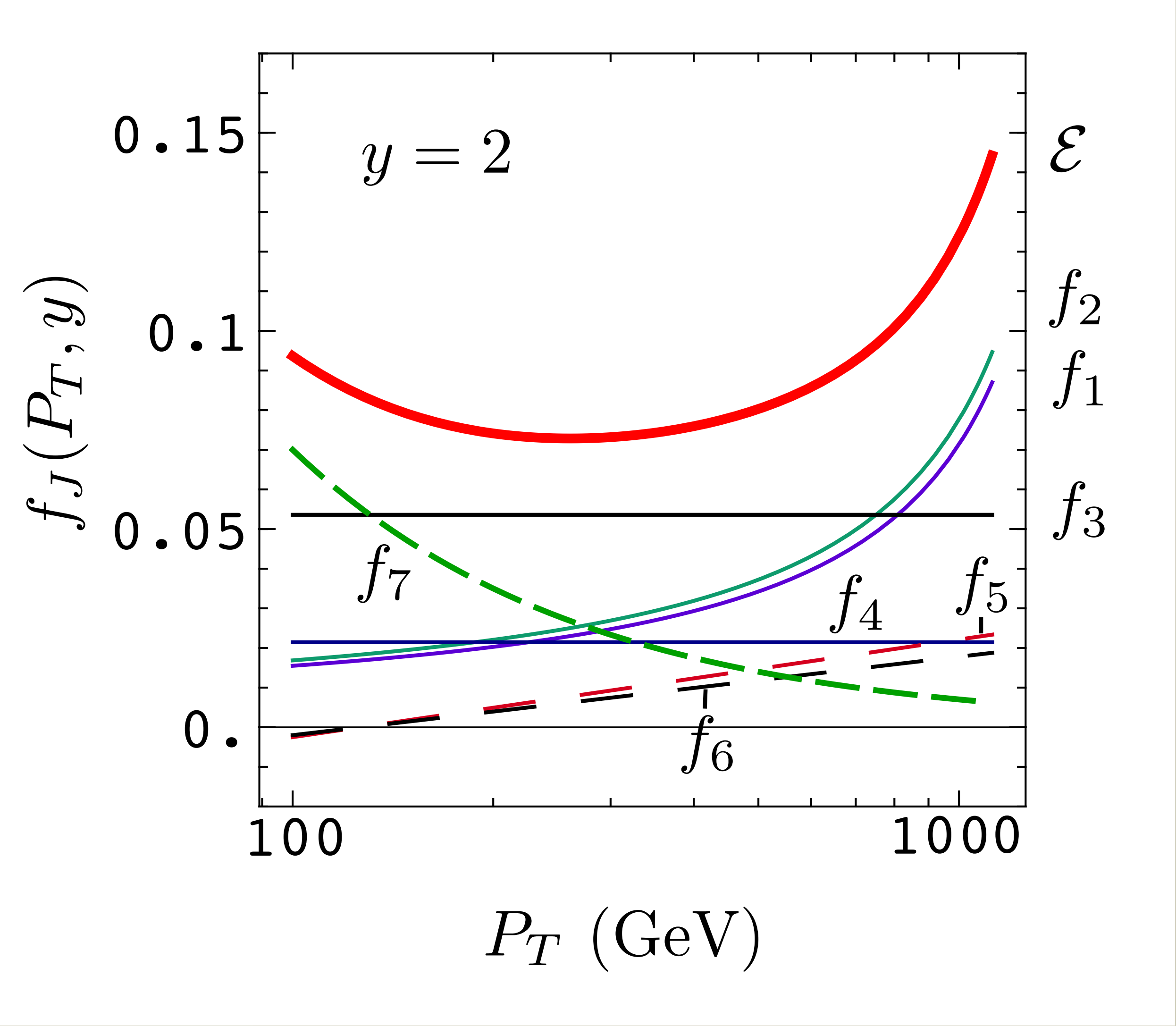
Mind the edge



There can be big fractional errors if you get near the edge of LO phase space.

Rapidity dependence





Estimated errors are bigger at large rapidity.

Inclusive is good

But too inclusive can be bad.

Can have fixed contribution from NLO virtual graphs

And big contributions from NLO real graphs

With E_T = total transverse energy in event,

for $d\sigma/dE_T$ at LHC at NLO at $E_T = 1600$ GeV, I get

$$\frac{\sigma(E_T/8, E_T/8)}{\sigma(E_T/4, E_T/4)} \approx 0.75 \qquad \frac{\sigma(E_T/2, E_T/2)}{\sigma(E_T/4, E_T/4)} \approx 0.89$$

We get a big estimated error because E_T may be too inclusive.