Fine Tuning

Standard Model and Beyond

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In collaboration with
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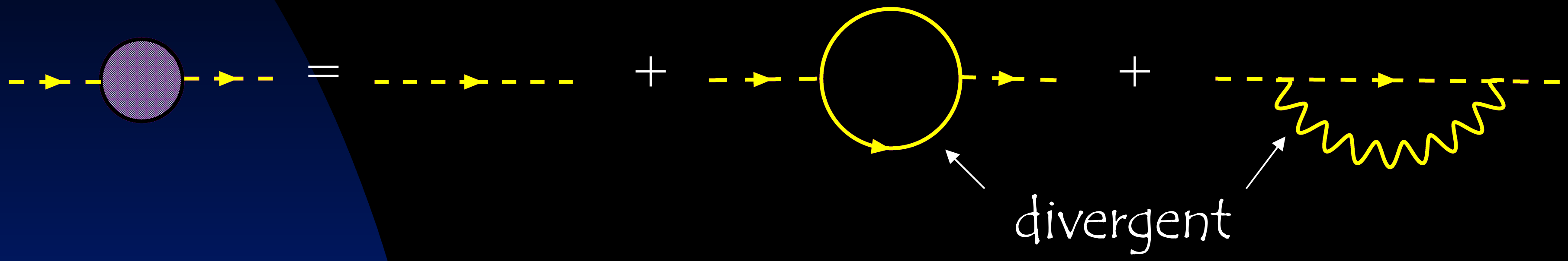
Hierarchy Problem

- physical mass = “bare mass” + “loops”



- Cut off integral at Planck Scale (Λ)

Hierarchy Problem

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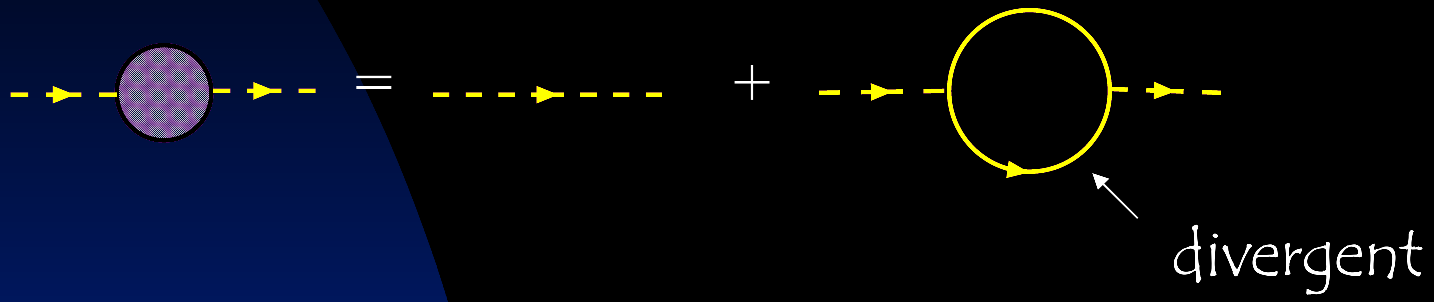

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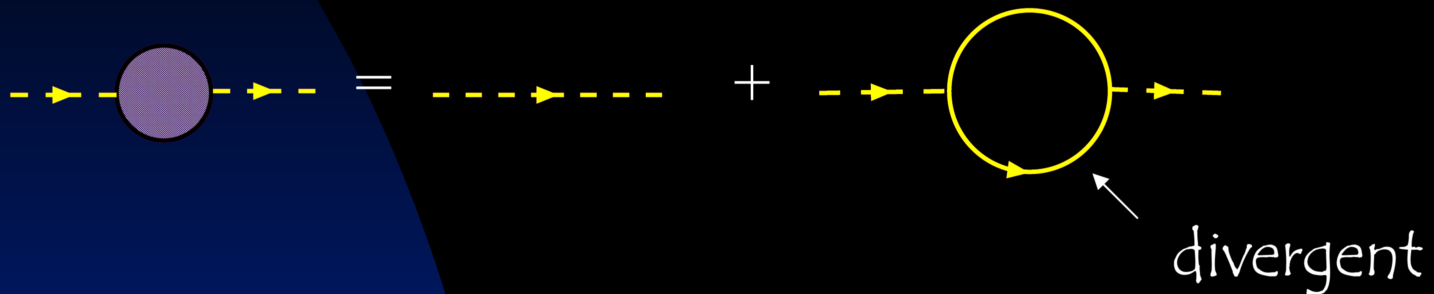


- Cut off integral at Planck Scale (Λ)

$$m_h^2 = m_0^2 - \frac{\lambda_f^2}{8\pi^2}\left(\Lambda^2 - \int_0^1 dx\, 2\triangle \ln \frac{\Lambda^2+\triangle}{\triangle}\right)$$

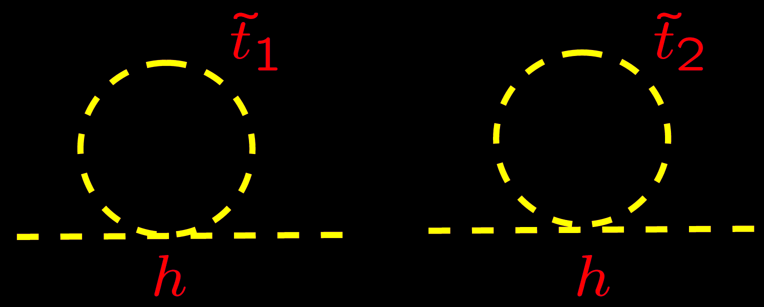
$$\Lambda \sim 10^{19} GeV, m_h \sim 100 GeV \Rightarrow$$ Fine tuning

Supersymmetry

- The only possible extension to space-time
- Unifies gauge couplings
- Provides Dark Matter candidates
- Baryogenesis in the early universe
- Essential ingredient for M-Theory
- Elegant solution to the Hierarchy Problem!

Bosonic degrees of freedom = Fermionic degrees of freedom.

$\Rightarrow$ Two scalar superpartners for each fermion



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$$+ \frac{\lambda_{\tilde{t}}}{16\pi^2}\left(2\Lambda^2 - m_{\tilde{t}_1}^2 \ln \frac{\Lambda^2 + m_{\tilde{t}_1}^2}{m_{s_1}^2} - m_{\tilde{t}_2}^2 \ln \frac{\Lambda^2 + m_{\tilde{t}_2}^2}{m_{\tilde{t}_2}^2}\right)$$

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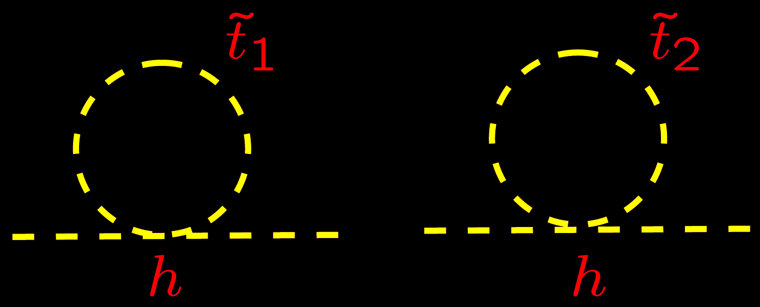

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In Susy $\lambda_{\tilde{t}} = \lambda_t^2$

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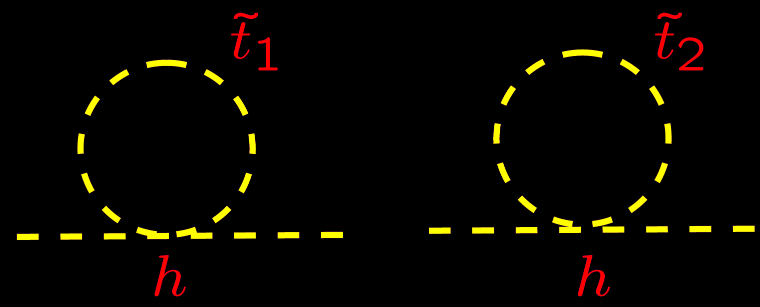

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In Susy $\lambda_{\tilde{t}} = \lambda_t^2$

Quadratic divergences cancelled!

$\Longrightarrow$ No Fine Tuning?

Exact Susy

$$m_{\tilde{t}_1} = m_{\tilde{t}_2} = m_t$$

LEP search fruitless!! $\Longrightarrow$ Lower bounds on sparticles

Softly Broken Susy

$$m_{\tilde{t}_1} \neq m_{\tilde{t}_2} \neq m_t$$

Fine Tuning reintroduced?

MSSM At Tree Level: $$M_z^2 = \frac{2(m_{H_d}^2 - m_{H_u}^2 \tan^2\beta)}{1 - \tan^2\beta} - 2|\mu|^2$$

Sparticle mass limits $\Rightarrow$ Parameters $\gtrsim (1TeV)^2$

But $M_z = 91.188GeV$

$\Longrightarrow$ Little Hierarchy Problem

Traditional Measure

- R. Barbieri & G.F. Giudice, (1988)

Observable

Define Tuning $$\triangle_{BG}(p_i) = |\frac{p_i}{O}\frac{\partial O}{\partial p_i}|$$

Parameter

% change in O from 1% change in p_i

$$\triangle_{BG} = max_{p_i}(\triangle_{BG}(p_i))$$

$\triangle_{BG} > 10$ is fine tuned

Criticisms of Traditional Measure

- Considers each parameter separately

  The fine tuning is about cancellations between parameters .
  A good fine tuning measure considers all parameters together.

- Takes infinitesimal perturbations about the point

  MSSM observables are complicated functions of many parameters. Many small isolated regions of parameter space may give the same value of the observable.

- Implicitly assumes a uniform distribution of parameters

  Parameters in $\mathcal{L}_{GUT}$ may be different to those in $\mathcal{L}_{SUSY}$
  Corresponds to choosing parameters from a different probability distribution

New Measure

Tuning occurs when variations in dimensionless parameters $\Rightarrow$ larger variations in dimensionless observables.

Parameter space point, $P = \{p_i'\}$

$F =$ the volume of parameter space, $a \leq \frac{p_i}{p_i'} \leq b$

$G =$ the subspace of F s.t. the observable $a \leq \frac{O(p_i)}{O(p_i')} \leq b$

Tuning is defined as:

$$\triangle = \frac{F}{G}$$

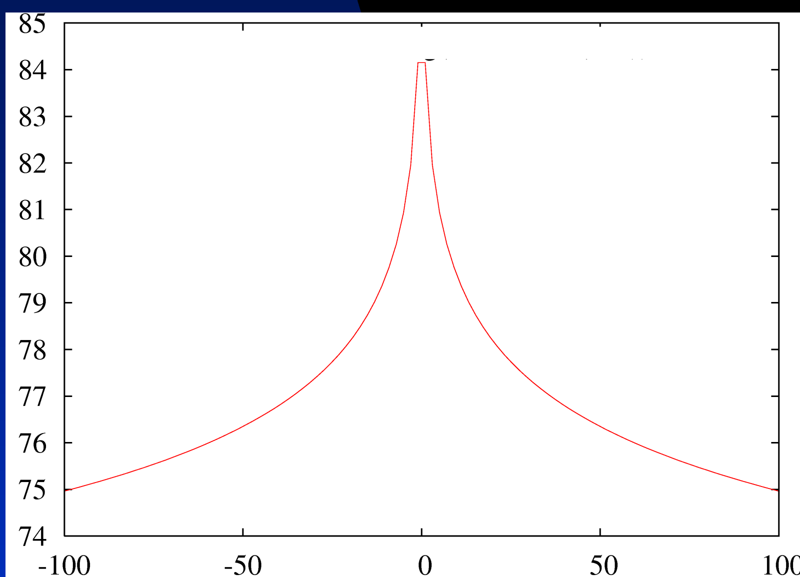
SM Revisited

$$F = \frac{9}{4}(\Lambda^2 m_h^2 + g^2\Lambda^4) \qquad G = \frac{9}{4}\Lambda^2 m_h^2$$

$$\triangle = 1 + \frac{g^2\Lambda^2}{m_h^2} = \triangle_{BG}$$

Fine tuning as a function of Higgs Mass

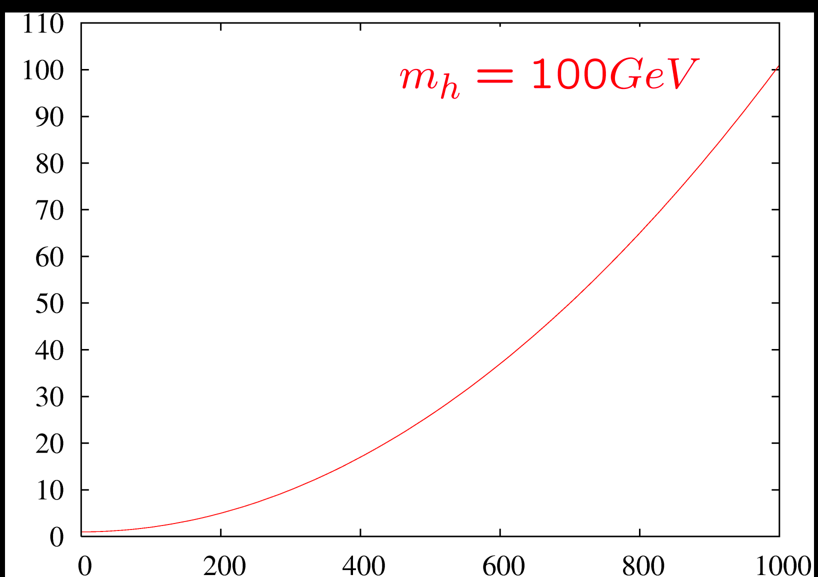
ln $\triangle$



$m_h(GeV)$

Fine tuning as a function of the New Physics scale

ln $\triangle$



$\Lambda(GeV)$

Fine Tuning in the MSSM

- Choose a point P in the parameter space at GUT scale
- Take random fluctuations about this point.
- Using a modified version of Softsusy (B.C. Allanach)
  - Run to Electro-Weak Symmetry Breaking scale.
  - Predict M_z and sparticle masses
- Count how often M_z is ok
- Apply fine tuning measure

$$\triangle_N = \frac{N_F}{N_G}$$

If the tuning in the MSSM is fine for flat probability distributions:

- Nature is fine tuned.
- EWSB by some other mechanism than the Higgs

  e.g. Technicolor
- The Hierarchy problem is solved other new physics

  e.g. Little Higgs, Large Extra Dimensions
- Extended Higgs sector SSM's are favoured
  e.g. NMSSM, nMSSM, ESSM
- The MSSM parameters are not all equally likely.

What probability distribution minimises tuning?
Is there a GUT with this distribution?

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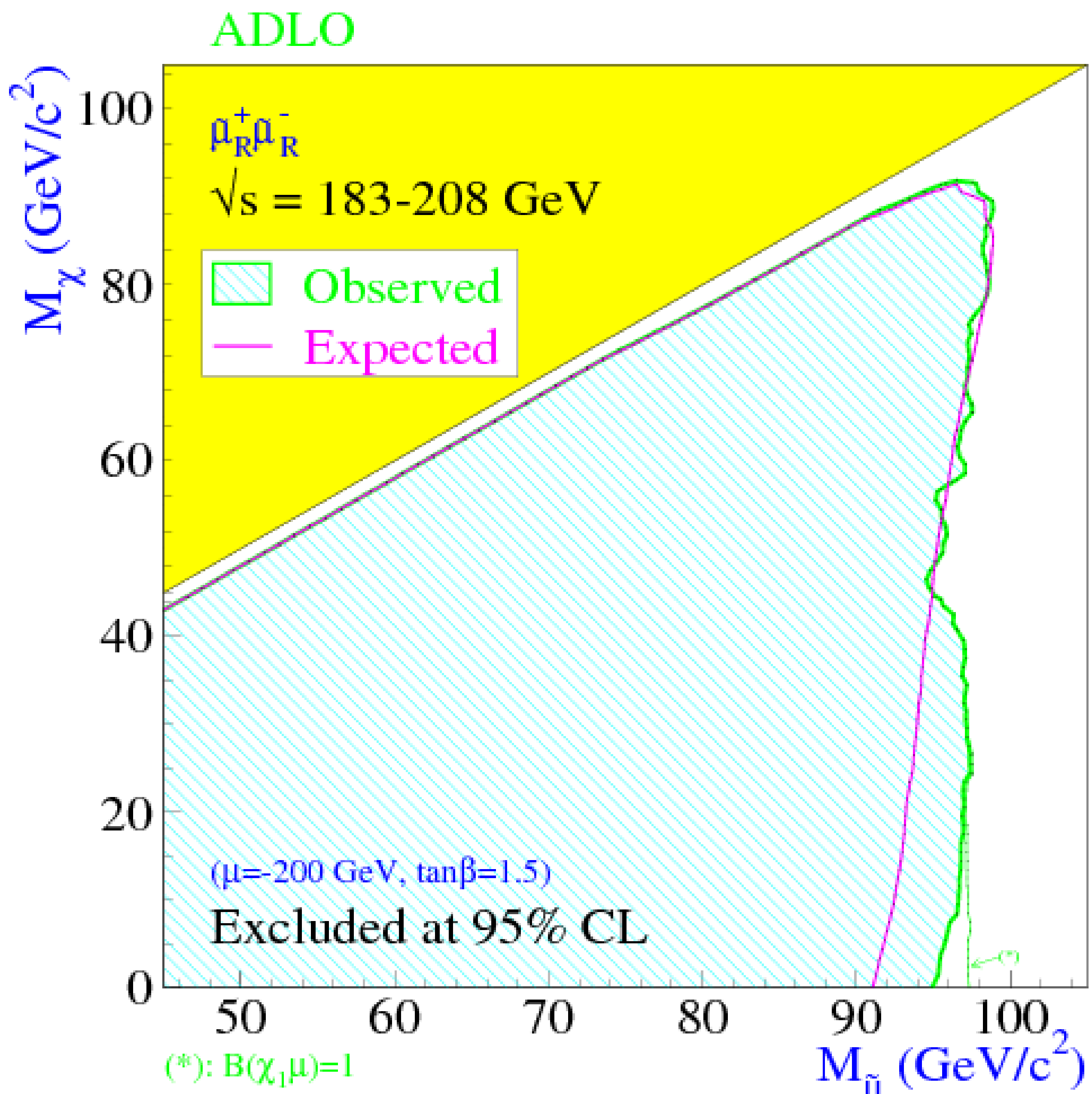
Also contains tunings

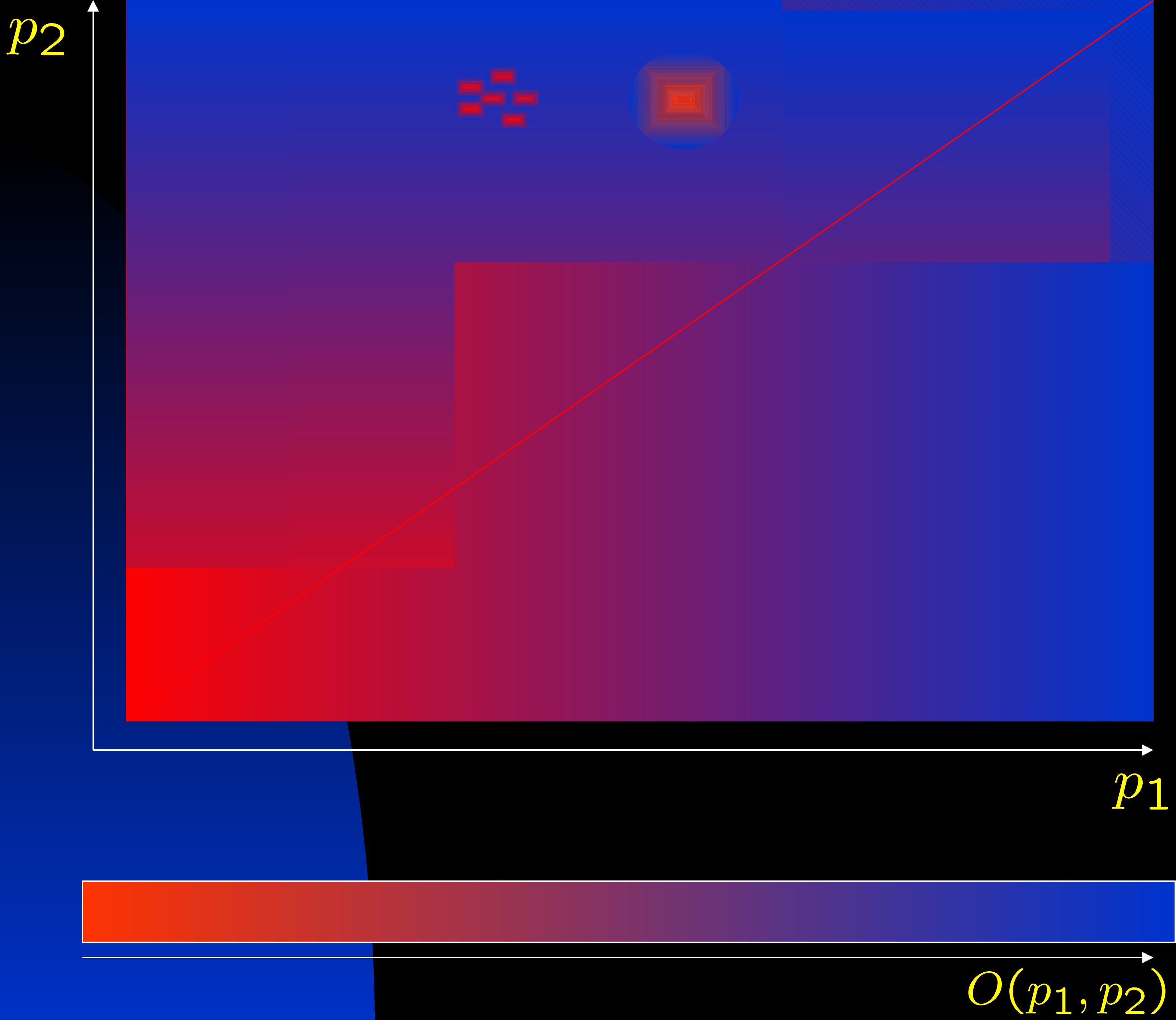
S.F. King, S. Moretti, R. Nevzarov,

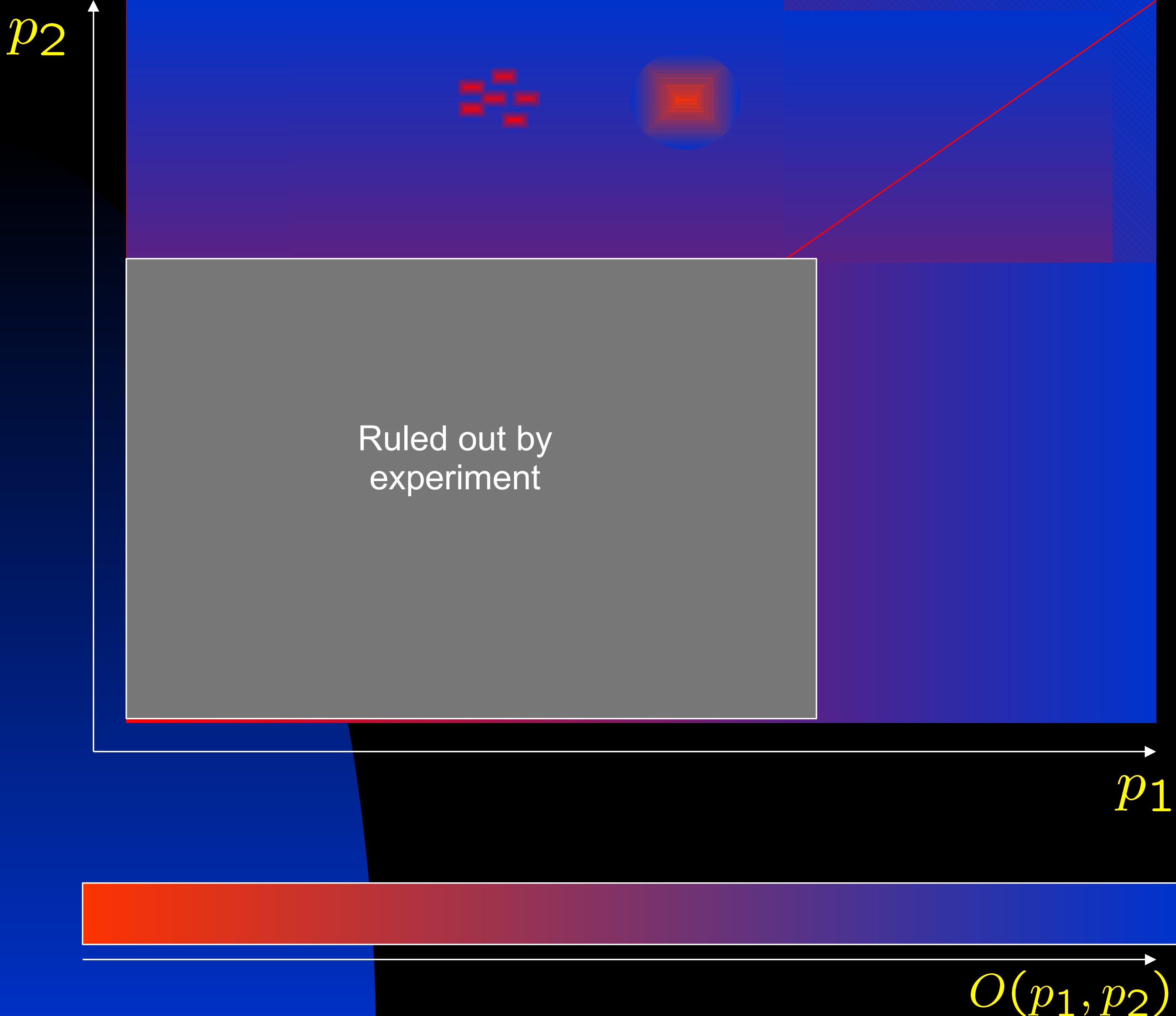
hep-ph/0510419, hep-ph/0511256

Conclusions

- Fine Tuning in the SM $\approx 10^{32}$
  - SUSY
- Broken SUSY appears fine tuned $\approx 10^{2}$
  - Little Hierarchy Problem
- Current measures of tuning neglect:
  - Probability distribution of parameters.
  - Many parameter nature of fine tuning
  - Cancellations a finite distance from point
- New measure addresses these issues
  - Results for MSSM coming soon

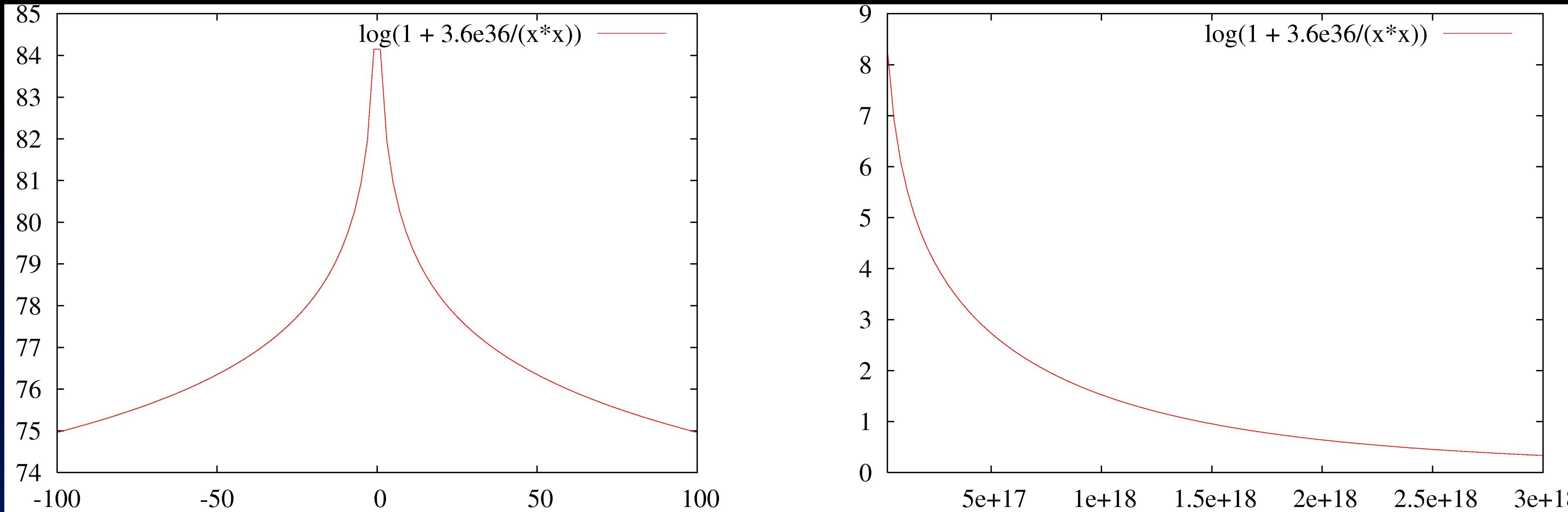

ADLO
M_χ (GeV/c^2)
100
80
60
40
20
0
$\tilde{\mu}^+_R\tilde{\mu}^-_R$
$\sqrt{s}$ = 183-208 GeV
Observed
Expected
(μ=-200 GeV, tanβ=1.5)
Excluded at 95% CL
(*)
50
60
70
80
90
100
$M_{\tilde{\mu}}$ (GeV/c^2)
(*): B($\chi_1\mu$)=1


p_2
p_1
$O(p_1, p_2)$



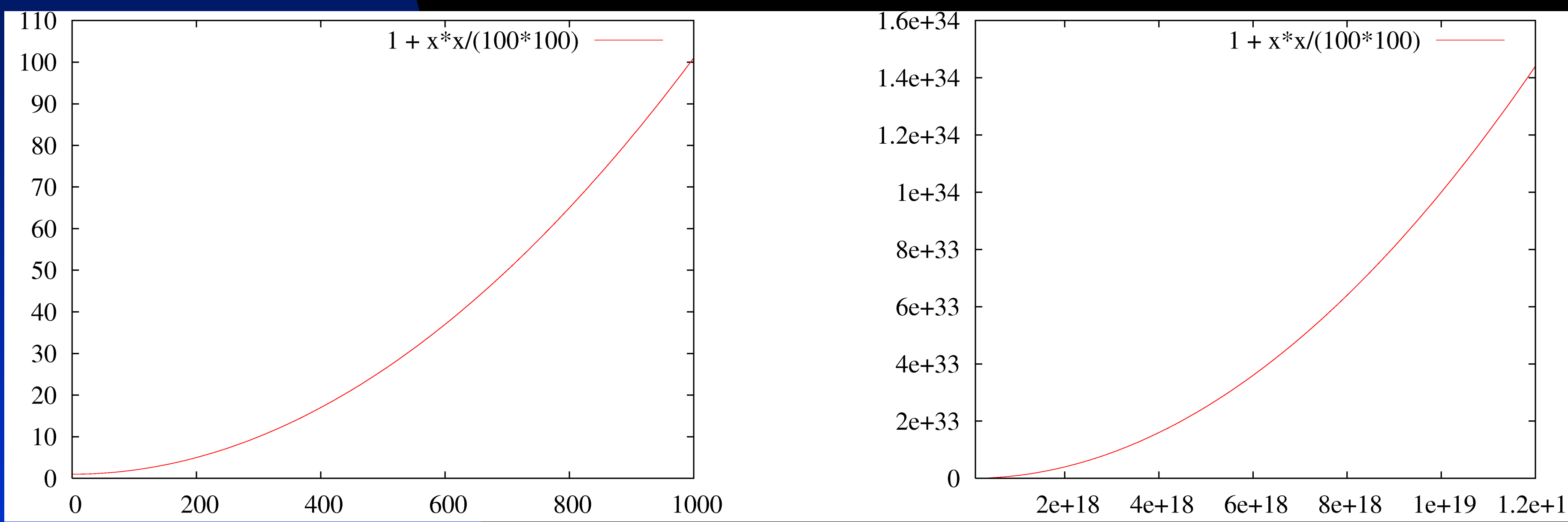
p_2
Ruled out by experiment
p_1
$O(p_1, p_2)$

Fine tuning as a function of Higgs Mass



LABEL these!!!

Fine tuning as a function of the New Physics scale



Is the Tuning Fine?

- Aesthetic question.
  - How much tuning are we prepared to tolerate?
- Intuitive physical notion.
  - How do we measure tuning?

Numerical Measure

- Applying numerical approximation of fine tuning measure

$$\triangle_N = \frac{N_F}{N_G}$$

Where N_S is the number of points in space S

$$\triangle = \lim_{N_F \to \infty} \triangle_N$$