Kaon Physics: Theory Overview

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Outline

V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, J. Portolés, Rev. Mod. Phys. 84 (2012) 399

1. **Theoretical Framework.**

   Short and long-distance physics

2. **Leptonic and Semileptonic Decays.**

   Lepton Universality. CKM determinations

3. **Nonleptonic Decays.**

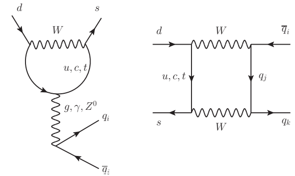
   Octet Enhancement. ε'/ε

4. **Rare and Radiative Decays.**

   $K \to \pi\nu\bar{\nu}\,,\ K \to \pi\ell^+\ell^-\,,\ K \to \pi\gamma\gamma\ \ldots$

1. Theoretical Framework

- **Sensitivity to Short-Distance Scales:**

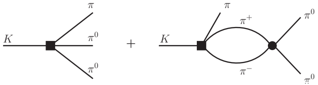


Charm mass prediction

Top quark

New Physics ?

- **Long-Distance Physics:**



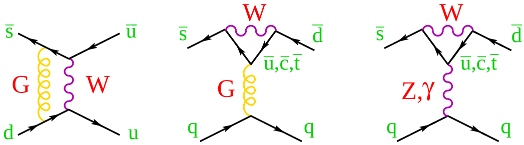
Chiral Dynamics

- **Multi-Scale Problem:**

$\log(M/\mu)$ **(OPE)** , $\log(\mu/m_\pi)$ **(χPT)**

Energy Scale	Fields	Effective Theory
M_W	W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Standard Model
	⬇ OPE	
$\lesssim m_c$	$\gamma, g\ ;\ \mu, e, \nu_i$ s, d, u	$\mathcal{L}_{\mathrm{QCD}}^{(n_f=3)}$, $\mathcal{L}_{\mathrm{eff}}^{\Delta S=1,2}$
	⬇ $N_C \to \infty$	
M_K	$\gamma\ ;\ \mu, e, \nu_i$ π, K, η	χPT

$\Delta S = 1$ TRANSITIONS



$$\mathcal{L}_{\mathrm{eff}}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i(\mu) \; Q_i(\mu)$$

$$Q_1 = (\bar{s}_\alpha u_\beta)_{\mathrm{V-A}} (\bar{u}_\beta d_\alpha)_{\mathrm{V-A}} \qquad Q_2 = (\bar{s}u)_{\mathrm{V-A}} (\bar{u}d)_{\mathrm{V-A}}$$

$$Q_{3,5} = (\bar{s}d)_{\mathrm{V-A}} \sum_q (\bar{q}q)_{\mathrm{V\mp A}} \qquad Q_4 = (\bar{s}_\alpha d_\beta)_{\mathrm{V-A}} \sum_q (\bar{q}_\beta q_\alpha)_{\mathrm{V-A}}$$

$$Q_{7,9} = \frac{3}{2}(\bar{s}d)_{\mathrm{V-A}} \sum_q e_q (\bar{q}q)_{\mathrm{V\pm A}} \qquad Q_{10} = \frac{3}{2}(\bar{s}_\alpha d_\beta)_{\mathrm{V-A}} \sum_q e_q (\bar{q}_\beta q_\alpha)_{\mathrm{V-A}}$$

$$Q_6 = -8 \sum_q (\bar{s}_L q_R)(\bar{q}_R d_L) \qquad Q_8 = -12 \sum_q e_q (\bar{s}_L q_R)(\bar{q}_R d_L)$$

$$Q_{11,12} = (\bar{s}d)_{\mathrm{V-A}} \sum_\ell (\bar{\ell}\ell)_{\mathrm{V,A}} \qquad Q_{13} = (\bar{s}d)_{\mathrm{V-A}} \sum_\nu (\bar{\nu}\nu)_{\mathrm{V-A}}$$

- $q > \mu$: $\quad C_i(\mu) = z_i(\mu) - y_i(\mu) \left(V_{td} V_{ts}^* / V_{ud} V_{us}^* \right)$

  $O(\alpha_s^n t^n)$, $O(\alpha_s^{n+1} t^n)$ $\qquad [t \equiv \log(M/m)]$ $\qquad$ (Munich / Rome)

- $q < \mu$: $\quad \langle \pi\pi | Q_i(\mu) | K \rangle$? $\qquad$ Physics does not depend on μ

CHIRAL PERTURBATION THEORY (χPT)

- **Expansion in powers of p^2/Λ_χ^2 :** $\mathcal{A} = \sum_n \mathcal{A}^{(n)}$ ($\Lambda_\chi \sim 4\pi F_\pi \sim 1.2$ GeV)
- **Amplitude structure fixed by chiral symmetry**

$$\mathrm{SU(3)_L} \otimes \mathrm{SU(3)_R} \to \mathrm{SU(3)_V}$$

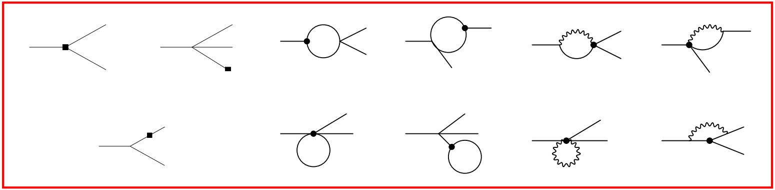
- **Short-distance dynamics encoded in Low-Energy Couplings**
- **$O(p^2)$ χPT:** (Goldstone interactions: π, K, η)

$$\mathcal{L}_2^{\Delta S=1} = G_8\, F^4\, \langle \lambda L_\mu L^\mu \rangle + G_{27}\, F^4 \left(L_{\mu 23} L^\mu_{11} + \frac{2}{3}\, L_{\mu 21} L^\mu_{13} \right)$$

$$G_R \equiv -\frac{G_F}{\sqrt{2}}\, V_{ud} V^*_{us}\, g_R \quad ; \quad L_\mu = -iU^\dagger D_\mu U \quad ; \quad \lambda \equiv \frac{1}{2}\lambda_{6-i7} \quad ; \quad U \equiv \exp\left\{ i\sqrt{2}\,\Phi/F \right\}$$

- **Loop corrections (χPT logarithms) unambiguously predicted**
- **LECs can be determined at $N_C \to \infty$ (matching)**
- **$O(p^2)$ LECs (G_8, G_{27}) can be phenomenologically determined**

$\mathbf{O}\left[\mathbf{p^4}, (\mathbf{m_u} - \mathbf{m_d})\,\mathbf{p^2}, \mathbf{e^2p^0}, \mathbf{e^2p^2}\right]$ χPT



- **Nonleptonic weak Lagrangian:** $\mathcal{O}(G_F\, p^4)$

$$\mathcal{L}_{\rm weak}^{(4)} = \sum_i G_8\, N_i\, F^2\, O_i^8 + \sum_i G_{27}\, D_i\, F^2\, O_i^{27} + \text{h.c.}$$

- **Electroweak Lagrangian:** $\mathcal{O}(G_F\, e^2 p^{0,2})$

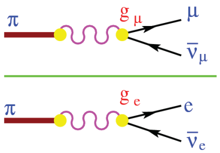
$$\mathcal{L}_{\rm EW} = e^2 F^6 G_8\, g_{ew}\, \langle \lambda U^\dagger \mathcal{Q} U \rangle + e^2 \sum_i G_8\, Z_i\, F^4\, O_i^{EW} + \text{h.c.}$$

- $\mathcal{O}(e^2 p^{0,2})$ **Electromagnetic** $+$ $\mathcal{O}(p^4)$ **Strong:** $Z\,,\ K_i\,,\ L_i$

2. (Semi) Leptonic Decays

Lepton Universality:

$$R_{e/\mu}^{(P)} \equiv \frac{\Gamma(P^- \to e^- \bar{\nu}_e)}{\Gamma(P^- \to \mu^- \bar{\nu}_\mu)}$$



$$R_{e/\mu}^{(\pi)} = (1.2352 \pm 0.0001) \cdot 10^{-4}$$

$$R_{e/\mu}^{(K)} = (2.477 \pm 0.001) \cdot 10^{-5}$$

Cirigliano-Rosell '07

$$R_{e/\mu}^{(\pi)}\Big|_{\mathrm{exp}} = (1.230 \pm 0.004) \cdot 10^{-4}$$

$$R_{e/\mu}^{(K)}\Big|_{\mathrm{exp}} = (2.488 \pm 0.012) \cdot 10^{-5}$$

(KLOE, NA62)

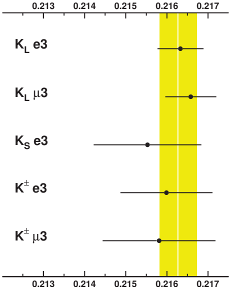
$$\frac{|g_\mu|}{|g_e|} = \begin{cases} 1.0021 \pm 0.0016 & \pi \to \mu/e \\ 0.9978 \pm 0.0024 & K \to \mu/e \\ 1.0010 \pm 0.0025 & K \to \pi\mu/e \\ 1.0018 \pm 0.0014 & \tau \to \mu/e \end{cases}$$

$K \to \pi\, \ell\, \nu_\ell$

$$|\mathbf{V_{us}\, f_+(0)}| \; = \; \mathbf{0.2163 \pm 0.0005}$$

Flavianet Kaon WG, arXiv:1005.2323 [hep-ph]

$$\langle \pi^- | \bar{s} \gamma_\mu u | K^0 \rangle = (p_\pi + p_K)_\mu \; f_+(t) + (p_K - p_\pi)_\mu \; f_-(t)$$



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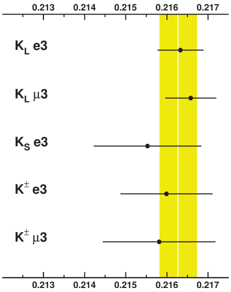

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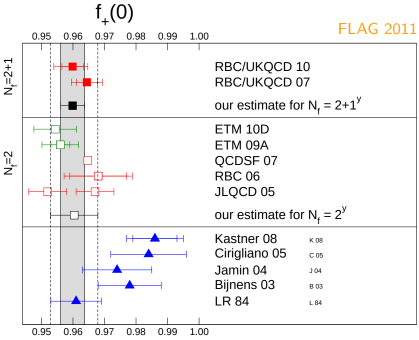

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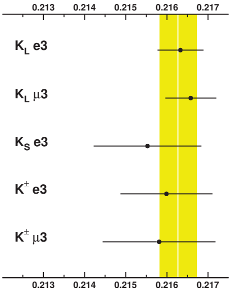
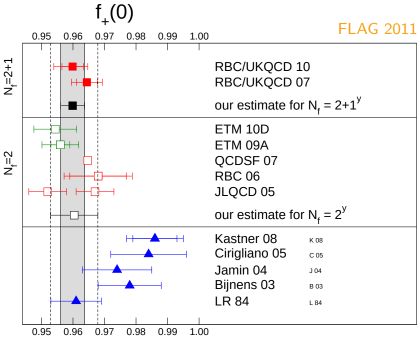




$$f_+(0) = 0.9599\,(34)(^{+31}_{-47})(14)$$ (RBC/UKQCD)

$$\longrightarrow \quad |V_{us}| = 0.2255\,(14)$$

$$f_+(0) = 0.97 \pm 0.01$$

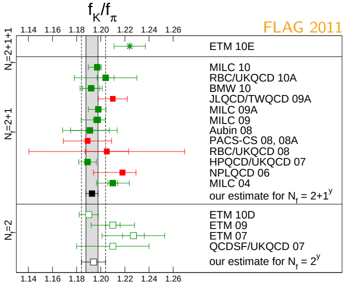
$$\longrightarrow \quad |V_{us}| = 0.2230\,(24)$$

Large $\mathcal{O}(p^6)$ χPT correction

$\Gamma(K^+ \to \mu^+\nu_\mu)/\Gamma(\pi^+ \to \mu^+\nu_\mu)$:

(Cirigliano-Neufeld 2011)

$$\frac{|V_{us}\, f_K|}{|V_{ud}\, f_\pi|} = 0.2763 \pm 0.0005 \quad \longrightarrow \quad \frac{|V_{us}|}{|V_{ud}|} = 0.2316 \pm 0.0012$$



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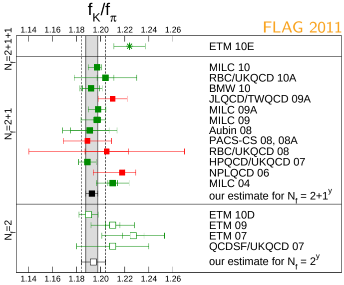
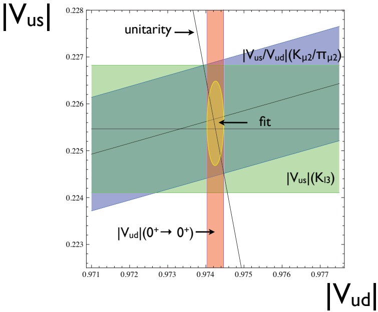


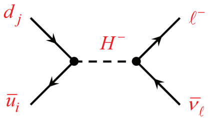


Fit: $|V_{ud}| = 0.97425 \pm 0.00022 \qquad |V_{ud}| = 0.2256 \pm 0.0009$

$$\Delta_{\rm CKM} \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = 0.0001 \pm 0.0006$$

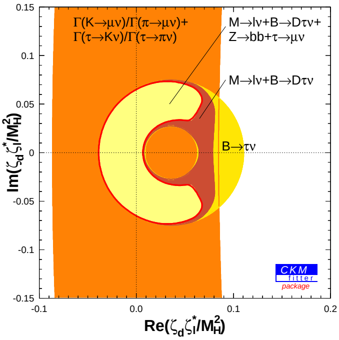
Constraints on the aligned 2-Higgs-doublet model:

(95% CL, Jung-Pich-Tuzón)

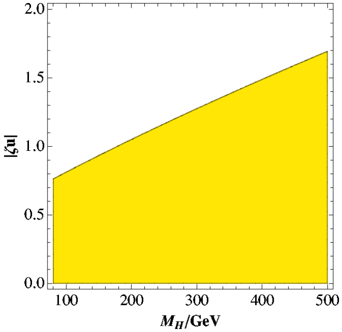


$$\mathcal{L}_Y = -\frac{\sqrt{2}}{v} H^+ \left\{ \bar{u} \left[\varsigma_d V_{\mathrm{CKM}} M_d \mathcal{P}_R - \varsigma_u M_u^\dagger V_{\mathrm{CKM}} \mathcal{P}_L \right] d + \varsigma_l \left(\bar{\nu} M_l \mathcal{P}_R l \right) \right\} + \mathrm{h.c.}$$

Semileptonic



ε_K



3. Nonleptonic Decays

- **Octet Enhancement:** $$\frac{A(K \to \pi\pi)_{I=0}}{A(K \to \pi\pi)_{I=2}} \approx 22$$

  - **Short-distance:** **gluonic corrections, penguins**
  - **Long-distance:** **large χPT corrections (FSI)**
  - **Ongoing Lattice efforts**

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  - **Ongoing Lattice efforts**

- **Direct CP Violation:** $\eta_{ij} \equiv \frac{A(K_L \to \pi^i \pi^j)}{A(K_S \to \pi^i \pi^j)}$

$$\mathrm{Re}\,(\epsilon'/\epsilon) = \frac{1}{3}\left(1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|\right) = (16.8 \pm 1.4) \cdot 10^{-4}$$

(NA31, E731, NA48, KTeV)

$$\mathrm{Re}\,(\epsilon'/\epsilon)_{\mathrm{SM}} = \left(19 \pm 2\,{}^{+9}_{-6} \pm 6\right) \cdot 10^{-4}$$

(Pallante-Pich-Scimemi)

Anatomy of ε'/ε calculation

$$\frac{\varepsilon'_K}{\varepsilon_K} \sim \left[\frac{105\ \mathrm{MeV}}{m_s(2\ \mathrm{GeV})}\right]^2 \left\{B_6^{(1/2)}\,(1-\Omega_{\mathrm{eff}}) - 0.4\ B_8^{(3/2)}\right\}$$

❶ $O(p^4)$ χPT Loops: **Large correction** (FSI) Pallante-Pich-Scimemi

❷ $O(p^4)$ LECs fixed at $N_C \to \infty$: **Small correction**

❸ Isospin Breaking $O\left[(m_u - m_d)\, p^2, e^2 p^2\right]$: **Sizeable corrections**

$\Omega_{\mathrm{eff}} = 0.06 \pm 0.08$ Cirigliano-Ecker-Neufeld-Pich

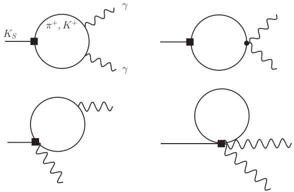
❹ $O(p^4)$ LECs $[\mathrm{Re}(g_8),\ \mathrm{Re}(g_{27})]$ and phase-shifts fitted to data

❺ $m_s(2\ \mathrm{GeV}) = 110 \pm 20$ MeV (quark condensate)

4. Rare and Radiative Decays

$K^0 \to \gamma\gamma$

Long-distance dynamics



Finite loop: $\mathrm{Br}_{\mathrm{LO}} = 2.0 \cdot 10^{-6}$

(D'Ambrosio-Espriu, Goity)

$$\mathrm{Br}(K_S \to \gamma\gamma) = (2.63 \pm 0.17) \cdot 10^{-6}$$

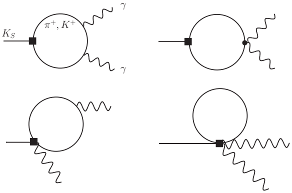
Agreement at $O(p^6)$ (FSI)

$$K_S \to \pi\pi \to \pi^+\pi^- \to \gamma\gamma$$

(Kambor-Holstein, Buchalla et al)

4. Rare and Radiative Decays

$K^0 \to \gamma\gamma$ — Long-distance dynamics



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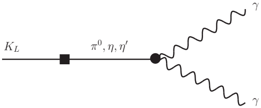


$\mathrm{Br}(K_L \to \gamma\gamma) = (5.47 \pm 0.04) \cdot 10^{-4}$

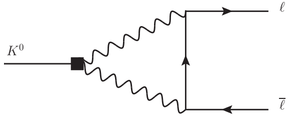
WZW Anomaly

$\mathbf{T}_{\mathrm{LO}} = \mathbf{0}$ [$\mathcal{O}(p^4)$, GMO cancel.]

$\mathcal{O}(p^6)$: SU(3) breaking, η–η' mixing

Well understood

$K^0 \to \ell^+\ell^-$



$K_S \to \ell^+\ell^-$

Long-distance dynamics

Finite 2-loop amplitude: (Ecker-Pich)

$$\mathrm{Br}(K_S \to e^+e^-)_{\mathrm{LO}} = 2.1 \cdot 10^{-14}$$

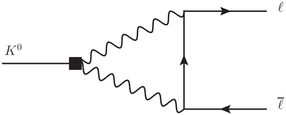
$$\mathrm{Br}(K_S \to \mu^+\mu^-)_{\mathrm{LO}} = 5.1 \cdot 10^{-12}$$

$$\mathrm{Br}(K_S \to e^+e^-)_{\mathrm{exp}} < 9 \cdot 10^{-9}$$

$$\mathrm{Br}(K_S \to \mu^+\mu^-)_{\mathrm{exp}} < 3.2 \cdot 10^{-7}$$

(90% CL)

$K^0 \to \ell^+\ell^-$



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(90% CL)

$K_L \to \ell^+\ell^-$

$$\mathrm{Br}(K_L \to \mu^+\mu^-) = (6.84 \pm 0.11) \cdot 10^{-9}$$

$$\mathrm{Br}(K_L \to e^+e^-) = (9\,^{+6}_{-4}) \cdot 10^{-12}$$

Saturated by absorptive contrib.

Local countertem ⟷ **SD**

LD extracted from $\pi^0, \eta \to \ell^+\ell^-$

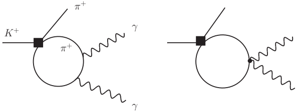
(Gomez-Dumm, Pich)

Fitted SD contrib. agrees with SM

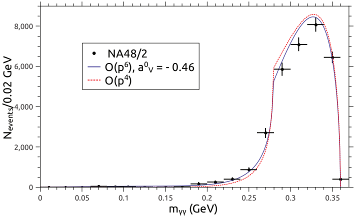
Longitudinal Polarization: (Ecker-Pich)

$$|\mathcal{P}_L| = (2.6 \pm 0.4) \cdot 10^{-3}$$

$K \to \pi\gamma\gamma$



$$\mathrm{Br}(K_L \to \pi^0\gamma\gamma) = (1.27 \pm 0.03) \cdot 10^{-6}$$



Finite 1-loop amplitude $[\mathcal{O}(p^4)]$**:**

$$\mathrm{Br}(K_L \to \pi^0\gamma\gamma)_{\mathrm{LO}} = 6.8 \cdot 10^{-7}$$

(Ecker-Pich-de Rafael, Cappiello-D'Ambrosio, Sehgal)

$\mathcal{O}(p^6)$ unitarity corrections needed

(Cohen et al, Cappiello et al, D'Ambrosio-Portolés)

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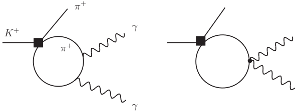

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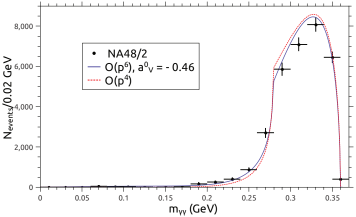

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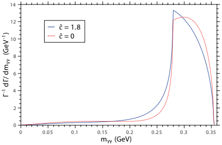
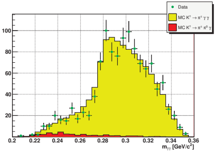
$$\mathrm{Br}(K^+ \to \pi^+\gamma\gamma) = (1.1\pm0.3)\cdot10^{-6}$$

(BNL-E787)



NA48



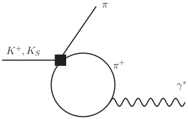
Local $\mathcal{O}(p^4)$ LEC: (Ecker-Pich-de Rafael)

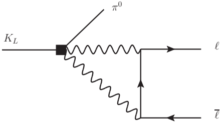
$\hat{c} = 1.6\,(1.8)\pm0.6$ at $\mathcal{O}(p^4)$ (p^6)

Small higher-order corrections

(D'Ambrosio-Portolés)

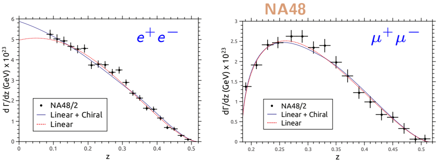
$K \to \pi\, \ell^+ \ell^-$





$$\mathrm{Br}(K^\pm \to \pi^\pm e^+ e^-) = 3.14\ (10) \cdot 10^{-7}$$

$$\mathrm{Br}(K^\pm \to \pi^\pm \mu^+ \mu^-) = 9.62\ (25) \cdot 10^{-8}$$

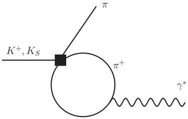


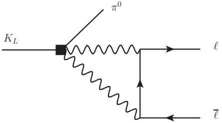
Local $\mathcal{O}(p^4)$ LECs (Ecker-Pich-de Rafael)

Electromagn. transition form factor

$\mathcal{O}(p^6)$ corrections (D'Ambrosio et al)

$K \to \pi \ell^+ \ell^-$





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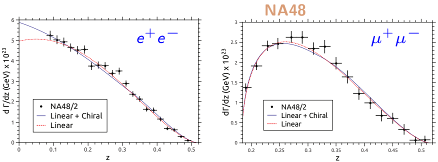


Local $\mathcal{O}(p^4)$ LECs (Ecker-Pich-de Rafael)

Electromagn. transition form factor

$\mathcal{O}(p^6)$ corrections (D'Ambrosio et al)

$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) < 2.8 \cdot 10^{-10}$

$\mathrm{Br}(K_L \to \pi^0 \mu^+ \mu^-) < 3.8 \cdot 10^{-10}$

(KTeV, 90% CL)

3 contributions: (Ecker-Pich-de Rafael)

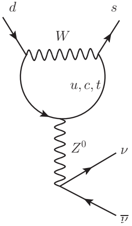
- **Direct** $\not{C}\not{P}$
- **Indirect** $\not{C}\not{P}$
- **CP conserving** (2γ)

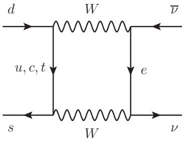
$\not{C}\not{P}$ **dominates for** $\mathbf{e^+e^-}$**:**

$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) = 3.1\ (0.9) \cdot 10^{-11}$

(Buchalla et al)

$K \to \pi\nu\bar{\nu}$





$$T \sim F\left(V_{is}^* V_{id},\, \frac{m_i^2}{M_W^2}\right) \left(\bar{\nu}_L \gamma_\mu \nu_L\right) \langle\pi|\,\bar{s}_L \gamma^\mu d_L\,|K\rangle$$

Negligible long-distance contribution

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu}) = (7.8 \pm 0.8)\cdot 10^{-11} \sim A^4 \left[\eta^2 + (1.4-\rho)^2\right]$$

$$\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu}) = (2.4 \pm 0.4)\cdot 10^{-11} \sim A^4\,\eta^2$$

Buras et al

$\mathcal{A}(K_L \to \pi^0\nu\bar{\nu}) \neq 0$ ➡ **Direct** $\not{C}\!\!\not{P}$

BNL-E949: few events! ➡ $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu}) = \left(1.73\,^{+1.15}_{-1.05}\right)\cdot 10^{-10}$

KEK-E391a: $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu}) < 2.6\cdot 10^{-8}$ (90% CL)

Ongoing Experiments: NA62, K0TO (ORKA, Project-X)

Summary

Kaons continue providing important physics information:

- Interesting interplay of short and long-distances
- Sensitive to heavy mass scales. New Physics?
- Superb probe of flavour dynamics and $\not{C}\not{P}$
- Excellent testing ground of χPT dynamics

Theoretical challenge: precise control of QCD effects

Increased sensitivities at ongoing experiments $(K \to \pi\nu\bar{\nu})$

Future data could bring interesting surprises