

# Course on Advanced Accelerator Physics at Bonn University Last Week

<https://indico.hiskp.uni-bonn.de/event/1111/timetable/#20251001>

- 4 x ~90 min Lectures on Plasma Wakefield Accelerators
  - Beam and Laser Driven
  - Audience: Master Students
- 2 x ~120 min of Exercises → Arthur (exercises are uploaded)

→ Selected some topics for discussion today

# Plasma Wakefield Acceleration

## Novel Concept on How to Accelerate Charged Particles

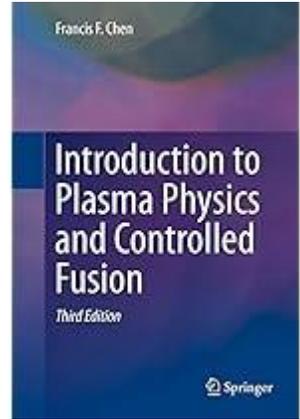
- Use high amplitude fields sustained in a plasma wave (plasma wakefields) to accelerate charged particles
- **Change of technology:** Increase the acceleration gradient  $\mathbf{G}_{\text{acc}}$  by a factor ~10-1000 over established technologies

→ Enable compact linear accelerators: **high risk – high reward R&D**

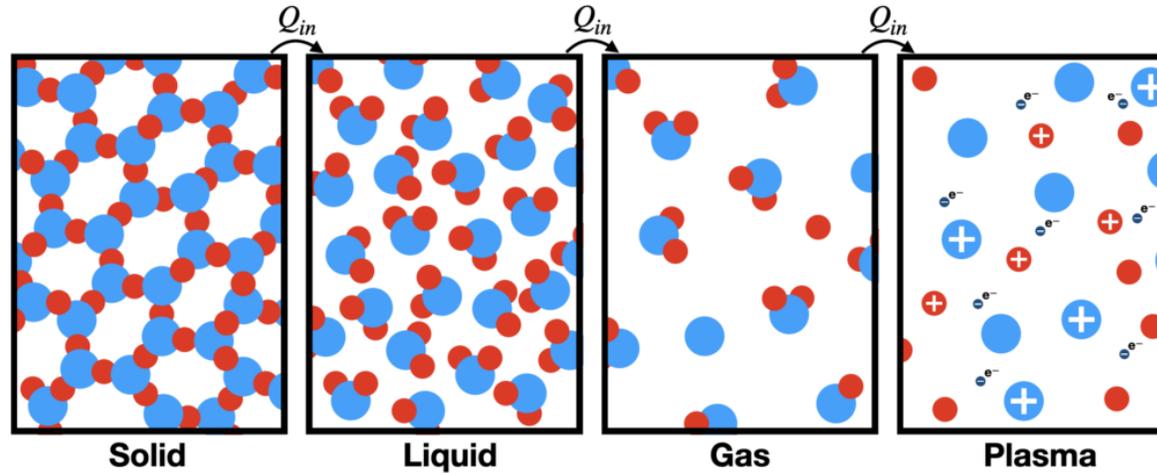
$$E_{WB} = \frac{m_e c \omega_{pe}}{e} \simeq 100 \frac{\text{eV}}{m} \sqrt{n_{pe} [\text{cm}^{-3}]}$$

# 1.1 Definition of a Plasma

From Francis F. Chen  
"Introduction to Plasma Physics and Controlled Fusion"



Plasma: ionized gas (4<sup>th</sup> state of matter)



From: <https://www.jessearodriguez.com/what-is-plasma/>

99% of the universe is plasma

- A plasma is a quasineutral gas of charged and neutral particles which exhibits collective behavior

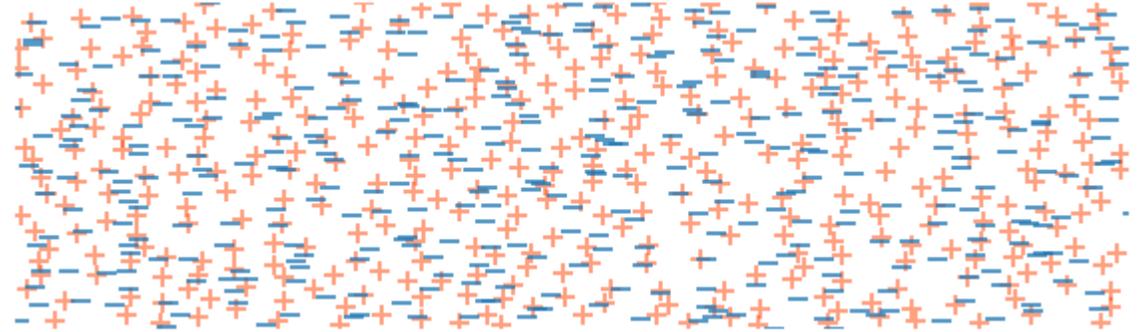
→ Three conditions

# 1.1 Definition of a Plasma

## 1) Quasi-neutrality

$$n_{pi} \cong n_{pe}$$

$n_{pi}$ : density of plasma ions  
 $n_{pe}$ : density of plasma electrons



- Plasma is overall **not** charged
- There can be local concentrations of charge
- $m_i \gg m_e$  (Hydrogen:  $m_i \cong 1836 m_e$ )



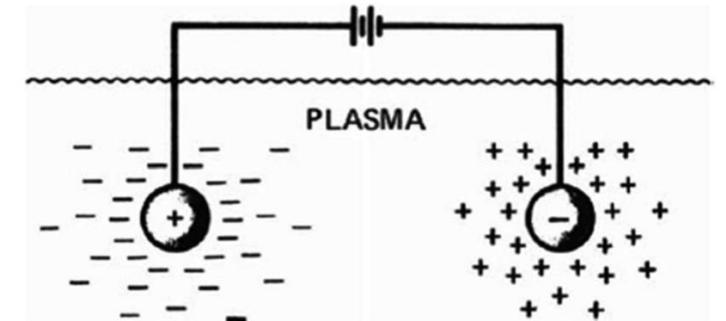
### Exercise session

Caution:  $n_{pe}$ ,  $n_{pi}$  is usually in  $[\text{cm}^{-3}]$  → convert to  $[\text{m}^{-3}]$  for calculations in SI units.

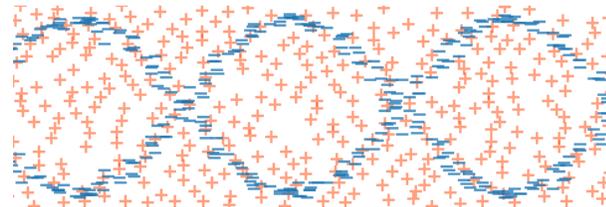
# 1.1 Definition of a Plasma

## 2) Plasma exhibits collective behavior

→ Charged particles are close enough to 'feel' each other and to respond



→ Necessary for wave-like behavior



# 1.1 Definition of a Plasma

## 3) Plasma particle motion is controlled by electromagnetic forces

- Rather than e.g. ordinary hydrodynamic forces, scattering, etc...

Candle flame



Partially/weakly ionized gas

**Not a plasma** → too many neutrals → collisions (with neutrals) dominate  
A candle flame doesn't get hot enough to ionize a significant number of atoms ( $\sim 10^{-15}$ )

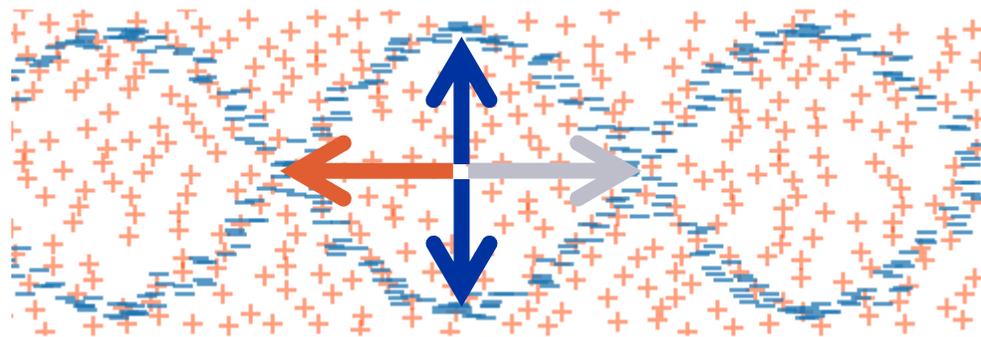
Fluorescent lamp



**Plasma** → sufficient fraction of atoms are ionized ( $\sim 10^{-7}$ )

# 1.2 Plasma Wakefield Acceleration

Intuitive description



wave in the wake of a boat

# 1.2 Plasma Wakefield Acceleration

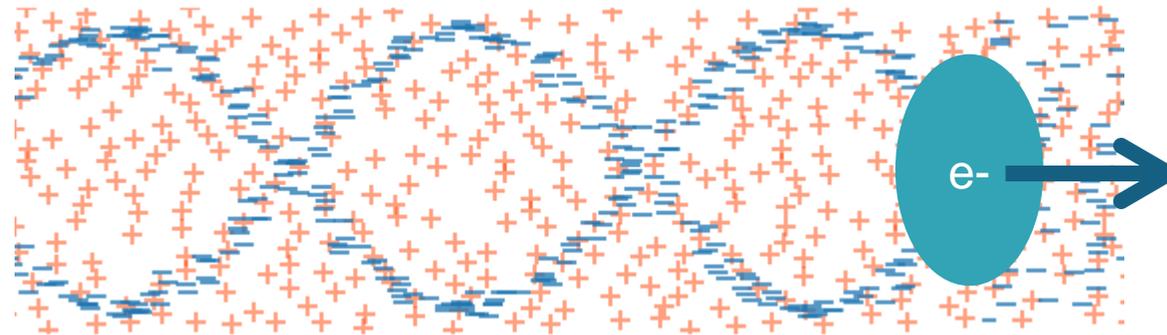
## Plasma

Quasi-neutral plasma in which **electrostatic interactions dominate** and charged particles are dense enough to support **collective behaviour**

## Drive bunch or pulse

Relativistic **charged particle bunch/es**  
or  
**laser pulse/s**

- + Plasma ion
- Plasma electron



- Electrostatic wave



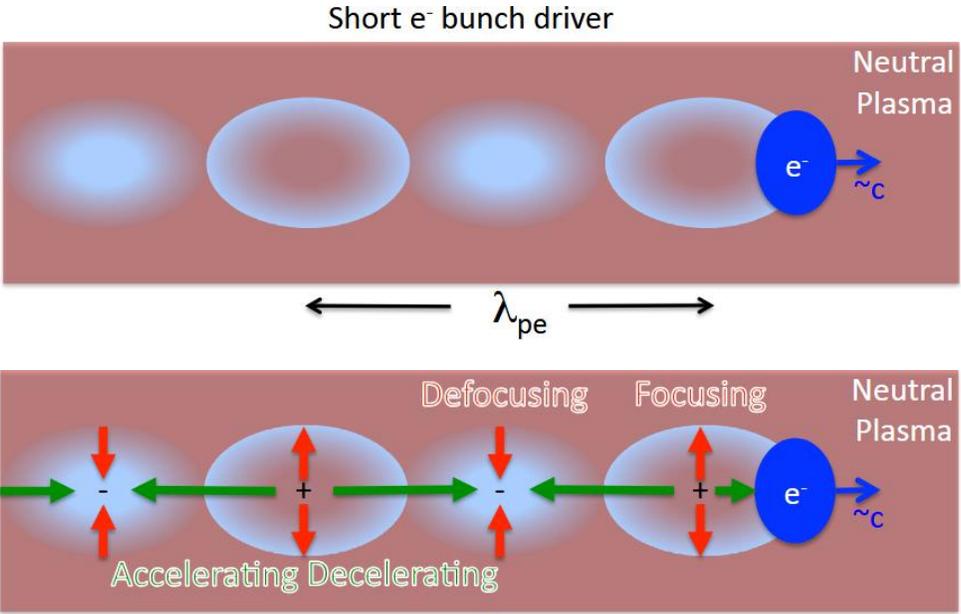
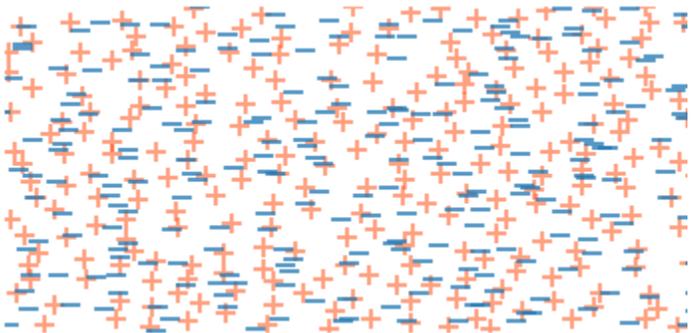
# 1.2 Plasma Wakefield Acceleration

## Plasma

Quasi-neutral plasma in which **electrostatic interactions dominate** and charged particles are dense enough to support **collective behaviour**

## Drive bunch or pulse

Relativistic **charged particle bunch/es**  
or  
**laser pulse/s**



From P. Muggli:  
<https://indico.cern.ch/event/759579/contributions/3184780/attachments/1814174/2964384/ERN-CAS2019Muggli.pdf>

# 1.3 Plasma Electron Oscillations

**Mathematical description**

**Langmuir wave**

# Background Knowledge

# 1.3 Key Equations

**Maxwells equations:**

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\oint_{\partial V} \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\mu_0 \epsilon_0 = \frac{1}{c^2}$$

**E:** electric field  
**B:** magnetic field  
 **$\rho$ :** electric charge density  
 **$\epsilon_0$ :** vacuum permittivity  
**J:** electric current density  
 **$\mu_0$ :** vacuum permeability  
**dA:** vector area element on the closed surface V  
 **$Q_{\text{enc}}$ :** enclosed electric charge by surface V

**Continuity equation:**

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = 0$$

→ in the absence of ionization or recombination the number of particles of each species is conserved

**n:** particle density  
**v:** particle velocity

**Newtons second law:**

$$\mathbf{F} = m\mathbf{a} = m \frac{d\mathbf{v}}{dt}$$

**F:** net force acting on particle  
**m:** particle mass  
**v:** particle velocity

**Lorentz force:**

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

**q:** particle charge  
**E:** electric field  
**B:** magnetic field  
**v:** particle velocity  
**u:** energy density

# 1.3 Wave Description

Euler's formula:

$$e^{i\theta} = \cos \theta + i \sin \theta$$

$$e^{i(kx - \omega t)} = \cos(kx - \omega t) + i \sin(kx - \omega t)$$

**k**: wave number  
 **$\omega$** : angular frequency

k describes how the wave evolves in space  
 $\omega$  describes how the wave evolves in time

$\omega$  and k are related by the dispersion relation

$$v_{ph} = \frac{\omega}{k}$$

$$v_g = \frac{d\omega}{dk}$$

Example: Electric field wave

$$\mathbf{E}(x, t) = E_0 e^{i(kx - \omega t)} \hat{y}$$

$$\mathbf{E}_{\text{real}}(x, t) = \Re\{\mathbf{E}(x, t)\} = E_0 \cos(kx - \omega t) \hat{y}$$

**$E_0$** : wave amplitude

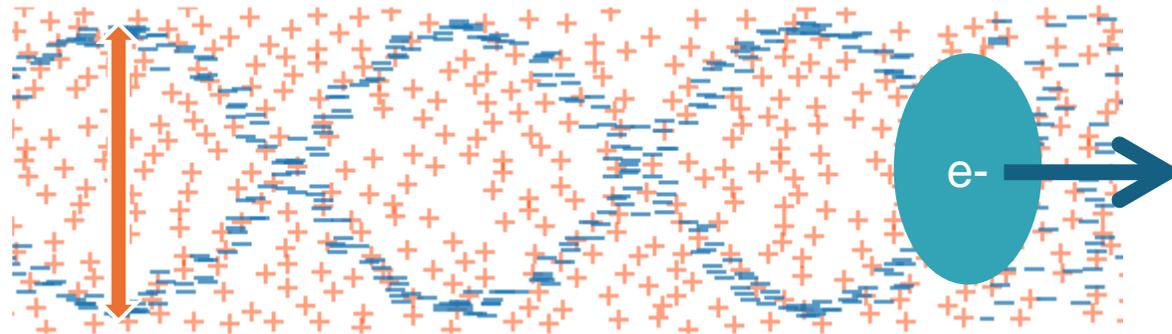
$$e^{i(kx - \omega t)} \quad \nabla \rightarrow ik$$
$$\frac{\partial}{\partial t} \rightarrow -i\omega$$

# 1.3 Langmuir Waves

A **Langmuir wave** (also called a **plasma oscillation** or **electron plasma wave**) is a **collective oscillation of the electron density** in a plasma.

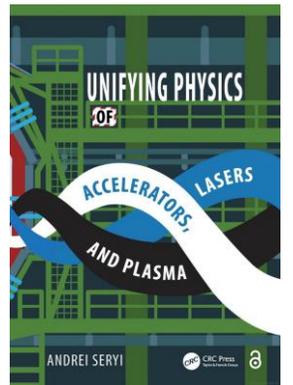
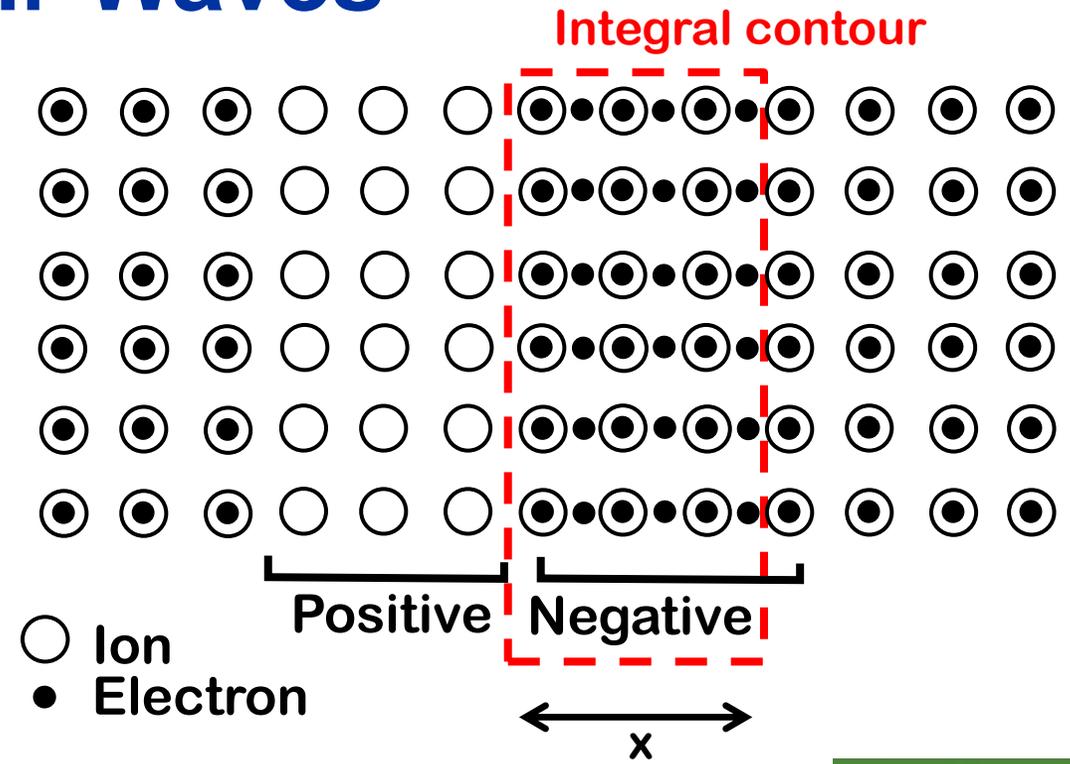
## Assumptions:

- Uniform plasma
- No magnetic field
- Stationary ions



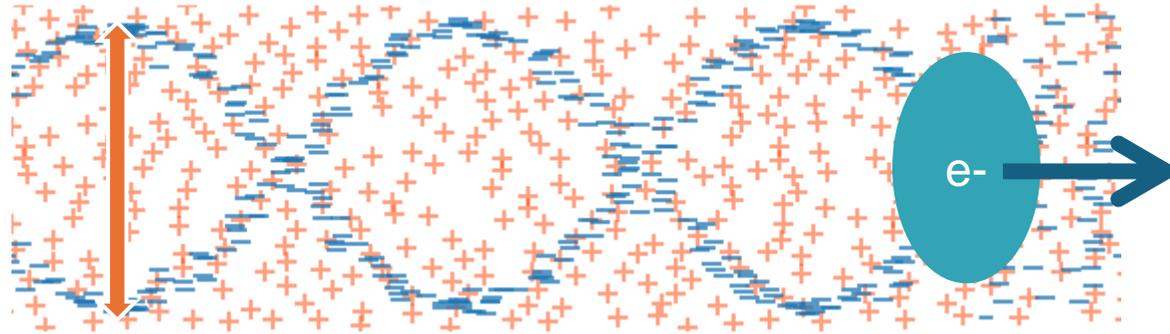
**Q1: At which period/frequency do plasma electrons oscillate?**

# 1.3 Langmuir Waves



# 1.3 Langmuir Waves

Group velocity of the driver:  $v_b$   
Common assumption:  $v_b = c$



Plasma electrons oscillate in place at  $(\omega_{pe})$

$$\omega_{pe}$$



Plasma electron wavelength  $(\lambda_{pe})$

$$\lambda_{pe}$$



**Exercise session**

**Assume  $v_b = c$**

- $n_{pe}$ : plasma electron density
- $e$ : particle charge
- $\epsilon_0$ : vacuum permittivity
- $m_e$ : particle mass

# 1.4 Dispersion Relation for Plasma Oscillations

# 1.4 Dispersion Relations

**Dispersion relations** are the **equations of motion for waves in plasma**. They tell us how disturbances behave — whether they **oscillate, propagate, or grow**.

→ Almost every plasma physics lecture starts with “*Let’s derive the dispersion relation...*”

## Assumption:

- 1D
- Uniform plasma
- No magnetic field
- Stationary ions
- Small perturbations
- Cold electrons

$$\nabla E = \frac{\rho}{\epsilon_0}$$

$$m_e \frac{dv}{dt} = -eE$$

$$\frac{dn}{dt} + \nabla(nv) = 0$$

## 1) Step 1: Linearize equations

$$E = E_0 + E_1$$

$$v = v_0 + v_1$$

$$n_{pe} = n_{pe,0} + n_{pe,1}$$

$$n_{pi} = n_{pi,0} =$$

$$\nabla E_1 = \frac{-en_{pe,1}}{\epsilon_0}$$

$$m_e \frac{dv_1}{dt} = -eE_1$$

$$\frac{dn_{pe,1}}{dt} + \nabla(n_{pe,0}v_1) = 0$$

# 1.4 Dispersion Relation for Plasma Oscillations

2) **Step 2:** Solutions of the form  $e^{i(kx-\omega t)}$

$$\nabla E_1 = \frac{-en_{pe,1}}{\epsilon_0}$$

$$m_e \frac{dv_1}{dt} = -eE_1$$

$$\frac{dn_{pe,1}}{dt} + \nabla(n_{pe,0}v_1) = 0$$

$$E_1 = \tilde{E}_1 e^{i(kx-\omega t)}$$

$$v_1 = \tilde{v}_1 e^{i(kx-\omega t)}$$

$$n_{pe,1} = \tilde{n}_{pe,1} e^{i(kx-\omega t)}$$

$$ikE_1 = \frac{-en_{pe,1}}{\epsilon_0}$$

$$i\omega m_e v_1 = -eE_1$$

$$-\omega n_{pe,1} + kn_{pe,0}v_1 = 0$$

# 1.4 Dispersion Relation for Plasma Oscillations

$\omega$ : angular frequency  
 $\omega_{pe}$ : plasma electron frequency

## Dispersion relation:

for plasma oscillations (Langmuir waves) in a cold, uniform, unmagnetized plasma

$$\omega = \omega_{pe}$$

Dispersion relation of beams:  $\omega = v_b k$



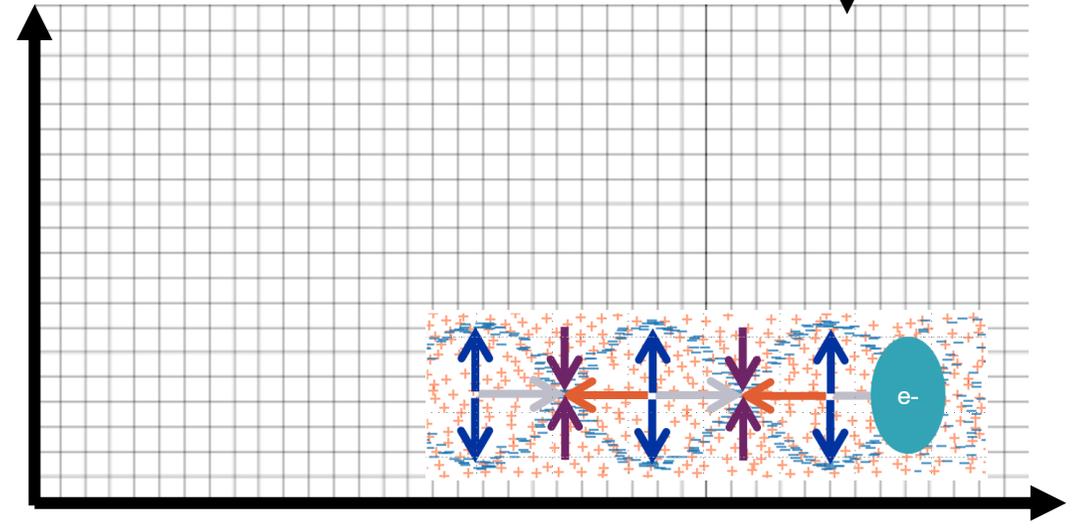
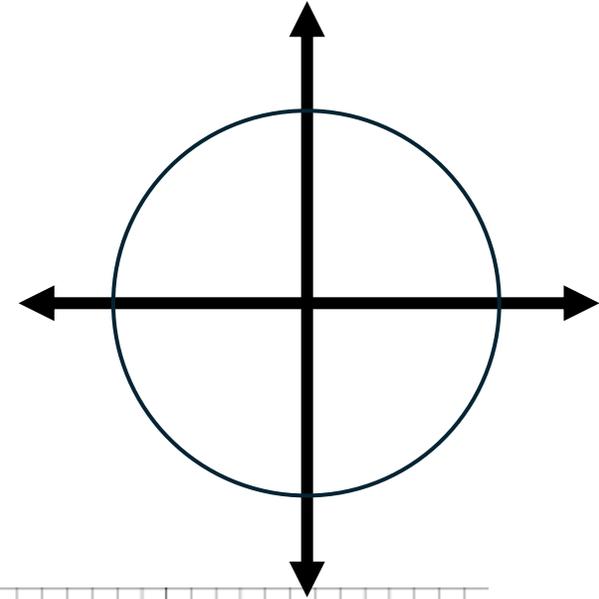
# 1.5 The 1D Cold Plasma Wavebreaking Field ( $E_{WB}$ )

# 1.5 Acceleration Gradient & Wakefield Phase

$$ikE_1 = \frac{-en_{pe,1}}{\epsilon_0}$$

$$i\omega m_e v_1 = eE_1$$

$$-\omega n_{pe,1} + kn_{pe,0}v_1 = 0$$



# 2.1 Driving Wakefields with Charged Particle Beams

The **amplitude** of the plasma wakefield depends on the drive beam density  $n_b$

$(n_b \ll n_{pe})$   
Linear regime

$$E_{acc} \sim \frac{n_b}{n_{pe}} E_{WB}$$

$(n_b \gg n_{pe})$   
Nonlinear regime

$$E_{acc} \sim \sqrt{\frac{n_b}{n_{pe}}} E_{WB}$$

To get a reasonably high  $E_{acc}$   
→ Beam density ( $n_b$ ) comparable or larger than the plasma density ( $n_{pe}$ )

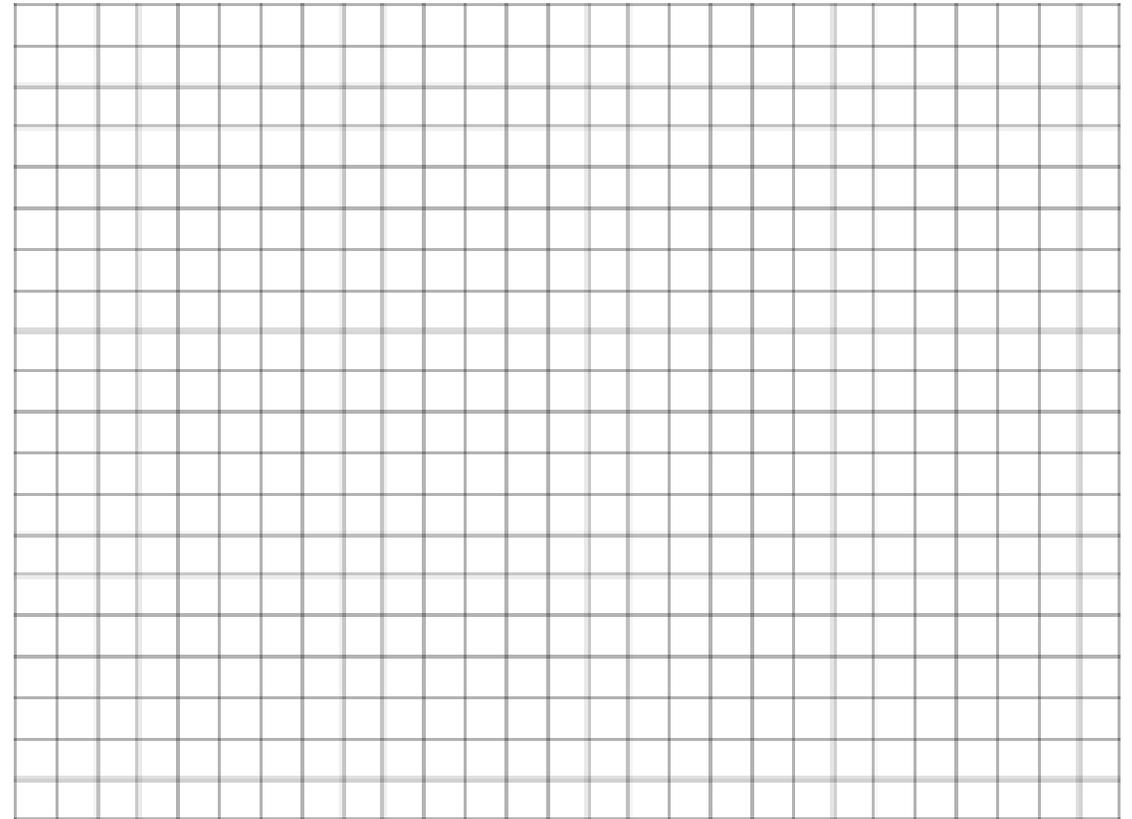
# 2.1 Optimum Drive Bunch Parameters

**Bunch length ( $\sigma_z$ ) on the order of the plasma wavelength ( $\lambda_{pe}$ )**

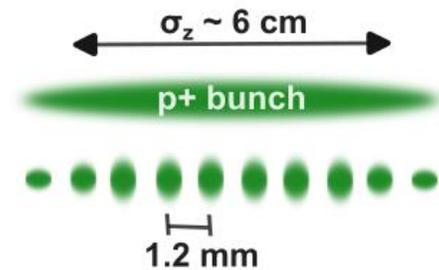
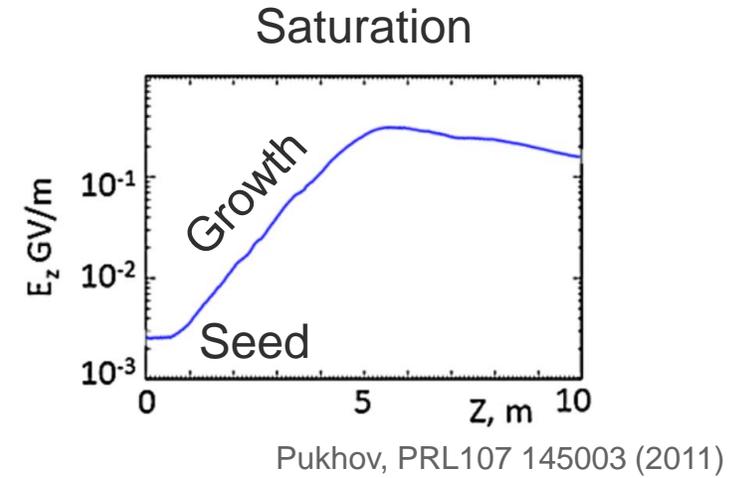
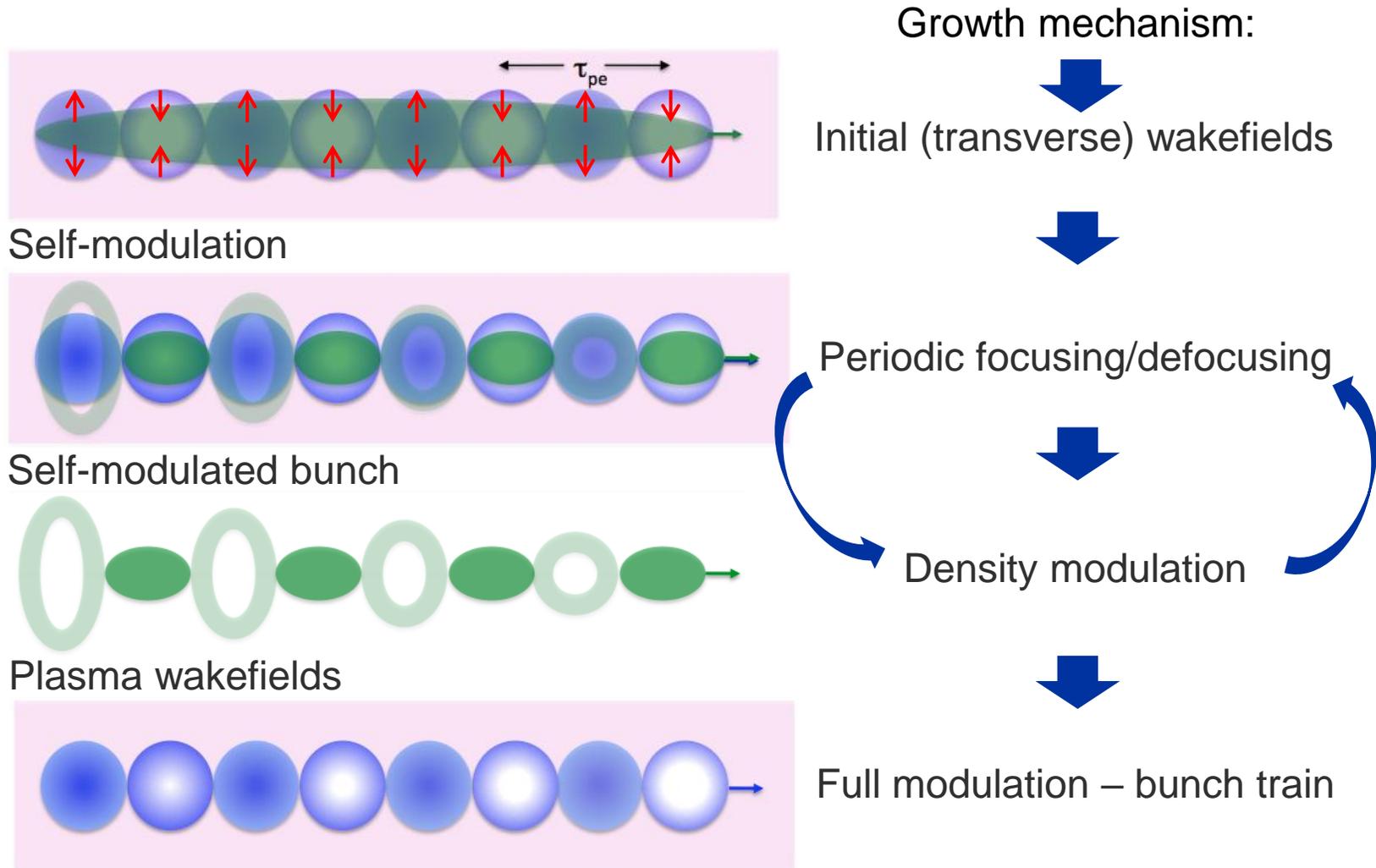
- For rectangular bunches:  $L_{z,opt} = \lambda_{pe}/2$
- For Gaussian bunches:  $\sigma_{z,opt} = \sqrt{2} c/\omega_{pe}$

**Beam radius ( $\sigma_r$ ) smaller than the plasma skin depth  $k_{pe}^{-1} = \omega_{pe}/c$**

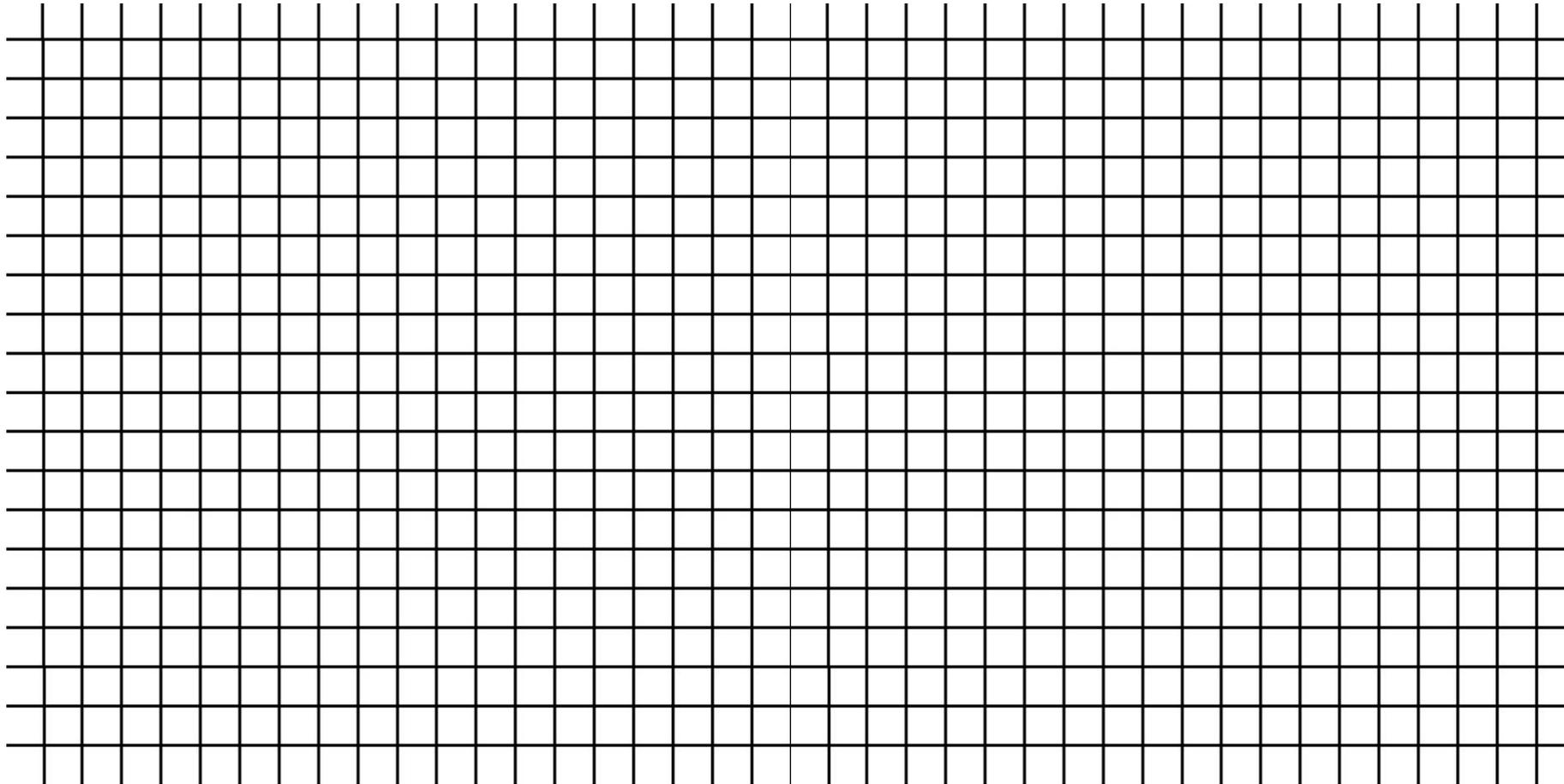
- To avoid e.g. transverse filamentation



# 4.3 Self-Modulation Process



# 4.3 Driving Wakefields With a Bunch Train



More than one driver, so  $N_\mu > 1$

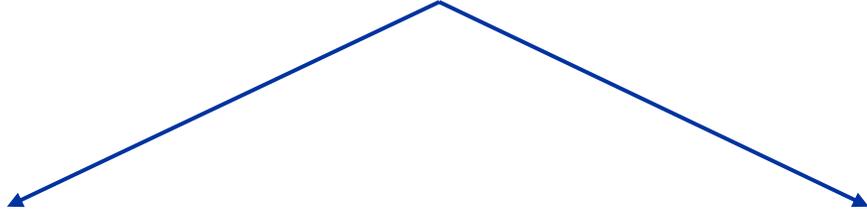
$$E_{acc} \sim N_\mu \frac{n_b}{n_{pe}} E_{WB}$$

Shorter microbunches  
→ choose higher  $n_{pe}$

$$n_{pe} \sim 7 \times 10^{14} \text{ cm}^{-3}$$
$$\rightarrow E_{acc} \sim \text{few GV/m}$$

→ To be aware: requires  $\Delta n_{pe}/n_{pe} \ll 1/N_\mu$  and a heavy enough ion

# 3.4.2 How to Accelerate Beams of Quality?

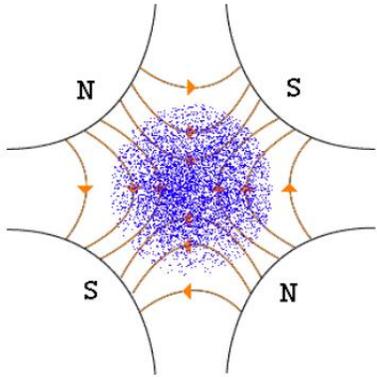


Small energy spread:  
→ Beam loading

Emittance preservation:  
→ Linear focusing force  
& matching

# 3.4.2 To Preserve Emittance During Acceleration

## Linear focusing fields



Reminder:  
Conventional beam optics →  
**Force is linear with radius! → Emittance preservation**

**Do we also have linear transverse fields in plasma wakefields?**

Linear regime ( $n_b \ll n_{pe}$ ,  $a_0 \ll 1$ ): → no

Nonlinear regime ( $n_b \gg n_{pe}$ ,  $a_0 \gg 1$ ): → yes

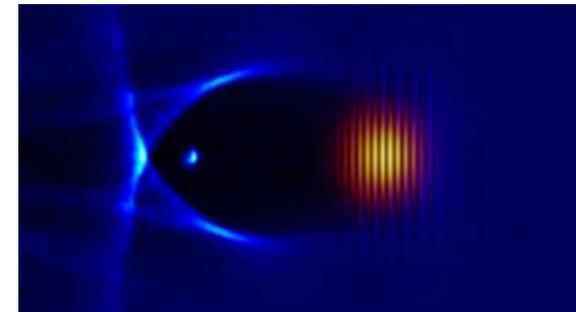
## Matching

Witness beams:

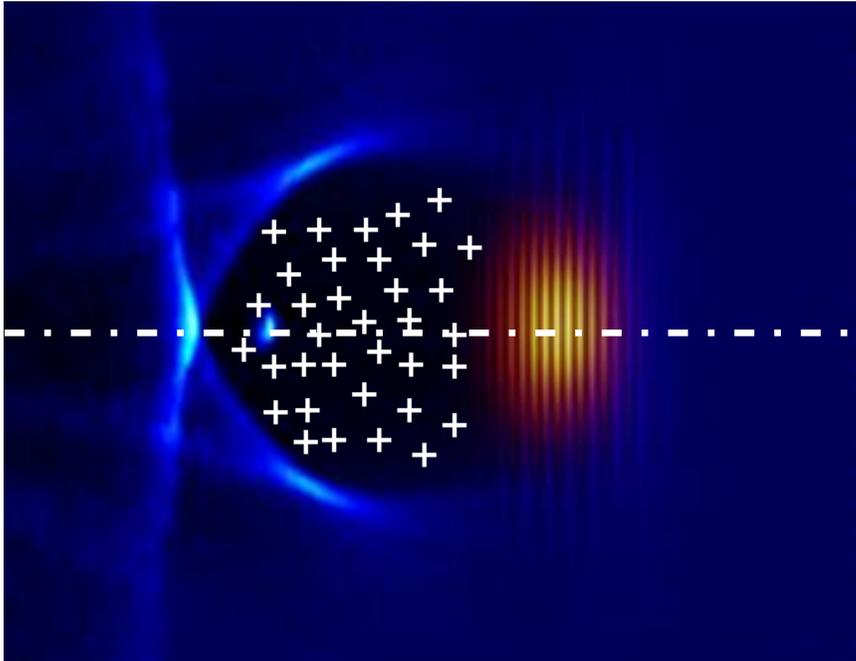
- 1) **Have a natural divergence**  
(depends on their emittance & spot size)
- 2) **Experience a focusing force** in the wakefields → wakefield acts like a focusing lens on the electron bunch

→ If the focusing force compensated for the natural divergence → **Matching**

→ Witness beam envelope stays constant



## 3.4.2 Linear Focusing Force



Gauss law:

$$F = -eE = -\frac{n_{pe}e^2}{2\epsilon_0}r$$

→ Focusing force is linear as long as  $n_{pe}$  is uniform

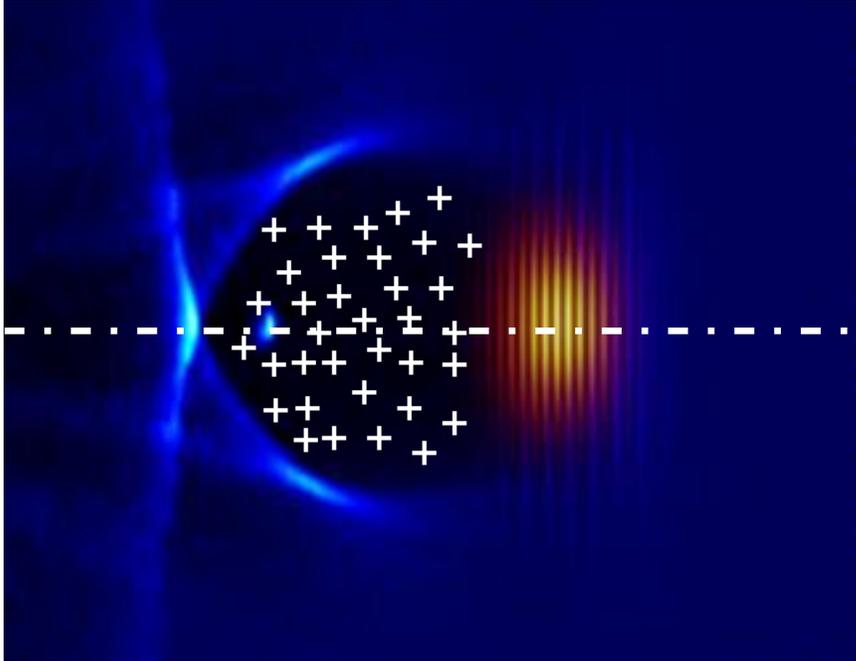
$$F = \gamma m_e a$$

$$r'' + \frac{n_{pe}e^2}{2\epsilon_0\gamma_e m_e c^2}r = 0$$

Betatron wavenumber:

which describes transverse oscillations of relativistic electrons in the focusing field

## 3.4.2 Matching Condition



Beam envelope equation:

$$\sigma_{r,w}'' + k^2 \sigma_{r,w} = \frac{\epsilon_N^2}{\gamma_e^2 \sigma_{r,w}^3}$$

Describes how  $\sigma_r$  evolves under the combined influence of external (plasma) focusing and emittance pressure

$$k^2 = \frac{\omega_{pe}^2}{2\gamma_e c^2}$$

$$\sigma_m = \sqrt{\sqrt{\frac{2}{\gamma_e} \frac{c}{\omega_{pe}} \epsilon_N}}$$

# Q&A

[marlene.turner@cern.ch](mailto:marlene.turner@cern.ch)



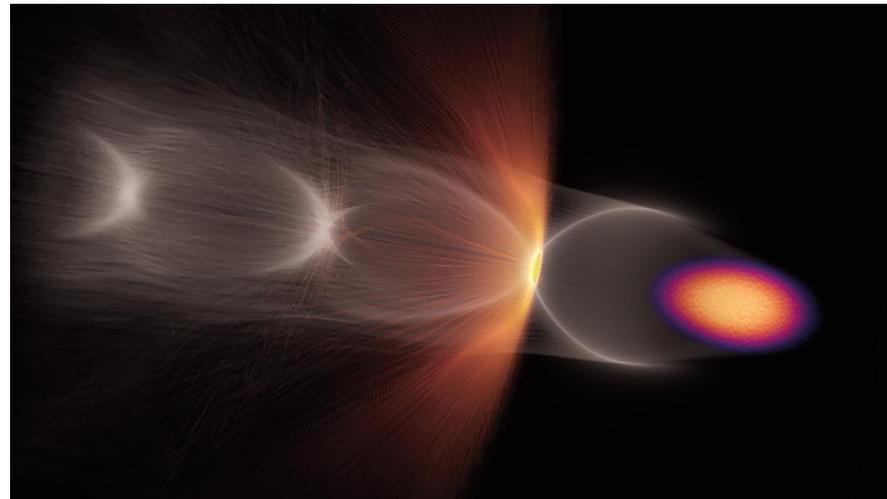
# 1.3 Others

**Taylor expansion:**

$$f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$\sqrt{1-x} \rightarrow 1 - \frac{1}{2x}$$

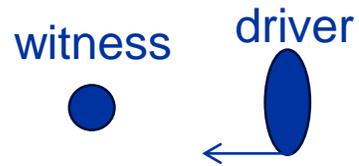
## 2.3 Acceleration Limits



# 2.3 Plasma Acts As a Transformer

Driver deposits energy, witness gains energy

Acceleration distance typically limited by either



**2.3.1 Depletion:** The drive bunch/pulse running out of energy. Solution: couple in a new driver or use a more energetic driver



**2.3.2 Dephasing:** The witness bunch outruns the driver. Solution: couple in a new driver or use change plasma target parameters



**2.3.3 Diffraction:** (mainly for laser pulses): Drive beam evolution. Solution: Guiding

# 2.3.1 Depletion

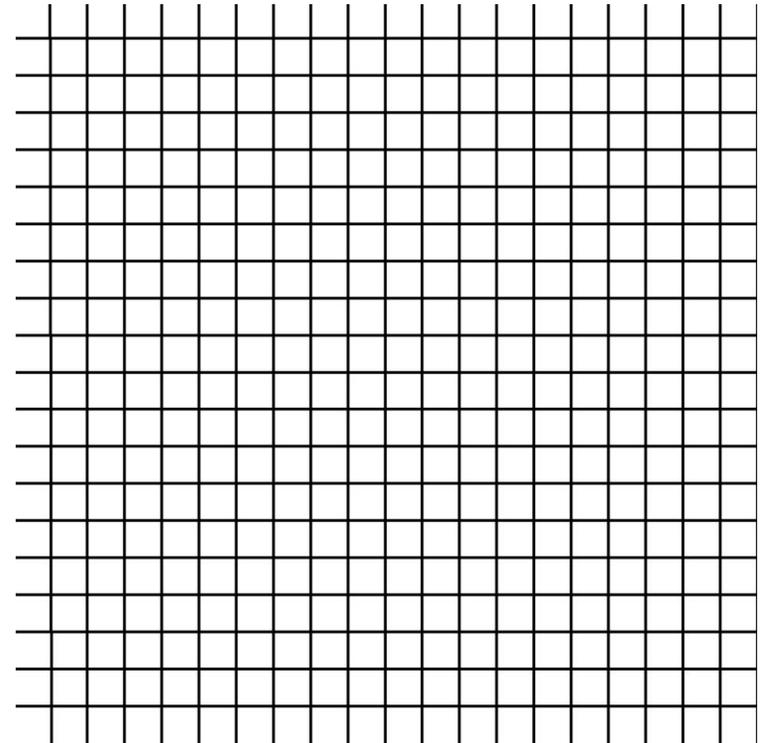
## 2.3.2 Dephasing

# 3.4.3 What About Efficiency?

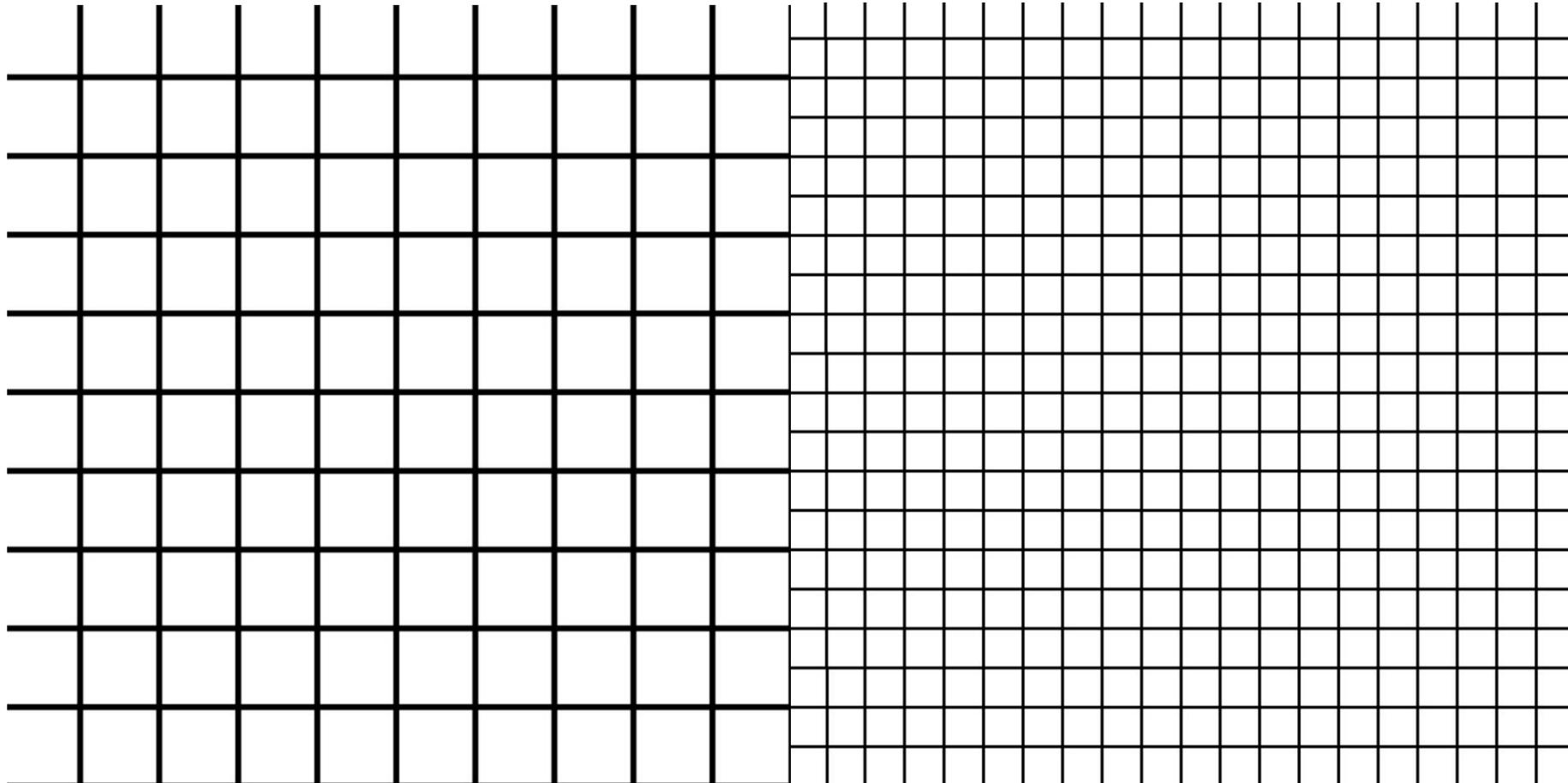
# 3.4.1 PWFA: Transformer Ratio

Transformer ratio:  $R = \frac{|E_{acc}|}{|E_{dec}|}$

- $R > 1$  means the witness can gain more energy per particle than the drive beam loses per particle
  - For a single symmetric drive bunch  $\rightarrow R \leq 2$
  - Shaped drive bunches (triangular, ramped) can reach  $R > 2$
- In plasma wakefield acceleration, achieving a large transformer ratio is desirable for:
  - Maximizing the single stage witness energy gain
  - Reducing the required drive beam energy



# Example: Shaping the Drive Bunch To Get $R > 2$

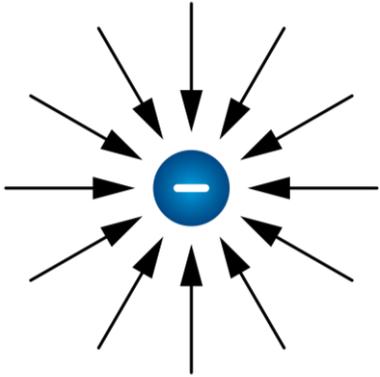


→ May want to shape your driver

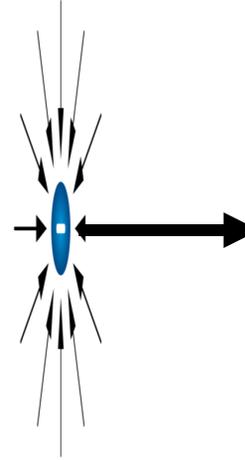
# **2.1 Driving Wakefields with Charged Particle Beams**

# 2.1 Driving Wakefields with Charged Particle Beams

E-field of charge(s) at rest (particle rest frame):



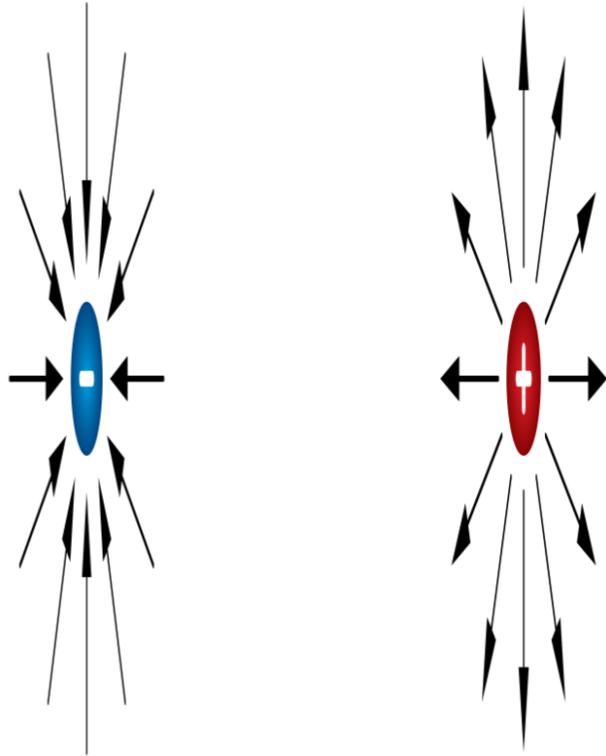
E-field of relativistic charge(s) :



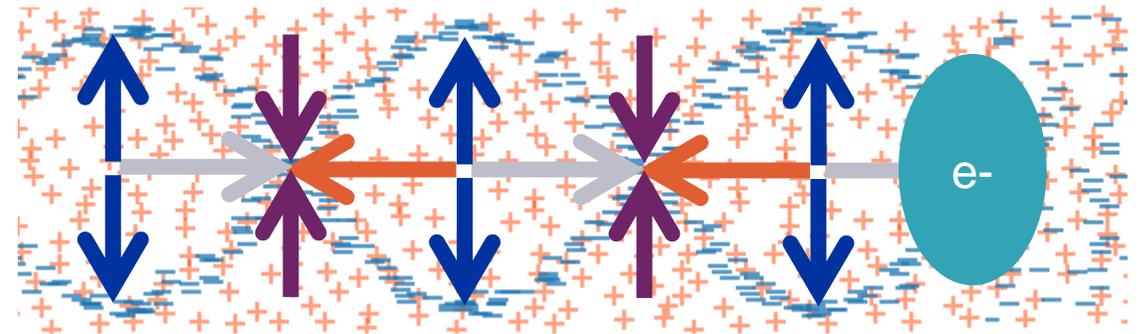
- **Field strength increases** by a factor  $\gamma$  in the perpendicular direction
- **E-field is compressed** into a narrow cone **perpendicular to motion**

# 2.1 Driving Wakefields with Charged Particle Beams

- **Acceleration requires an electric field**
- Particles **only accelerate** while **interacting** with the field



- Plasma allows to convert part of the **transverse field** of the bunch into a **longitudinal field**
- The longitudinal field propagates nearly at the speed of light ( $c$ )

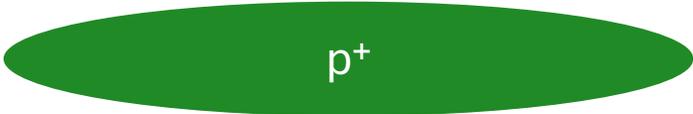




# 4.3 Plasma Wakefield Acceleration @ CERN

**AWAKE**

# 4.3 CERN Proton Bunches are Not Ideal for Driving Wakefields

$\sigma_r \sim 200 \mu\text{m}$    $p^+$   $N_{p^+} \sim 3 \times 10^{11}$   
 $n_b \sim 10^{13} \text{ cm}^{-3}$

$\sigma_z \sim 5 \text{ cm}$

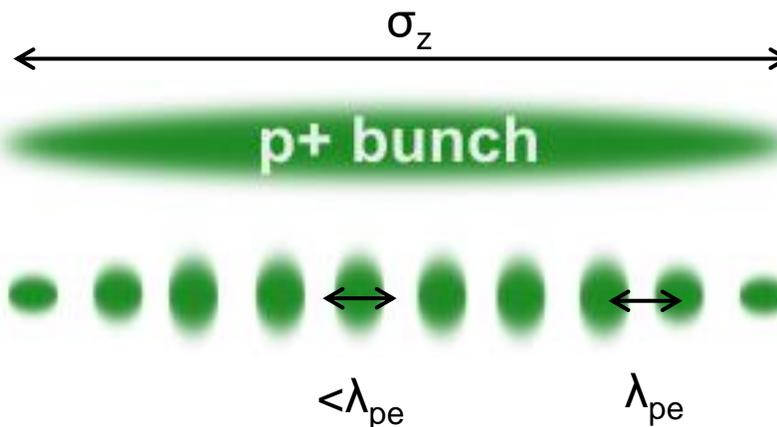
Ideal  $\sim \lambda_{pe}$   
That would mean...



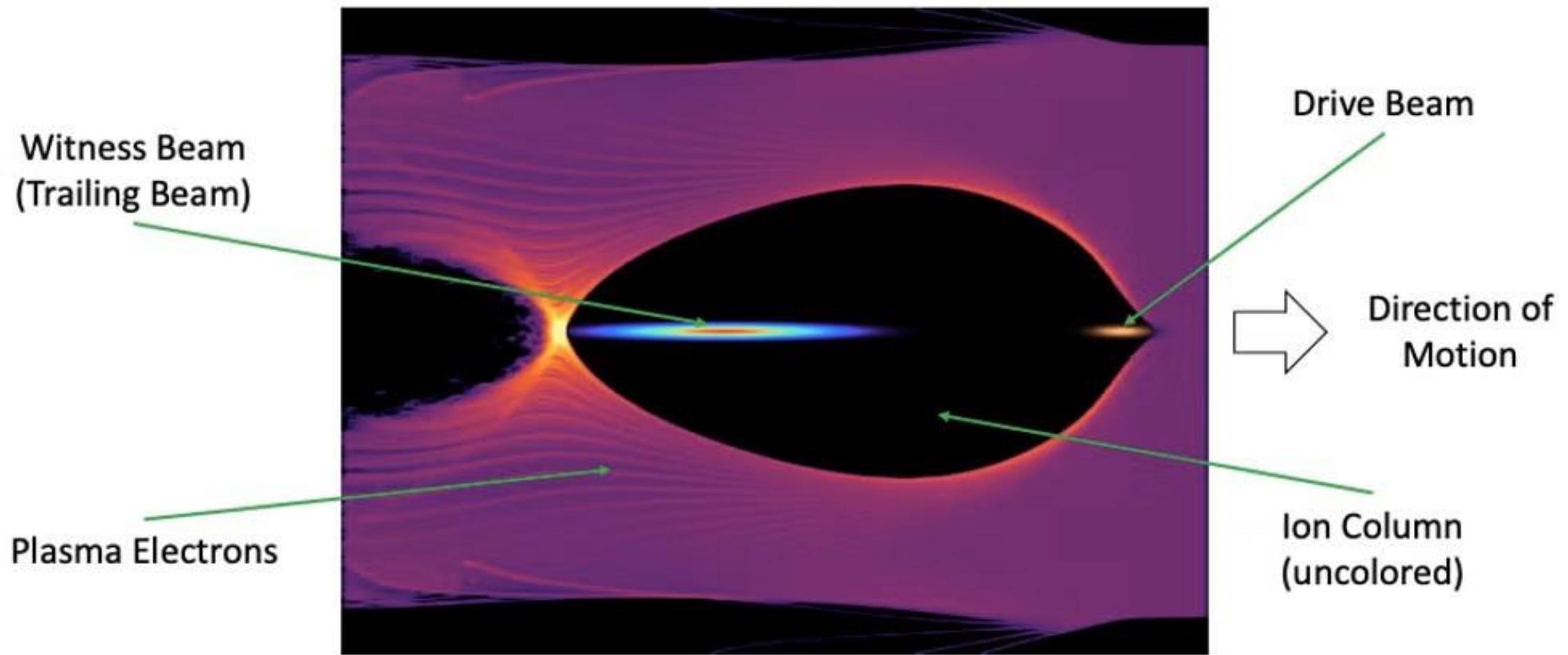
Exercise session

→ Not so interesting

Instead: Split the bunch into many shorter bunches  
→ Microbunches

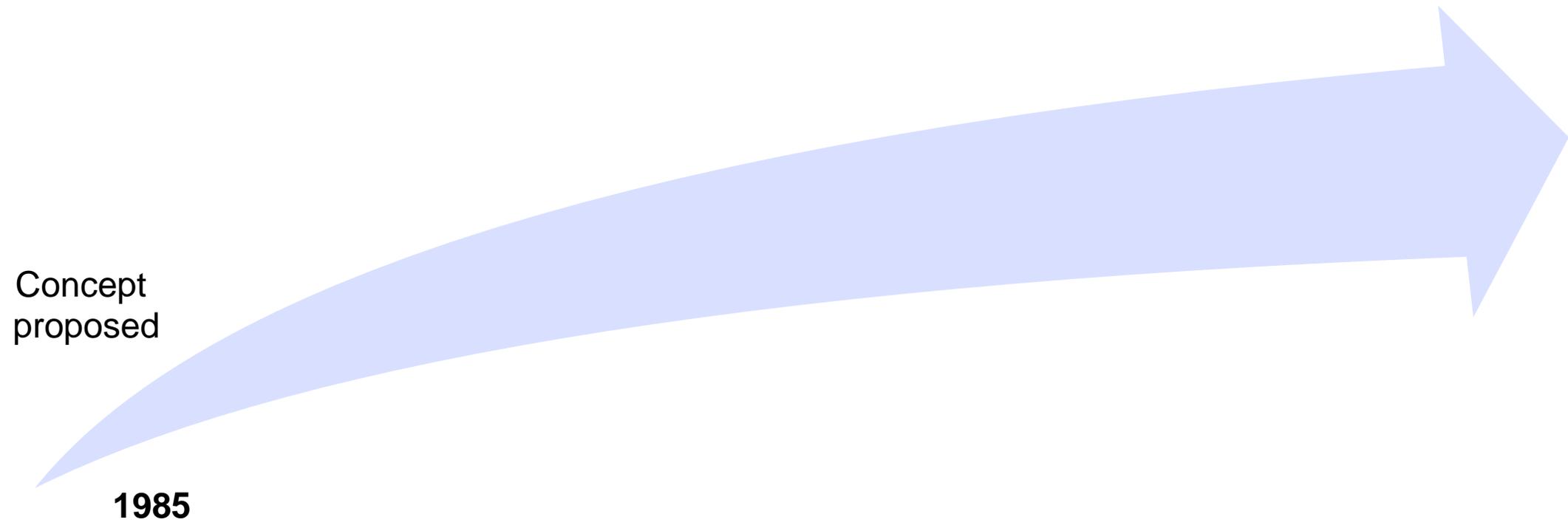


# 3.3 Some History of PWFA (Beam Driven Plasma Wakefield Acceleration)



From: <https://www.colorado.edu/lab/warg/>

# 3.3 PWFA



# 3.3 Concept Proposed (1985)

- Chen et al., propose using a high-current charged particle beam to drive plasma wakefields.
- P. Chen, J. M. Dawson, R. W. Huff & T. Katsouleas, *Phys. Rev. Lett.* **54**, 693 (1985).

Physics:

→ Derived scaling laws for wakefield excitation by relativistic particle beams

$$E_{acc} \sim \frac{n_b}{n_{pe}} E_{WB}$$

→ Showed that accelerating gradients could reach the GeV/m scale

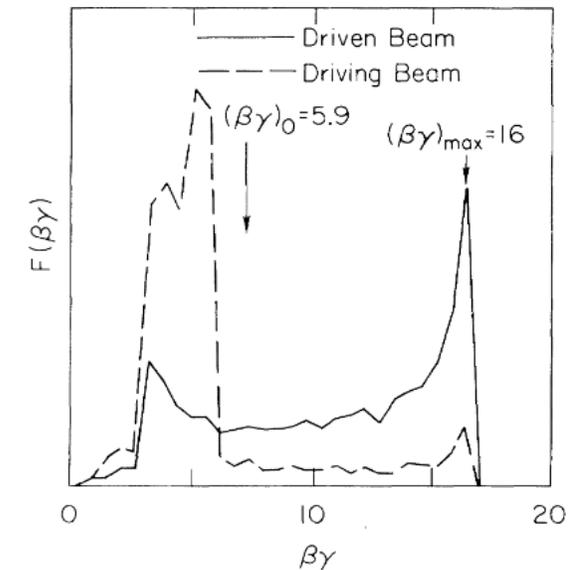
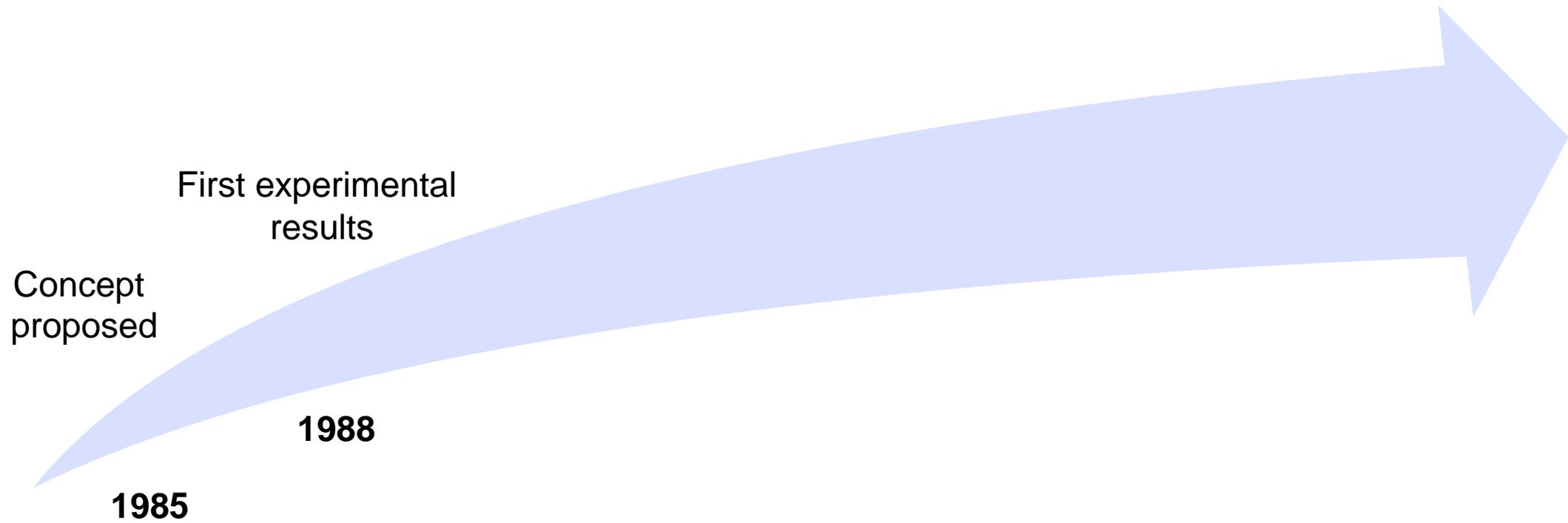


FIG. 2. Momentum distribution of the driving- and driven-electron beams when the latter has attained its maximum upper limit.

# 3.3 PWFA



# 3.3 First Experimental Results

## Late 1980s–1990s – First experimental results

- Early experiments demonstrate plasma wake excitation by, **short** low energy electron beams.
- Exploration of linear vs nonlinear regimes and beam loading in theory/simulations.

J. B. Rosenzweig *et al.*, *Phys. Rev. Lett.* **61**, 98 (1988):

→ First experimental demonstration of beam-driven plasma wakefield acceleration (21 MeV driver,  $n_{pe} \sim 10^{13}/\text{cm}^{-3}$ )

→ The plasma wakefields generated were on the order of **MeV/m**, plasma length  $\sim 30$  cm

→ Observed also the transverse fields → indicating that the plasma wakefield can accelerate and focus particles

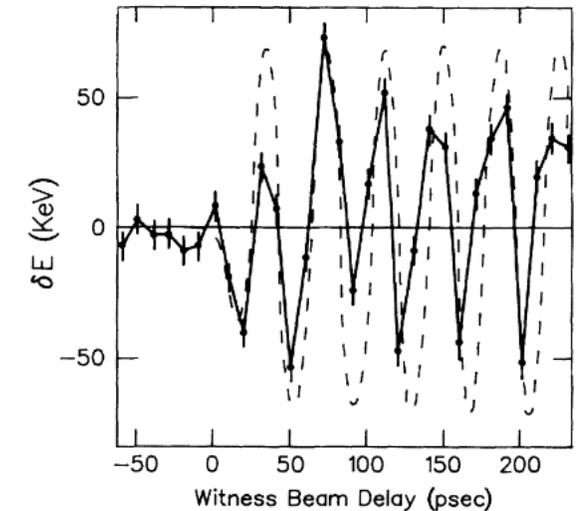
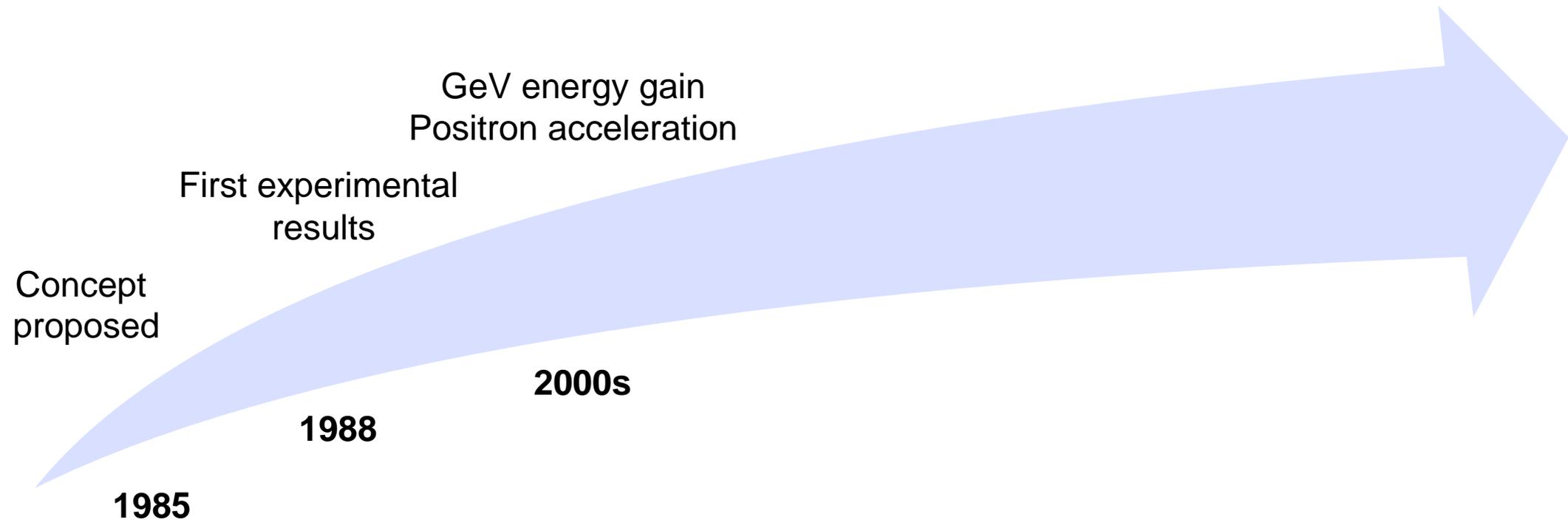


FIG. 2. Scan 1: Witness-beam energy-centroid change  $\delta E$  vs time delay behind driver. Total driver-beam charge  $Q = 2.1$  nC; plasma parameters  $L = 28$  cm and  $n_e = 8.6 \times 10^{12} \text{ cm}^{-3}$ . Theoretical predictions are given by the dashed line.

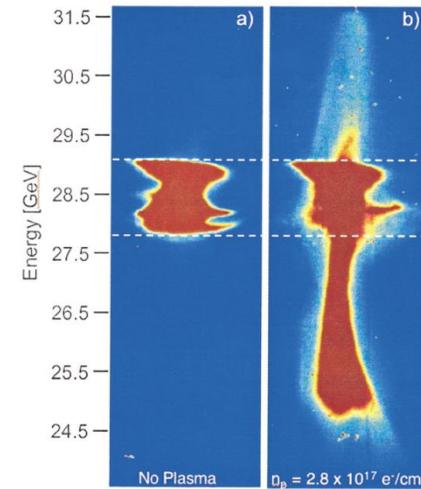
# 3.3 PWFA



# 3.3 GeV e<sup>-</sup> Energy Gain; e<sup>+</sup> Acceleration

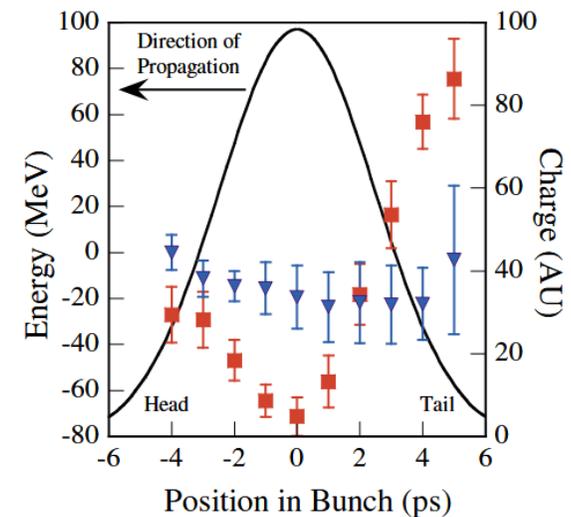
## 2005 – Multi-GeV Energy Gain in a Plasma-Wakefield Accelerator

- ~30 GeV, short (20 μm) electron beams become available
- *M. J. Hogan et al., Phys. Rev. Lett. 95, 054802 2005*

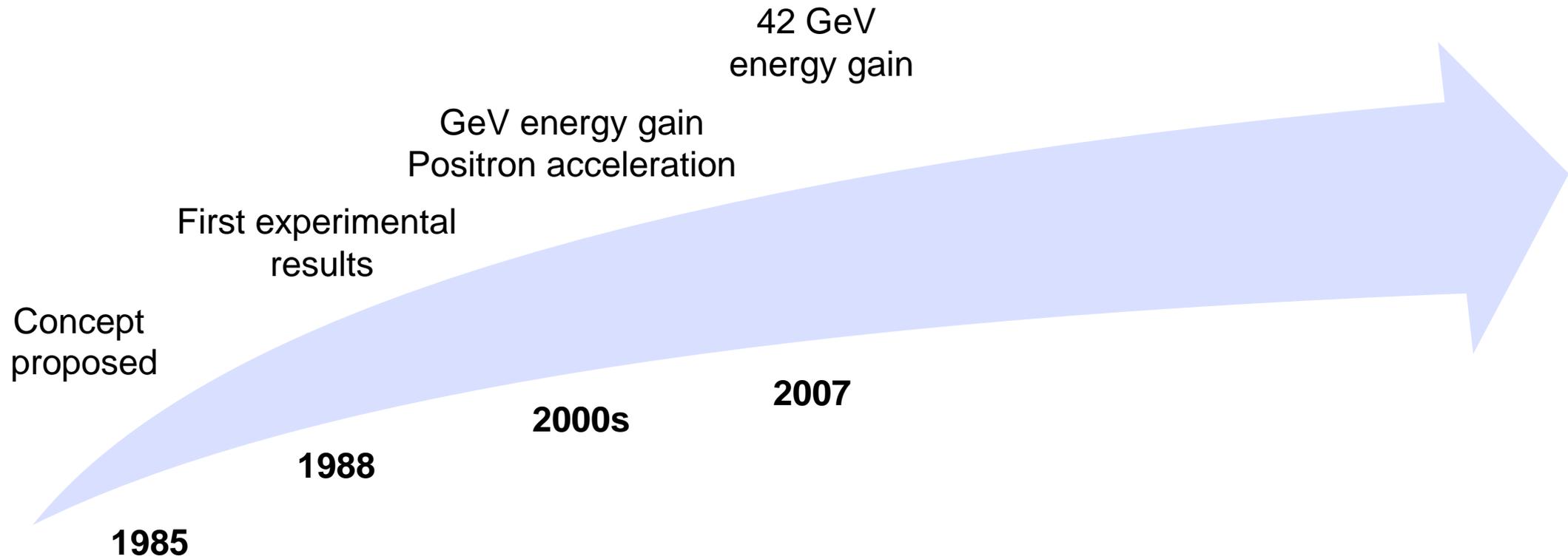


## 2003 – Positron acceleration

- ~30 GeV, short (200 μm) positron beams become available
- The bunch **suffers emittance growth**
- *P. Muggli et al., Phys. Rev. Lett. 101, 055001 2008*



# 3.3 PWFA



# 3.3 42 GeV Energy Gain

## 2007 – Multi 10's of GeV energy gain

- Short, dense multi-GeV beam becomes available
- Demonstrates PWFA as a viable high-gradient, high-energy accelerator concept.

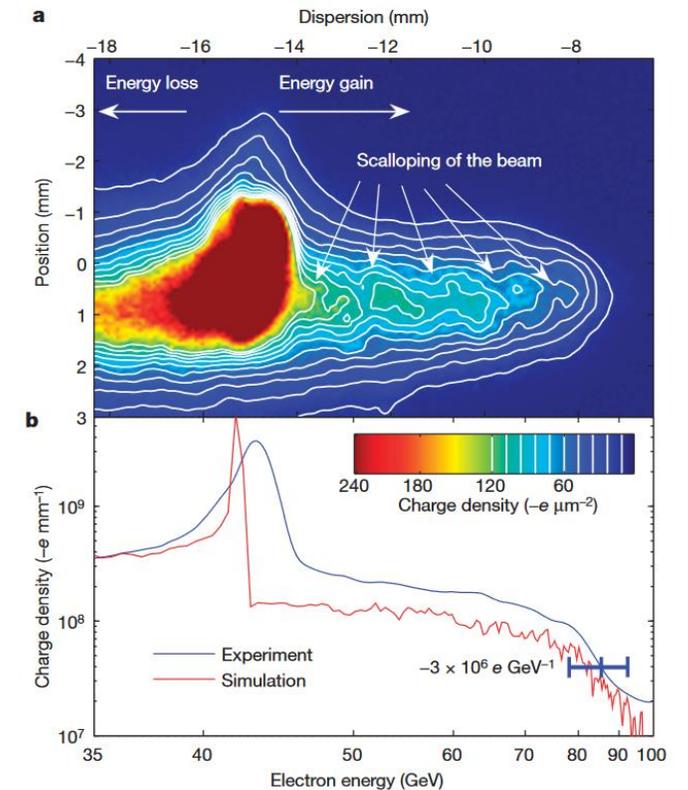
I. Blumenfeld *et al.*, *Nature* **445**, 741 (2007):

→ Driver: 42 GeV electron bunch

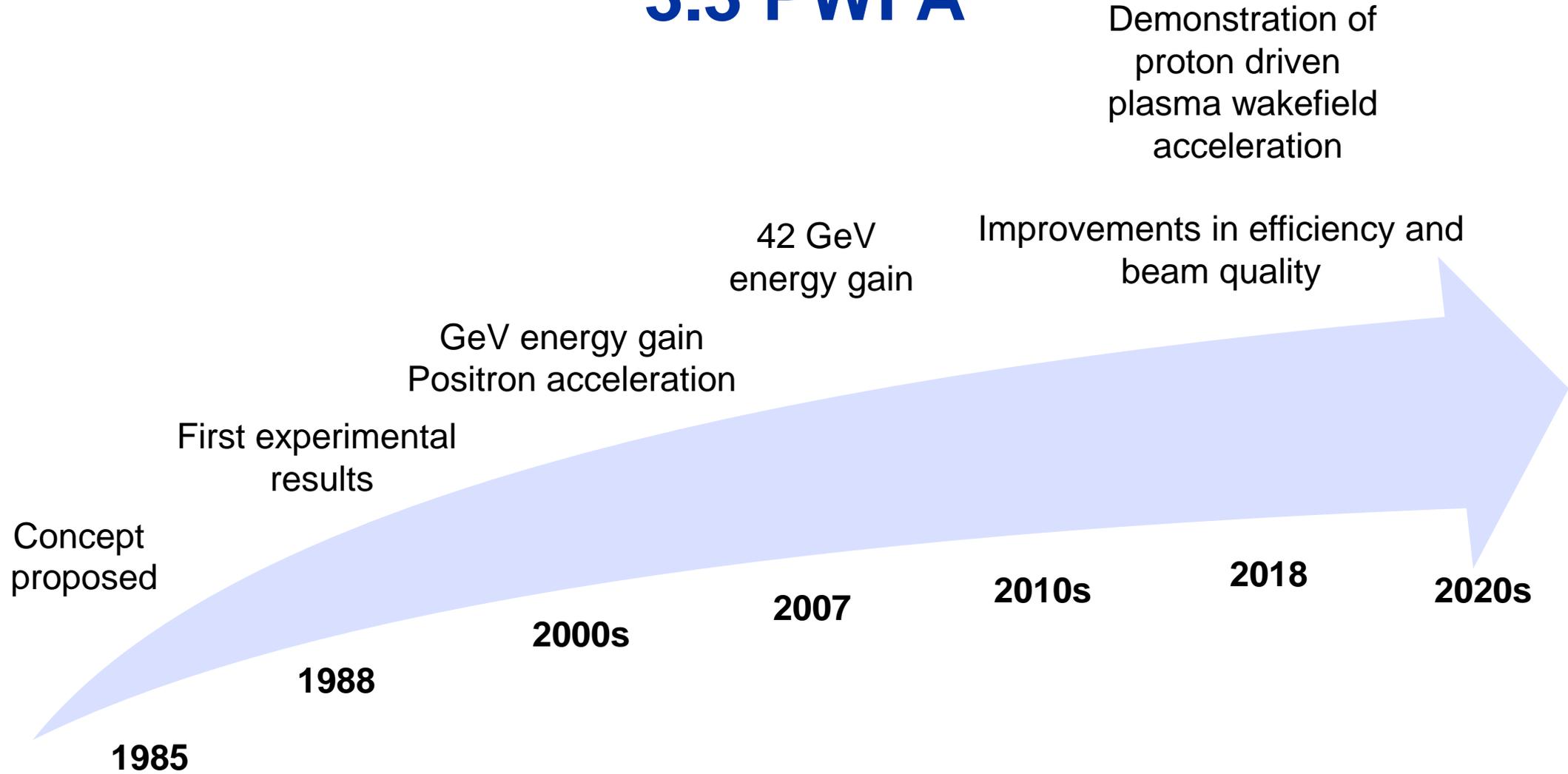
→ 85 cm long lithium vapor plasma

→ Acceleration Gradient: approximately 52 GeV/m

→ Energy Gain: Some electrons at the rear of the bunch experienced energy gains exceeding 42 GeV



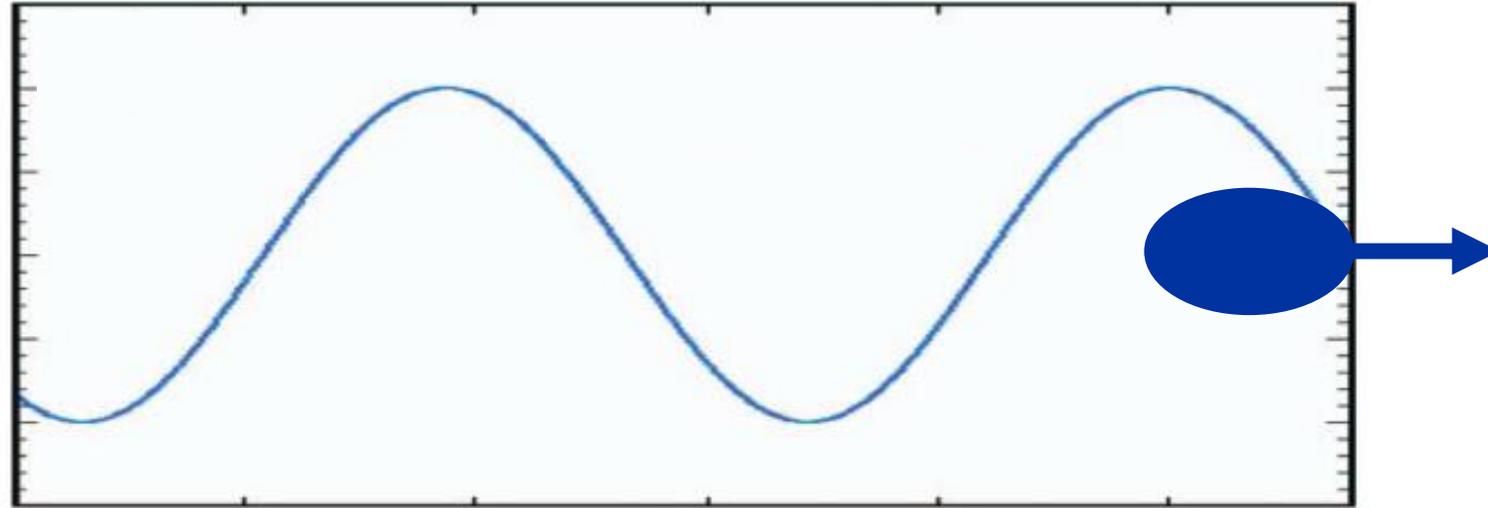
# 3.3 PWFA



# 3.4.2 Beam Loading

→ Allows to Minimize Energy Spread

The situation:

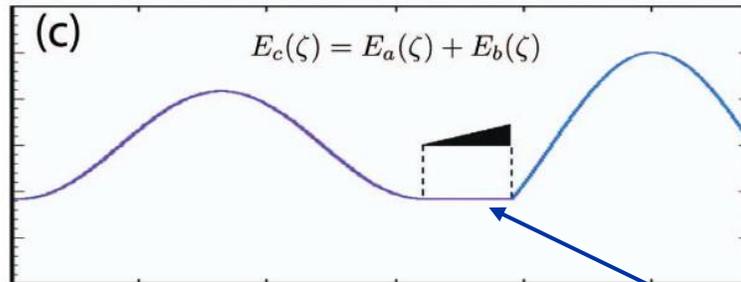
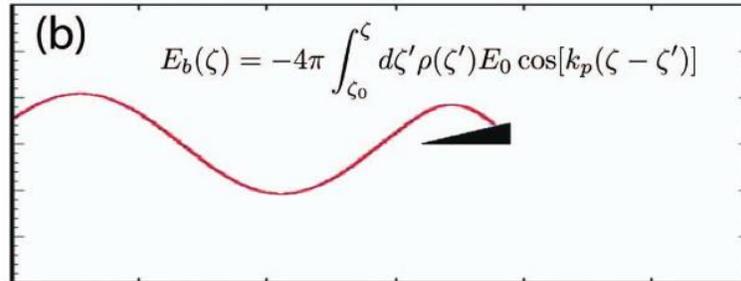
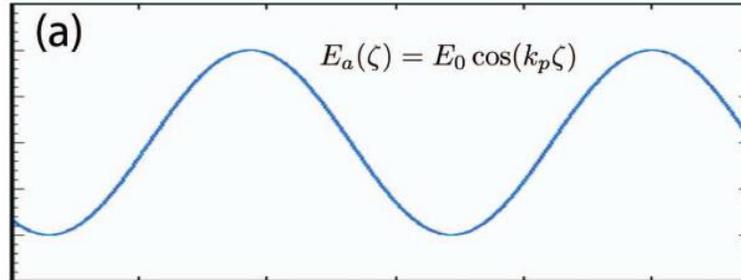


- The witness beam modifies (loads) the wakefield
- The goal is to load the wake just right, so the field is flat → uniform acceleration
- Higher loading → lower accelerating field

# 3.4.2 Beam Loading

→ Allows to Minimize Energy Spread

The situation:



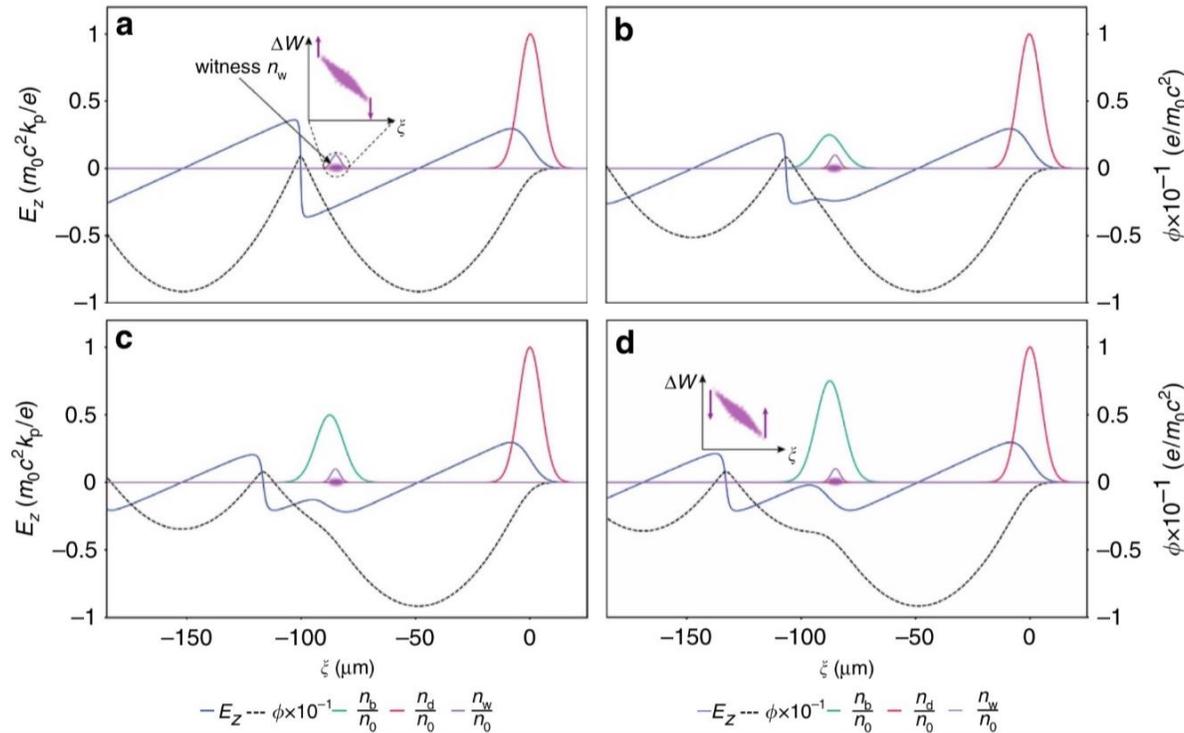
Every particle of the witness bunch accelerates at the same gradient → No increase in energy spread

- Beam loading
- The goal is t

T. Katsouleas et al. (1987)

# 3.4.2 Beam Loading

- Requires the correct witness charge density and shape
  - (a) underloading → energy spread
  - (b) proper loading flattens the accelerating field → minimizes energy spread
  - (c,d) overloading → energy spread



Exercise session

Estimate for optimum loading:

Nonlinear regime:

$$Q_W [\text{nC}] \sim \frac{E_{acc}}{E_{WB}} 0.047 \sqrt{\frac{10^{16} [\text{cm}^{-3}]}{n_{pe} [\text{cm}^{-3}]}} (k_{pe} R_b)^4$$

Beam Driven:

$$k_{pe} R_b \simeq 2 \sqrt{\frac{n_b}{n_{pe}}} (k_{pe} \sigma_r)^2$$

Laser Driven:

$$k_{pe} R_b \simeq 2 \sqrt{a_0}$$

From: [https://www.stonybrook.edu/com/mcims/case/\\_Phy\\_s694PDFs/Lec%20Set%207%20-%20Beam%20loading.pdf](https://www.stonybrook.edu/com/mcims/case/_Phy_s694PDFs/Lec%20Set%207%20-%20Beam%20loading.pdf)

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