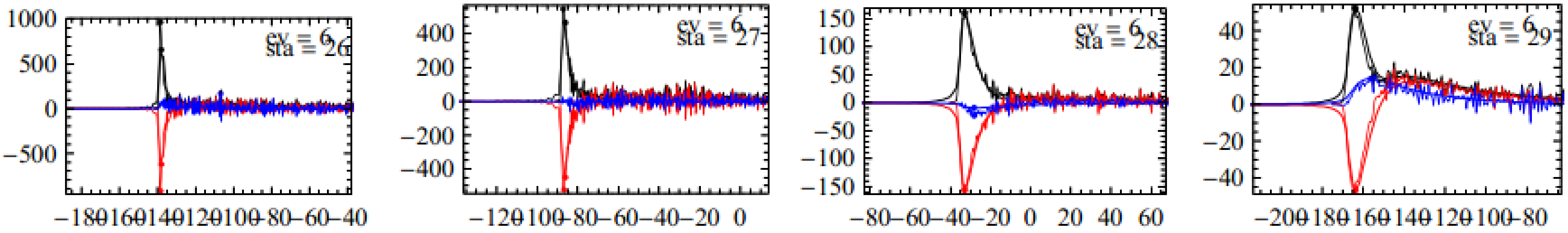
Semi-analytic emission from transient currents

David Seckel



June 22, 2012

Topics

- History
- General framework
  - $E(\omega)$, $A(\omega)$, $J(x,t)$
  - components
  - phases
  - separation of variables, convolution
- Details
  - Normalization, spectrum of secondaries
  - Longitudinal
  - Transverse
  - Shower front
- Some results
- Topics of interest
  - CROME
  - n(z) and inclined showers
  - Proposal for fitting

History

- 2009/2010: ANITA – 16 hpol events
    - energy uncertain, models for lower f, reflection, geometry

- Attempt semi-analytic model with n(z)
- Apply to Haverah Park (Allan)
- Apply to some LOPES events
- Get lost on shower detail … pause

- Resolve shower details
    - Compare to REAS-3. [Remove n(z), disable some terms]
- Use for RASTA … pause

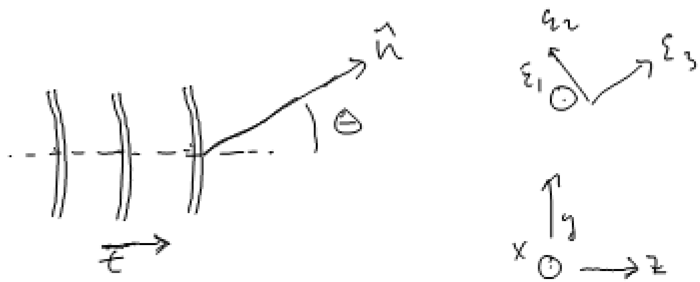
- Add in non-REAS, n(z) features
- Test higher freq - ANITA, CROME ?

E and A

General

$$\vec{E} = \frac{\partial \vec{A}}{\partial t} = i\,\omega\,\vec{A}$$

$$\vec{A} = \int d^3x\, dt \left[\hat{n} \times \left(\hat{n} \times \vec{J}(\vec{x},\, t)\right)\right] e^{i\phi(\vec{x},t)}$$

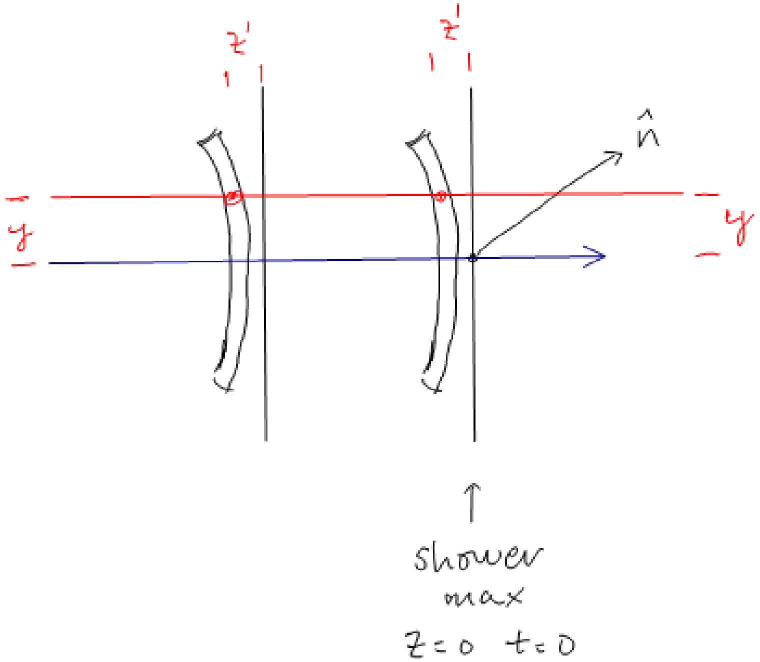


In-Ice

$$\vec{J}(\vec{x},\, t) = \rho(t)\,\delta(z - c\,t)\,\vec{j}(x,\, y)$$

$$\phi(\vec{x},\, t) = 2\,\pi\, f\left[t(1 - n\cos(\theta)) + n\,\frac{y}{c}\,\sin(\theta)\right]$$

Separation and Convolution



$$\int d^3x\, dt \rightarrow \int dz \int dx\, dy\, dz' \qquad z = ct$$

$$\vec{J}(x,t) \rightarrow \rho(z)\, \vec{J}(x,y,z')$$

$$e^{i\phi} \rightarrow e^{i(\phi_z + \phi_y + \phi_{z'})}$$

$$\int d^4x\, J\, e^{i\phi} \Rightarrow g_0\, G_z\, G_y\, G_{z'}$$

$$G_z = \int \rho(z)\, e^{i\phi_z}$$

$$\int \rho\, dz \equiv 1 \Rightarrow G_z \leq 1$$

Convolution over secondary energies

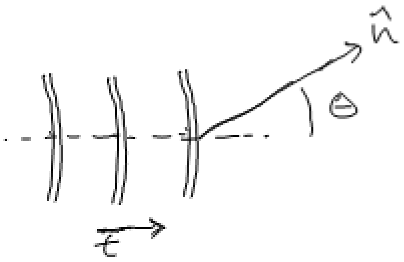
Components of j

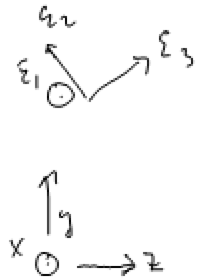
$$\hat{n}\times\left(\hat{n}\times\vec{j}\right) = -\vec{j}_{\perp} = -\left(\vec{j} - \left(\hat{n}\cdot\vec{j}\right)\hat{n}\right) = -\left(\vec{j} - \vec{j}_{\parallel}\right)$$

$$\hat{n}\times(\hat{n}\times\hat{z}) = \sin(\theta)\,\hat{n}\times\hat{x} = \sin(\theta)\,\hat{\epsilon}_2$$

$$\hat{n}\times(\hat{n}\times\hat{y}) = -\cos(\theta)\,\hat{n}\times\hat{x} = -\cos(\theta)\,\hat{\epsilon}_2$$

$$\hat{n}\times(\hat{n}\times\hat{x}) = \hat{n}\times\hat{\epsilon}_2 = -\hat{\epsilon}_1$$

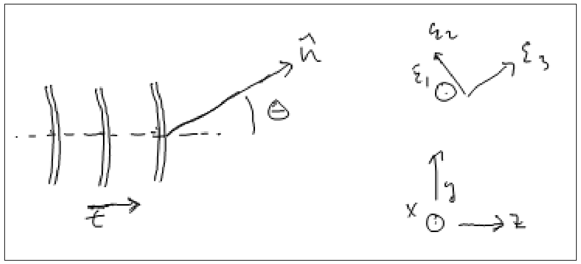




$$\vec{j}_{\perp} = [\sin(\theta)\; j_z - \cos(\theta)\; j_y]\,\hat{\epsilon}_2 - j_x\,\hat{\epsilon}_1$$

$$\simeq \theta\; j_z\,\hat{\epsilon}_2 - \vec{j}_{xy}$$

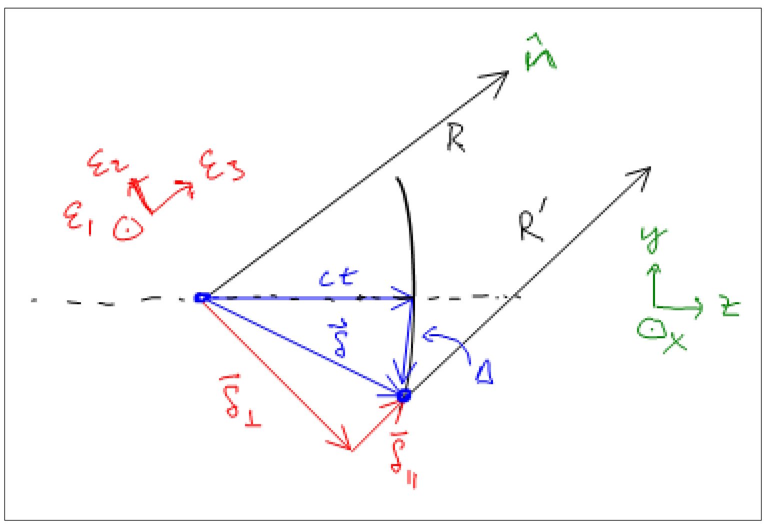
Co-ordinates & Displacements



$$R' = \left|\vec{R} - \vec{\delta}\right| = \sqrt{R^2 - 2\,R\,\hat{n}\cdot\vec{\delta} + \delta^2}$$

$$= R\left[1 - \tfrac{1}{R}\,\hat{n}\cdot\vec{\delta} + \tfrac{1}{2}\,\tfrac{1}{R^2}\left(\delta^2 - \left(\hat{n}\cdot\vec{\delta}\right)^2\right)\right]..$$

$$= R - \delta_{\parallel} + \tfrac{1}{2\,R}\,\delta_{\perp}^2$$



$$\delta_{\parallel} = (ct + z')\cos\theta + y\sin\theta$$

$$\delta_{\perp}^2 = [y\cos\theta - (ct + z')\sin\theta]^2 + x^2$$

Charge Excess and Dipole

$$\int \rho(y)\, e^{iky} = \int \rho_+(y-y_+)\, e^{iky} - \rho_-(y-y_-)\, e^{iky}$$

$$= \frac{1}{2} \int (1+\Delta)\tilde{\rho}(y)\, e^{ik(y+y_B)} - (1-\Delta)\tilde{\rho}(y)\, e^{ik(y-y_B)}$$

Coulomb scattering

$$= \frac{1}{2} \int \tilde{\rho}(y)\, e^{iky} \left[(1+\Delta)\, e^{iky_B} - (1-\Delta)\, e^{-iky_B} \right]$$

$$= G_y \left[\Delta \cos(ky_B) + i \sin(ky_B) \right]$$

⇑ charge excess

⇑ Dipole

$$y_B = a_{yB}\, \underbrace{\Theta_{By}\, z_{rad}}_{\text{magnetic deflection}}\, \Theta_{ant}$$

Transverse currents

$$J_y = \rho(y)\, v_y(y) \qquad v_y = \left(\Theta_{By} + a_{\Theta_s} \frac{y}{z_{rad}}\right) c$$

$$J_x = \rho(y)\, v_x(y) \qquad v_x = \Theta_{Bx}\, c$$

$$G_y\, \Theta_B \left[\cos(k y_B) + i \Delta \sin(k y_B)\right]$$

$$+\, G_y'\, a_{\Theta_s} \left[\Delta \cos(k y_B) + i \sin(k y_B)\right]$$

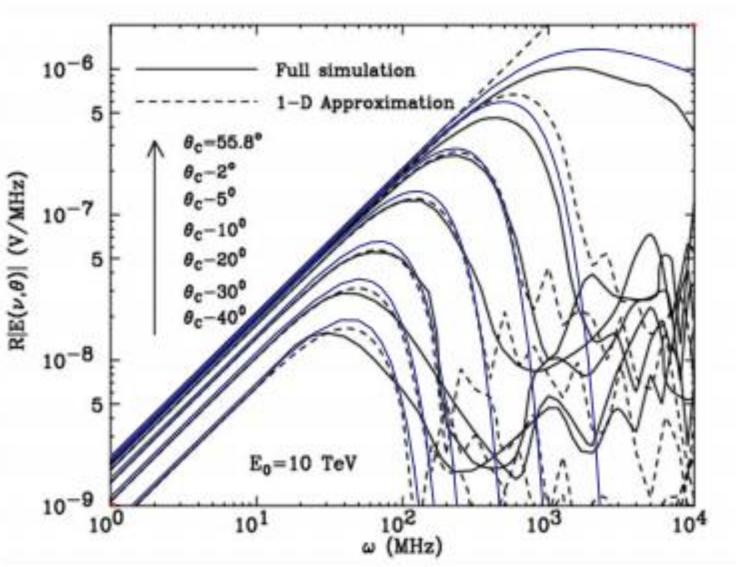
Terms in current

$\hat{\epsilon}_i$	$\hat{j}_i$	N	α	J	G_y	$n \times n \times j$	Comments
1	x	1 Δ	Cos Sin	B_y	g_y	-1	Geosynch
2	y	1 Δ	Cos Sin	B_x	g_y	$-\text{Cos}(\theta)$	
		1 Δ	Sin Cos	θ_s	$\beta\, g_y$	$-\text{Cos}(\theta)$	Diffusion
	z	1 Δ	Sin Cos	v_z	g_y	$\text{Sin}(\theta)$	Charge sep (Dipole) Askaryan

Semi-analytic. ARENA '12 (Seckel)

Normalization

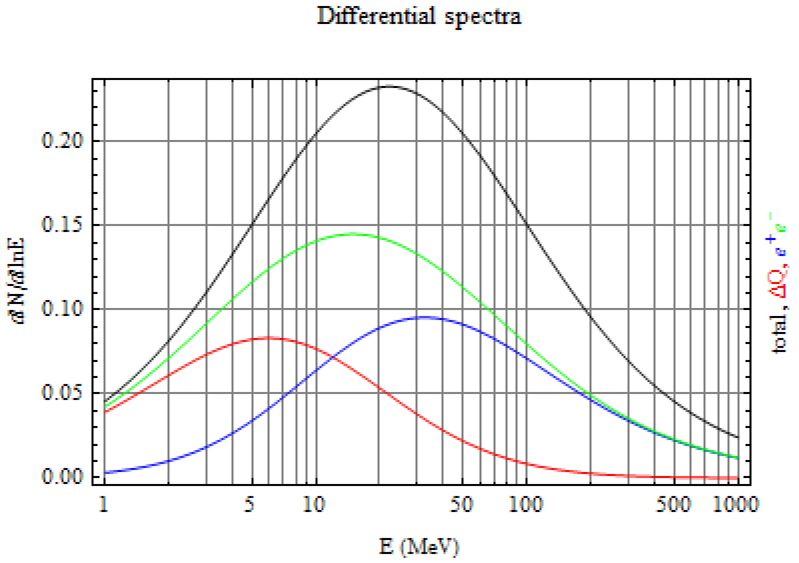
- Scale to 1 PeV shower in-ice



$$g00 = amp0\ f0ice\ \frac{1}{eice}\ \frac{1}{dqice}\ \frac{zrair00}{zrice}\ \frac{dice}{dair00};$$

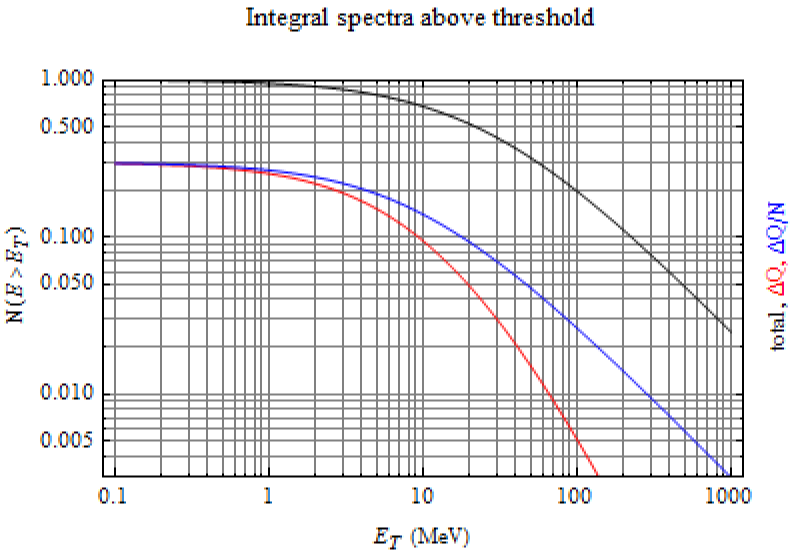
Semi-analytic. ARENA '12 (Seckel)

Spectrum of secondaries



Shape is by eye to Fluka/Geant 100 GeV showers – not crazy, but could do better.

$$N(E > E_T) \simeq \frac{0.25}{E_{100}}$$



Semi-analytic. ARENA '12 (Seckel)

Longitudinal Integral

Ground!

$$G_z = \int_{z_i}^{0} \frac{1}{R^a} \rho(z)\, e^{i\phi(z)} = \int \frac{1}{R^a} \rho(x)\, e^{i\phi(z(x))} \frac{dz}{dx}\, dx$$

$\frac{1}{n(z)}$

for charge excess + Dipole $\quad \sin\theta \Rightarrow \frac{1}{R} \qquad G \sim \frac{1}{R^2} \quad a=2$

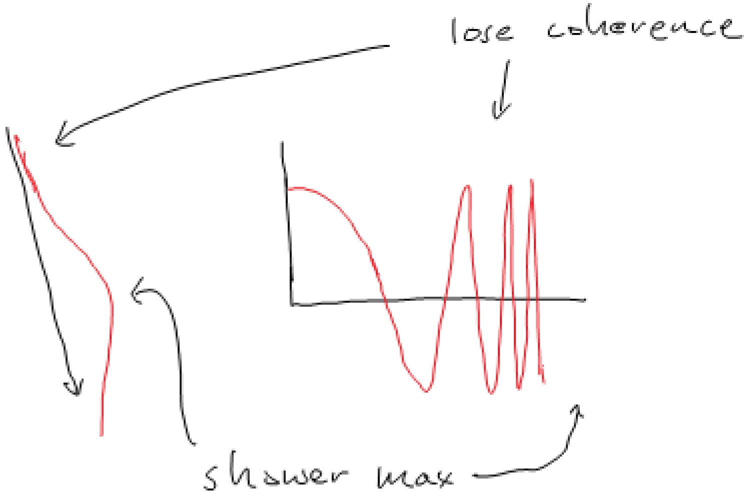
for $J_\perp \qquad G \sim \frac{1}{R} \quad a=1$

Now using $\rho \sim x^a e^{-bx}$ $\quad a, b$ adjusted for X_{max}

what about ϕ?

Effective emission height

- At high freq, phase dominates over shower profile.



Semi-analytic. ARENA '12 (Seckel)

Transverse profile

- started with Gaussian profiles
- tried to reproduce 50 yrs of work on shower physics
- give up and use NKG

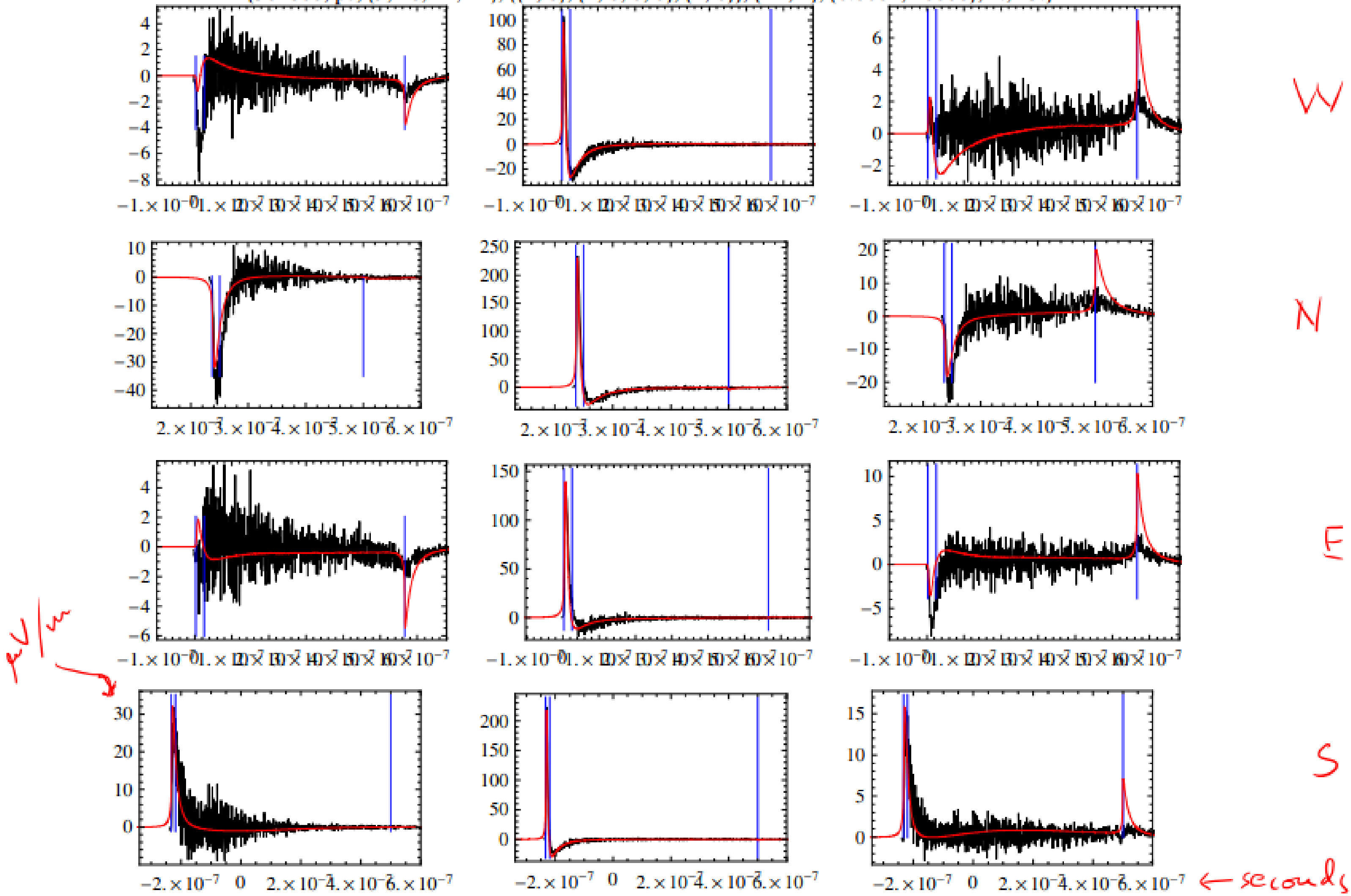
- Integrate over secondary particle energy from 10 MeV – 100 GeV

Shower Front

- time delays taken from Lafebre, etal
- Also scales with secondary energies

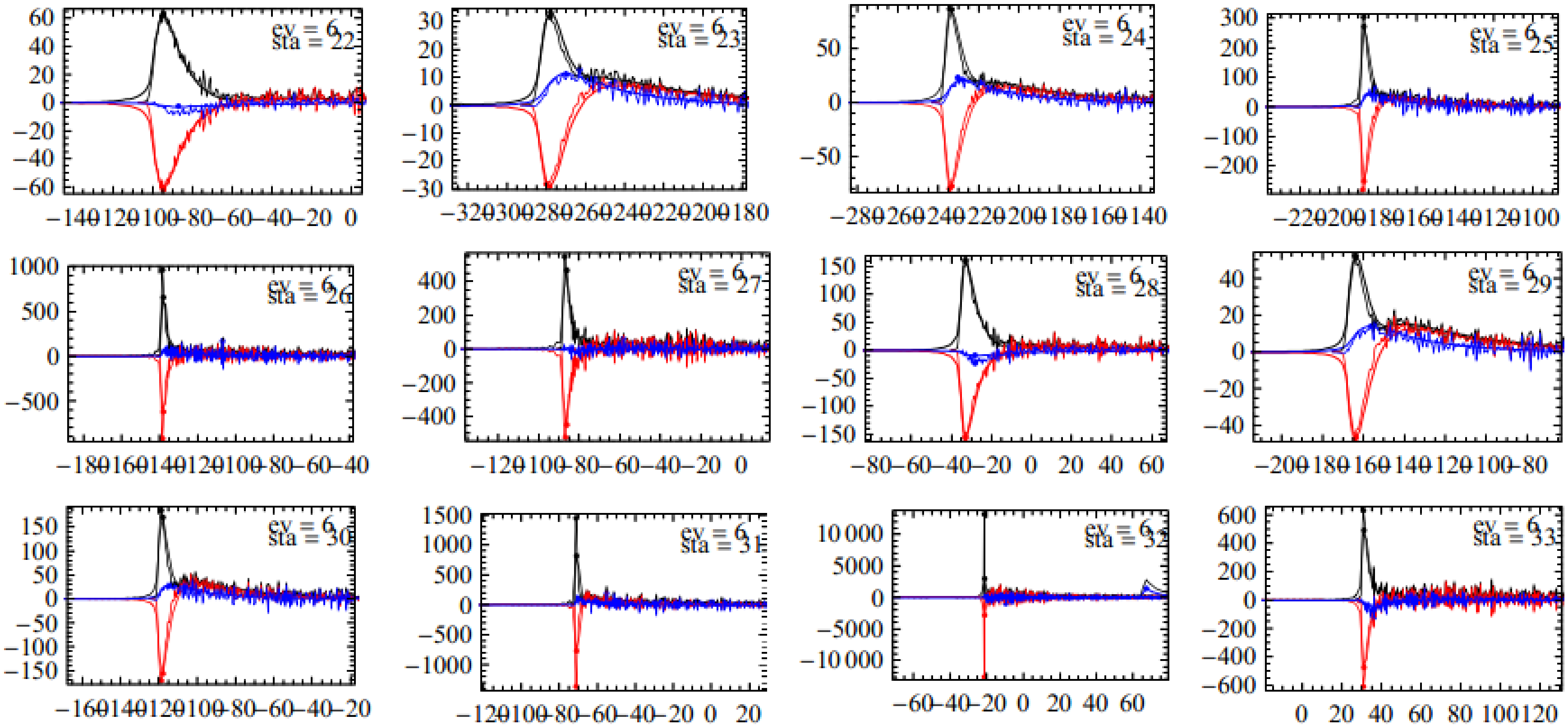
Some results: A south pole shower

{502800, pe, {9, 10, 11, 12}, {{1, 0}, {1, 0, 0, 0}, {1, 0}}, {−2, 2}, {0.0001, 10000}, 1., 25.}



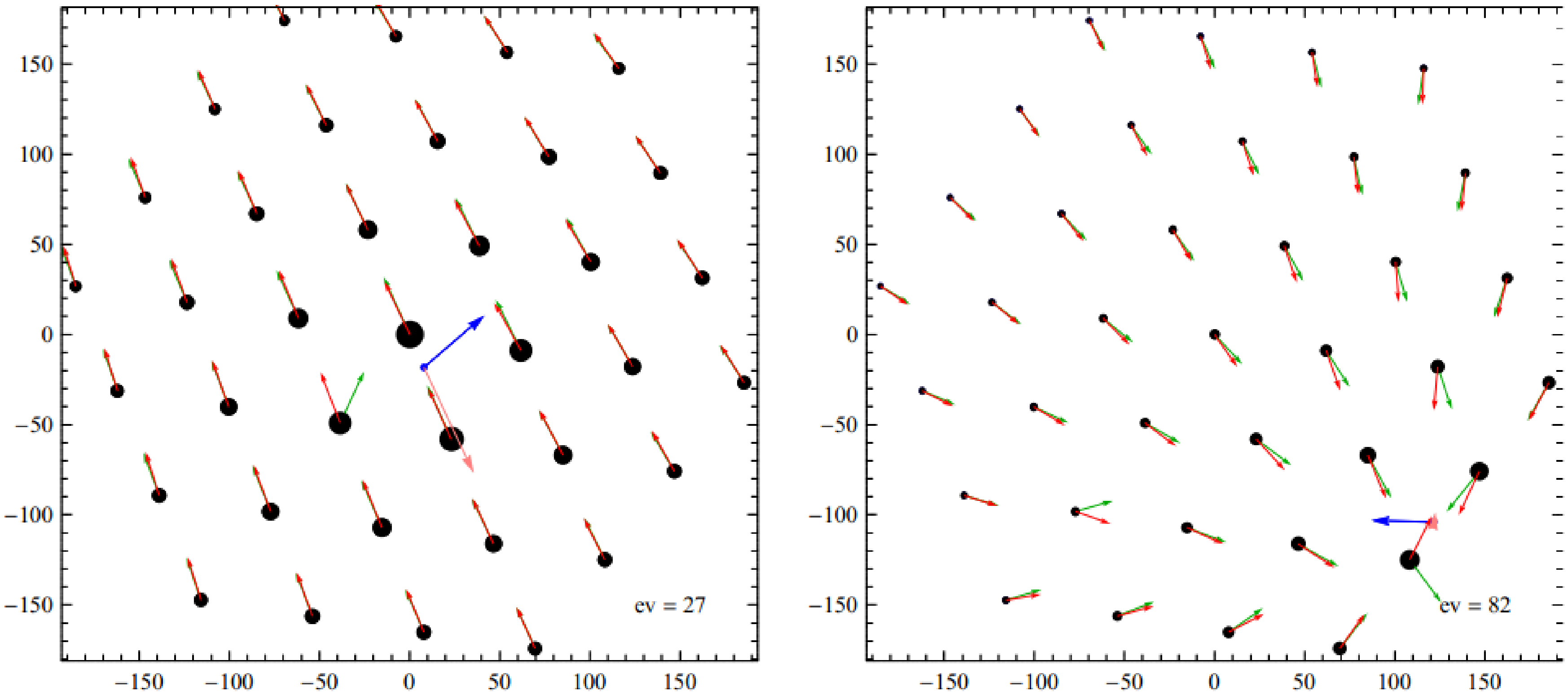
Semi-analytic. ARENA ‘12 (Seckel)

Part of a rasta event



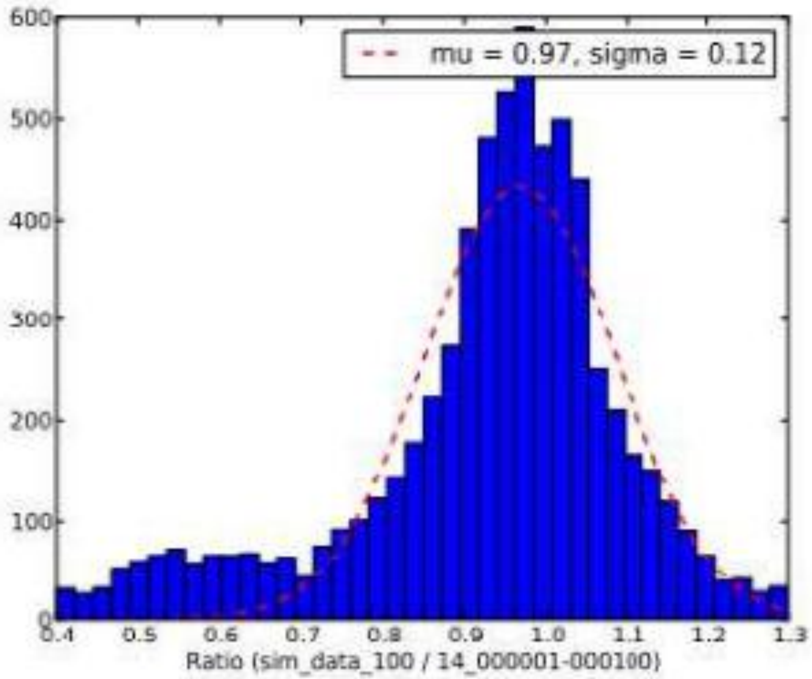
Semi-analytic. ARENA '12 (Seckel)

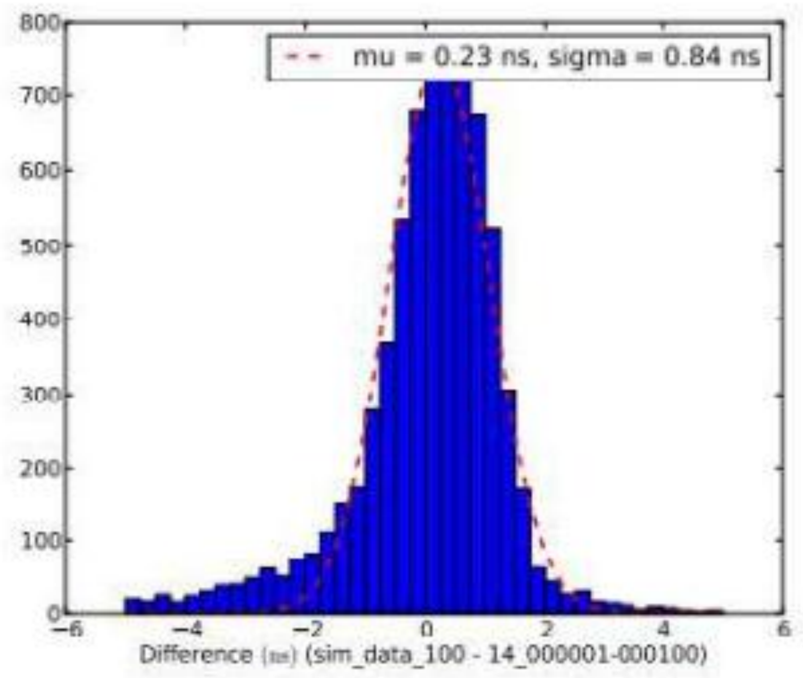
overall event comparison



Semi-analytic. ARENA ‘12 (Seckel)

100 events … 74 antennas

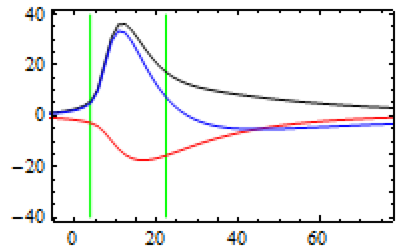




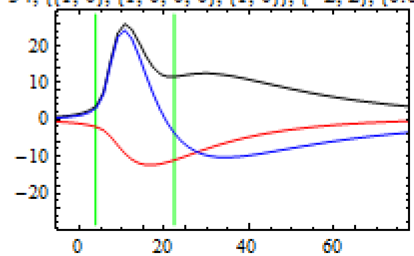
Semi-analytic. ARENA '12 (Seckel)

Adding in other terms

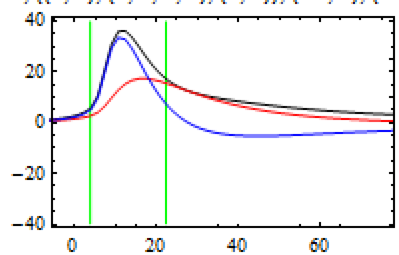
33, {{1, 0}, {1, 0, 0, 0}, {1, 0}}, {−2, 2}, {0.001, :



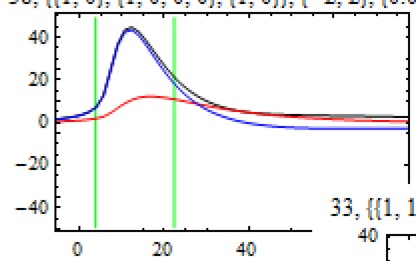
34, {{1, 0}, {1, 0, 0, 0}, {1, 0}}, {−2, 2}, {0.001,



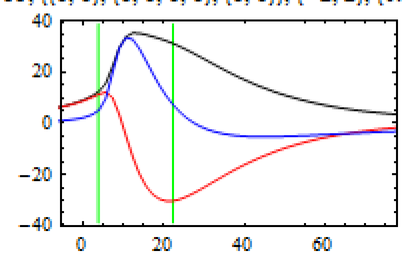
37, {{1, 0}, {1, 0, 0, 0}, {1, 0}}, {−2, 2}, {0.001, :



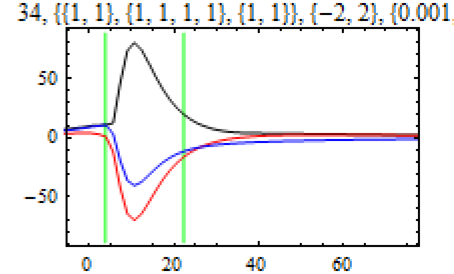
38, {{1, 0}, {1, 0, 0, 0}, {1, 0}}, {−2, 2}, {0.001,



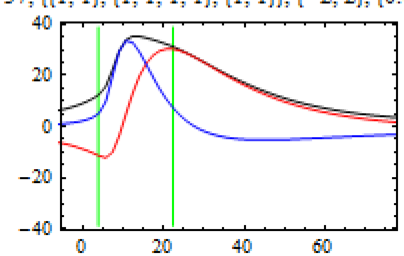
33, {{1, 1}, {1, 1, 1, 1}, {1, 1}}, {−2, 2}, {0.001, :



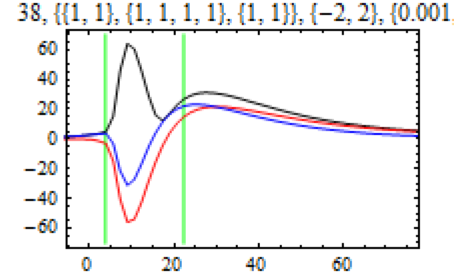
34, {{1, 1}, {1, 1, 1, 1}, {1, 1}}, {−2, 2}, {0.001,



37, {{1, 1}, {1, 1, 1, 1}, {1, 1}}, {−2, 2}, {0.001, :



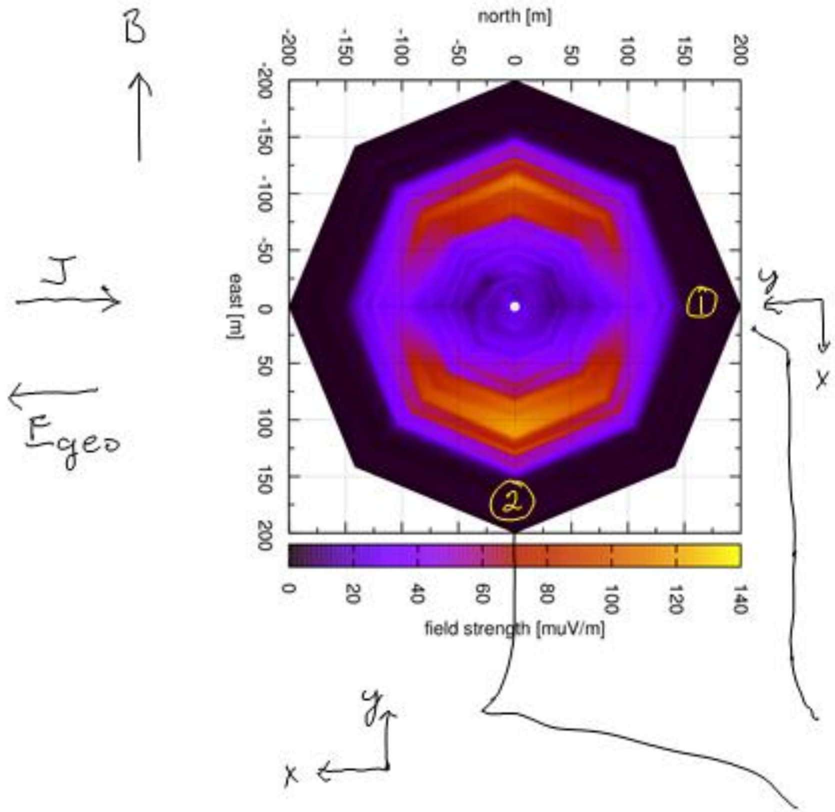
38, {{1, 1}, {1, 1, 1, 1}, {1, 1}}, {−2, 2}, {0.001,



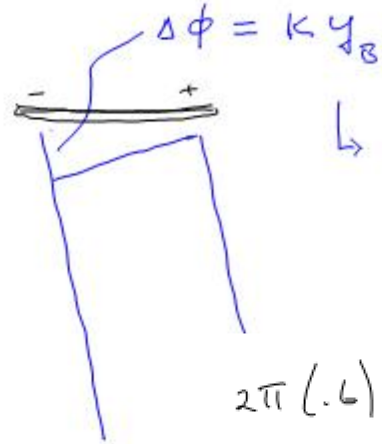
Semi-analytic. ARENA ‘12 (Seckel)

CROME

$$G_y \, \Theta_B\left[\cos(k y_B) + i\Delta \sin(k y_B)\right]$$



From above



↳ $a_{yB}\,\Theta_{By}\,Z_{rad}\,\Theta_{Ant}$

↑ depends on B_x

$2\pi(.6)\,\frac{f}{3\,GHz}\,\frac{1\,GeV}{E}$ @ 8000 m

B = 0.5 G

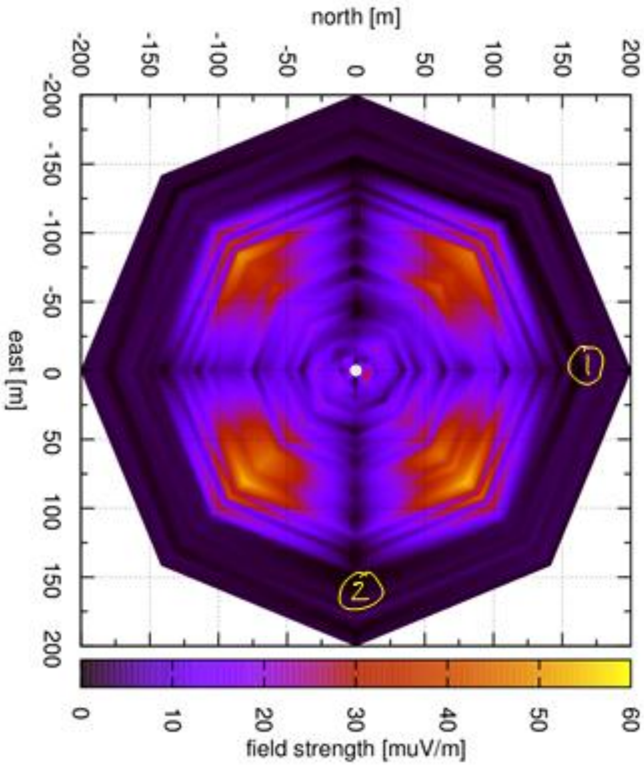
NB: y is toward shower axis

For ① $B_x = B$ $\langle\cos(k y_B)\rangle \to 0$ at high freq

For ② $B_x = 0$ $\cos(k y_B) = 1$

CROME-II

$\uparrow B$



① vanishes at high freq.

② vanishes if E_y vanishes at $\alpha = 0$

$\hat{\epsilon}_i$	$\hat{j}_i$	N	α	J	G_y	$n \times n \times j$	Comments
1	x	1 Δ	Cos Sin	B_y	g_y	-1	Geo~~synch~~ mag!
2	y	1 Δ	Cos Sin	B_x	g_y	$-\text{Cos}(\theta)$	
		1 Δ	Sin Cos	θ_z	~~g~~ g_y'	$-\text{Cos}(\theta)$	Diffusion
	z	1 Δ	Sin Cos	v_z	g_y	$\text{Sin}(\theta)$	Charge sep (Dipole) Askaryan

Dipole, Diffusion comparable to geomagnetic

$\Theta_S = \frac{13\ \text{MeV}}{\text{GeV}} = .013$ $\quad \Theta_B = \frac{z_{Rad}}{R_B} \simeq \frac{1000}{3.3\ 10^4} .5 = .015$ $\quad \Theta_C = \sqrt{2\ \delta n} = .014$

Semi-analytic. ARENA '12 (Seckel)

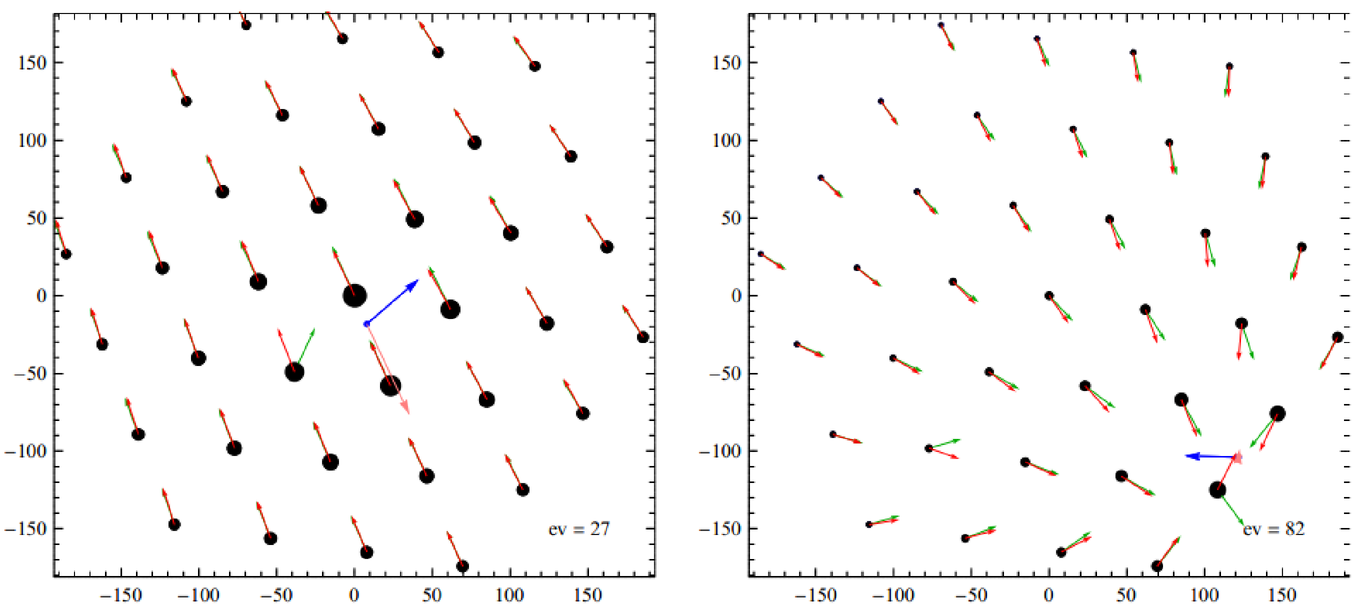
Proposal to describe shower

$$\vec{E} = \left[\vec{E}_{geo} + \Delta \vec{E}_r (A r - D)\right] f(r)$$

Due to sin Θ

Askaryan

Diffusion



Semi-analytic. ARENA ‘12 (Seckel)

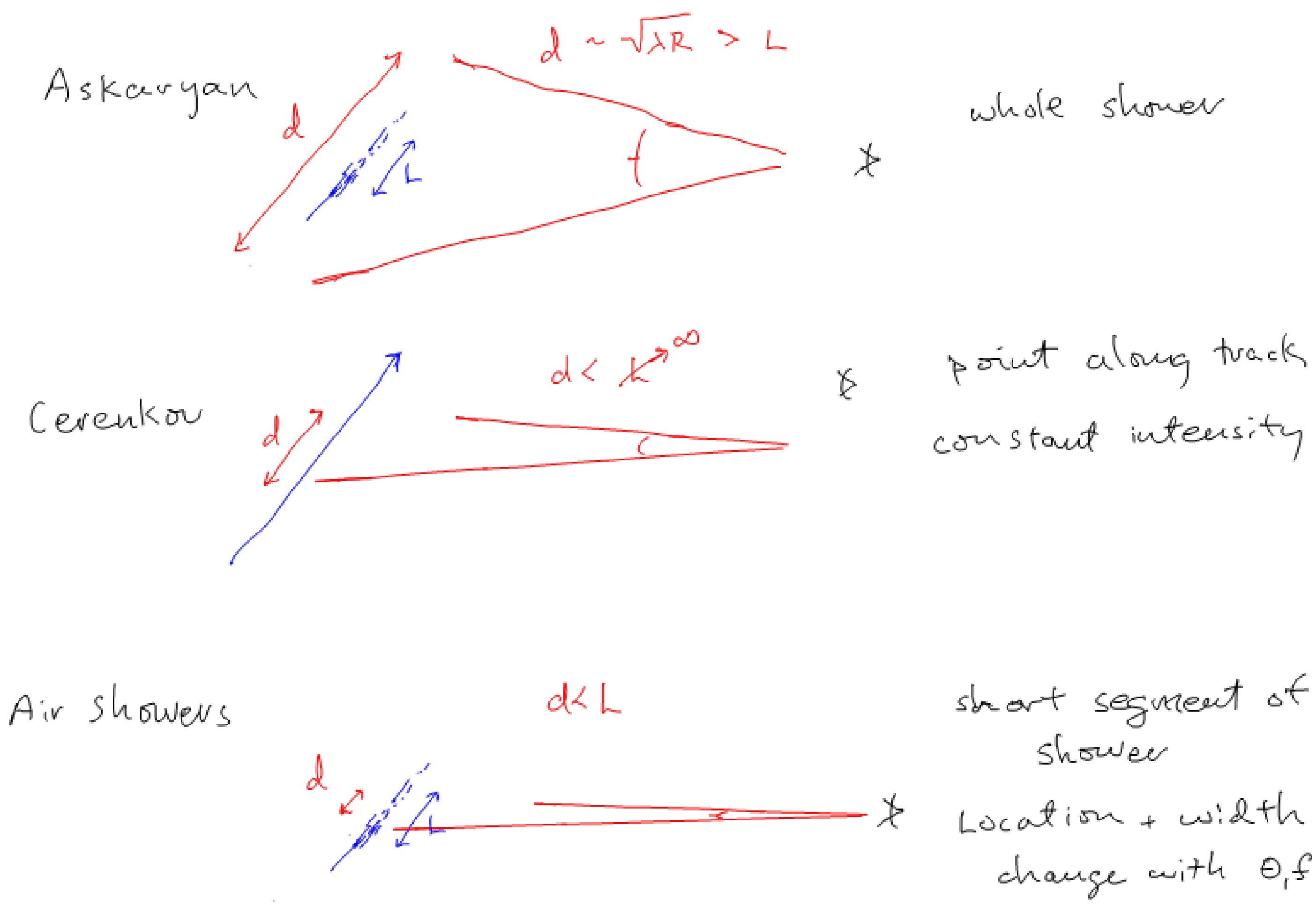
Summary

- semi-analytic with B
- components of radiation
- phases to second order
- shower profiles matter
  - scale with and integrate over secondary energy

- tested against reas for south pole atm.
- include additional terms (test against coreas …)

- qualitative features of CROME simulation
- propose vector E as intermediate for reconstruction

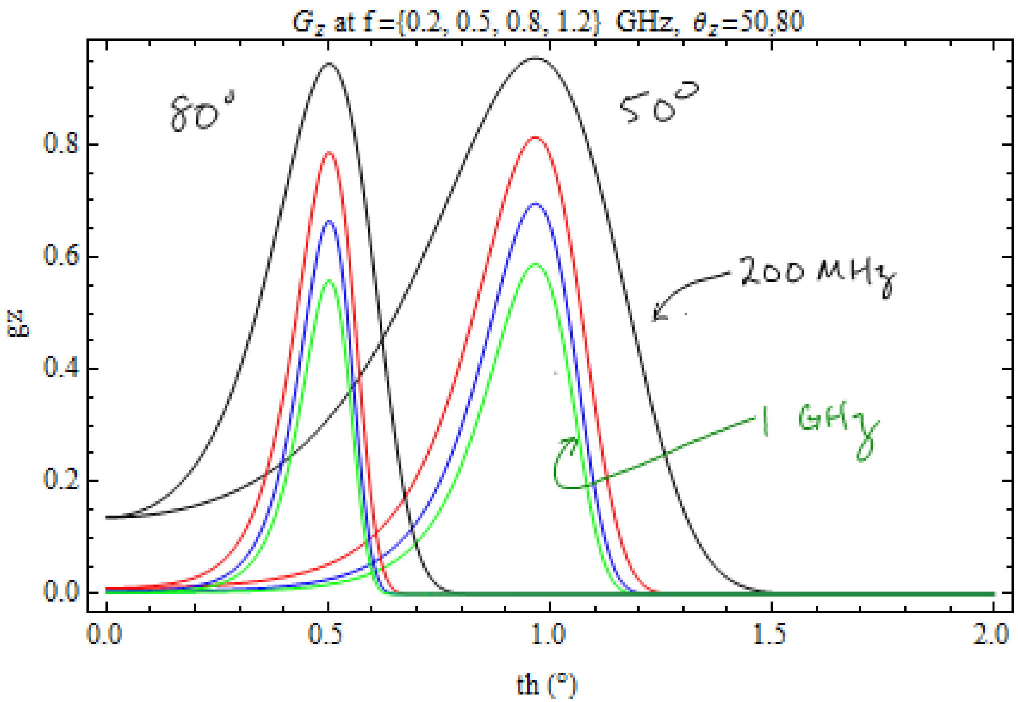
Extra Slides



Semi-analytic. ARENA ‘12 (Seckel)

“true” Cherenkov

Semi-analytic. ARENA ‘12 (Seckel)



$$\Delta\Omega = 2\pi\,\theta\,\Delta\theta = 2\pi \cdot 1 \cdot 0.3 \approx 2 \text{ sq deg.}$$

$$\approx 6 \cdot 10^{-4}\ \text{sr}$$

Semi-analytic. ARENA '12 (Seckel)

Phases to 2nd order in displacement

$$c\tau = ct - n\delta_{||} + \tfrac{1}{2}\, n\, \frac{\delta_\perp^2}{R} - \tfrac{1}{2}\, n' \cos\theta\, z^2$$

$$= z(1 - n\cos\theta) - \tfrac{1}{2}\, n' \cos\theta\, z^2 - z'\, n\cos\theta + y\, n\sin\theta + \frac{n}{2R}\left([y\cos\theta - (z+z')\sin\theta]^2 + x^2\right)$$

$$c\tau = \left(\tfrac{1}{2}\,\theta^2 - \delta n\right) z - \tfrac{1}{2}\, n'\, z^2 - z' + y\theta + \frac{1}{2R}\left([y - z\,\theta]^2 + x^2\right)$$

$$= \left(\tfrac{1}{2}\,\theta^2 - \delta n\right) z + \tfrac{1}{2}\left(\tfrac{1}{R}\,\theta^2 - n'\right) z^2 - z' + y\theta$$

$$\delta n = \tfrac{1}{2}\,\theta_c^2$$

$$n' = \frac{\delta n}{h}\cos(\theta_z) = \theta_c^2\, \frac{\cos(\theta_z)}{2h}$$

$$\phi = 2\pi\frac{f}{c}\, c\tau = \phi_z + \phi_{z'} + \phi_y$$

$$\phi_z = k\left(az + \frac{1}{2b}\, z^2\right) \qquad k = 2\pi\frac{f}{c}, \quad a = \tfrac{1}{2}(\theta^2 - \theta_c^2), \quad \frac{1}{b} = \frac{1}{R}\left(\theta^2 - \theta_c^2\, \frac{R\cos\theta_z}{2h}\right)$$

$$\phi_{z'} = -kz'$$

$$\phi_y = k\, y\, \theta$$

CPU efficiency

- 64 antenna test events … about 50 mins
  – in Mathematica on my laptop
  – estimate to compiled version ~1sec/antenna

- This is with about 150 frequency points from 100 KHz to 3 GHz, sparsely sampled and interpolated for FFT.

- Most time spent in longitudinal integrals