Theory Overview

of Semileptonic B decays

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Theoretical Tools

Several tools in the toolbox:

For inclusive decays:

- local Operator Product Expansion
- Factorization theorems
- Effective Field Theories (e.g. HQET, SCET-1)

For exclusive decays:

- Effective Field Theories (e.g. HQET, SCET-2)
- QCD sum rules (also on the light-cone)
- Lattice

Precision methods without invoking models

Inclusive $B \to X_c \ell \bar{\nu}$

$$d\Gamma \sim \mathrm{Im} \int d^4x \, e^{-im_b v\cdot x} \langle B(v) | T\{\widetilde{\mathcal{H}}_{eff}(x) \widetilde{\mathcal{H}}^\dagger_{eff}(0)\} | B(v) \rangle$$

perform a local OPE

Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstein, Manohar, Wise, Neubert, ...

$$\int d^4x e^{-im_b vx} T\{\widetilde{\mathcal{H}}_{eff}(x) \widetilde{\mathcal{H}}^\dagger_{eff}(0)\} = \sum_{n=0}^{\infty} \left(\frac{1}{2m_Q}\right)^n C_{n+3}(\mu) \mathcal{O}_{n+3}$$

The rate can be expressed in a (power) series:

$$\Gamma = \Gamma_0 + \frac{1}{m_Q}\Gamma_1 + \frac{1}{m_Q^2}\Gamma_2 + \frac{1}{m_Q^3}\Gamma_3 + \cdots$$

Each coefficient is a perturbative series.

Inclusive $B \to X_c \ell \bar{\nu}$

- Γ_0 is the decay of the free quark
- $\Gamma_1 = 0$ vanishes due to Heavy-Quark Symmetry
- Γ_2 is expressed in terms of two matrix elements,

$$\mu_\pi^2, \mu_G^2$$

"kinetic energy" and "chromomagnetic moment"

- Γ_3 is expressed in two more parameters:

$$\rho_D^3, \rho_{LS}^3$$

"Darwin term" and "spin-orbit term"

- Γ_4, Γ_5 have also been computed recently

Bigi, Uraltsev, Turczyk, Mannel 2011

Inclusive $B \to X_c \ell \bar{\nu}$

State of the art

- At tree level, terms up to (and including) $1/m_b^5$ known

  Bigi, Zwicky, Uraltsev, Turczyk, Mannel, ...

- $\mathcal{O}(\alpha_s)$ and full $\mathcal{O}(\alpha_s^2)$ for the partonic rate are computed.

  Melnikov, Czarnecki, Pak

- $\mathcal{O}(\alpha_s)$ for the kinetic term μ_π^2/m_b^2 known.

  Becher, Boos, Lunghi, Gambino

- We are working on the completion of the full α_s/m_b^2 term, including the chromomagnetic term $\mathcal{O}(\alpha_s)\mu_G^2/m_b^2$

  also on "intrinsic charm" at 3rd order in p.c.

$\Longrightarrow$ A theo. uncertainty of 1% in Vcb, incl looks plausible.

Exclusive $B \to D^{(*)}\,\ell\bar{\nu}$

The rates are expressed in the kinematic variable $\omega = v \cdot v'$

$$\frac{d\Gamma}{d\omega}(B \to D^*\ell\bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3}|V_{cb}|^2 m_{D*}^3(\omega^2-1)^{1/2}P(\omega)(\mathcal{F}(\omega))^2$$

$$\frac{d\Gamma}{d\omega}(B \to D\ell\bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3}|V_{cb}|^2(m_B+m_D)^2 m_D^3(\omega^2-1)^{3/2}(\mathcal{G}(\omega))^2$$

Here, P(w) is a phase-space factor (known exactly), and the hadronic physics is encoded in the form factors

Exclusive $B \to D^{(*)} \ell\bar{\nu}$

From **Heavy-Quark Symmetry** we know the normalization of the form factors in the infinite mass limit.

$$\mathcal{F}(\omega) = \eta_{\mathrm{QED}}\eta_A \left[1 + \delta_{1/\mu^2} + \cdots\right] + (\omega - 1)\rho^2 + \mathcal{O}((\omega-1)^2)$$
$$\mathcal{G}(1) = \eta_{\mathrm{QED}}\eta_V \left[1 + \mathcal{O}\left(\frac{m_B - m_D}{m_B + m_D}\right)\right]$$

Corrections by the breaking scale $\frac{1}{\mu} = \frac{1}{m_c} - \frac{1}{m_b}$

$$\eta_A = 0.960 \pm 0.007,\ \eta_V = 1.022 \pm 0.004,$$
$$\delta_{1/\mu^2} = -(8 \pm 4)\%,\ \eta_{\mathrm{QED}} = 1.007$$

Exclusive $B \to D^{(*)}\,\ell\bar{\nu}$

From **Lattice QCD** the deviation from unity is calculated.

The Fermilab-MILC Collaboration presented the only LQCD calculation with realistic 2+1 flavour sea quarks.

The current value includes the electroweak corrections:

Bailey et al., 2010

$$\eta_{\rm QED}\mathcal{F}(1) = 0.908 \pm 0.017$$

The quark masses are larger than in Nature, and $m_l > 0.1\, m_s$ extrapolation guided by chiral PT.

Exclusive $B \to D^{(*)} \ell\bar{\nu}$

From **zero-recoil sum rules** a connection between form factors and a structure function is explored

$$|h_{A_1}(1)|^2 + \frac{1}{2\pi}\int_0 d\epsilon\, w(\epsilon) = 1 - \Delta_{1/m_Q^2} - \Delta_{1/m_Q^3},$$

The structure function w is integrated over a domain μ that is carefully chosen. Combined with known 2-loop calculations one derives an upper bound and an estimate

$$h_{A_1}(1) < 0.93 \qquad \mathcal{F}(1) = 0.86 \pm 0.02$$

Gambino, Mannel, Uraltsev

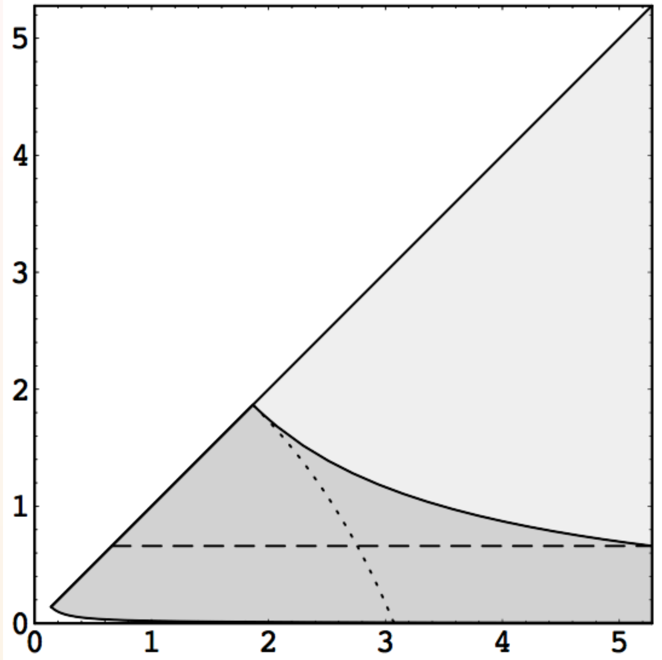
similarly

$$\mathcal{G}(1) = \begin{cases} 1.02 \ \ \pm 0.04 & \text{sum rule} \\ 1.074 \pm 0.024 & \text{LQCD,} \end{cases}$$

Uraltsev

Inclusive $B \to X_u \ell \bar{\nu}$

$P_+ = E_H - |\vec{P}_H|$



$P_- = E_H + |\vec{P}_H|$

The fully differential rate is expressed in three structure functions:

$$\frac{d^3\Gamma_u}{dP_+\, dP_-\, dP_l} = \frac{G_F^2 |V_{ub}|^2}{16\pi^3}\, U_y(\mu_h,\mu_i)\, (M_B - P_+) \Big[(P_- - P_l)(M_B - P_- + P_l - P_+)\, \mathcal{F}_1 + (M_B - P_-)(P_- - P_+)\, \mathcal{F}_2 + (P_- - P_l)(P_l - P_+)\, \mathcal{F}_3 \Big],$$

Note: no partonic variables.

In the "shape-function region" these functions factorize, e.g. at leading power:

$$\mathcal{F}_1^{(0)}(P_+, y) = H_u(y,\mu_h) \int_0^{P_+} d\hat{\omega}\, y m_b\, J(y m_b (P_+ - \hat{\omega}), \mu_i)\, \hat{S}(\hat{\omega}, \mu_i)$$

Inclusive $B \to X_u \ell \bar{\nu}$

Moments of the shape function(s) can be calculated in an OPE, provided the integration domain is large enough.

- strategies for the shape function(s)
  - From direct measurement of $B \to X_s \gamma$ — problematic, different subleading effects
  - From OPE of spectral moments
  - From modeling — see below.
- QCD based calculations
  - BLNP — Bosch, Lange, Neubert, Paz, (Greub, Neubert, Pecjak) — NNLO, but only LO in 1/mb
  - GGOU — Gambino, Giordano, Ossala, Uraltsev — no resummation, non-universal SF, BLM calc.
  - SIMBA — Tackmann, Tackmann, Lacker, Ligeti, Stewart — expansion in function basis.
- QCD inspired models
  - Dressed Gluon Exponentiation — Andersen, Gardi
  - Analytic Coupling — Aglietti et al.
- relation to the photon spectrum $B \to X_s \gamma$
  Neubert, Lange, Rothstein, ...

Exclusive $B \to \pi \ell \bar{\nu}$

Focus is on pions -- rho semileptonic available, but nothing new. Ball, Zwicky, 2005

Differential rate

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} \mathbf{p}_\pi^3 |f_+^{B\pi}(q^2)|^2$$

with vanishing lepton mass.

- Shape constrained by analyticity, but need norm $f_+^{B\pi}(0)$
- with tau leptons we also need f0.
- As example: Becirevic Kaidalov Parameterization: 1-pole and 1-parameter
- z parameterization

$$P_+(q^2)\phi_+(q^2, t_0) f_+(q^2) = \sum_{k=0}^{\infty} a_k(t_0) z(q^2, t_0)^k.$$

Arnesen et al., Boyd, Grinstein, Lebed, ...

with

$$z(q^2, t_0) = \frac{\sqrt{1 - q^2/t_+} - \sqrt{1 - t_0/t_+}}{\sqrt{1 - q^2/t_+} + \sqrt{1 - t_0/t_+}},$$

Exclusive $B \to \pi \, \ell \bar{\nu}$

From **Lattice QCD** : Fermilab-MILC presents form factors as coefficients in the BGL series.

Alternatively one may state in terms of

$$\Delta\zeta \equiv \frac{G_F^2}{24\pi^3} \int_{q_i^2}^{q_f^2} dq^2 \mathbf{p}_\pi^3 |f_+^{B\pi}(q^2)|^2$$

different ranges:
LQCD: high q2
LCSR: low q2

	$q^2(\mathrm{GeV}^2)$	$\Delta\zeta(\mathrm{ps}^{-1})$
HPQCD [(Dalgic et al., 2006)]	> 16	2.02(0.55)
Fermilab/MILC [(Bailey et al., 2009)]	> 16	$2.21\left(^{+0.47}_{-0.42}\right)$
LCSR [(Khodjamirian, Mannel, Offen, and Wang, 2011)]	< 12	$4.59\left(^{+1.00}_{-0.85}\right)$

HPQCD relies on the Ball-Zwicky parameterization.

Exclusive $B \to \pi \ell \bar{\nu}$

From **Light-cone sum rules** : form factors expressed i.t.o.

$$f_+^{B\pi}(q^2) = \left(\frac{e^{m_B^2/M^2}}{2m_B^2 f_B}\right) \frac{1}{\pi} \int_{m_b^2}^{s_0^B} ds \, \mathrm{Im}\, F^{(OPE)}(s,q^2)\, e^{-s/M^2} .$$

where light-cone OPE is performed in a twist expansion

$$\mathrm{Im}\, F^{OPE} = \sum_{t=2,3,\ldots} T^{(t)} \otimes \phi_\pi^{(t)}$$

currently results include:

- NLO for 2-particle DA at twist 2 and 3
- LO for 2- and 3-particle DA up to twist 4.

Khodjamirian, Mannel, Offen, Wang, 2011

complementary to LQCD and competitive.

Summary

of Semileptonic B decays

Vcb is in good shape, excl. and incl. values are consistent.

$$|V_{cb}|_{excl} = [39.64\ (1 \pm 0.014_{exp} \pm 0.015_{th})] \times 10^{-3}$$
$$|V_{cb}|_{incl} = [41.88\ (1 \pm 0.010_{exp} \pm 0.014_{th})] \times 10^{-3}$$

Vub is still a puzzle, the tension persists.

$$|V_{ub}|_{excl} = [3.28\ (1 \pm 0.05_{exp} \pm 0.08_{th})] \times 10^{-3}$$
$$|V_{ub}|_{incl} = [4.41\ (1 \pm 0.034_{exp} \pm 0.039_{th})] \times 10^{-3}$$

values from private discussion with
Th. Mannel and A. Khodjamirian.
Thank you.