Infrared Finite Scattering Amplitudes for Massless Gauge Theories

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Outline

- Defining a physical observable
- The infrared problem
- Old solution in perturbation theory
- New solution - The Asymptotic Interaction Picture
- Progress and summary

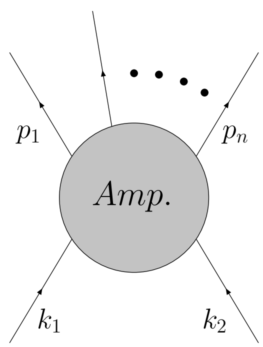
Physical Obervables

- The goal of theoretical physics, at least phenomenology, is to make predictions for physical observables

$$\hat{O} \;=\; \int dLIPS\,(k_i,\cdots)\,|\mathcal{A}|^2\,\mathcal{J}\,(k_i,\cdots)$$

- The S-matrix defines the interactions of a particular theory, mapping the initial states onto the final states

$$\mathcal{A} = \langle \Psi_{out}|S|\Psi_{in}\rangle = \sqrt{Z}^{\,n}$$



The Infrared Problem

- Consider Coulomb scattering of a non-relativistic particle

$$H = \frac{\vec{p}^{\,2}}{2m} + \frac{g}{|\vec{r}\,|} = H_0 + V$$

- The asymptotic behaviour of $V(t)$ is given by,

$$\lim_{t \to \pm\infty} V(t) = \frac{mg}{|\vec{p}\,||t|} + O\left(|t|^{-2}\right)$$

- Asymptotic dynamics not described by H_0 alone, long ranged force cannot be neglected at infinity (photon is massless)

Infrared Finite Theories

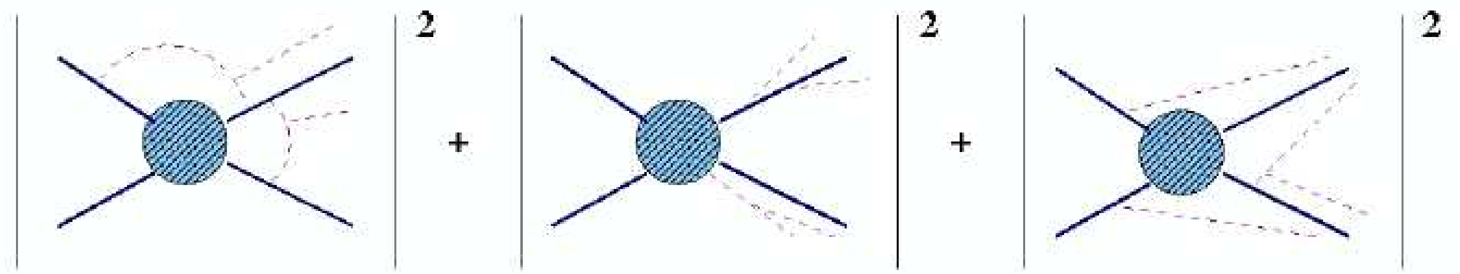
- For this simple system we can solve the asymptotic schrödinger equation

$$
\begin{aligned}
|\Psi_S(t)\rangle &= U_A(t,t_0)|\Psi_S(t_0)\rangle \\
&= e^{-iH_0 t}\exp\left\{-i\frac{mg}{|\vec{p}|}\ln\frac{t}{t_0}\right\}|\Psi_S(t_0)\rangle
\end{aligned}
$$

- For these states the S matrix is well defined
- QCD: Asymptotic schrödinger equation cannot be solved!
- Use free states $\Rightarrow$ S matrix becomes ill-defined $\Rightarrow$ IR divergences
- Note: Massive QED can be solved asymptotically, P.P.Kulish & L.D.Faddeev (1970)

The Cross-Section Method

- In massless QED/QCD the S matrix between free states is rendered finite via dimensional regularisation ($D = 4 - 2\epsilon$)
- Then ϵ may be taken to zero at the cross-section level after summing over physically indistinguishable (unresolved) processes to form infrared safe observables, F.Bloch & A.Nordsieck (1937)



Is There A Better Solution?

- Cancellation of IR poles is technically very hard at NNLO!
- Initial state radiation introduces futher problems - collinear singularities are factorised into PDFs
- Current theory not aesthetically pleasing - pick a better, finite basis
- IR finite amplitudes would allow automisation - merging fixed order calculations with parton showers, Z.Nagy & D.E.Soper (2003)

New Solution - AIP

- New split of Lagrangian used to compute correlation functions

$$\mathcal{L} = \mathcal{L}_{asym} + \mathcal{L}_{hard} = \mathcal{L}_0 + \mathcal{L}_{IR} + \mathcal{L}_{hard}$$

- S_A is defined through this split and we obtain correlation functions in the asymptotic interaction picture, H.F.Contopanagos and M.B.Einhorn (1992)
- Unfortunately we cannot solve for the asymptotic states $|\Xi\rangle$, we are forced to relate them perturbatively to free states $|\phi\rangle$
- This relation requires further theoretical work

AIP Feynman rules

- Asymptotic propagators calculated using free states

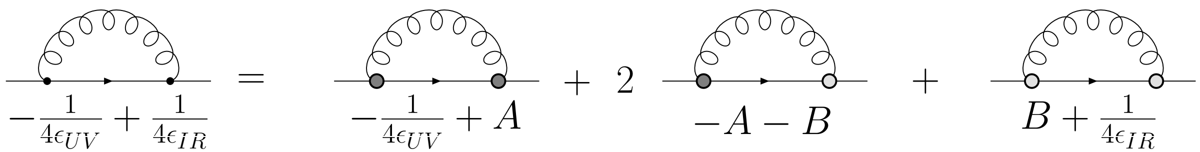
$$S_\Xi = S_0 + S_0\Sigma_\Xi S_0 + S_0\Sigma_\Xi S_0\Sigma_\Xi S_0 + \cdots$$
$$= \frac{S_0}{1 - S_0\Sigma_\Xi}$$

- The 3-point vertex is modified by a hard factor preventing the flow of soft momenta through it

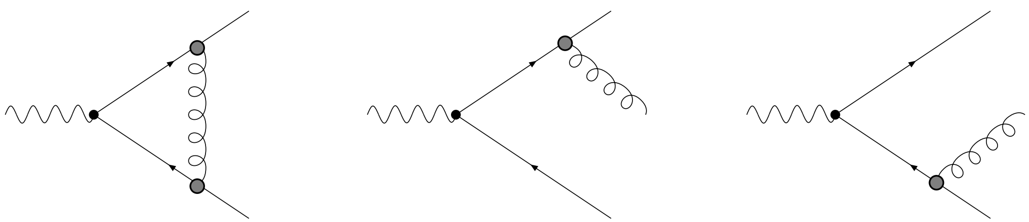
$$ig\gamma^\mu t^a h_h = ig\gamma^\mu t^a \frac{p_1^2 + p_2^2 + (p_1+p_2)^2}{p_1^2 + p_2^2 + (p_1+p_2)^2 + \Delta}$$

AIP: $e^+e^- \to jets$ at $O(\alpha_s)$

- Using AIP rules compute hard wavefunction renomalization factor



- Usually zero due to a cancellation between UV and IR poles - take only the first term on the RHS for our Z_2
- Now calculate process using hard vetices only



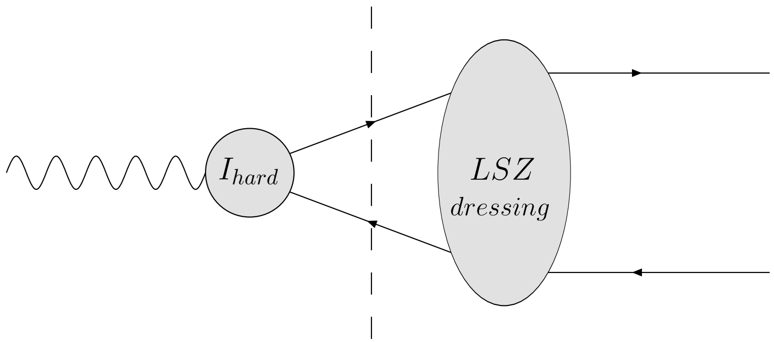
AIP: $e^+e^- \to jets$ at $O(\alpha_s)$

- In addition to standard procedure we include soft correction terms
- Soft interaction vertices required to ‘dress’ the free external states

$$|\Xi\rangle = \Omega_\Delta |\Phi\rangle$$

I_{hard} - hard vertices

LSZ - soft dressing for external states



- Soft diagrams represent unresolved inclusive processes $\Rightarrow$ may sum over all cuts
- Thanks to Cutkosky corrections will be finite

IRF amplitudes

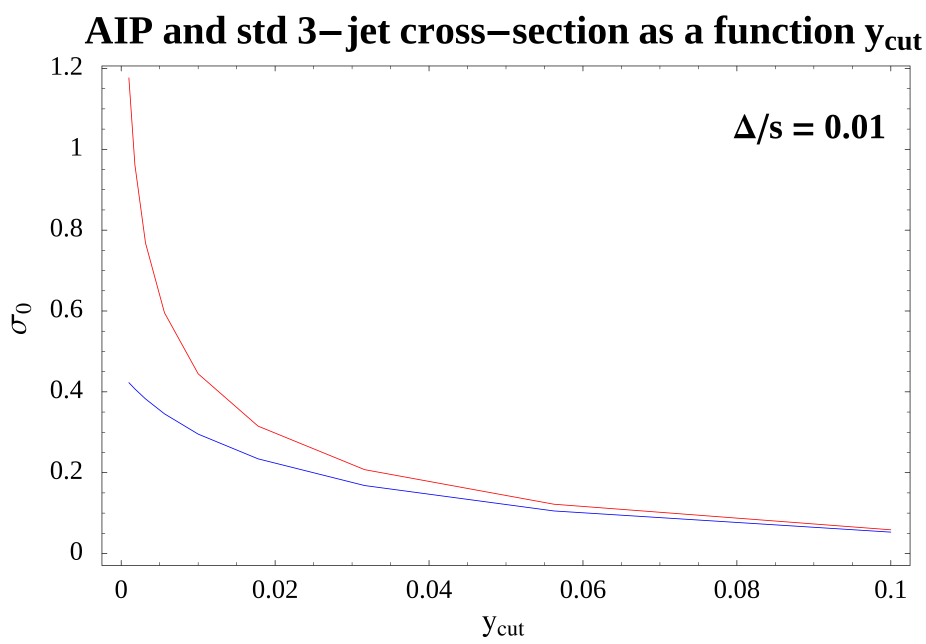
- Explicitly the virtual and real amplitudes are given by

$$i\mathcal{M}^v = \frac{i\alpha\alpha_s}{s}\langle p_1|\gamma^\mu|p_2\rangle\langle q_1|\gamma_\mu|q_2\rangle\left(\frac{-s}{\mu^2}\right)^{-\epsilon}\left(\frac{1}{\epsilon} - 2\ln^2\left(\frac{\Delta}{s}\right) - 4\ln\left(\frac{\Delta}{s}\right) + finite\right)$$

$$\begin{aligned} i\mathcal{M}^r = \quad & \frac{4\pi\alpha g}{s^2}\langle q_1|\gamma_\mu|q_2\rangle \\ & \left\{ -\frac{\langle p_1|\gamma^\mu \not{p}_3 \not{\varepsilon}^*(p_3)|p_2\rangle}{(y_{23}+\frac{\Delta}{s})} - \frac{2\langle p_1|\gamma^\mu|p_2\rangle p_2\cdot\varepsilon^*(p_3)}{(y_{23}+\frac{\Delta}{s})} \right. \\ & \left. \frac{\langle p_1|\not{\varepsilon}^*(p_3)\not{p}_3\gamma^\mu|p_2\rangle}{(y_{13}+\frac{\Delta}{s})} + \frac{2\langle p_1|\gamma^\mu|p_2\rangle p_1\cdot\varepsilon^*(p_3)}{(y_{13}+\frac{\Delta}{s})} \right\} \end{aligned}$$

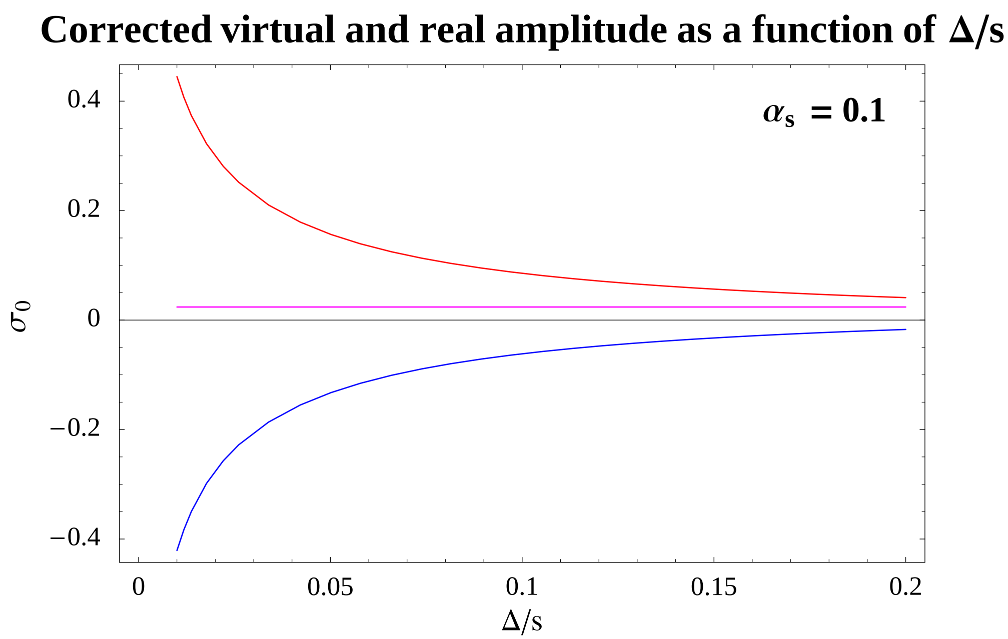
The 3-jet cross-section

- Using a simple jet definition: $\mathcal{S}_3 = \Theta(y_{12} - y_{cut})\Theta(y_{13} - y_{cut})\Theta(y_{23} - y_{cut})$ the real contribution (blue) can be compared to the standard result (red)



Total cross-section

- Hard real + hard virtual + soft corrections reproduce the standard $O(\alpha_s)$ correction as we would hope



Future work

- Does a systematic method exist for computing the soft correction terms in a manifestly finite way?
- Can this be understood in terms of the asymptotic LSZ formalism?
- Develop theory and compute more observables!
- Ultimately we want a completely numerical approach (Z.Nagy & D.E.Soper 2003)