Puzzles in Heavy-Quarkonium Production

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Theoretical Framework

NRQCD Factorization of the Inclusive Production Cross Section

- Conjecture (GTB, Braaten, Lepage (1995)):
  The inclusive cross section for producing a quarkonium at large momentum transfer (p_T) can be written as a sum of "short-distance" coefficients times NRQCD matrix elements.

$$\sigma(H) = \sum_n F_n(\Lambda)\langle 0|\mathcal{O}_n^H(\Lambda)|0\rangle .$$

- The "short-distance" coefficients $F_n(\Lambda)$ are essentially the process-dependent partonic cross sections to make a $Q\bar{Q}$ pair convolved with the parton distributions.
- The NRQCD matrix elements $\langle 0|\mathcal{O}_n^H(\Lambda)|0\rangle$ are the probability for a $Q\bar{Q}$ pair to evolve into a heavy quarkonium.
- They are matrix elements of four-fermion operators in NRQCD, but with a projection onto an intermediate state of the quarkonium H plus anything:

$$\mathcal{O}_n^H(\Lambda) = \langle 0|\chi^\dagger \kappa_n \psi \left(\sum_X |H+X\rangle\langle H+X|\right)\psi^\dagger \kappa'_n \chi|0\rangle .$$

- κ_n and κ'_n are combinations of Pauli and Color matrices.

- The short-distance coefficients have expansions in powers of α_s.
- The operator matrix elements are nonperturbative, but they are conjectured to be universal (process independent).
  - Only the color-singlet production and decay matrix elements are simply related.
- The matrix elements have a known scaling with v.
  $v^2 \approx 0.23$ for the J/ψ.
- The current phenomenology of J/ψ, $\psi(2S)$, and Υ production uses matrix elements through relative order v^4:

$$\begin{array}{ll}
\langle \mathcal{O}^H({}^3S_1^{[1]}) \rangle & (O(v^0)), \\
\langle \mathcal{O}^H({}^1S_0^{[8]}) \rangle & (O(v^3)), \\
\langle \mathcal{O}^H({}^3S_1^{[8]}) \rangle & (O(v^4)), \\
\langle \mathcal{O}^H({}^3P_J^{[8]}) \rangle & (O(v^4)).
\end{array}$$

- A key feature of NRQCD factorization:
  Quarkonium production can occur through color-octet, as well as color-singlet, $Q\bar{Q}$ states.
- If we drop all of the color-octet contributions and retain only the leading color-singlet contribution, then we have the color-singlet model (CSM).
  - Inconsistent at higher orders in v and for P-wave production: IR divergent.

Status of a Proof of Factorization

- A proof is complicated because gluons can dress the basic production process in ways that apparently violate factorization.
- A proof of factorization would involve a demonstration that diagrams in each order in α_s can be re-organized so that
  - All soft singularities cancel or can be absorbed into NRQCD matrix elements,
  - All collinear singularities and spectator interactions can be absorbed into parton distributions.
- Nayak, Qiu, Sterman (2005, 2006): The color-octet NRQCD matrix elements must be modified by the inclusion of Wilson lines (path integrals of the gauge field) to make them gauge invariant.
  - Contributions involving the Wilson lines first appear in NNLO and are essential to allow soft contributions to be absorbed into the NRQCD matrix elements.
  - If they are to be universal, the NRQCD matrix elements must be independent of the direction of the Wilson lines.
  - At NNLO, a “miracle” occurs and the dependence on the direction of the Wilson lines cancels.
  - It is not known if this generalizes to all orders.
  - An all-orders proof is essential because the α_s associated with soft gluons is not small.

The Fragmentation Approach

(Kang, Qiu, and Sterman (2011))

- Writes the cross section in terms of
  - single-parton production cross sections convolved with the fragmentation functions for a single parton into a quarkonium

    $$d\hat{\sigma}_{A+B\to i+X} \otimes D_{i\to H}$$

    Gives $1/p_T^4$ behavior.
  - $Q\bar{Q}$ production cross sections convolved with fragmentation functions for a $Q\bar{Q}$ pair into a quarkonium

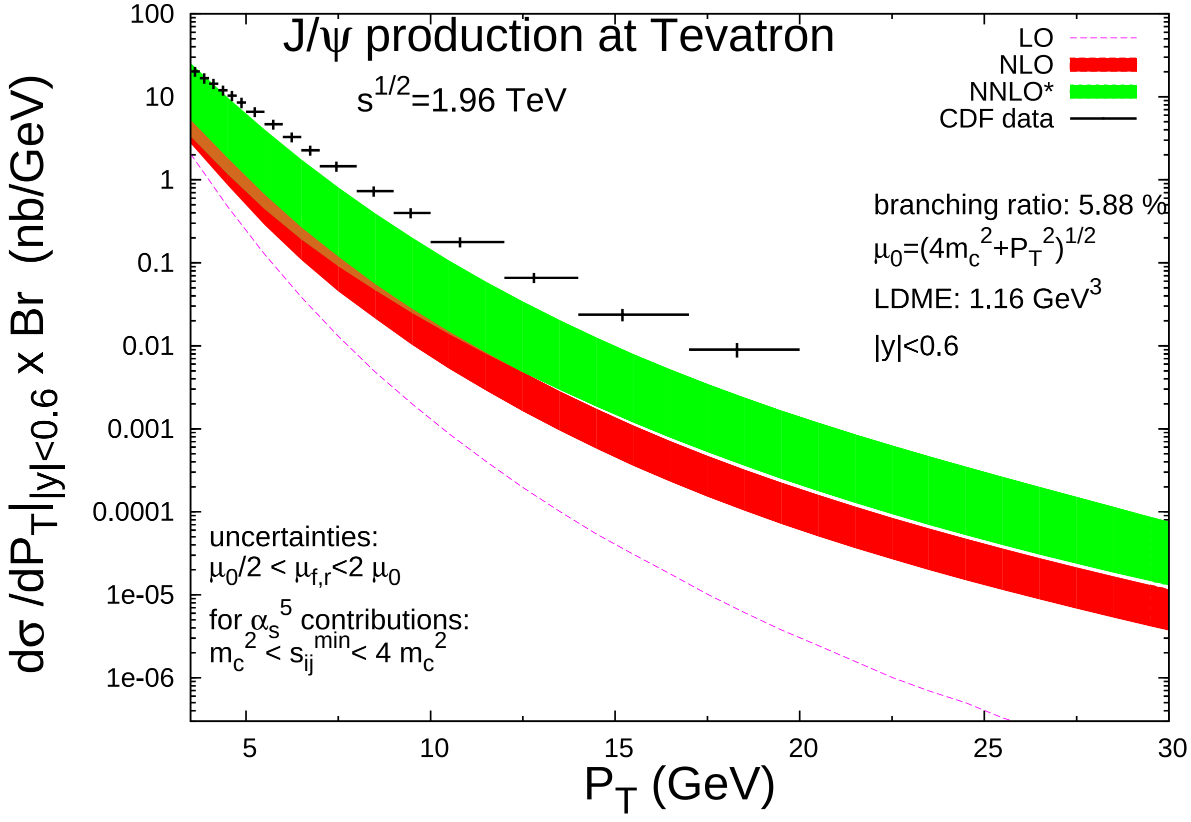
    $$d\hat{\sigma}_{A+B\to Q\bar{Q}+X} \otimes D_{Q\bar{Q}\to H}$$

    Gives $1/p_T^6$ behavior.
- Re-organizes the perturbation expansion as an expansion in powers of $1/p_T$.
- Believed to hold to all orders in perturbation theory up to corrections of order m_Q^4/p_T^4.
- If NRQCD factorization holds, then the fragmentation functions can be written as a sum of NRQCD matrix elements times perturbatively calculable short-distance coefficients.
- See Qiu's talk at QWG2010 (conferences.fnal.gov/QWG2010) and Eur. Phys. J. C **71**, 1534 (2011).

The Problem of Large k Factors

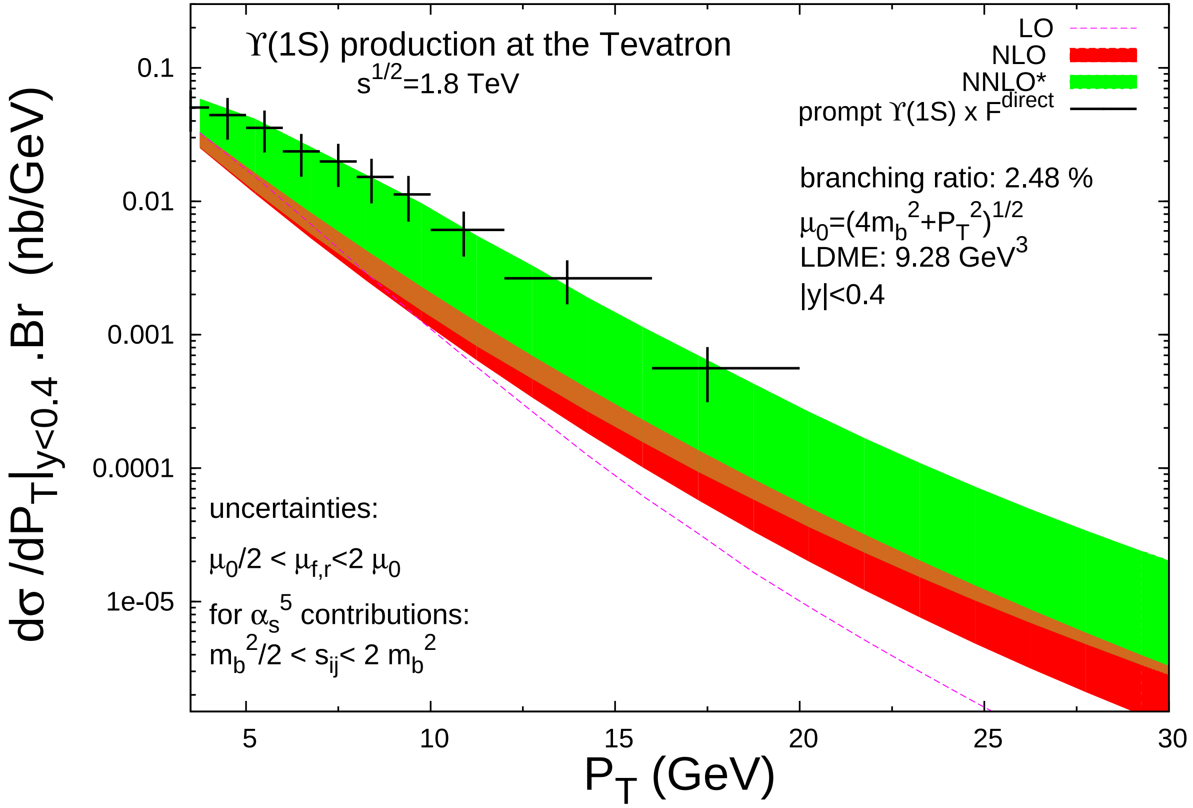
- Higher-order corrections to color-singlet quarkonium production at the Tevatron are unexpectedly large. (Campbell, Maltoni, Tramontano(2007); Artoisenet, Lansberg, Maltoni (2007))

NLO and NNLO* Color-Singlet J/ψ Production



- Plot from Pierre Artoisenet, based on work by Artoisenet, Campbell, Lansberg, Maltoni, Tramontano.
- The NNLO* calculation is an estimate based on real-emission contributions only.
- The data still seem to require a color-octet contribution.

NLO and NNLO* Color-Singlet Υ Production



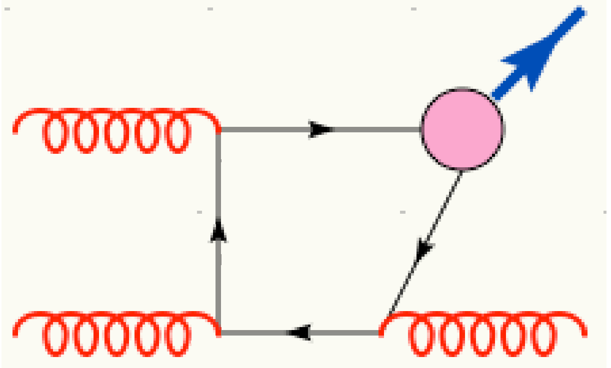
- Plot from Pierre Artoisenet, based on work by Artoisenet, Campbell, Lansberg, Maltoni, Tramontano (2008)
- NLO results confirmed by Gong and Wang (2007).
- The data could be explained by color-singlet production alone.
- There is still room for a substantial amount of color-octet production.

- A large k factor ~ -10 is also seen in the ${}^3P_J$ color-octet channel.
  (Ma, Wang, and Chao (2010); Butenschön and Kniehl (2010))
- NLO corrections to the S-wave channels are small.
  (Gong, Li, and Wang (2008, 2010))
  - k factors at the Tevatron are about 1.235 for the ${}^1S_0$ channel and 1.139 for the ${}^3S_1$ channel.
- Does the perturbation series converge?
- How do we understand the different k factors for different channels?

Explanation of Large k Factors

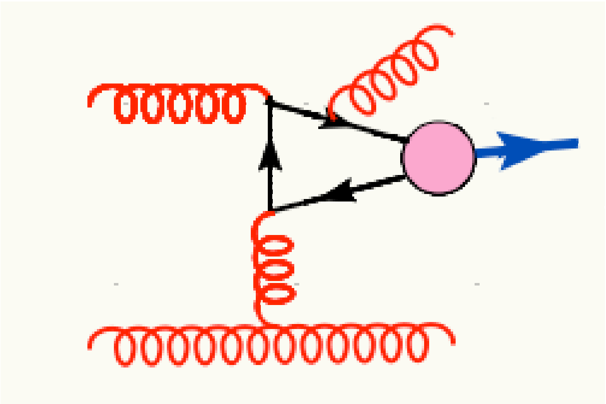
- At high p_T, higher powers of α_s can be offset by a less rapid fall-off with p_T.
  (Campbell, Maltoni, Tramontano(2007); Artoisenet, Lansberg, Maltoni (2007))

Color-singlet LO:

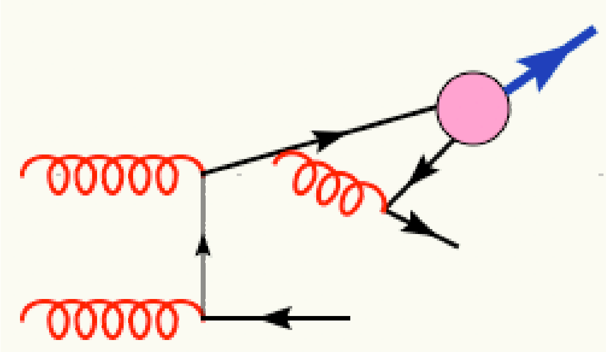


$$\sim \alpha_s^3 \frac{(2m_c)^4}{p_T^8}$$

Color-singlet NLO:

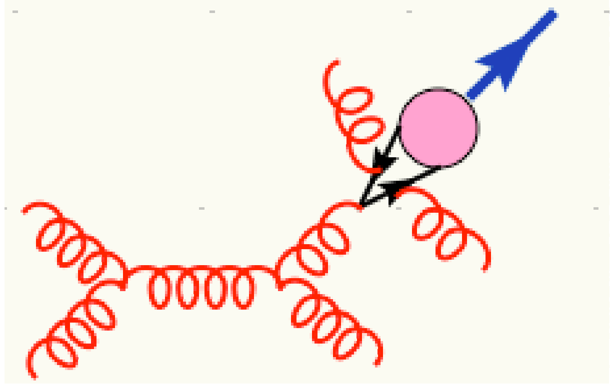


$$\sim \alpha_s^4 \frac{(2m_c)^2}{p_T^6}$$



$$\sim \alpha_s^4 \frac{1}{p_T^4}$$

Color-singlet NNLO:



$$\sim \alpha_s^5 \frac{1}{p_T^4}$$

- Similar explanations account for the k factors in the color-octet channels.

Discussion

- The fragmentation approach of Kang, Qiu, and Sterman allows one to focus on the leading and first subleading power corrections, which seem to account for the large k factors.
    - It should be possible to calculate higher-order corrections and resum logs of p_T^2/m_c^2 for these contributions.
- It is important to check that the fragmentation contributions really do account for all of the large corrections.
    - Confirmed for the color-singlet NLO correction. (Kang, Qiu, Sterman (2011))
    - Preliminary confirmation for the ${}^3P_J$ color-octet NLO correction. (GTB, Jungil Lee)
- The color-singlet NNLO* correction seems be dominated by contributions proportional to $\log^2(p_T^2/p_{T\text{cut}}^2)$. (Ma, Wang, Chao (2011)).
    - These should cancel when virtual corrections are included, making the complete NNLO contribution smaller than the NNLO* contribution.

Comparisons of NRQCD Factorization with Experiment

Summary

- NLO corrections have been computed for many quarkonium production processes:
  - J/ψ and $\psi(2S)$ production at the Tevatron, RHIC, the LHC;
  - J/ψ photoproduction and polarization at HERA;
  - $J/\psi + \eta_c$ production, $J/\psi + c\bar{c}$ production, and $J/\psi + X$(non-$c\bar{c}$) in e^+e^- annihilation at the B factories.
- For quarkonium polarization in hadron-hadron collisions, an NLO calculation exists for the ${}^3S_1^{[1]}$, ${}^3S_1^{[8]}$, and ${}^1S_0^{[8]}$ channels, but not for the ${}^3P_J^{[8]}$ channel.
- Data and theory for quarkonium production generally agree within errors.
  - See the global fit of NRQCD matrix elements (Butenschön and Kniehl (2011)).
- There are two important exceptions:
  - quarkonium polarization in hadron-hadron collisions.
  - $J/\psi + X$(non-$c\bar{c}$) in e^+e^- annihilation at the B factories.
- There is also an important issue regarding the extraction of NRQCD matrix elements from fits of NLO to the data.

NLO Calculations for Hadroproduction

- In NLO, all of the color-octet channels develop $1/p_T^4$ dependence through gluon fragmentation.
  - The ${}^3S_1$ color-octet channel is no longer dominant at large p_T.
    It receives no further kinematic enhancement in NLO.
  - The gluon-fragmentation contribution to the ${}^1S_0$ color-octet channel is numerically small.
    The fragmentation function is not strongly peaked at $z = 1$ because spin-flip interactions do not produce IR divergences.
  - The ${}^3P_J$ color-octet channel is enhanced by a gluon-fragmentation contribution.
    The fragmentation function is strongly peaked at $z = 1$ because of remnants of IR divergences.
- As expected,
  - NLO corrections to the color-octet S-wave channels are small.
    (Gong, Li, Wang (2008,2010))
  - NLO corrections to the ${}^3P_J$ channel are large (k factor ~ -10).
    (Ma, Wang, and Chao (2010); Butenschön and Kniehl (2010))

First Complete NLO Calculations for Hadro-Production

(Ma, Wang, and Chao (2010); Butenschön and Kniehl (2010))

- The results the two groups for the short-distance coefficients agree.
- However, the fitted NRQCD matrix elements are very different.
- Using the CDF data, Ma and Chao could fit only two linear combinations of matrix elements unambiguously:

$$\begin{aligned} M_{0,r_0} &= \langle O^\psi({}^1S_0^{[8]})\rangle + (r_0/m_c^2)\langle O^\psi({}^3P_0^{[8]})\rangle = (7.4 \pm 1.9) \times 10^{-2}\ \text{GeV}^3, \\ M_{1,r_1} &= \langle O^\psi({}^3S_1^{[8]})\rangle + (r_1/m_c^2)\langle O^\psi({}^3P_0^{[8]})\rangle = (0.05 \pm 0.02) \times 10^{-2}\ \text{GeV}^3. \end{aligned}$$

  $r_0 = 3.9$ and $r_1 = -0.56$ chosen on the basis of approximate relations between the short-distance coefficients.

- Using both the CDF and HERA data, Butenschön and Kniehl fit all three color-octet matrix elements:

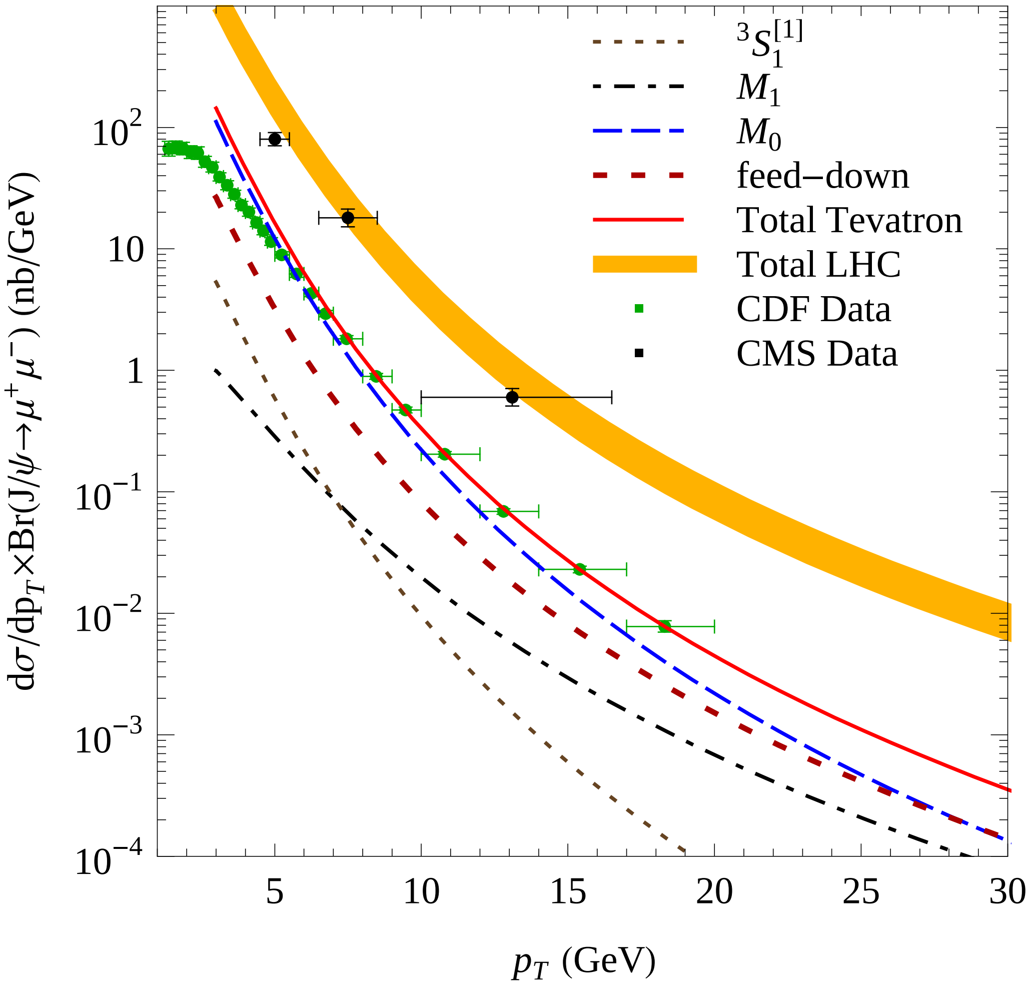
$$\begin{aligned} \langle O^\psi({}^1S_0^{[8]})\rangle &= (4.76 \pm 0.06) \times 10^{-2}\ \text{GeV}^3, \\ \langle O^\psi({}^3S_1^{[8]})\rangle &= (0.265 \pm 0.014) \times 10^{-2}\ \text{GeV}^3, \\ \langle O^\psi({}^3P_0^{[8]})\rangle/m_c^2 &= (0.587 \pm 0.013) \times 10^{-2}\ \text{GeV}^3, \end{aligned}$$

  which implies that

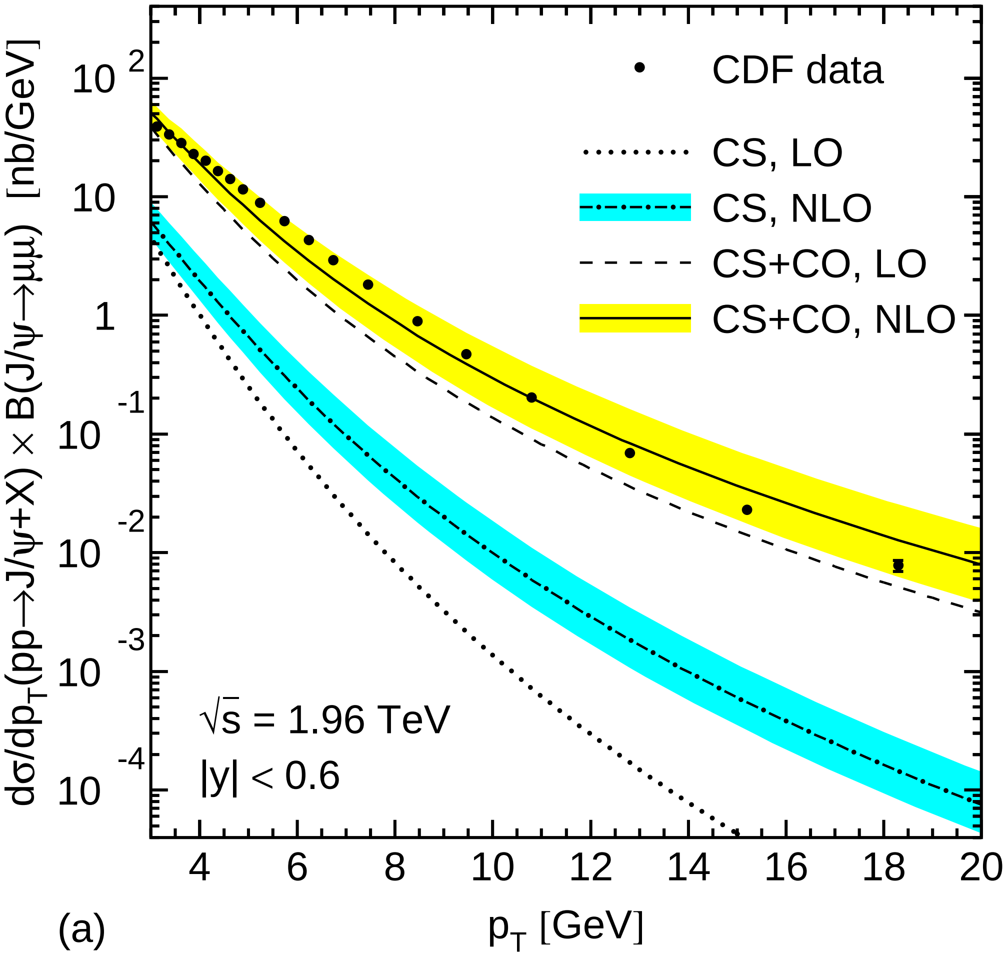
$$\begin{aligned} M_{0,r_0} &= (2.5 \pm 0.08) \times 10^{-2}\ \text{GeV}^3, \\ M_{1,r_1} &= (0.59 \pm 0.14) \times 10^{-2}\ \text{GeV}^3. \end{aligned}$$

- There are many small differences in the fitting procedures.
  - Inclusion of feeddown from $\psi(2S)$ and χ_{cJ} states (Ma and Chao),
  - Use of 2-parameter constrained fits (Ma and Chao),
  - Different p_T cuts:
    Ma and Chao: $p_T > 7$ GeV,
    Butenschön and Kniehl: $p_T > 3$ GeV.
- The most important difference is the use of low-p_T HERA (H1 (2002, 2005)) data by Butenschön and Kniehl.
  (1 GeV $\lesssim p_T \lesssim 3$ GeV).

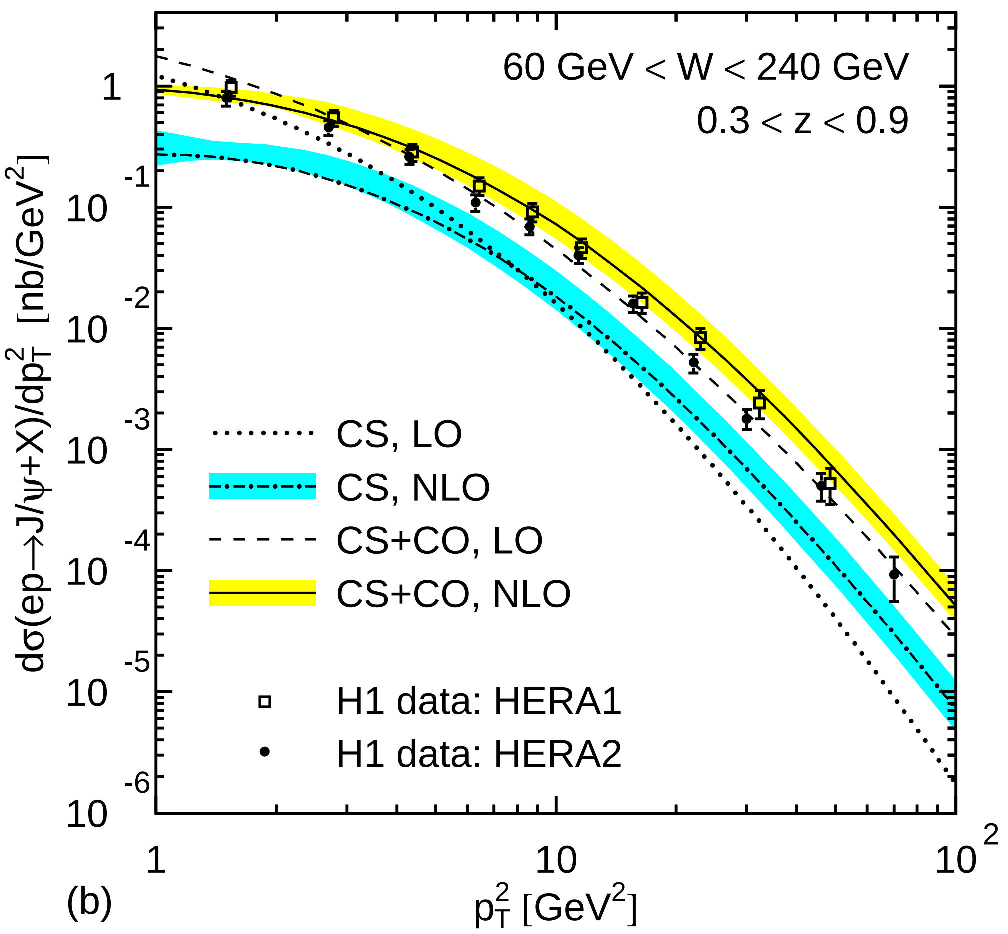
- Both predictions fit the data within errors, but the shape of the Ma and Chao fit agrees with the CDF data better than the shape of the Butenschön and Kniehl fit.



Ma and Chao

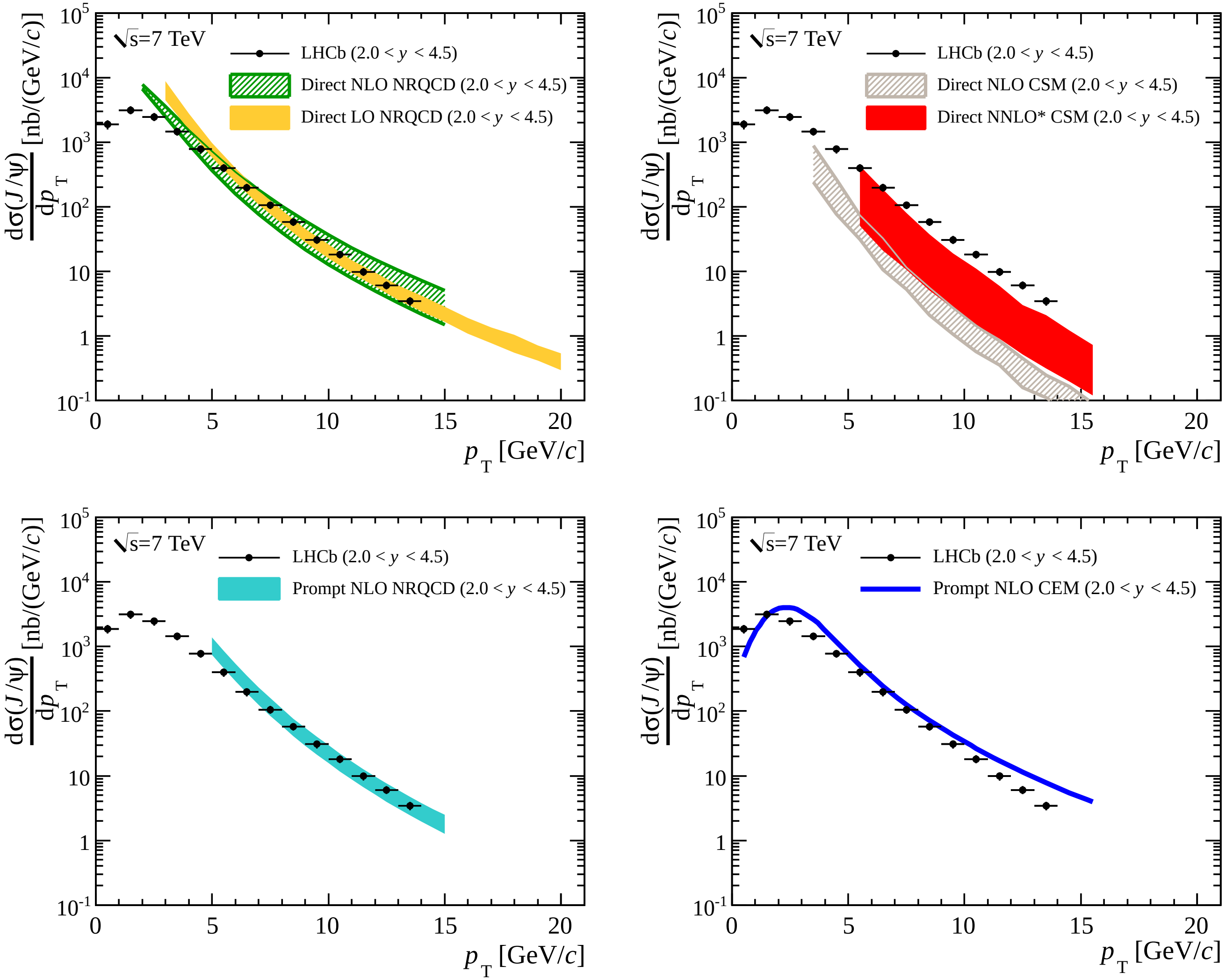


Butenschön and Kniehl



There is a slight discrepancy in shape between the Butenschön and Kniehl NLO prediction and the H1 data.

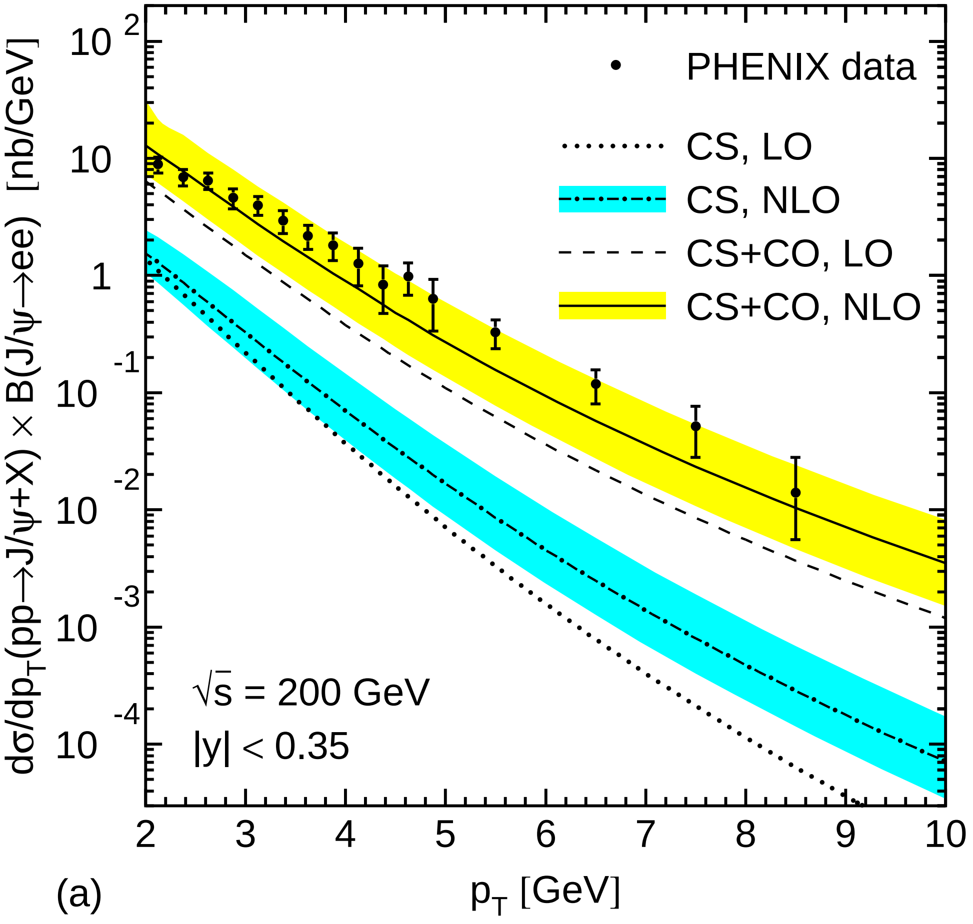
- There is also a slight discrepancy in shape between the Butenschön and Kniehl NLO prediction and the LHCb data. (There are also CMS, Atlas, and Alice measurements.)



Top left: Butenschön and Kniehl (2010). Top right: Artoisenet *et al.* (2008), Lansberg (2009).
Bottom left: Ma, Wang, and Chao (2010). Bottom right: Frawley, Ullrich, and Vogt (2008).

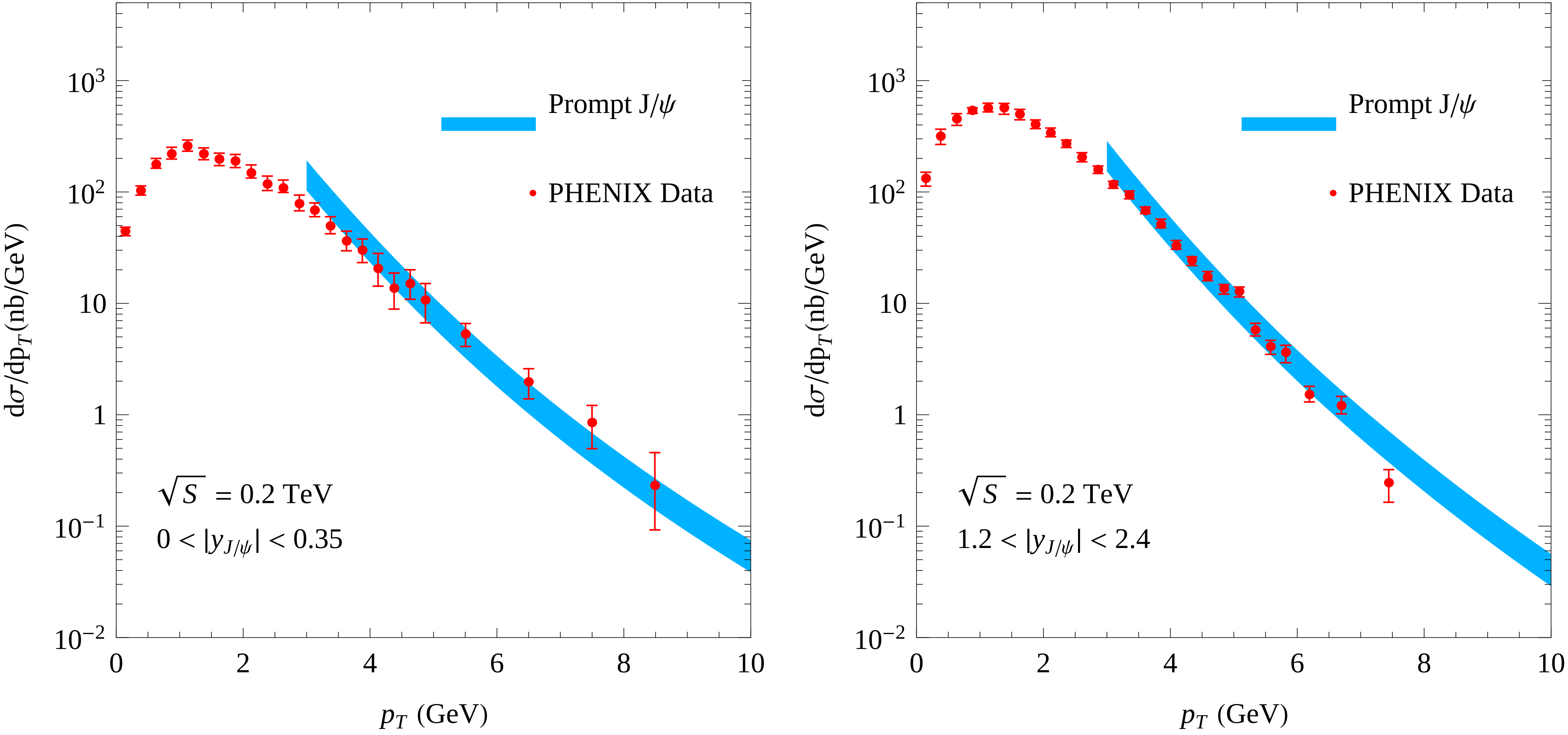
- The NNLO* CSM prediction lies below the data. The CEM prediction lies above the data.

- The Butenschön and Kniehl prediction lies slightly below the PHENIX (2011) data at high p_T.



- Feeddown is not included in the fit to the CDF data or in the prediction for PHENIX.
- The NLO color-singlet contribution is well below the PHENIX data.

- The Ma, Wang, and Chao prediction fits the PHENIX data slightly better at high p_T. The fit and the prediction include feeddown.



Discussion

- The analysis of Kang, Qiu, and Sterman suggests that we can predict only the leading and first subleading power in m_c^2/p_T^2.
- We need more information than just $d\sigma/dp_T$ for one process in order to fix the three important color-octet NRQCD matrix elements.
  - Are rapidity distributions sufficient to do this?
- For $p_T \lesssim m_c$, subsubleading power corrections may be important and factorization may break down.
  - Comparisons between theory and experiment in this region may not be valid.

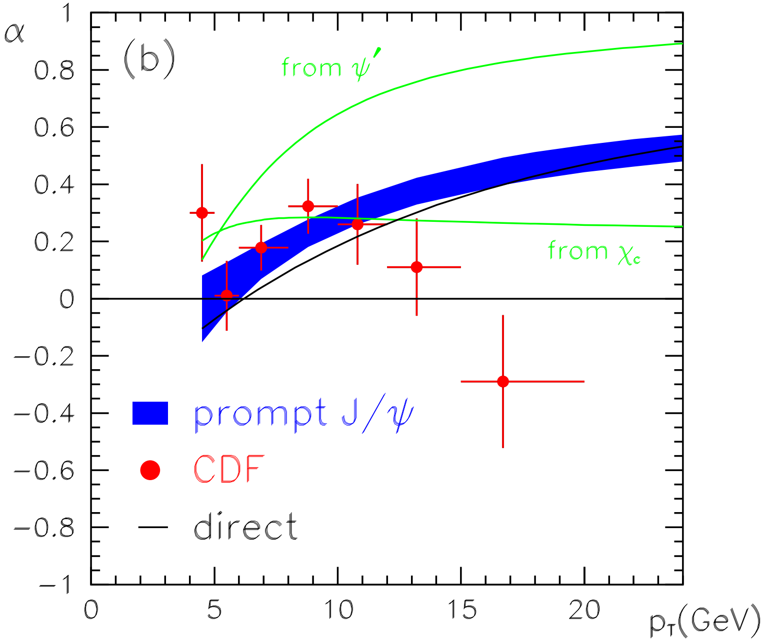
Polarization

Polarization in Leading Order

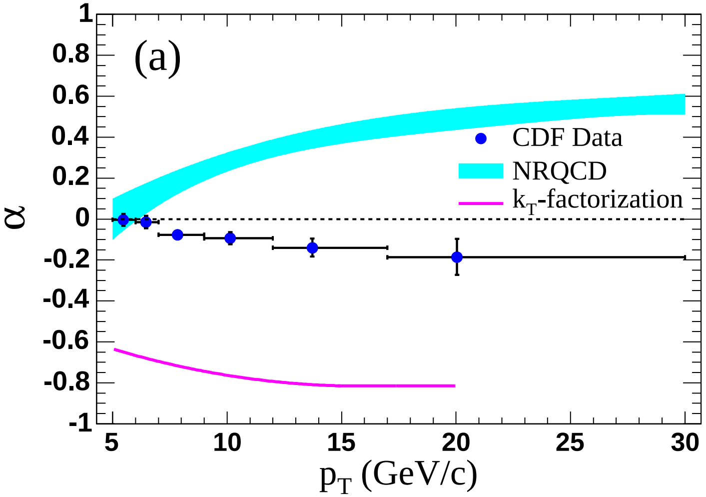
- In LO quarkonium production at large p_T ($p_T \gtrsim 4m_c$ for J/ψ), gluon fragmentation via the color-octet $^3S_1$ channel dominates.
- At large p_T, the gluon is nearly on mass shell, and, so, is transversely polarized.
- In color-octet gluon fragmentation, most of the gluon's polarization is transferred to the quarkonium (Cho, Wise (1994)).
  - Spin-flip interactions are suppressed as v^3.
  - Verified in a lattice calculation of decay matrix elements (GTB, Lee, Sinclair (2005)).

J/ψ Polarization in LO

Run I:



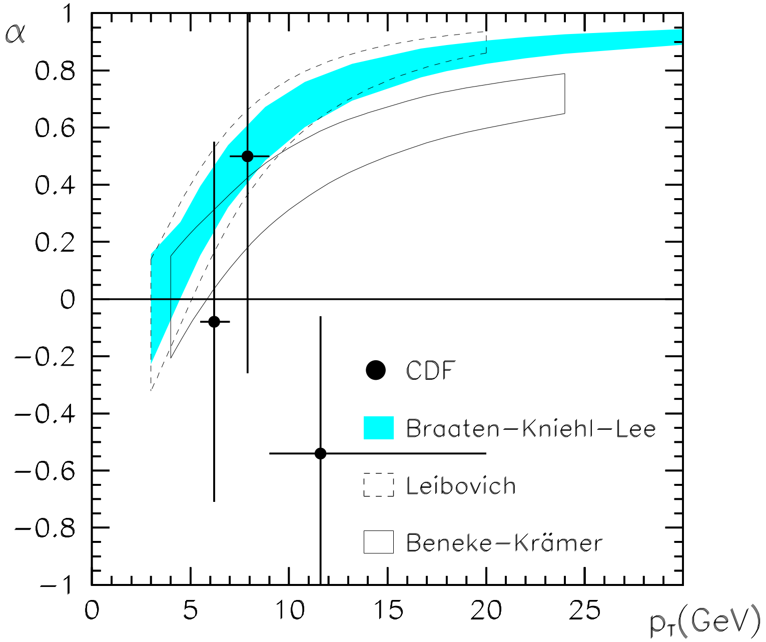
Run II:



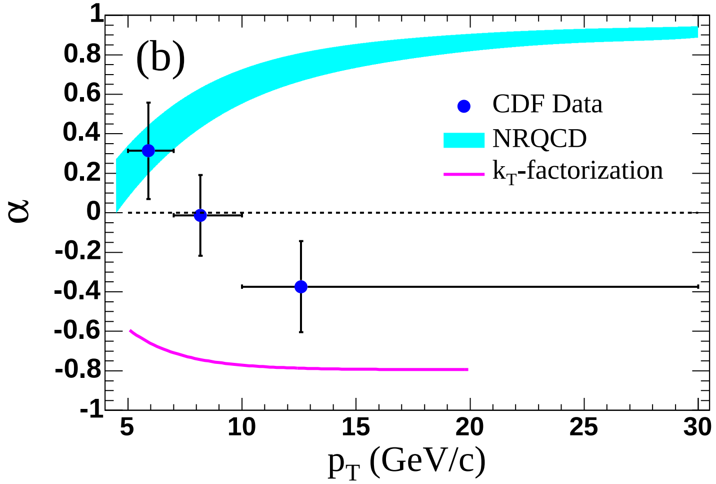
- $d\sigma/d(\cos\theta) \propto 1 + \alpha\cos^2\theta$.
  - $\alpha = 1$ is completely transverse;
  - $\alpha = -1$ is completely longitudinal.
- NRQCD prediction from Braaten, Kniehl, Lee (1999).
  - Feeddown from χ_c states is about 30% of the J/ψ sample and dilutes the polarization.
  - Feeddown from $\psi(2S)$ is about 10% of the J/ψ sample and is largely transversely polarized.
- Run I results are marginally compatible with the NRQCD prediction.
- Run II results are inconsistent with the NRQCD prediction.
- Also inconsistent with the Run I results. CDF was unable to track down the source of the Run I-Run II discrepancy.

$\psi(2S)$ Polarization in LO

Run: I



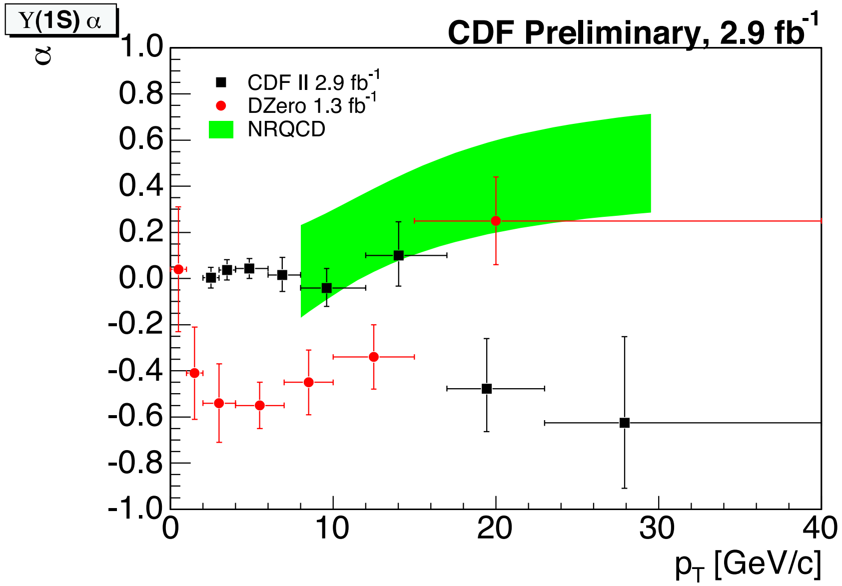
Run: II



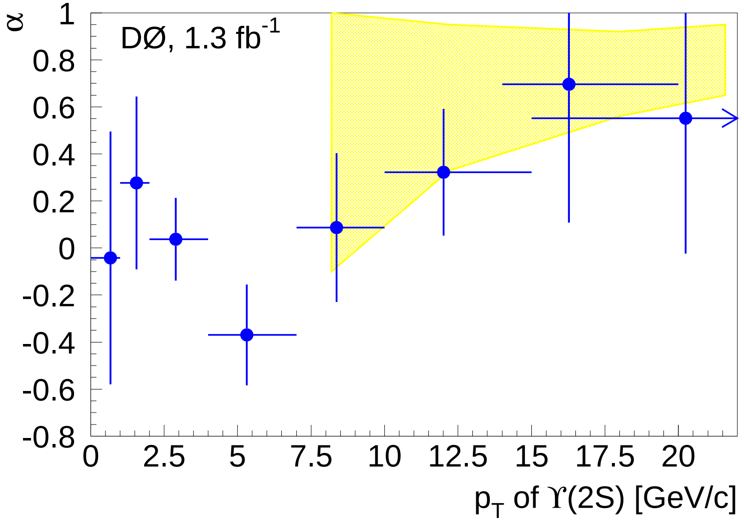
- The Run II data are incompatible with the LO NRQCD prediction.

Υ Polarization in LO

$\Upsilon(1S)$ Polarization:



$\Upsilon(2S)$ Polarization:



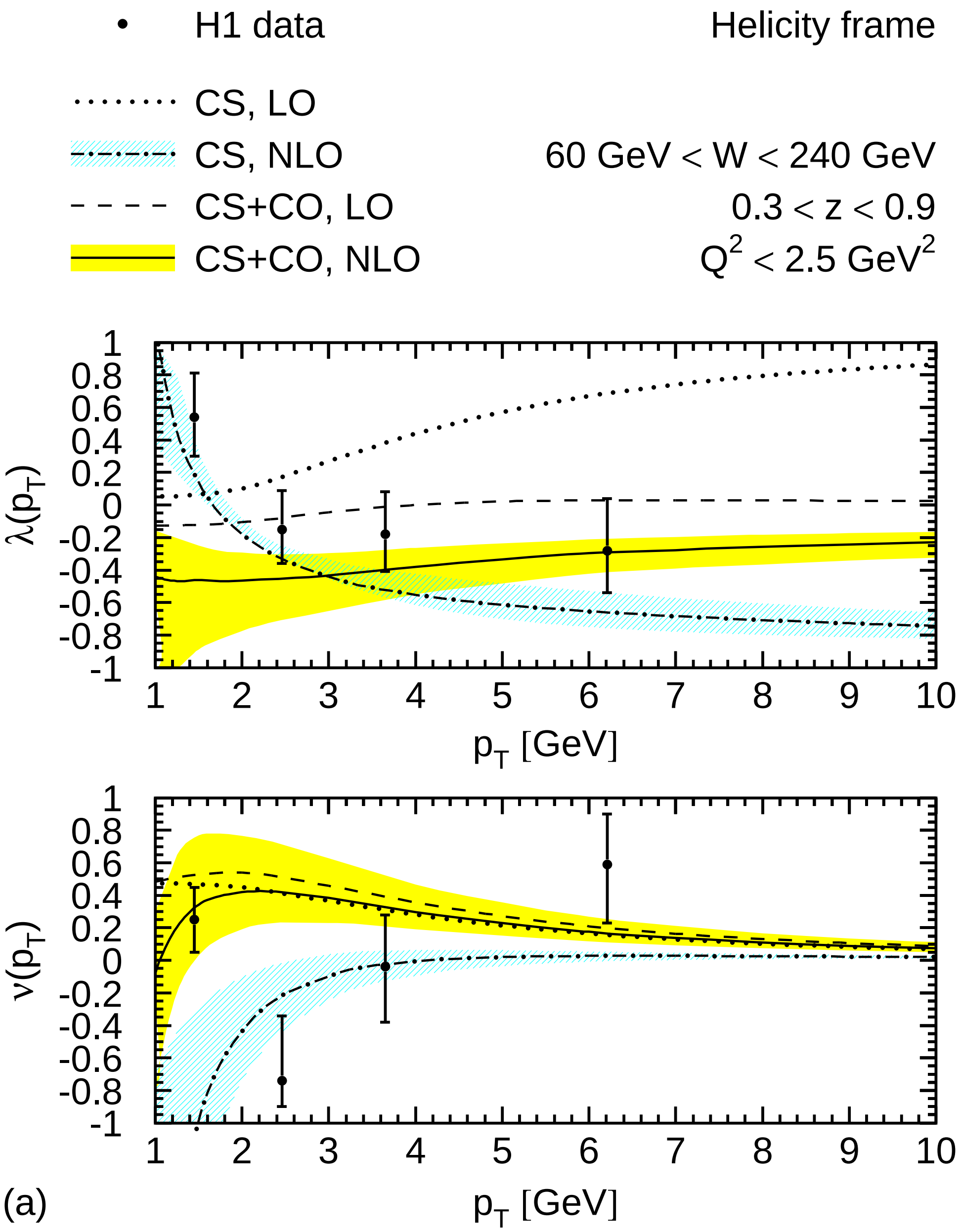
- In the $\Upsilon(1S)$ case, the D0 results (red) are incompatible with the CDF results (black).
- Both the CDF and D0 results are incompatible with the LO NRQCD prediction of Braaten and Lee (2000) (green), but in different regions of p_T.
- In the $\Upsilon(2S)$ case, the theoretical and experimental error bars are too large to make a stringent test.

Polarization in Hadroproduction at NLO

- Color-singlet J/ψ polarization at the Tevatron changes from transverse to longitudinal when NLO corrections are included.
  (Gong and Wang (2008))
- Color-singlet Υ polarization at the Tevatron changes from transverse to longitudinal when NLO and NNLO* corrections are included.
  (Artoisenet, Campbell, Lansberg, Maltoni, Tramontano (2008))
- As expected, the predictions for the J/ψ polarization (Gong, Li, and Wang (2008)) and Υ polarization (Gong, Wang, and Zhang (2010)) are little affected by NLO corrections to the color-octet ${}^1S_0$ and ${}^3S_1$ channels.
- In NLO, there are large corrections to the ${}^3P_J$ color-octet channel at large p_T.
- Hence, in NLO, the ${}^3S_1$ color-octet channel is no longer dominant at large p_T.

- A complete NLO calculation of polarization in hadroproduction does not exist.
  The color-octet ${}^3P_J$ contribution has not been calculated.
- A preliminary result (GTB, Jungil Lee) indicates that gluon fragmentation to a ${}^3P_J$ color-octet $Q\bar{Q}$ state dominates the ${}^3P_J$ channel for $p_T \gtrsim 10$ GeV.
  - This fragmentation contribution is almost 100% transversely polarized.
- Then, at large p_T, the color-octet ${}^3S_1$ and ${}^3P_J$ channels are transversely polarized, while the color-octet ${}^1S_0$ channel is unpolarized.
- The color-octet ${}^1S_0$ NRQCD matrix element is well determined in the Butenschön and Kniehl fit.
  Implies a large transverse polarization at large p_T.
- The color-octet ${}^1S_0$ NRQCD matrix element is determined with $\pm 129\%$ uncertainty in the Ma and Chao fit.
  The polarization could range from largely transverse to slightly longitudinal.
- Fits to high-p_T data involving additional observables are needed in order to determine the NRQCD matrix elements reliably and make a solid prediction for the polarization in hadroproduction.

Polarization in Inelastic J/ψ Photoproduction at HERA



- Complete NLO calculation (Butenschön and Kniehl (2011)) is compatible with the HERA data, but the error bars are large and the p_T range is low.

$e^+e^- \to J/\psi + X(\text{non-}c\bar{c})$

- Belle (2009):

$$\sigma(e^+e^- \to J/\psi + X(\text{non-}c\bar{c})) = 0.43 \pm 0.09 \pm 0.09 \text{ pb}.$$

- NLO calculation (Zhang, Ma, Wang, Chao (2009), Butenschön and Kniehl (2011)):

$$\sigma(e^+e^- \to J/\psi + X(\text{non-}c\bar{c})) = 0.99^{+0.35}_{-0.17} \text{ pb} \qquad (\mu = \sqrt{s}/2).$$

  - NRQCD matrix elements from the Butenschön-Kniehl global fit to KEKB, LEP II, RHIC, HERA, Tevatron, and LHC data.
  - Includes feeddown estimate of 0.29 pb from Zhang, Ma, Wang, Chao (2009).

- The comparison with the Belle data favors the Butenschön-Kniehl value of M_{0,r_0}.

Comments

- Note that BaBar (2001) obtained

$$\sigma(e^+e^- \to J/\psi + X) = 2.52 \pm 0.21 \pm 0.21 \text{ pb},$$

in contrast with the value that can be inferred from the latest Belle (2009) measurements

$$\sigma(e^+e^- \to J/\psi + X) = \sigma(e^+e^- \to J/\psi + c\bar{c} + X) + \sigma(e^+e^- \to J/\psi + X(\text{non-}c\bar{c})) = 1.17 \pm 0.12^{+0.13}_{-0.12} \text{ pb}.$$

- It is important for BaBar to check the Belle results for $\sigma(e^+e^- \to J/\psi + X(\text{non-}c\bar{c}))$.

Wish List

- Theory
  - Proof or disproof of NRQCD factorization.
  - Calculations of all of the leading (in $1/p_T^2$) and first subleading fragmentation contributions to quarkonium production at NLO and with resummation of logarithms of p_T^2/m_c^2.
  - Complete NLO calculation of quarkonium polarization in hadroproduction.
- Experiment
  - Measurement of $d\sigma/dp_T dy$ at large p_T with high statistics for direct production of J/ψ, $\psi(2S)$, χ_{cJ}, $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$.
  - Measurement of quarkonium polarization at large p_T with high statistics for direct production of J/ψ, $\psi(2S)$, χ_{cJ}, $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$.
  - Measurement of all three polarization parameters in the helicity, Collins-Soper, and target frames.