IoP 2006

Lattice QCD meets experiment

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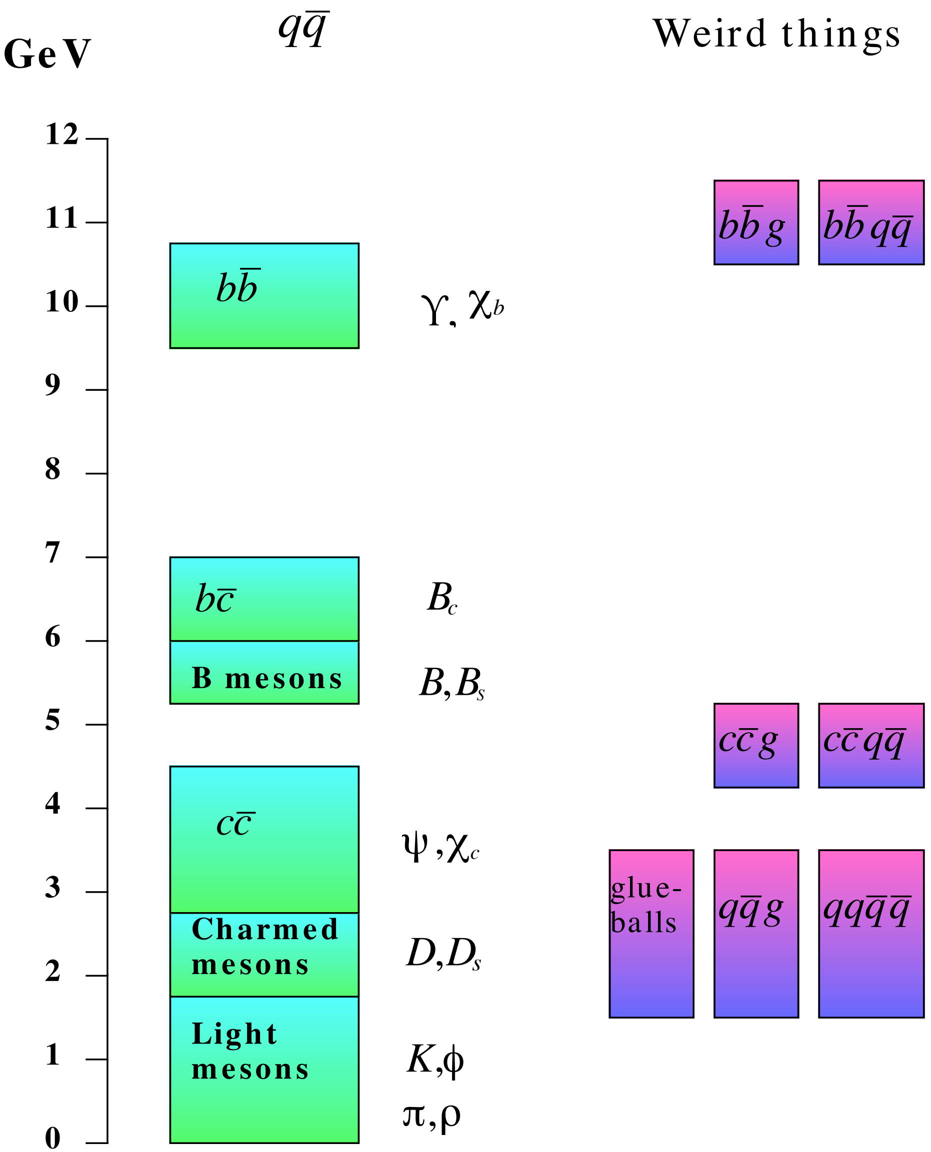
Strong intns/QCD a key part of the Standard Model.

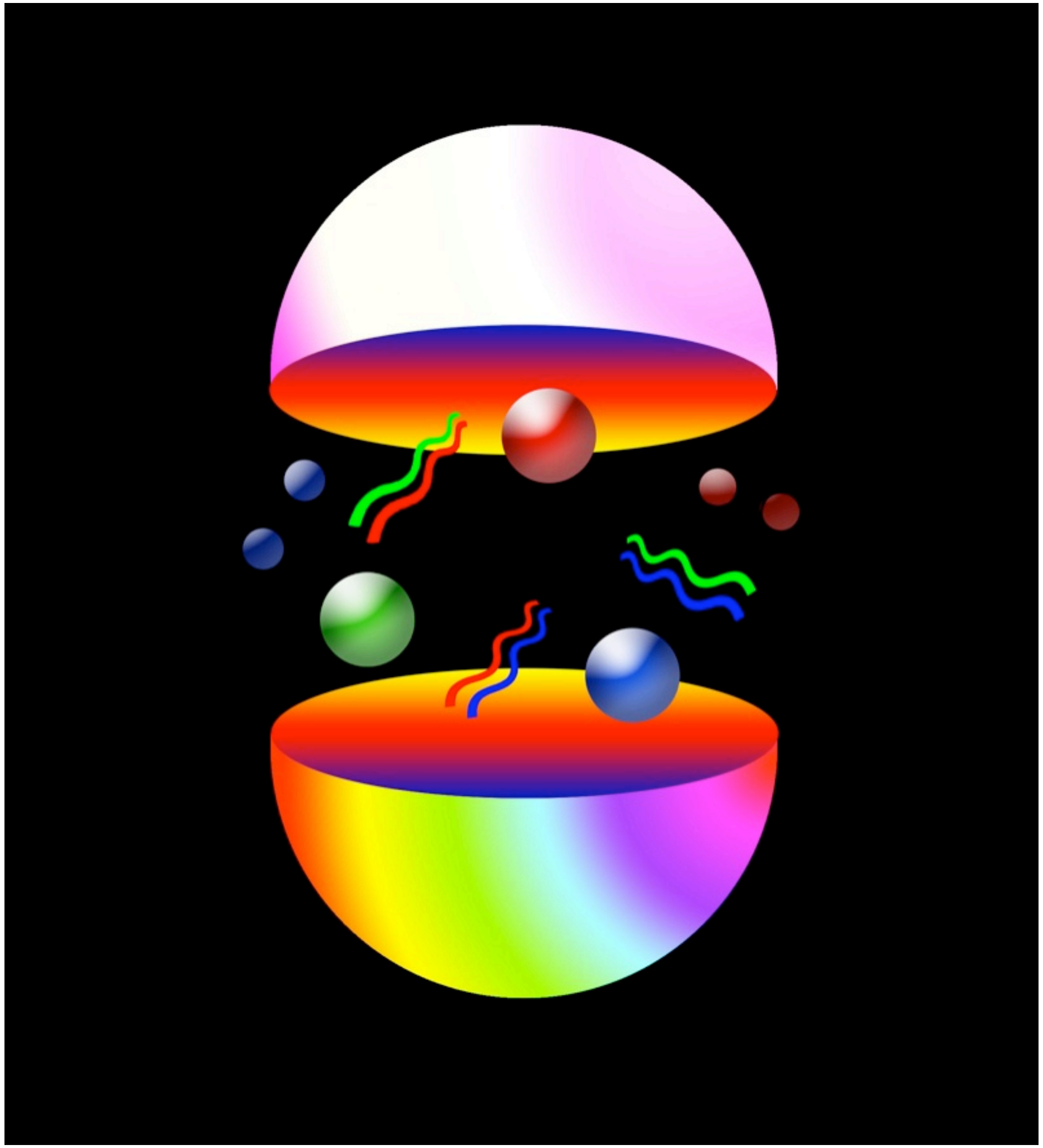


The Meson Spectrum

Confinement
Rich spectrum of states - masses and some properties calculable in QCD if we can solve theory.
All info. about quarks indirect - from calcs on hadrons.

‘Weird things’ not yet unambiguously seen.
Theory predcns?

Lattice QCD

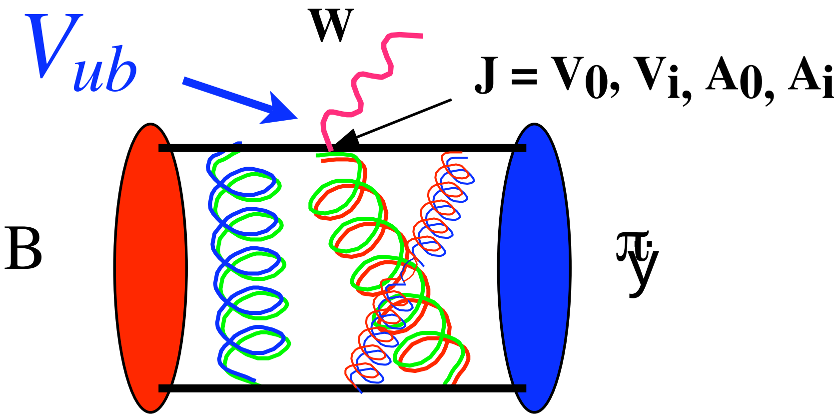


- Solve QCD by numerical evaluation of path integral using a space-time lattice

- Can calculate stable hadron masses and simple matrix elements (with errors)

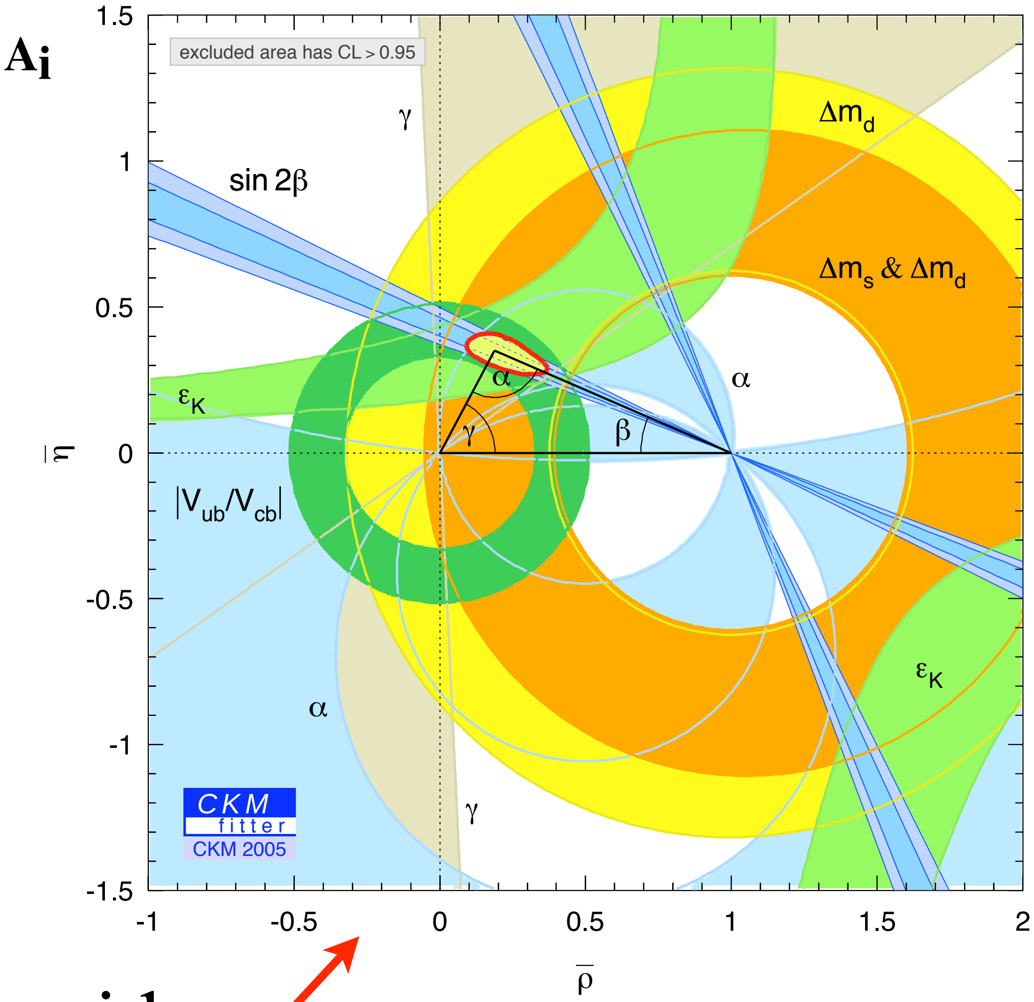
- How good do we need these calcns to be?

Determination of CKM matrix



weak decay of quark inside a hadron - calculate in lattice QCD

expt = CKM x lattice



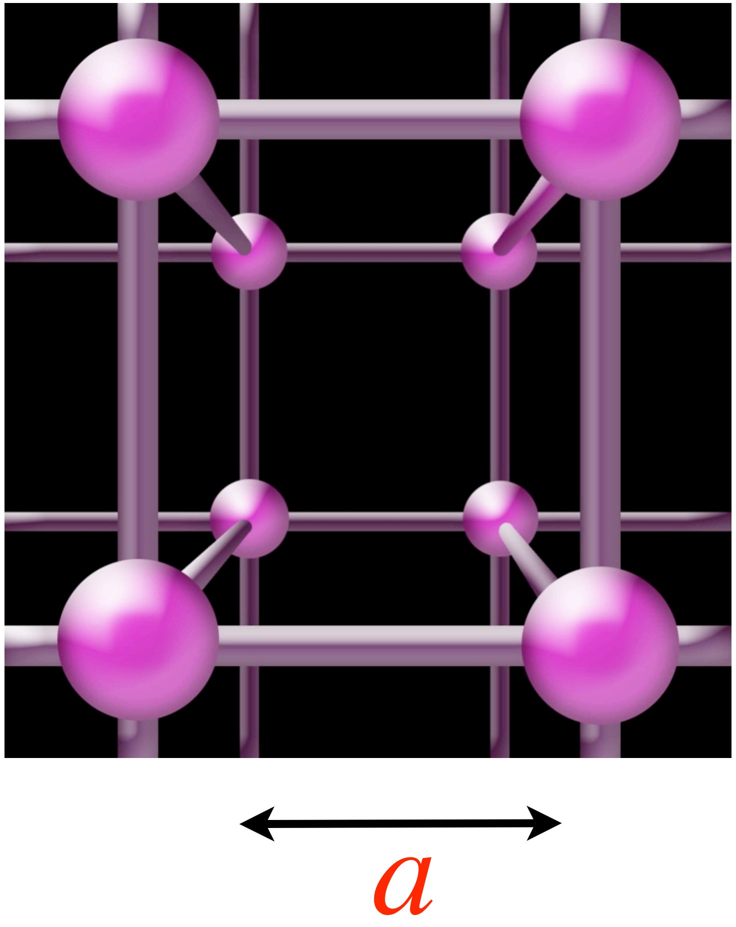
unitarity triangle sides

Precision (3%) lattice QCD needed

Good news : this now looks possible

Key advances during 1990s:

- Accurate discretisations of QCD using systematic ‘improvement’ programme.
- Effective field theories for heavy b and c quarks, also systematically improved.
- Understanding lattice QCD perturbation theory to match short-distance lattice physics to continuum.



Cost grows as a^{-7} so huge savings if can increase a and still have a small error.

Take-home message

- There has been a revolution in lattice QCD since 2003
- Quenched approximation (ignores sea quarks) is dead - stop quoting results from it
- Lattice QCD now delivering fully unquenched results: hadron masses that agree with expt; precise parameters of QCD. Matrix elements relevant to CKM physics on the way.

Handling light quarks is a big headache

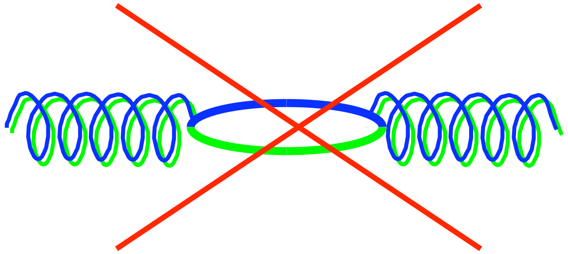
$$L_{q,QCD} = \overline{\psi}(\gamma \cdot D + m)\psi \equiv \overline{\psi} M \psi$$

For valence quarks, need to calculate M^{-1}

For sea quarks need to include $det(M)$ in making gluon configs

Very costly as $m_q \rightarrow 0$

Key problem is to simulate realistic u, d and s sea quarks

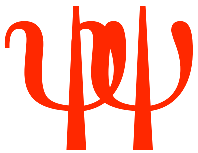


Early calculations:

Quenched Approximation - omitted sea quarks or inc 2 flavours (u, d no s) with masses 10-20 times too big.

This is wrong. Need to unquench with real s and light u/d.

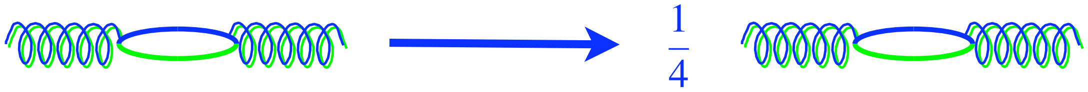
Issues for sea light quarks are speed, disc. errors and chiral symmetry. Doubling problem → controversy over how to handle them



Improved staggered quarks do not solve doubling problem

4 ‘tastes’ of quark instead of 1 - if all same, ‘divide by 4’ :

$det(M) \rightarrow det^{1/4}(M)$



‘Taste-changing’ interactions mess this up, but vanish as a^2

Advantage of this formalism is speed - allows us to do calculations on existing computers with light u, d sea quarks. Also good disc. errors and (part of) chiral symm.

What is now possible for lattice QCD?

MILC collaboration gluon configurations have:

- 2+1 flavors of sea quarks, down to $m_{u/d} = m_s/10$.
- Many $m_{u/d}$ values; 2 m_s values
- 3 values of lattice spacing: 0.18fm, 0.12fm and 0.09fm
- Spatial volume exceeding $(2.5fm)^3$

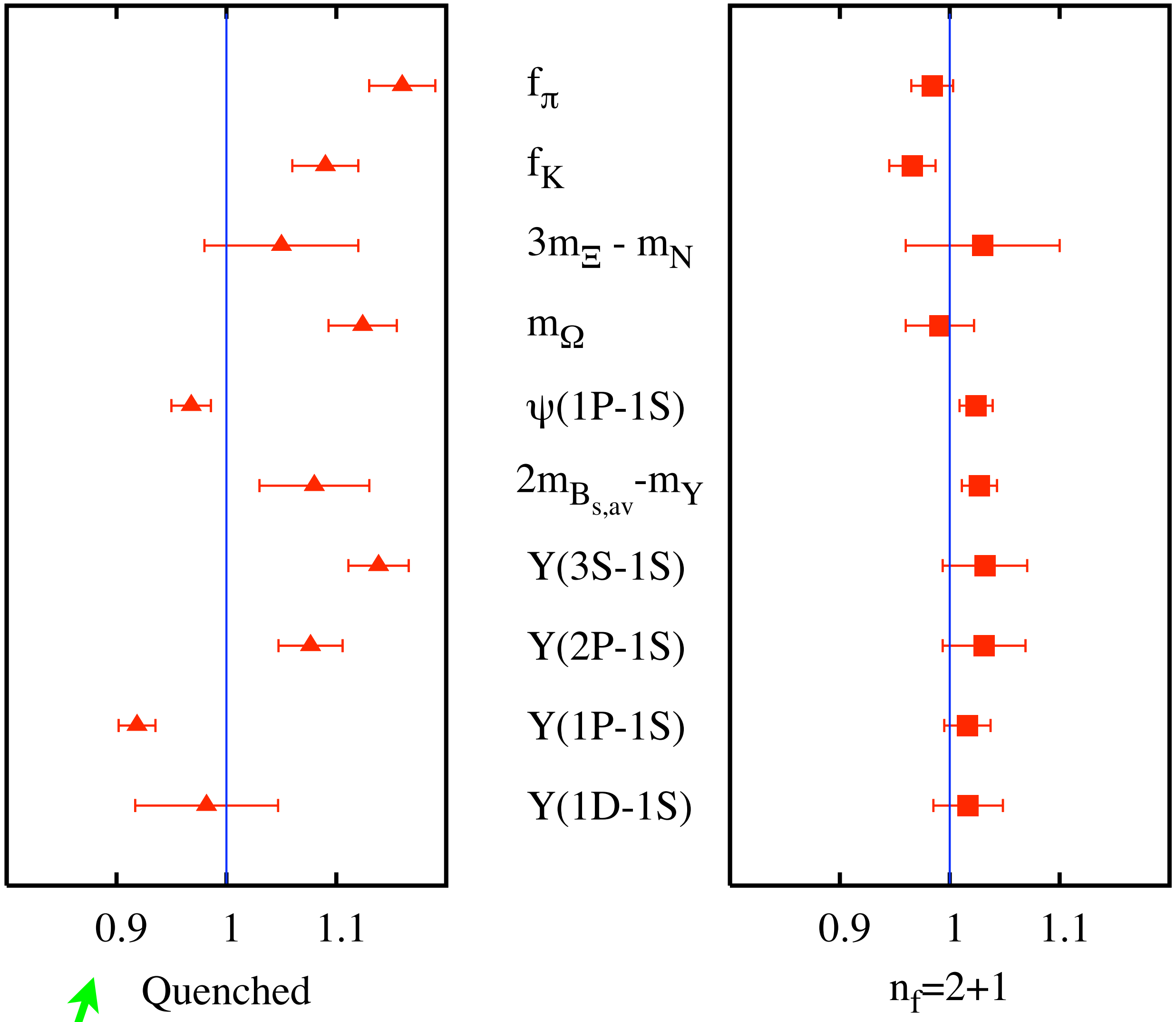
QCD has 5 parameters : 4 quark masses and a bare coupling. Must fix these using ‘gold-plated’ (i.e. stable) hadron masses. (i.e *not* ρ, K^* etc.)

HPQCD/MILC/FNAL/UKQCD analysis of MILC configs. Fix: $a : M_{\Upsilon'} - M_{\Upsilon}$

$m_{u/d} : M_\pi \quad m_s : M_K \quad m_c : M_{D_s} \quad m_b : M_\Upsilon$

Calculate other gold-plated hadron masses as test. Results that follow from this analysis (but not all configs).

2005 Updated summary of results



Results including u,d and s sea quarks agree with experiment *across the board* -from light to heavy hadrons. Parameters of QCD are *unambiguous*

Quenched results are both *wrong* and *ambiguous*

Davies et al, hep-lat/0304004,
Aubin et al, hep-lat/0407028,
Toussaint+Davies, hep-lat/0409129,
Gray et al, hep-lat/0507013,

Lattice QCD prediction! : mass of B_c meson

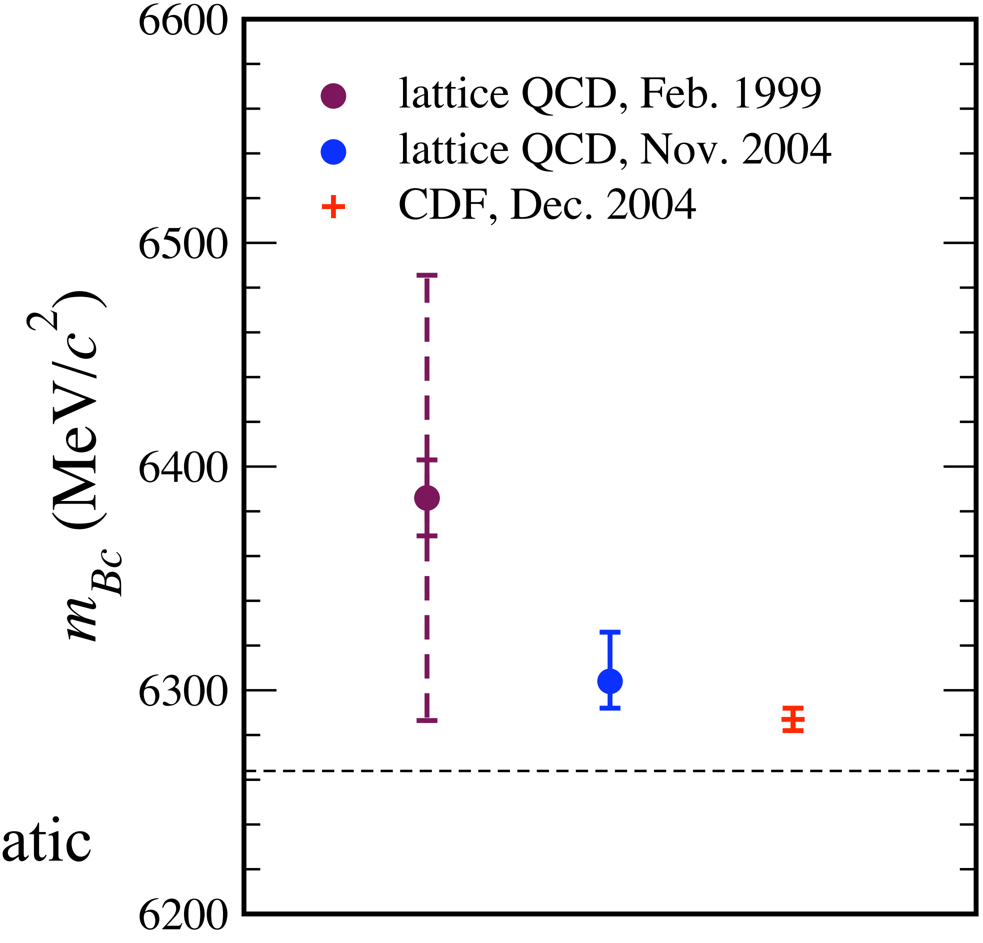
Using NRQCD formalism for *b* and FNAL formalism for *c*, calculate splitting

$$M_{B_c} - \frac{1}{2}(M_\Upsilon + M_{J/\psi})$$

Result:

$$M_{B_c} = 6.304(20)\mathrm{GeV}$$

Lattice systematic errors

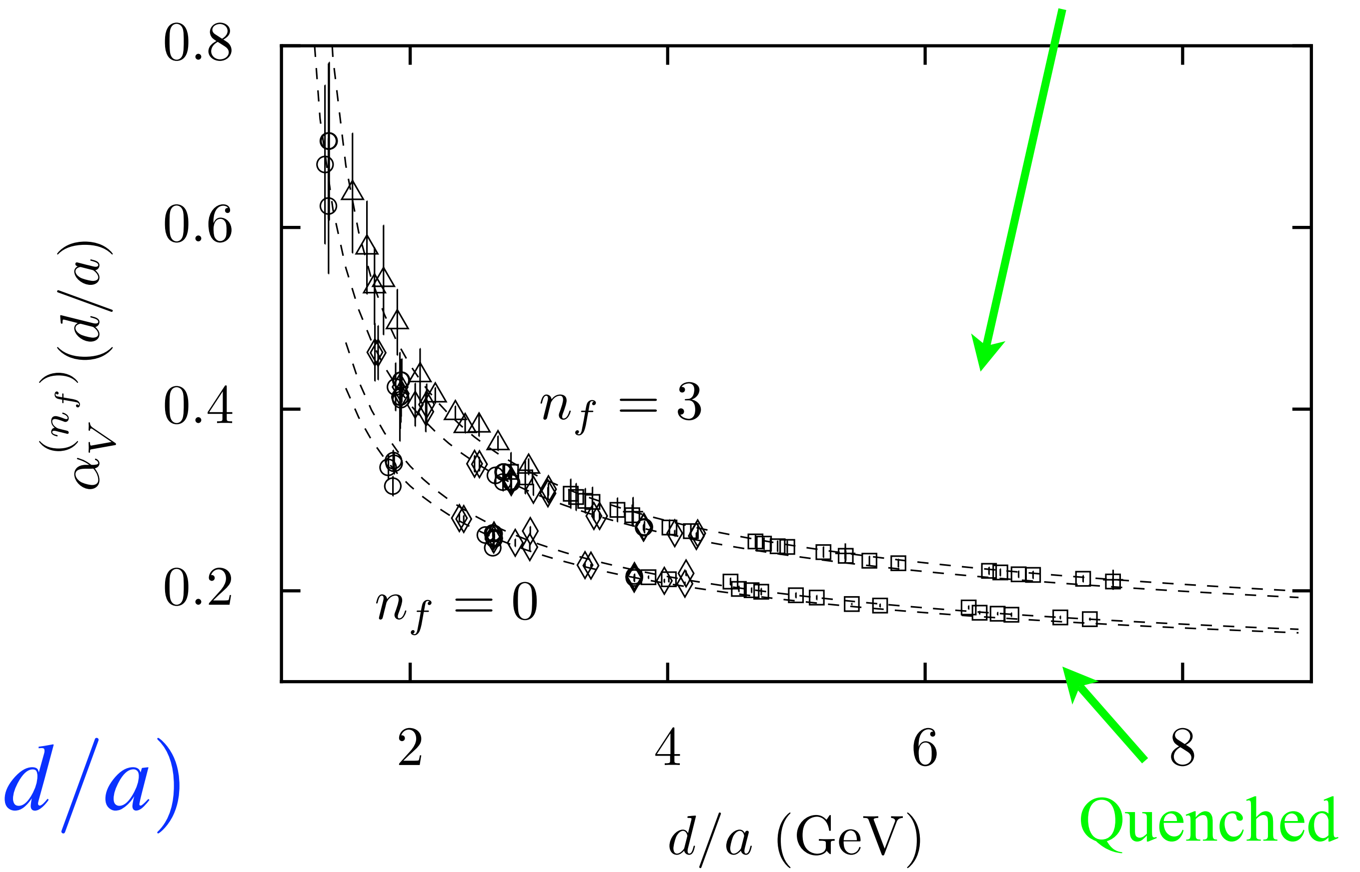


CDF (2005): 6.287(5) GeV

HPQCD/FNAL /UKQCD Allison et al, hep-lat/0411027; CDF, Acosta et al, hep-ex/0505076.

Determining Parameters of QCD : α_s

Combine 3-loop pert. th. for 28 different quantities (mainly Wilson loops) with numerical calc. on lattice



$$W_{latt} = \sum_{n=1}^{3} c_n \alpha_V^n(d/a)$$

Results at 3 values of *a* allows estimates of 4-loop terms. *d/a* is BLM scale - differs for each W, so see running of α_s .

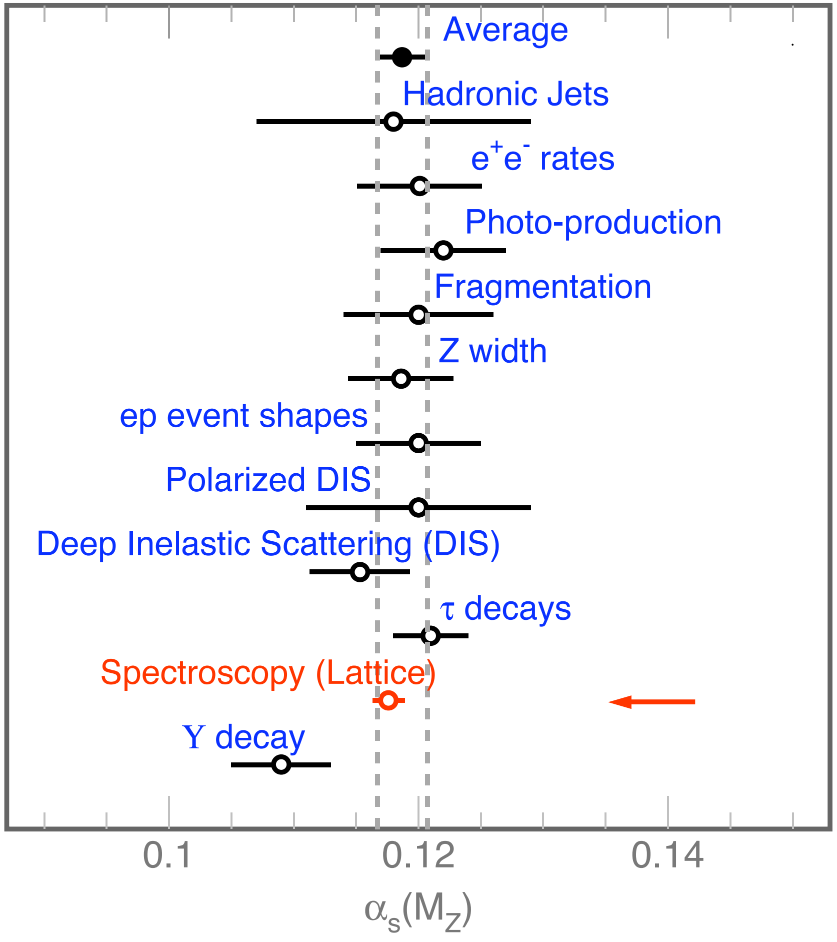
HPQCD, Mason et al, hep-lat/0503005

Convert to $\alpha^{(3)}_{\overline{MS}}$ and run to M_Z

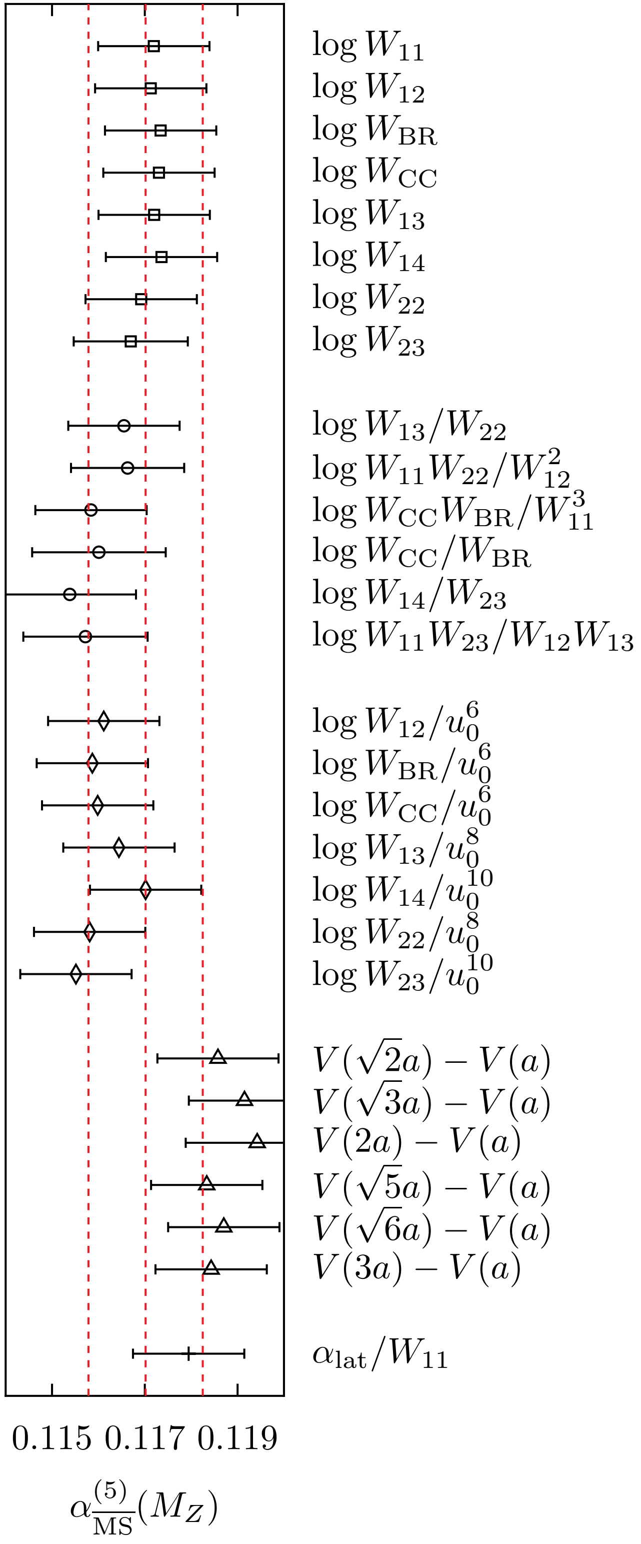
$$\alpha^{(5)}_{\overline{MS}}(M_Z) = 0.1170(12)$$

Mainly c_5 (8) and a (7)

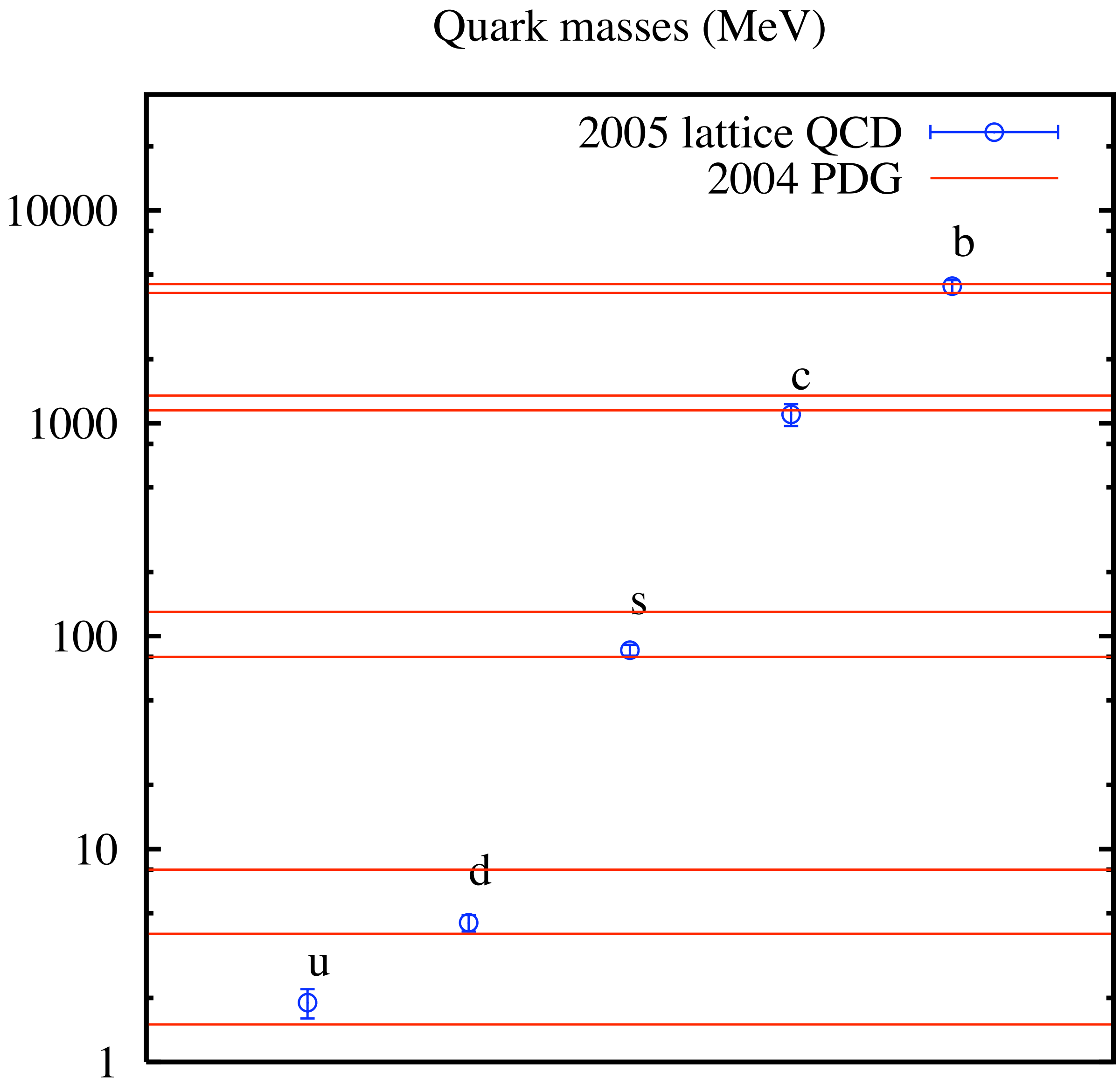
2004 PDG = 0.1187(20)



NEW- most accurate to date



Determining parameters of QCD: m_q



Use pert. th. to convert lattice bare mass to $\overline{MS}$

New b/c masses (GeV):

$$\overline{m}_b(\overline{m}_b) = 4.4(3)$$

$$\overline{m}_c(\overline{m}_c) = 1.10(13)$$

HPQCD, Gray et al, hep-lat/0507013,
Nobes et al, LAT05;
N.B. 3-loop nf=2 result mb = 4.21(7) from
Direnzo and Scorzato, hep-lat/0408015

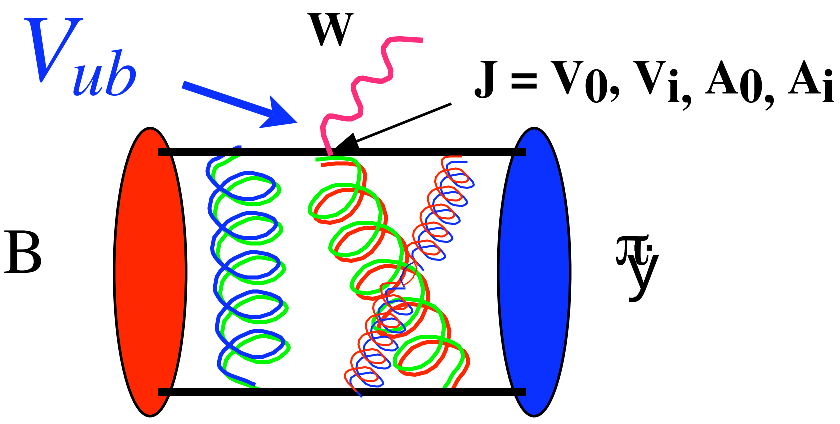
Light quark masses now have 2-loop matching!

$\overline{m}_s = 86(5); \overline{m}_u = 1.9(3); \overline{m}_d = 4.5(4)$ in MeV at 2 GeV

HPQCD/MILC, Aubin et al, hep-lat/0405022; HPQCD, Mason et al, hep-lat/0510000

Weak decay matrix elements for CKM matrix

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \pi \rightarrow l\nu & K \rightarrow l\nu & B \rightarrow \pi l\nu \\ & K \rightarrow \pi l\nu & \\ V_{cd} & V_{cs} & V_{cb} \\ D \rightarrow l\nu & D_s \rightarrow l\nu & B \rightarrow Dl\nu \\ D \rightarrow \pi l\nu & D \rightarrow Kl\nu & \\ V_{td} & V_{ts} & V_{tb} \\ \langle B_d|\overline{B}_d\rangle & \langle B_s|\overline{B}_s\rangle & \end{pmatrix}$$



Lattice QCD calc. gives m. e. of weak current between *hadrons*. CKM and lepton kinematics outside calc.

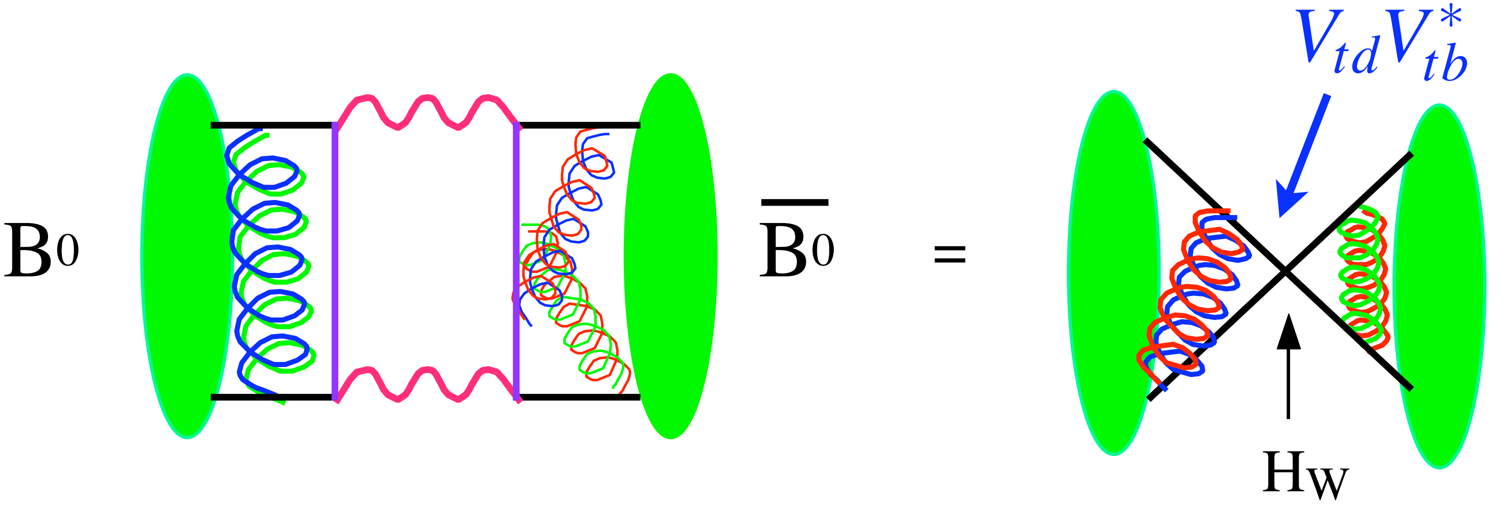
Lattice QCD can calculate decay matrix elements for at most one gold-plated hadron in final state.

Possible for almost every element of CKM matrix.

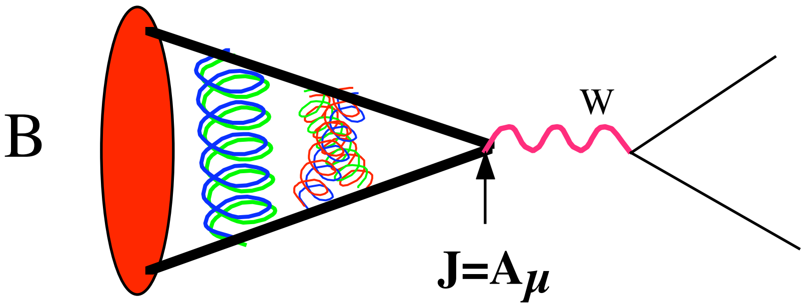
* Need multiple cross-checks of lattice calcs in different systems e.g. Υ, B, D, ψ etc

B meson decay constants and oscillation rates

B/B$_s$ oscillation rate determined by box diagram. Calculate in lattice QCD as 4-q operator.

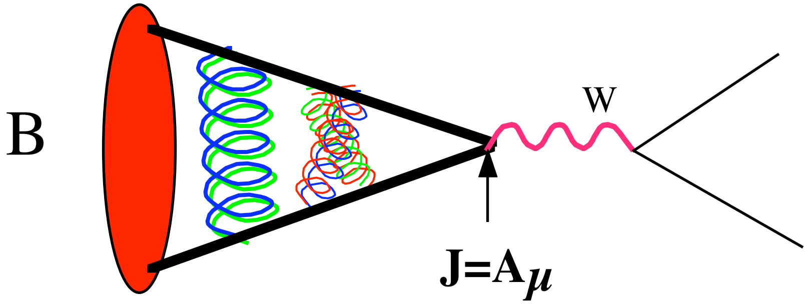


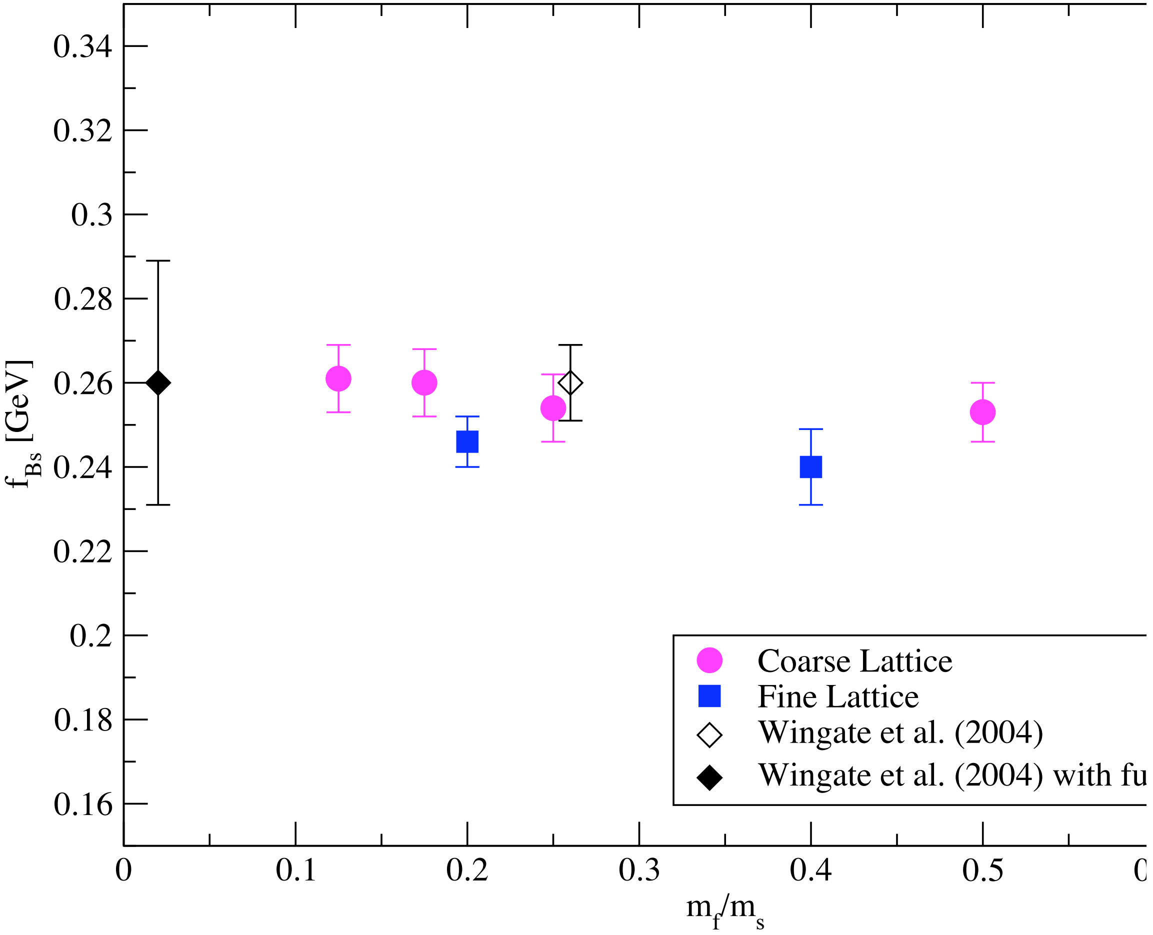
Parameterise as $f_B^2 B_B$ where f_B is decay constant.
Calculate this first.



New determination of f_B, f_{B_s}

NRQCD b quarks





Pert. matching to contnm reqd. Complete thru' $O(\alpha_s)$

$$f_{B_s} = 260(29)\mathrm{MeV}$$

Error mainly from missing α_s^2

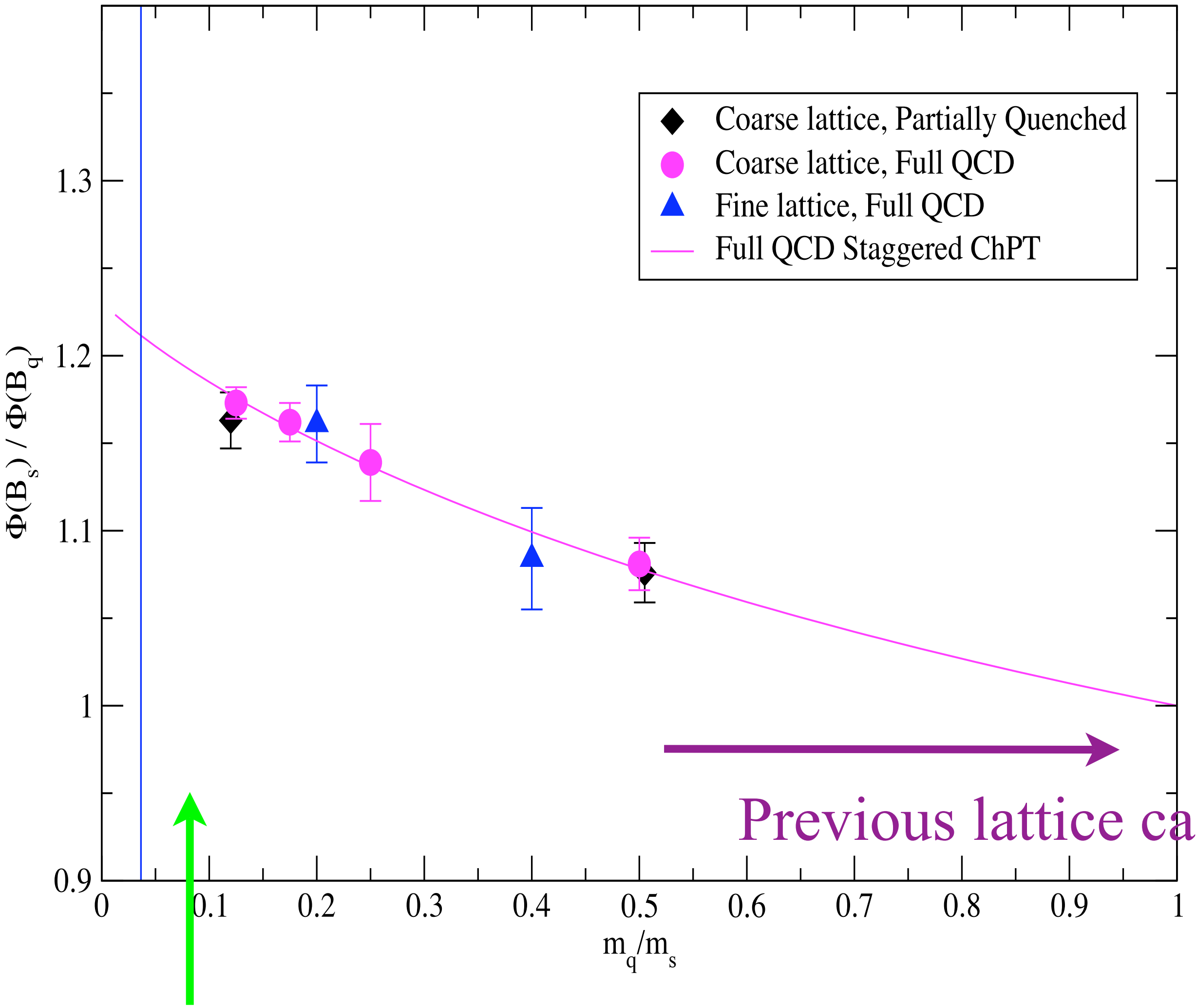
Dependence on $m_{u/d}$ mild

HPQCD, Wingate et al, hep-lat/0311130, Gray et al, hep-lat/0507015;

$$f_{B_s}\sqrt{B_{B_s}}(m_b) = 0.244(39)\mathrm{GeV}$$

PRELIMINARY - no 1/mb corrns yet

New determination of f_B, f_{B_s}



Previous lattice calcs

Chiral log with coeff

$1 + 3g^2_{B^*B\pi}$

$$f_B = 216(22)\,\mathrm{MeV}$$

9% error from pert matching

$$\frac{f_{B_s}}{f_B} = 1.20(3)$$

pert. error cancels. 3% from extrap. in u/d mass

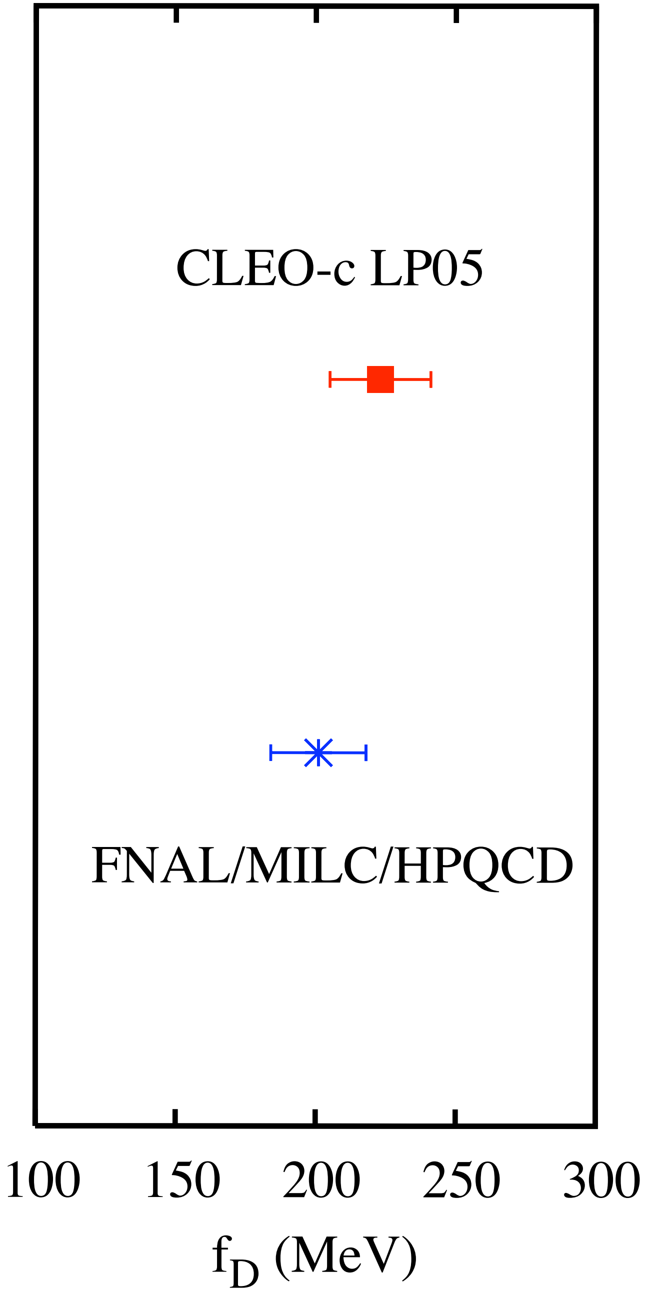
HPQCD, Gray et al, hep-lat/0507015;

Exptl result for $\mathrm{Br}(B \to \tau\nu)$ possible from Belle/BaBar

CLEO-c experiment at Cornell looking at weak decays of D mesons as a test of QCD calculations for B mesons

c quarks lighter than b quarks so use different methods for discretising c quark action, but analysis of errors same

Important checks using D mesons FNAL c quarks



NEW lattice result

7% from disc. errors

$$f_{D^+} = 201(17)\text{MeV}$$

NEW exptl result (CLEO-c)

$$\text{Br}(D^+ \to \mu^+\nu) = 4.45(76) \times 10^{-4}$$

Assume $V_{cd} = V_{us}$ gives

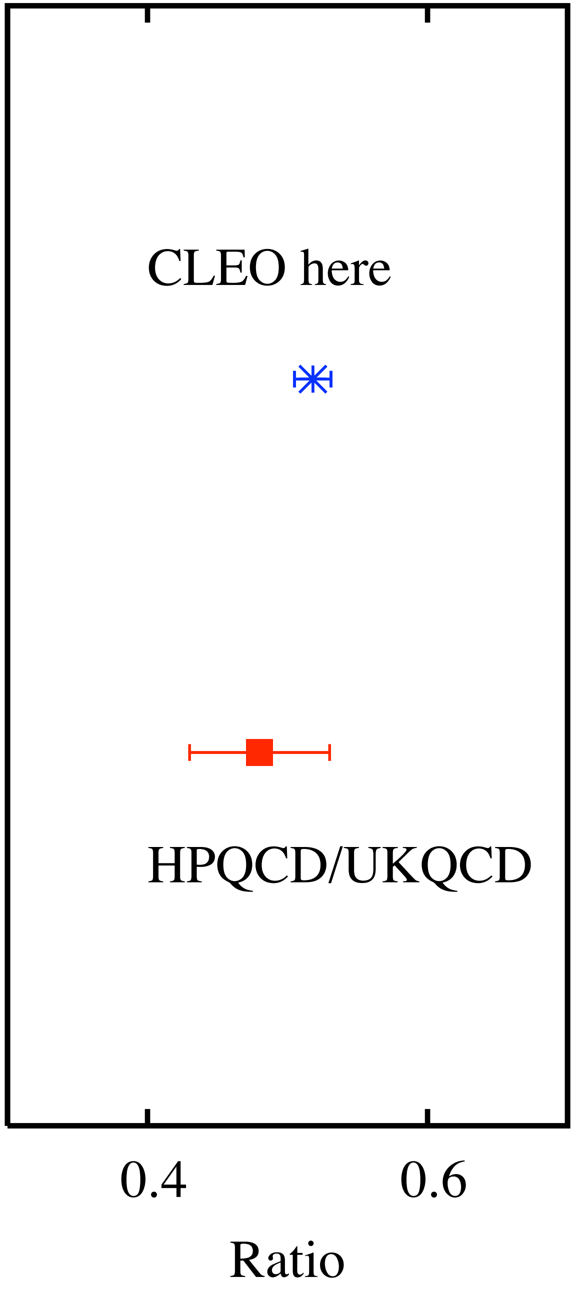
$$f_{D^+} = 223(18)\text{MeV}$$

8% errors theory and expt!

Need better c quark action to improve this - in progress! 5% errors this summer

FNAL/MILC/HPQCD Aubin et al, hep-lat/0506030;

Further checks : Υ leptonic width



$$\frac{\Gamma_{ee}^{(2S)} M_{2S}^2}{\Gamma_{ee}^{(1S)} M_{1S}^2} = \frac{\langle \Upsilon' | J_{em} | 0 \rangle^2}{\langle \Upsilon | J_{em} | 0 \rangle^2}$$

Calc. in lattice QCD

Pert. matching largely cancels in ratio. Lattice QCD result 2005 with NRQCD b =0.48(5)

mainly lattice disc.errors

Improvements on the way!

New CLEO result

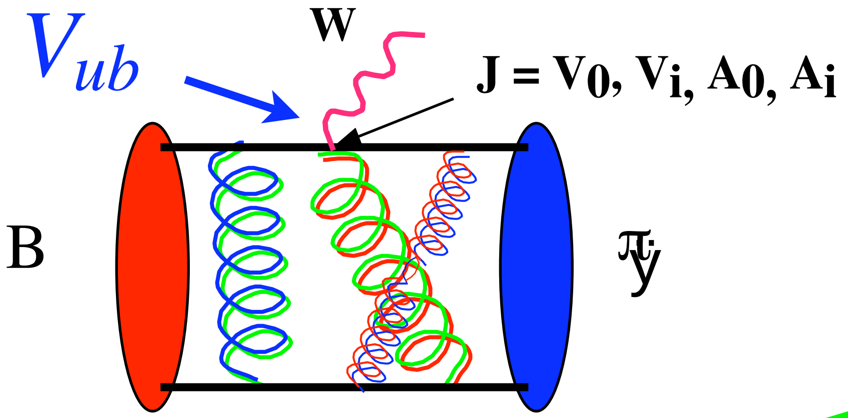
$$\Gamma_{ee}^{(1S)} = 1.336(21)\mathrm{keV}$$

$$\Gamma_{ee}^{(2S)} = 0.616(13)\mathrm{keV}$$

Ratio above = 0.518(13)

HPQCD/UKQCD Gray et al, hep-lat/0507013

Semileptonic form factors e.g. exclusive $B \rightarrow \pi l \nu$



Calculate decay matrix element for various B and π momenta. Determine form factors

$$\langle \pi | J_V^\mu | B \rangle = f_+(q^2)\left(p_B^\mu + p_\pi^\mu - \frac{m_B^2 - m_\pi^2}{q^2} q^\mu\right) + f_0(q^2)\left(\frac{m_B^2 - m_\pi^2}{q^2} q^\mu\right)$$

Compare to expt - normln gives CKM, shape is a test

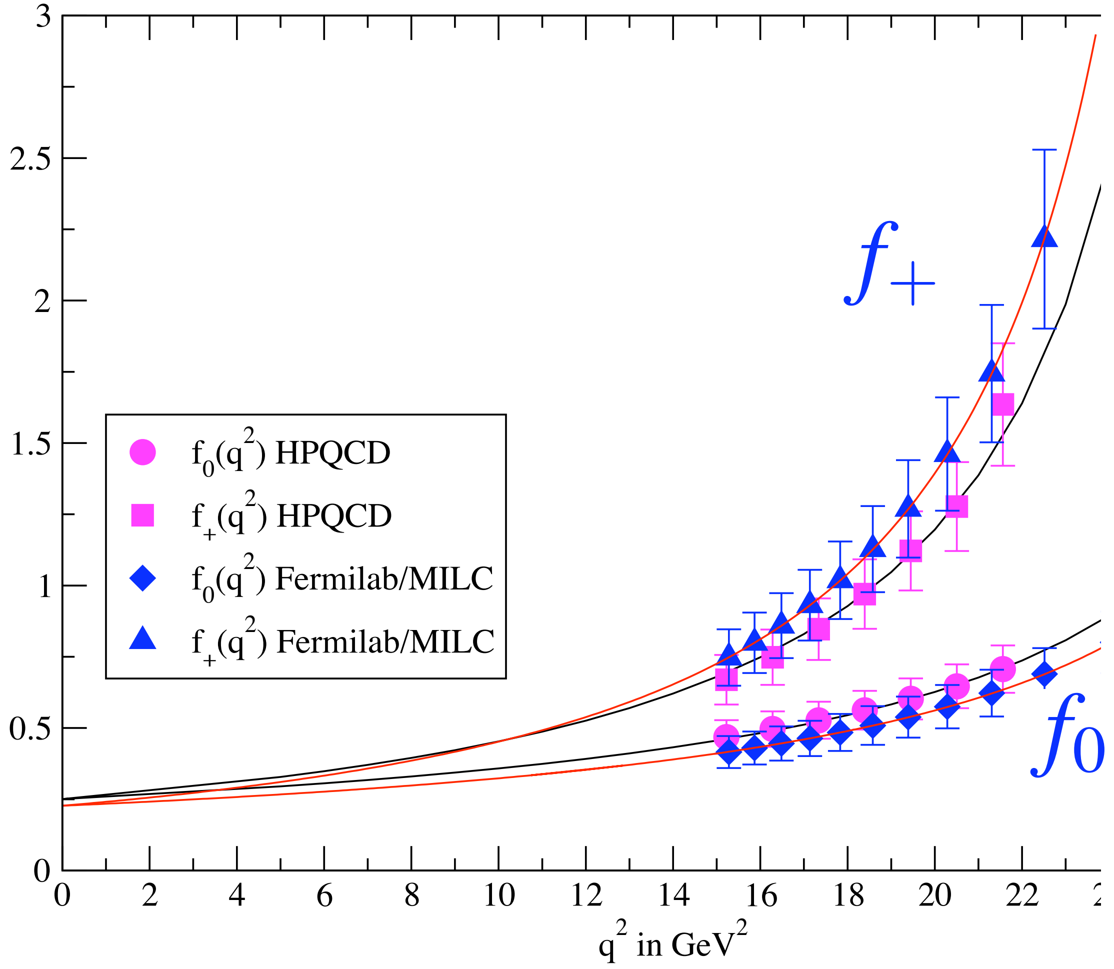
$$\frac{d\Gamma}{dq^2}(B \longrightarrow \pi l \nu) = \frac{G_F^2 |\vec{p}_\pi|^3}{24\pi^3} |V_{ub}|^2 |f_+(q^2)|^2$$

Expt

Lattice QCD

$B \to \pi l \nu$ and V_{ub}

Lattice calcs with NRQCD b and with FNAL b



Currently restricted to high q^2 where $\vec{p}_\pi$ not too large. Use B-K param. to cover whole range.

2005 av. over Babar, Belle, Cleo for $Br(B \to \pi l \nu)$ has 8% error.

Av. over HPQCD and FNAL form factors gives

$$V_{ub} = 4.1(7) \times 10^{-3}$$

11% of error from lattice

Stewart, LP05; HPQCD. hep-lat/0601021

Conclusions

- Lattice QCD is now giving real accuracy, making predictions and passing real tests. Precise parameters of QCD available and 10% errors on decay matrix elements for CKM. Will soon have better errors here for impact on unitarity triangle.

Future - next 2-3 years

- Work with improved staggered sea quarks will continue - beat errors down on f_B, B_B, $B \rightarrow \pi l \nu$ etc with improved matching. Improved c quark action will give corresponding c quark results to a few %.
- First look at 'silver-plated' particles, inc. flavour singlets
- a = 0.06fm possible on QCDOC.

Future - next 2-3 years

- More powerful computers will allow 2+1 simulns using other quark formalisms.

e.g. RBC/UKQCD running 2+1 domain wall on QCDOC

- 1 year running for $m_{u/d} = m_s/4$ at a =0.1fm

In Japan 50 Tflops computer will run overlap quarks.

Italy/Germany testing twisted mass for APEnext computers.

Faster algorithms for Wilson/clover look promising too.