Energy correlations of spinning sources

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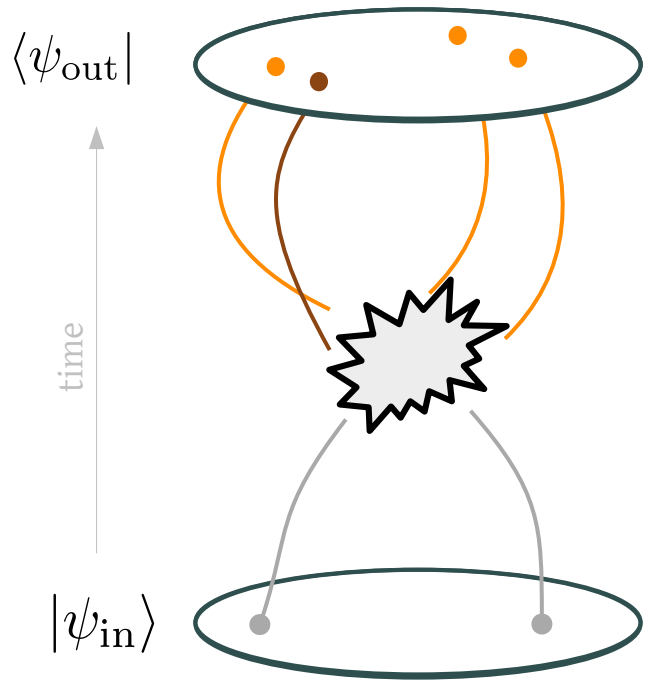
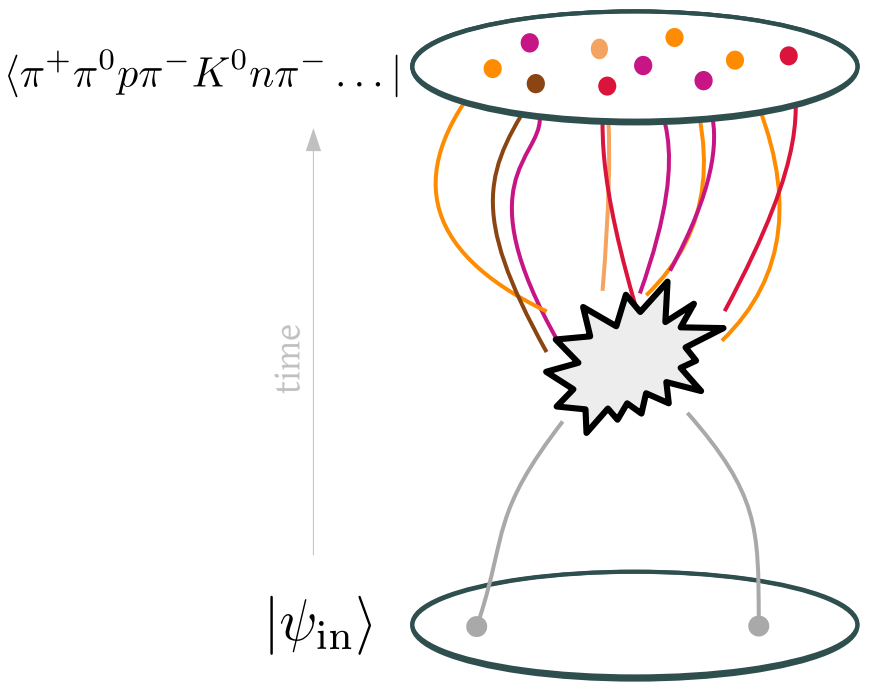
CERN-CKC Busan – November 2025



Collider experiments transform an initial state, e.g. pp, into a final state.

(Almost) all we know is based on the different production rates of different states.

Fine for theories with a mass gap and suppressed multiparticle production.



Collider experiments transform an initial state, e.g. pp, into a final state.

(Almost) all we know is based on the different production rates of different states.

Fine for theories with a mass gap and suppressed multiparticle production.

Not the case of real world!

Worse at high energies:
what does “diboson” even mean at √s=10TeV?

What do you do when multiparticle production is unsuppressed?
Need to “coarse grain” your Hilbert space into jets… matching, merging…

Jets from Quantum Chromodynamics

George Sterman and Steven Weinberg
Phys. Rev. Lett. **39**, 1436 – Published 5 December 1977

Discovery of Three-Jet Events and a Test of Quantum Chromodynamics at PETRA

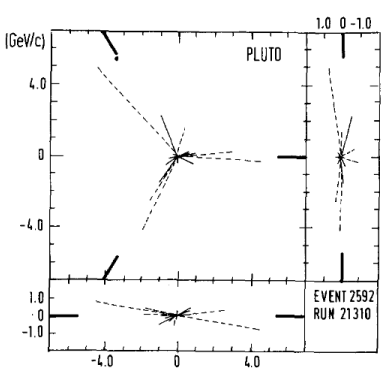
D. P. Barber *et al.*
Phys. Rev. Lett. **43**, 830 – Published 17 September 1979

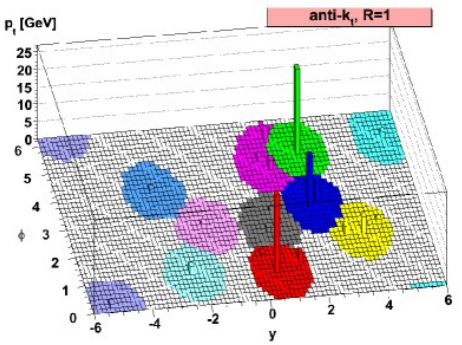
ELSEVIER

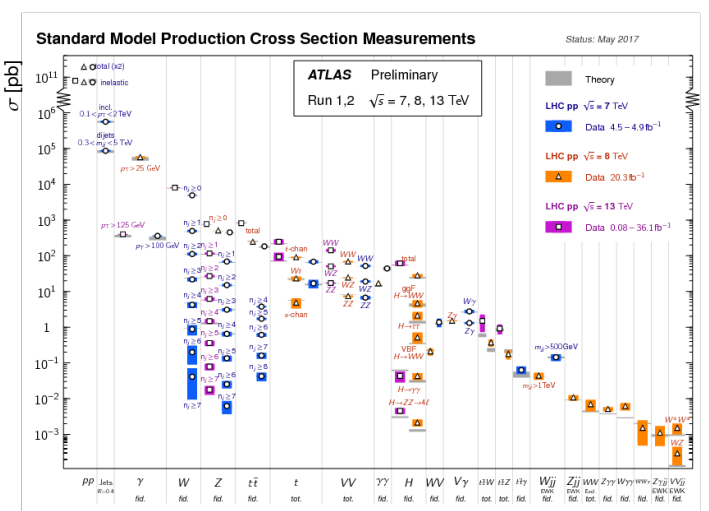
Physics Letters B
Volume 86, Issues 3–4, 8 October 1979, Pages 418-425

Evidence for gluon bremsstrahlung in e^+e^- annihilations at high energies

PLUTO Collaboration, Ch. Berger, H. Genzel, R. Grigull, W. Lackas, F. Raupach, A. Klovning, E. Lillestöl, E. Lillethun, J.A. Skard, H. Ackermann, G. Alexander [3], F. Barreiro, J. Bürger, L. Criegee, H.C. Dehne, R. Devenish [4], A. Eskreys [5], G. Flügge, G. Franke…K. Wacker [13]



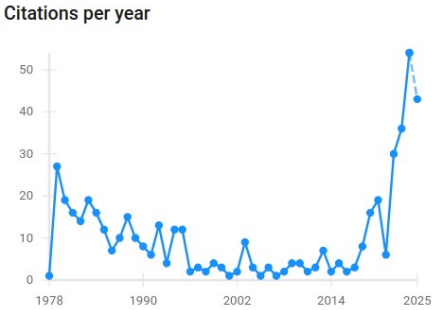




A different set of observables: correlators

Energy Correlations in Electron-Positron Annihilation: Testing Quantum Chromodynamics

C. Louis Basham, Lowell S. Brown, Stephen D. Ellis, and Sherwin T. Love
Department of Physics, University of Washington, Seattle, Washington 98195
(Received 21 August 1978)



- Energy weights have an operatorial definition

Sveshnikov, Tkachov ‘95

$$\mathcal{O}_n = \lim_{r\to\infty} \int dt r^2 n_i T_{i0}(t, r\hat{n}) \quad \Rightarrow \quad \mathcal{O}_n \sim \int d^4k \delta(k^2)\delta^{(2)}(\Omega_{\vec{k}} - \Omega_{\vec{n}})\, k^0\, a_k^\dagger a_k$$

In hindsight, this was a breakthrough. As long as the operator is well defined, as is the case of the energy operator, this gives a perfectly robust definition of observables in a gauge theory,
avoiding the theoretical nuance of defining an S-matrix for a gauge theory, or gravity.

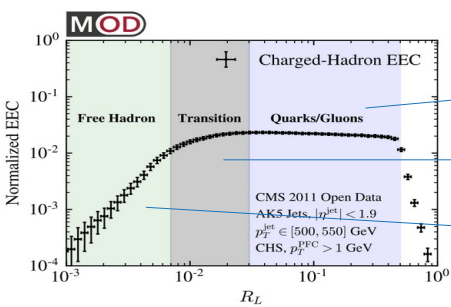
- Energy weights have an OPE

Hoffman, Maldacena ‘08

$$\mathcal{E}(n_1)\mathcal{E}(n_2) = \frac{1}{\theta_{12}^2} \int dt\, r^2\, \bar{\Psi}\gamma^+ D^+ D^+ \Psi + \dots$$

- Measured in CMS open data

Komiske, Moult, Thaler, X. Zhu ‘22



$$1 \gg \theta \gg \Lambda_{QCD}/E_{jet}$$

$$\theta \sim \Lambda_{QCD}/E_{jet}$$

$$\Lambda_{QCD}/E_{jet} \gg \theta$$

Despite the recent explosive progress, many important questions are still unanswered.
e.g:

How is information stored in the correlators?
How do I differentiate two S-matrices from their correlators?

WIP w/Durieux, Son, Lo Chiatto

In this talk, I will focus on a more elementary, but necessary, question:

What is the kinematics of the N-point correlator?

251X.XXXXX MR, M. Son

Why energy?

- For a global charge with an associated conserved current J^μ,

$$\mathcal{D}_n = \lim_{r\to\infty} \int_0^\infty dt r^2 n_i J^i(t, r\vec{n})$$

is a *detector* that measures the asymptotic flux of charge in the direction $\vec{n}$,

acting on physical multiparticle states as

$$\mathcal{D}_n|\alpha\rangle = \sum_i q_i \delta^{(2)}(\Omega_n - \Omega_i)|\alpha\rangle$$

- In the presence of an operator exciting the QCD vacuum, the one point correlator is given by

$$\langle \mathcal{D}_n \rangle = \frac{1}{\mathcal{N}} \int d^4x e^{ip\cdot x} \langle\, \mathcal{O}^\dagger(x) \mathcal{D}_n\, \mathcal{O}(0)\rangle$$

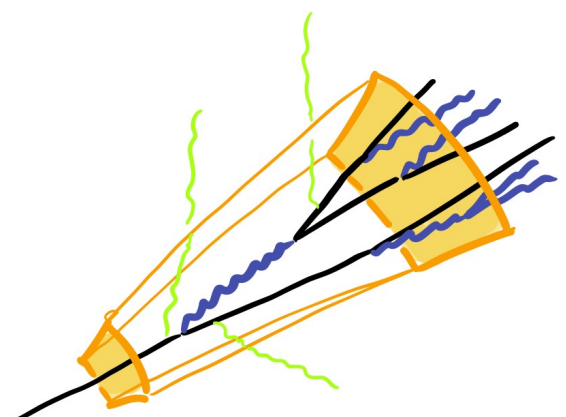
$$= \frac{1}{\mathcal{N}} \sum_{\alpha,i} \delta^{(d)}(p - p_\alpha)\, \omega_i \delta^{(2)}(\Omega_n - \Omega_i) |\langle \alpha|\, \mathcal{O}|0\rangle|^2$$

- That J^μ is conserved has two important consequences:

  i) IR finiteness against soft and collinear radiation.

  Since the conserved charge is preserved under collinear radiation, the detector acts homogeneously on the set of collinear particles, measuring the total charge.

  It is fully inclusive on the rest of the event, ensuring the cancellation of collinear IR divs.



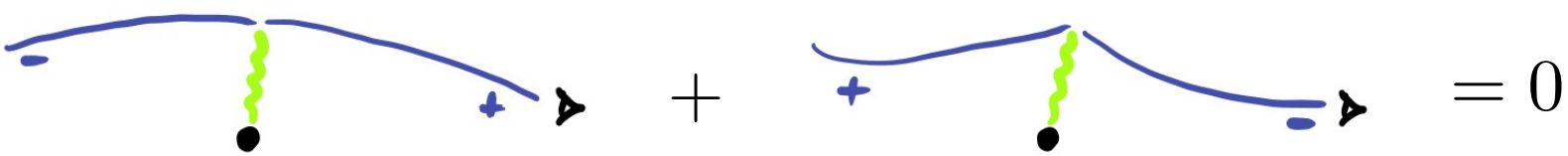
- That J^μ is conserved has two important consequences:

  i) IR finiteness against soft and collinear radiation.

  Soft radiation is instead annihilated by detectors of conserved charges.
  Using universality of soft interactions, a soft photon/gluon contributes as

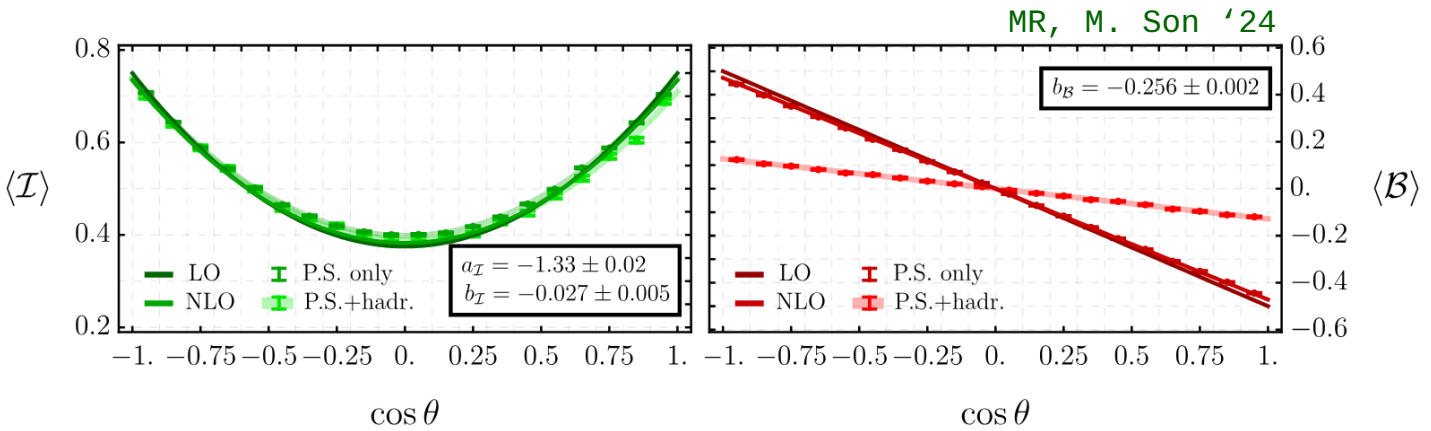
$$\int d\Phi_2 \frac{p_q^\mu p_{\bar{q}}^\nu + p_q^\nu p_{\bar{q}}^\mu - q^2/2\eta^{\mu\nu}}{q^2} \sum_{i=q\bar{q}} w_i \delta^{(2)}(\Omega_i - \Omega_n)$$

  which vanishes due to opposite fermion anti-fermion charges



Previous statements require (often underappreciated/ignored) clarifications:

- They are required for the cancellation of IR divergences order by order in perturbation theory, but there might be large non-perturbative corrections. e.g.: isospin and baryon number densities:



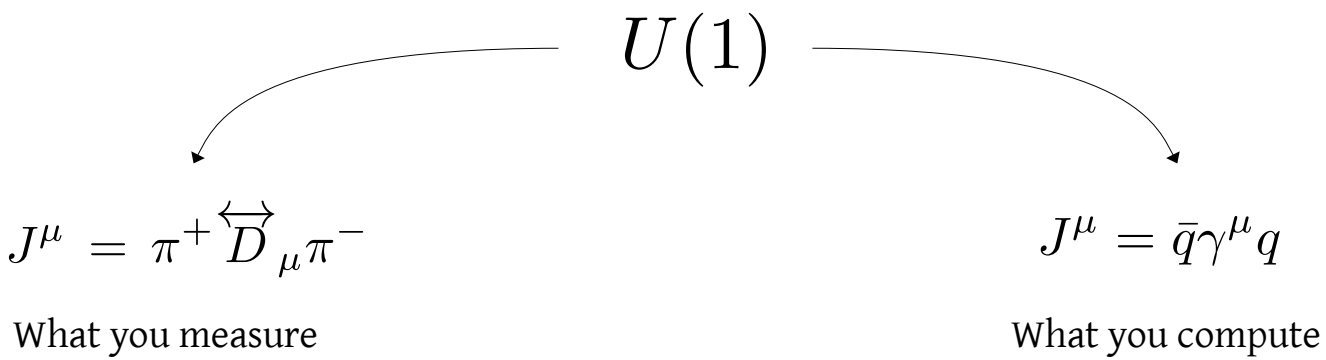
- Annihilation of soft sector by energy might be spoiled if there is a large number of soft quanta, of order $\sim Q/\Lambda_{IR}$.

  Ok(-ish) for QCD.

  A relevant counterexample is BH evaporation. All energy radiated by soft quanta.

- That J^μ is conserved has two important consequences:

  ii) Vanishing of anomalous dimension



In general, operators get anomalous dimensions. Non-conserved quantities at hadron level are mapped to unknown operators at quark level.

Symmetry allows to map between scales.

MR, M. Son ‘24

- Beyond one point:

  As long as detectors measure conserved charges,

  $$\langle \mathcal{D}_{n_1} \mathcal{D}_{n_2} \cdots \mathcal{D}_{n_m} \rangle$$

  is collinear safe for sufficiently separated directions.

- IR-safety under soft emissions puts restrictions on the operators, as they are forced to annihilate the soft sector.

- This happens if one of the detectors measures the energy, or a quark flavour highly suppressed during parton shower, like *b* or *c*.

$\langle \mathcal{Q}_{n_1} \mathcal{Q}_{n_2} \rangle$ ⊗ $\langle \mathcal{E}_{n_1} \mathcal{Q}_{n_2} \rangle$ ✓ $\langle \mathcal{E}_{n_1} \mathcal{E}_{n_2} \mathcal{Q}_{n_3} \rangle$ ✓

$\langle \mathcal{I}_{n_1} \mathcal{B}_{n_2} \rangle$ ⊗ $\langle b_{n_1} \mathcal{B}_{n_2} \rangle$ ✓

*In the back-to-back limit, *QQC* becomes convolution of two one-point, so IR-safe. Monni, Vita, Xu, Zhu ‘25

Back to the N-point kinematics:

- All information of the correlator encoded in the (fully differential) density matrix:

$$\langle \mathcal{E}_{n_1} \dots \mathcal{E}_{n_N} \rangle_{hh'} \;=\; \frac{1}{\mathcal{N}} \int d^4x \, e^{iq\cdot x} \, \langle \epsilon^*_{h'} \cdot J(x) \, \mathcal{E}_{n_1} \dots \mathcal{E}_{n_N} \, \epsilon_h \cdot J \rangle$$

- In the rest frame, $q^\mu = (Q, \vec{0})$, $n_i^\mu = (1, \vec{n}_i)$,

$$\langle \mathcal{E}_{n_1} \dots \mathcal{E}_{n_N} \rangle_{h'h} \;=\; \frac{1}{\mathcal{N}} \, \epsilon^{*a}_{h'} H^{ab}_{\mathcal{E}^{(N)}} \epsilon^b_h$$

The hadronic tensor can be split into the trace and the symmetric traceless part,

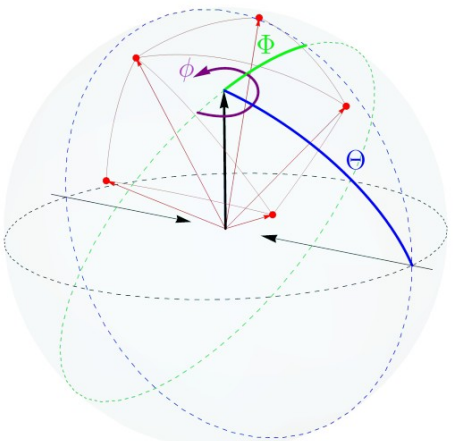
$$H^{ab}_{\mathcal{E}^{(N)}} \;=\; H^{\mathbb{1}}_{\mathcal{E}^{(N)}}(z_{ij}) \, \delta^{ab} \;+\; \sum_k H^{(k)}_{\mathcal{E}^{(N)}}(z_{ij}) \, S^{ab}_{(k)}$$

Inclusive correlator

Spinning energy correlator

- The density matrix of correlators can be written as

$$\langle \mathcal{E}_{n_1} \dots \mathcal{E}_{n_N} \rangle_{h'h} = \sum_{J=0}^{J=2} \sum_{\lambda,\lambda'} c^J(h,h',\lambda,\lambda') \; (H_{\mathcal{E}^{(N)}})_{\lambda'\lambda}(z_{ij}) \; D^J_{h'-h,\lambda'-\lambda}(\Phi,\Theta,\phi)$$



- Phase of the Wigner D-matrix depend on the polarization of the source and the polarization of the *rigid body* of detectors.
- Spinning part does depend nontrivially on the internal angles.
- Information lost for inclusive observables.

- The hadronic tensor can be written as

$$H^{ab}_{\mathcal{E}^{(N)}} = \sum_{\alpha} \sum_{i_1 \ldots i_N \in \alpha} (2\pi)^4 \delta^{(4)}(p - p_\alpha) w_{i_1,1} \cdots w_{i_N,N} \langle 0|J^{a\dagger}|\alpha\rangle\langle\alpha|J^b|0\rangle$$

  Where the weights are the energies measured by the detectors $w_{i_k,k} = E_{i_k} \delta^{(2)}(\Omega_{i_k} - \Omega_{n_k})$ and therefore

$$H^{ab}_{\mathcal{E}^{(N)}} \succ 0$$

- This bounds the spinning correlators in terms of the inclusive one

- Inclusive and spinning correlators depend on the internal angles

$$z_{ij} \equiv \frac{1-\cos\theta_{ij}}{2}$$

Out of the *N(N-1)/2* combinations, only *2N-3* independent.

- System characterized by the Gram matrix

$$G_{ij} = \vec{n}_i \cdot \vec{n}_j = 1 - 2z_{ij}$$

It is positive definite. Positiveness of *2x2* and *3x3* minors implies

$$z_{ij}(1-z_{ij}) \geq 0 \quad \text{, and} \quad -4e_3 + 4e_2 - e_1^2 \geq 0 \quad \text{with}$$

$$e_1 = z_{ij} + z_{jk} + z_{ki}$$
$$e_2 = z_{ij}z_{jk} + z_{jk}z_{ki} + z_{ki}z_{ij}$$
$$e_3 = z_{ij}z_{jk}z_{ki}$$

- Gram matrix is rank 3, and therefore only 2N-3 entries are indeed independent.

Note not all 2N-3 subset of coordinates fully specify the rest.

- The hadronic tensor of the one-point correlator is

$$H_{\mathcal{E}}^{ab} \;=\; H_{\mathcal{E}}^{\mathbb{1}} \left(\delta^{ab} \,+\, a_{\mathcal{E}} \left(3n^a n^b - \delta^{ab}\right)\right)$$

Positivity implies

$$-\frac{1}{2} \leq a_{\mathcal{E}} \leq 1$$

Hofman, Maldacena '08

- The previous discussion allows to better understand this result:

$$\frac{H_{\mathcal{E}}}{H_{\mathcal{E}}^{\mathbb{1}}} \;=\; 3/2(\epsilon_+^*\epsilon_+ + \epsilon_-^*\epsilon_-)\left(1 - \frac{a_{\mathcal{E}} + 1/2}{3/2}\right) \,+\, 3\epsilon_0\epsilon_0 \left(\frac{a_{\mathcal{E}} + 1/2}{3/2}\right)$$

Hadronic matrix is a convex sum of a $\lambda = \lambda' = 0$ state and a $\lambda = -\lambda' = \pm$ state.

$a_{\mathcal{E}} = 1$ ⟶ $J^\mu = \phi^* \overleftrightarrow{D}^\mu \phi$, $\mathcal{L} \supset \bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu}$

$a_{\mathcal{E}} = -\frac{1}{2}$ ⟶ $J^\mu = \bar{\psi} D^\mu \psi$, $\mathcal{L} \supset \phi F^{\mu\nu}\widetilde{F}_{\mu\nu}$, $J^\mu \sim \epsilon^{\mu\nu\rho\sigma} p_{1\nu} p_{2\rho} p_{3\sigma}$

Matter content is irrelevant, the relevant feature is the spin along the detector.

- Generalization to arbitrary spin:

$$\epsilon^{*a_1\cdots a_J}_{\lambda'} H^{a_1\ldots a_J b_1\ldots b_J}_{\mathcal{E}} \epsilon^{b_1\cdots b_J}_{\lambda} = \langle Jm_1 Jm_2|H_{\mathcal{E}}\rangle = \sum_{j=0}^{J} H_{2j}\langle Jm_1 Jm_2|2j,0\rangle$$

- Recovery of the one-point constraints: instead of asking positivite definiteness of the hadronic tensor, we ask for positivity of any polarized source:

$$\frac{\langle 1010|H_{\mathcal{E}}\rangle}{\langle 1010|00\rangle}\frac{1}{H_0} = 1 + \frac{H_2}{H_0}\frac{\langle 1010|20\rangle}{\langle 1010|00\rangle} = 1 + 2\left(-\frac{1}{\sqrt{2}}\frac{H_2}{H_0}\right) \geq 0$$

$$\frac{\langle 111\text{-}1|H_{\mathcal{E}}\rangle}{\langle 111\text{-}1|00\rangle}\frac{1}{H_0} = 1 + \frac{H_2}{H_0}\frac{\langle 111\text{-}1|20\rangle}{\langle 111\text{-}1|00\rangle} = 1 - \left(-\frac{1}{\sqrt{2}}\frac{H_2}{H_0}\right) \geq 0$$

$$\rightarrow \quad -\frac{1}{2} \leq -\frac{1}{\sqrt{2}}\frac{H_2}{H_0} = a_{\mathcal{E}} \leq 1$$

- This language allows to trivially extend to any spin:

$$\frac{\langle 2020|H_{\mathcal{E}}\rangle}{\langle 2020|00\rangle}\frac{1}{H_0} = 1 - \sqrt{\frac{10}{7}}\frac{H_2}{H_0} + 3\sqrt{\frac{2}{7}}\frac{H_4}{H_0} \geq 0$$

$$\frac{\langle 212\text{-}1|H_{\mathcal{E}}\rangle}{\langle 212\text{-}1|00\rangle}\frac{1}{H_0} = 1 - \sqrt{\frac{5}{14}}\frac{H_2}{H_0} - 2\sqrt{\frac{2}{7}}\frac{H_4}{H_0} \geq 0$$

$$\frac{\langle 222\text{-}2|H_{\mathcal{E}}\rangle}{\langle 222\text{-}2|00\rangle}\frac{1}{H_0} = 1 + \sqrt{\frac{10}{7}}\frac{H_2}{H_0} + \sqrt{\frac{1}{14}}\frac{H_4}{H_0} \geq 0$$

- For spin-2 sources,

$$\frac{\langle 2020|H_{\mathcal{E}}\rangle}{\langle 2020|00\rangle}\frac{1}{H_0} = 1 - \sqrt{\frac{10}{7}}\frac{H_2}{H_0} + 3\sqrt{\frac{2}{7}}\frac{H_4}{H_0} \geq 0$$

$$\frac{\langle 212\text{-}1|H_{\mathcal{E}}\rangle}{\langle 212\text{-}1|00\rangle}\frac{1}{H_0} = 1 - \sqrt{\frac{5}{14}}\frac{H_2}{H_0} - 2\sqrt{\frac{2}{7}}\frac{H_4}{H_0} \geq 0$$

$$\frac{\langle 222\text{-}2|H_{\mathcal{E}}\rangle}{\langle 222\text{-}2|00\rangle}\frac{1}{H_0} = 1 + \sqrt{\frac{10}{7}}\frac{H_2}{H_0} + \sqrt{\frac{1}{14}}\frac{H_4}{H_0} \geq 0$$

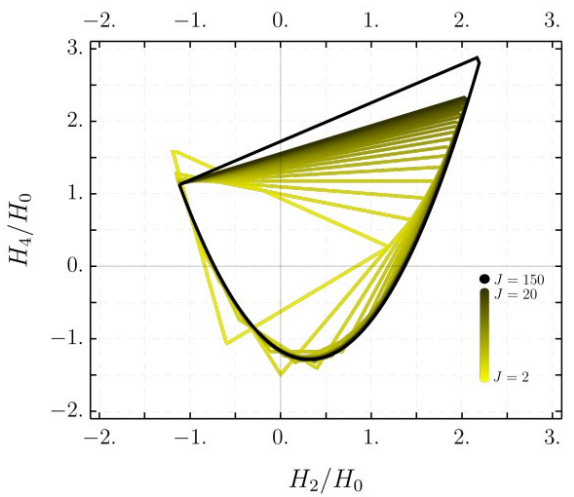
This is equivalent to known results.

Interpretation is the same for the vector source: allow space is convex sum of states with *m=0,1,2*.

- Spin-J generalization is straightforward.

  CFT representation of the bounds?

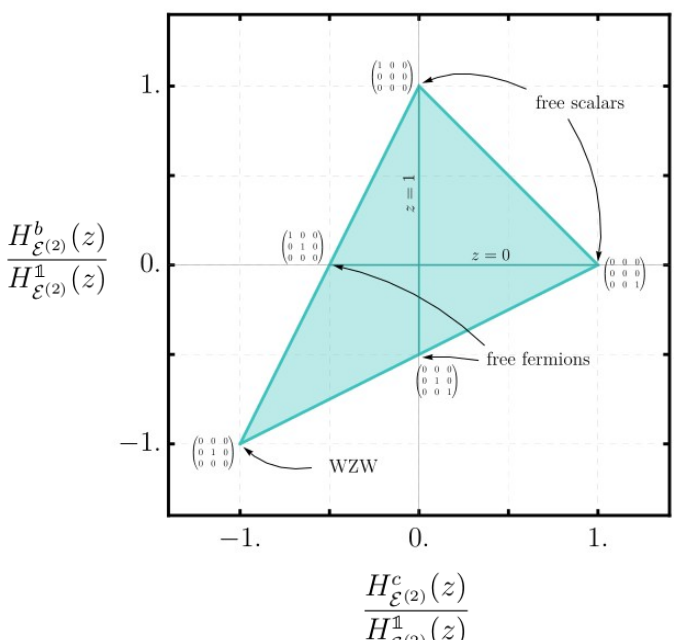
  Large J boundary is a conic: analytic approach?



- The most general hadronic tensor for the two-point correlator is

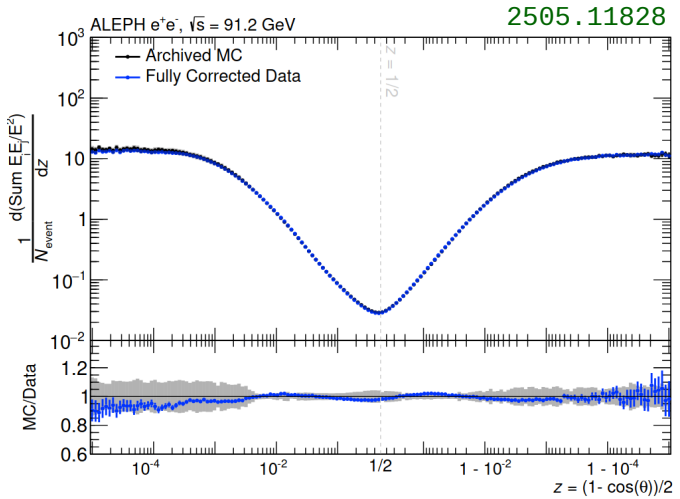
$$H^{ab}_{\mathcal{E}^{(2)}} = H^{\mathbb{1}}_{\mathcal{E}^{(2)}}(z)\,\frac{\delta^{ab}}{3} + H^{c}_{\mathcal{E}^{(2)}}(z)\left(\frac{1}{1-z}\,\frac{n_1^a + n_2^a}{2}\,\frac{n_1^b + n_2^b}{2} - \frac{\delta^{ab}}{3}\right) + H^{b}_{\mathcal{E}^{(2)}}(z)\left(\frac{1}{z}\,\frac{n_1^a - n_2^a}{2}\,\frac{n_1^b - n_2^b}{2} - \frac{\delta^{ab}}{3}\right) .$$

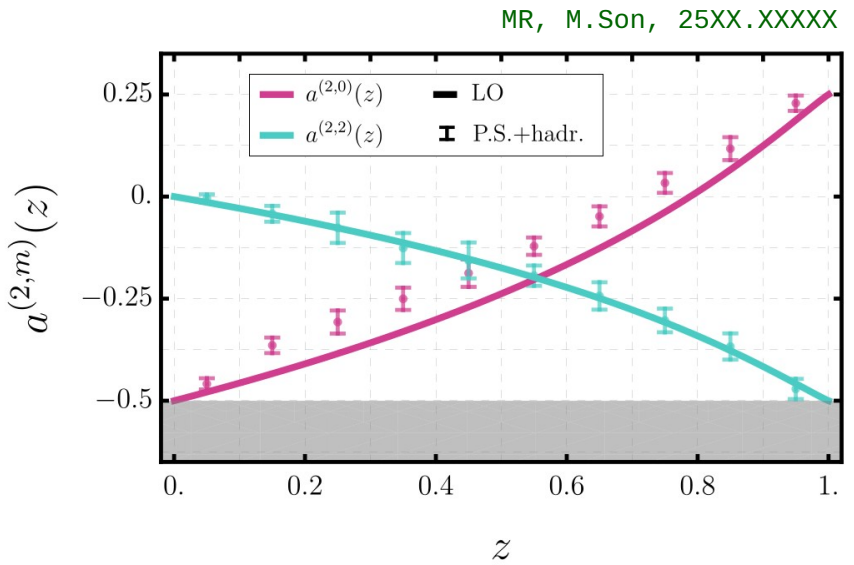
$$z = \frac{1 - \vec{n}_1 \cdot \vec{n}_2}{2}$$



- For an unpolarized transverse source,

$$\langle \mathcal{E}_{n_1}\mathcal{E}_{n_2}\rangle = \frac{1}{4\pi}\mathrm{EEC}(z) \times \left[1 + a_{\mathcal{E}\mathcal{E}}^{(2,0)}(z)\left(\frac{3}{2}\sin^2\Theta - 1\right) - a_{\mathcal{E}\mathcal{E}}^{(2,2)}(z)\frac{3}{4}\sin^2\Theta\cos 2\phi\right]$$





- Due to soft&coll. factorization, spinning correlators have no endpoint divergences. No need for resummation, direct probe of the hard scattering.

  Compared with the inclusive correlator, higher order and NP effects MUCH smaller. Better convergence of perturbative calculation.

  At the Z-pole, chiral structure can be recovered only via the spinning correlators.

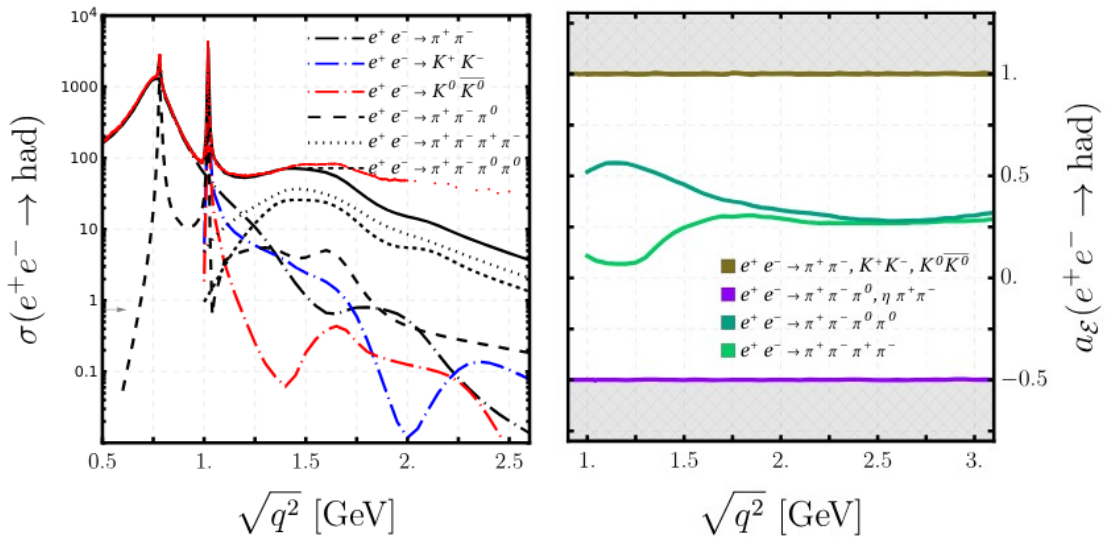
MR, M.Son '25

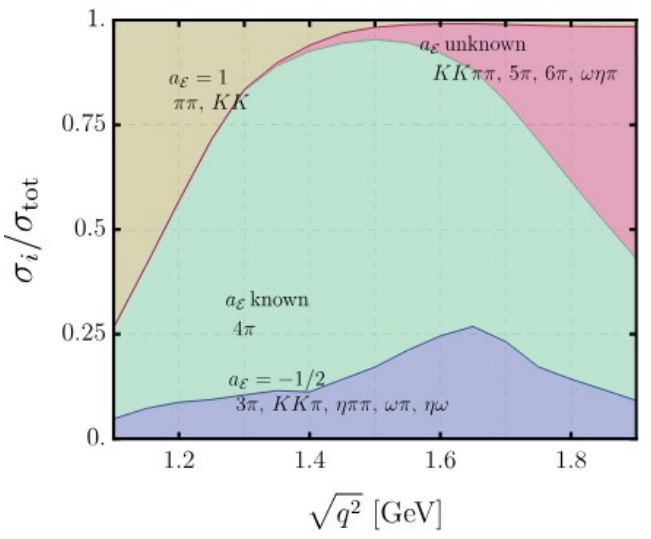
- Back to the one-point correlator of a vector current,
  in QCD it is close to the extremal fermionic boundary... at high energies.
  At low energies, the current interpolates scalars instead and it is close to the opposite boundary.
  So it defines a flow from the UV to the IR.
- The flow from the UV to the IR is controlled via perturbation theory in the gauge coupling

$$
\begin{aligned}
a_{\mathcal{E}} \;=\; & -\frac{1}{2}+\frac{\alpha_s}{\pi}\frac{9C_F}{8}+\left(\frac{\alpha_s}{\pi}\right)^2\left[-C_F^2\frac{99}{64}+C_FC_A\left(\frac{2023}{320}-\frac{9}{20}\zeta_3\right)-C_Fn_F\frac{37}{32}\right] \\
& +\left(\frac{\alpha_s}{\pi}\right)^3\Bigg[\; C_F^3\left(-\frac{3}{1280}-\frac{537}{40}\zeta_3+\frac{69}{4}\zeta_5\right)+C_F^2C_A\left(-\frac{14737}{960}-\frac{165}{16}\zeta_2+\frac{3803}{160}\zeta_3+\frac{605}{32}\zeta_4-\frac{231}{8}\zeta_5\right) \\
& \qquad +C_FC_A^2\left(\frac{2599751}{57600}+\frac{319}{64}\zeta_2-\frac{307}{25}\zeta_3-\frac{209}{20}\zeta_4+\frac{87}{16}\zeta_5\right) \\
& \qquad +C_F^2n_f\left(\frac{6163}{3840}+\frac{15}{8}\zeta_2-\frac{71}{40}\zeta_3-\frac{55}{16}\zeta_4+3\zeta_5\right)+C_Fn_f^2\left(\frac{461}{360}-\frac{1}{16}\zeta_2+\frac{3}{40}\zeta_3\right) \\
& \qquad +C_FC_An_f\left(-\frac{3622}{225}-\frac{9}{16}\zeta_2+\frac{259}{400}\zeta_3+\frac{19}{10}\zeta_4+\frac{3}{8}\zeta_5\right)+\frac{d_F^{abc}d_F^{abc}}{N_c}\left(\frac{69}{40}+\frac{42}{5}\zeta_3-12\zeta_5\right)\;\Bigg]
\end{aligned}
$$

MR, M.Son ‘25

- The flow from the IR to the UV is controlled via chiral perturbation theory.
  Two and three particle channels determined exactly by symmetry.





- 4 pion channel contribution determined via MonteCarlo simulations using data-based pheno model.
- Reliable prediction for other channels unavaliable. Main source of uncertainty.
- Relative weight between channels extracted from low energy data.

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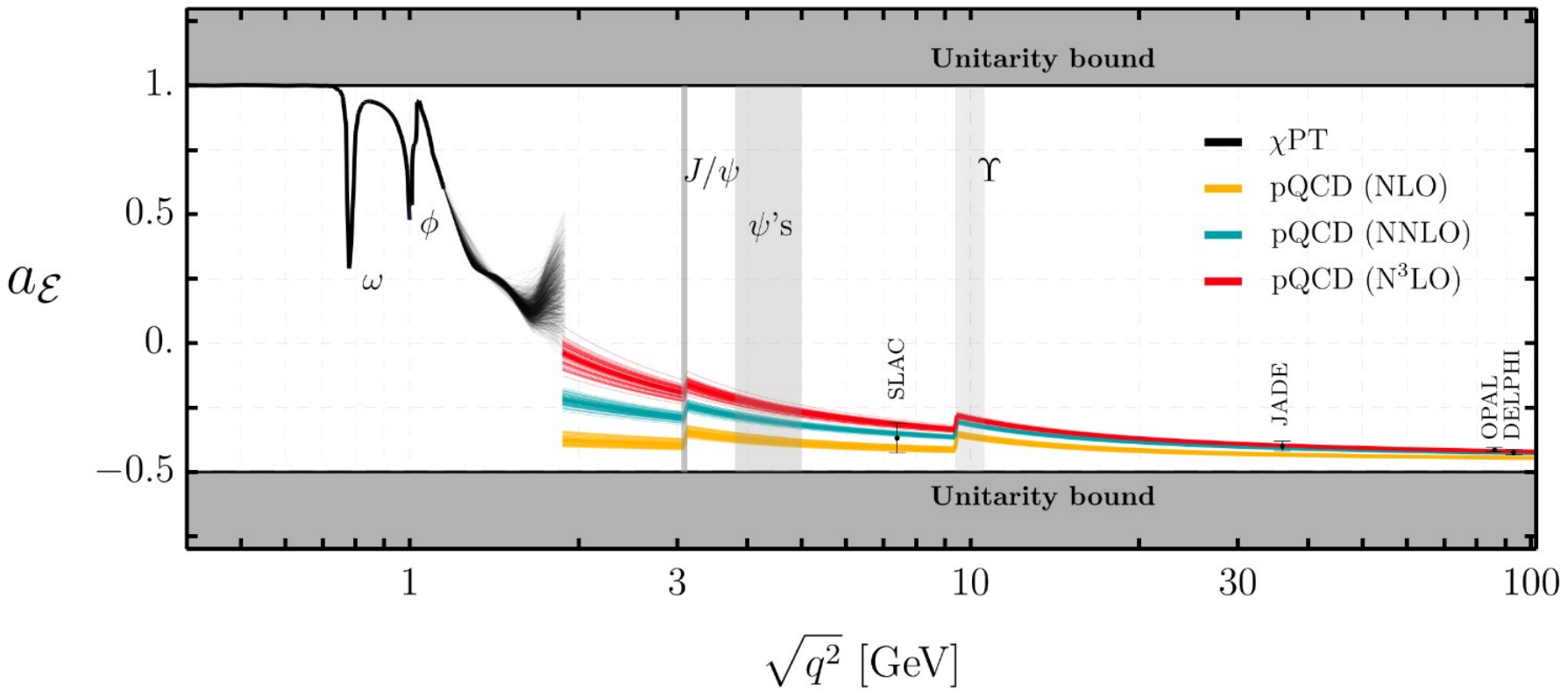
Unitarity bound
Unitarity bound
1.
0.5
0.
−0.5
$a_{\mathcal{E}}$
ω
ϕ
J/ψ
ψ's
Υ
SLAC
JADE
OPAL
DELPHI
χPT
pQCD (NLO)
pQCD (NNLO)
pQCD (N^{3}LO)
1
3
10
30
100
$\sqrt{q^2}$ [GeV]

Outlook & conclusions

- S-matrix calculation and measurement is unfeasible for theories with large multiparticle production. Correlations of conserved charges seem the correct approach.
- Full information requires spinning correlators. Necessary for W/Z/top spin structure, chiral couplings, higher twist pdf in DIS, etc.
- Spinning correlators constrained by positivity. Boundary saturated by states with different spin along the detector axis.
- Theories that transmute the matter content via strong dynamics flow from one boundary to another. QCD is a striking example of this phenomenon.
- Future improvements on theoretical predictions at both high and low energies should be made to resolve the transition region around 2 GeV.