Dynamic structural analysis of absorbers with spectral-element code ELSE

Yacine Kadi, Roberto Rocca, Wim Weterings - CERN
Luca Massidda - CRS4

Workshop on Materials for Collimators and Beam Absorbers
CERN - September, 3-5 2007

Table of Contents

- Overview of the Spectral Element Method
- TCDS absorber dynamic stress analysis
- TPSG absorber dynamic stress analysis
- Some thoughts on materials and numerical simulations

The beam dump problem

- Particles are dumped in the solid with internal heat generation, calculated by FLUKA
- Temperature increases suddenly giving origin to thermal stresses
- Elastic waves are propagated through the structure

$$\begin{cases} \rho \ddot{u}_i + \cancel{\beta \dot{u}_i} - \sum_{j=1}^{d} \tau_{ij,j} = f_i \\ \rho c_v \dot{\theta} - \left(\cancel{k \theta_{,i}}\right)_{,i} = q \end{cases}$$

$$\tau_{ij} = C_{ijlm}\left(\varepsilon_{lm} - \delta_{ij}\alpha_i\theta\right)$$

$$\varepsilon_{ij} = \tfrac{1}{2}\left(u_{i,j} + u_{j,i}\right)$$

SEM overview: problem formulation

- To give a short introduction to the methodology let us assume we have to solve the following simple PDE problem over a computational domain Ω with Dirichlet boundary Γ_D and Neumann boundary Γ_N

Find $u \in C^2(\Omega)$, such that $u\big|_{\Gamma_D} = g$ and

$$-\nabla \cdot (c\nabla u) = f \quad \text{in } \Omega$$

$$\vec{n} \cdot (c\nabla u) = h \quad \text{on } \Gamma_N$$

Weak form

- The PDE problem is turned in its weak form

Find $u \in C^2(\Omega)$, such that $u|_{\Gamma_D} = g$ and

$$-\nabla \cdot (c\nabla u) = f \quad \text{in } \Omega \qquad \vec{n} \cdot (c\nabla u) = h \quad \text{on } \Gamma_N$$

⇩

Find $u \in H^1(\Omega)$, such that $u|_{\Gamma_D} = g$ and

$$\int_\Omega -v\nabla \cdot (c\nabla u)d\Omega = \int_\Omega vfd\Omega \quad \forall v \in H_0^1(\Omega)$$

Numerical approximation

- The functional space $H^1(\Omega)$ is infinite dimensional

$$H^1(\Omega) = \left\{ v \in L^2(\Omega) \mid \frac{\partial u}{\partial x_j} \in L^2(\Omega), j = 1, \ldots, n \right\}$$

- Two ideas are adopted
  - Approximate $H^1(\Omega)$ with the finite dimensional space $V_h(\Omega)$ with dimension N, such that $V_h(\Omega) \subset H^1(\Omega)$
  - Choose the test function v among the basis of the space Ψi

$$u(\vec{x}) \cong u_h(\vec{x}) \equiv \sum_{j=1}^{N} u_j \Psi_j(\vec{x})$$

Numerical approximation

- The variational problem may be rewritten as follows for the finite dimensional space $V_h(\Omega)$, and it easily turns to a linear system of equations

$$\text{Find } u_j \text{ with } j = 1,\ldots,N \text{ such that } u\big|_{\Gamma_D} = g \text{ and}$$

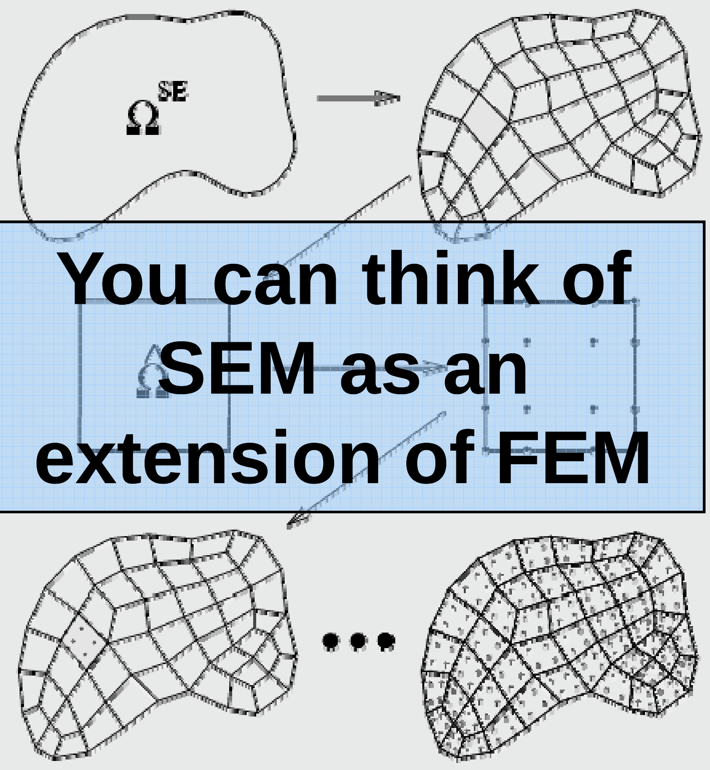
$$\sum_{j=1}^{N} u_j \int_{\Omega} \nabla\Psi_i \cdot c\nabla\Psi_j \, d\Omega = \int_{\Omega} \Psi_i f d\Omega + \int_{\Gamma_N} \Psi_i h d\Gamma$$

$$\forall \Psi_i \in V_h(\Omega) \text{ with } i = 1,\ldots,N$$

$$\mathbf{Ax} = \mathbf{b}$$

SE in a nutshell

- The domain is split into quad or hexa
- Each element is mapped onto a reference element
- LGL nodes are introduced (N=3)
- Spectral elements are mapped onto the domain



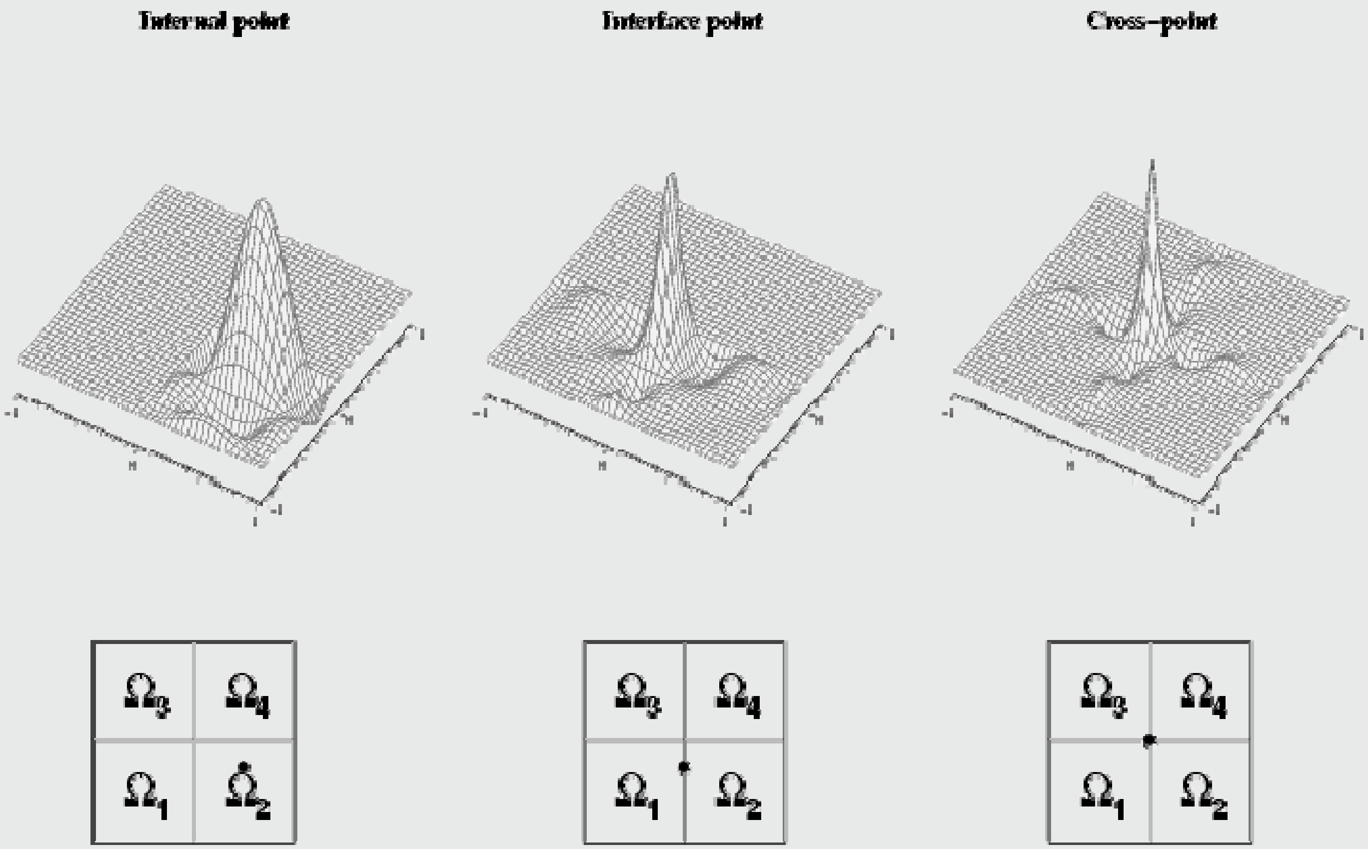
SE in a nutshell

- The computational domain Ω is split in a finite set of smaller elements Ω_k and each element is obtained by a 1 to 1 mapping from a reference element $\underline{\Omega}$
- The nodes in $\underline{\Omega}$ are placed in the LGL positions $\underline{\xi}_P$, in 1D the zeros of L'_N plus -1 and +1 (L_N is the Legendre polynomial of degree N)
- Shape functions ψ on $\underline{\Omega}$ are the Lagrange polynomials through the LGL nodes $\underline{\xi}_P$

$$\Psi_I = \frac{(\xi-\xi_1)\ldots(\xi-\xi_{I-1})(\xi-\xi_{I+1})\ldots(\xi-\xi_{N+1})}{(\xi_I-\xi_1)\ldots(\xi_I-\xi_{I-1})(\xi_I-\xi_{I+1})\ldots(\xi_I-\xi_{N+1})}$$

$$\Psi_I(\xi_J) = \delta_{IJ}$$

High order functions



Numerical integration

- Integration is performed element-wise

$$\int_{\Omega} f d\Omega = \sum_{k} \int_{\Omega_k} f d\Omega$$

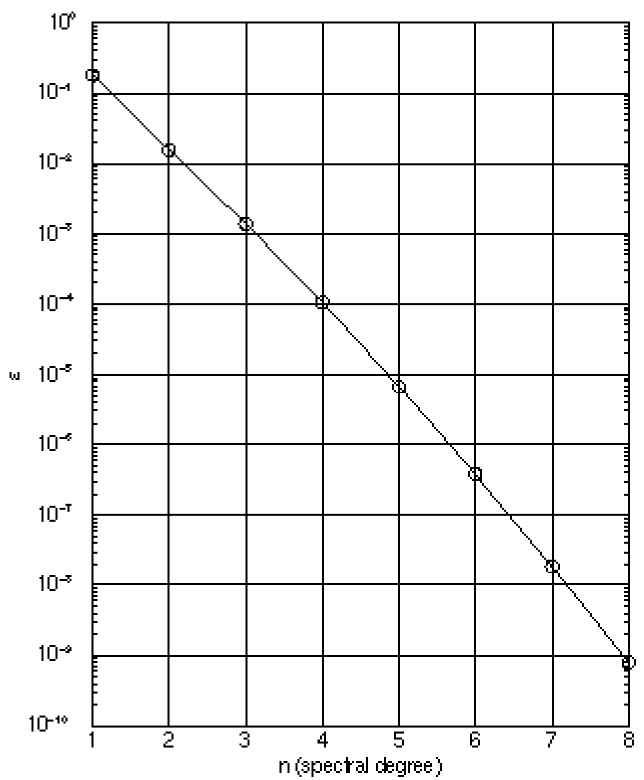
- Integrations are evaluated numerically over the reference element by the Legendre-Gauss-Lobatto (LGL) quadrature formula

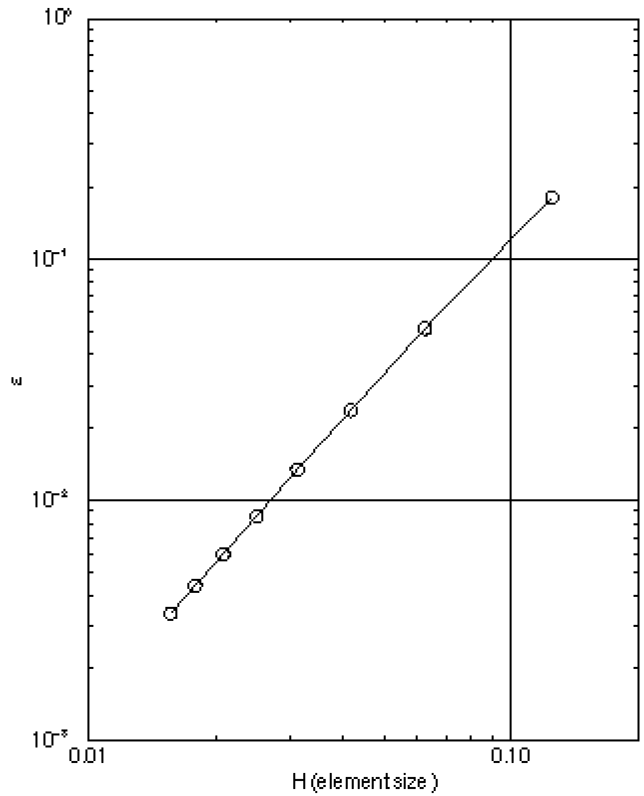
$$\int_{\Omega} f d\Omega \approx \sum_{q} \int_{\Omega_k} f(\hat{\mathbf{x}}_q) J_k(\hat{\mathbf{x}}_q) \hat{\omega}_q$$

- The choice of the particular position of the internal nodes assures the spectral accuracy

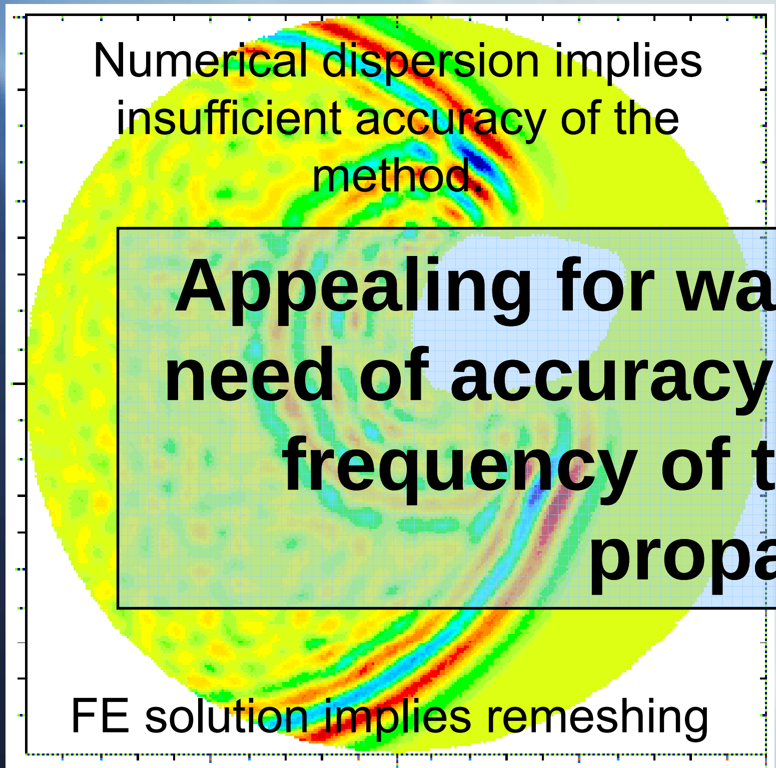
Spectral accuracy

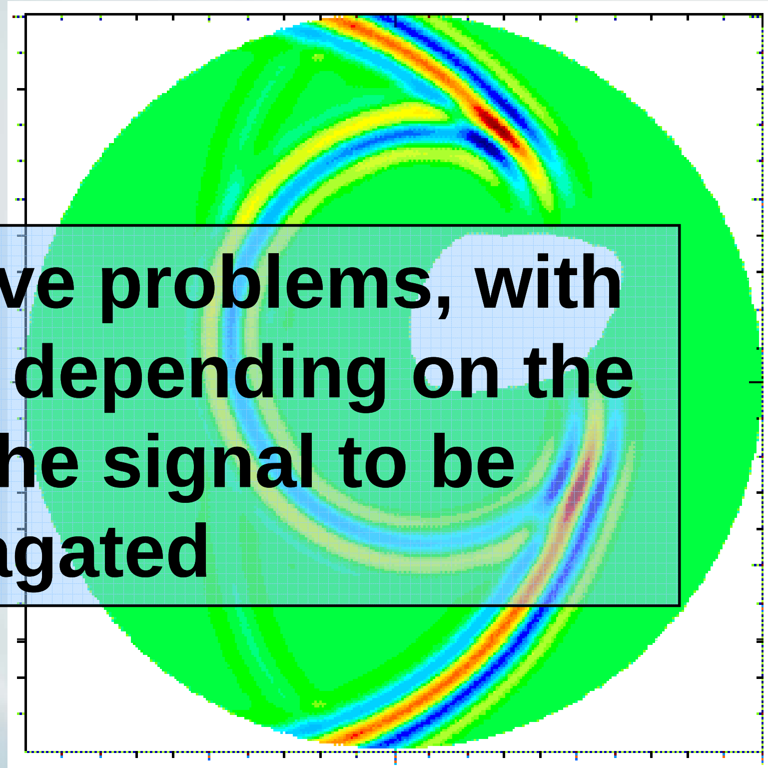
$$\left\| u - u_{N,h} \right\| \leq C e^{-N} h^{N}$$





Run-time high order





Appealing for wave problems, with need of accuracy depending on the frequency of the signal to be propagated

Run with spectral degree 1.

Analysis with increased spectral degree set to 2 at run-time.

Reasons for SEM

- Geometric flexibility of the Finite Elements
- High computational efficiency
- Spectral accuracy
- Run-time high order

Failure criteria: Stassi

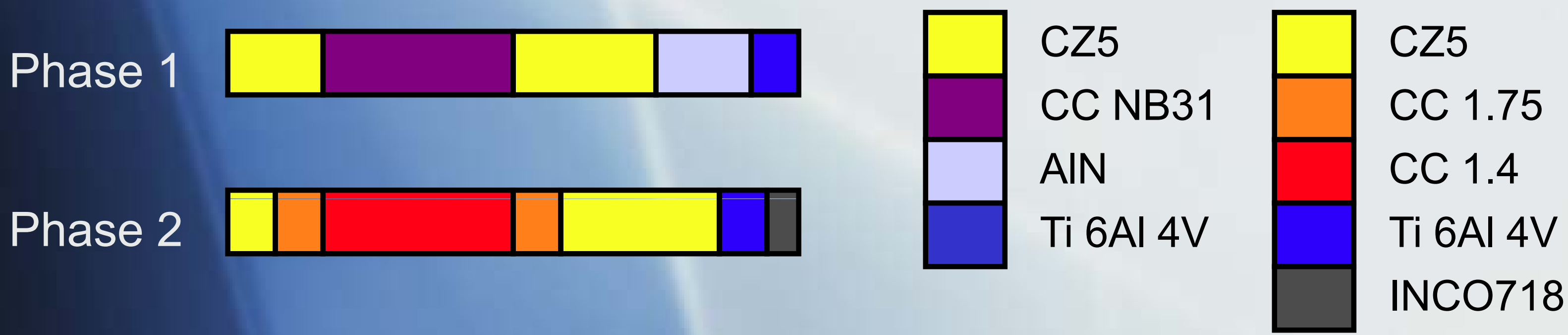
- Isotropic materials (graphite, titanium, steel, Inconel) are evaluated with the Stassi criterion
- The Stassi criterion is suitable for **isotropic brittle materials** having different tensile and compressive strength (whereas von Mises is only applicable for ductile materials)
- The equivalent Stassi stress is calculated on the base of the Von Mises equivalent stress, the ratio between the compressive and tensile strength and the hydrostatic pressure
- It is equivalent to the Von Mises stress when the tensile and compressive strengths are equal
- Structural safety is assessed on the base of the ratio between the Stassi equivalent stress and the tensile strenght of the material

Failure criteria: Maximum Stress

- For **anisotropic materials** (carbon composites) the maximum stress criterion is adopted
- Each component of the stress tensor is compared with the corresponding material strength, either compressive or tensile
- The maximum ratio between stress and strength is used to assess structural resistance

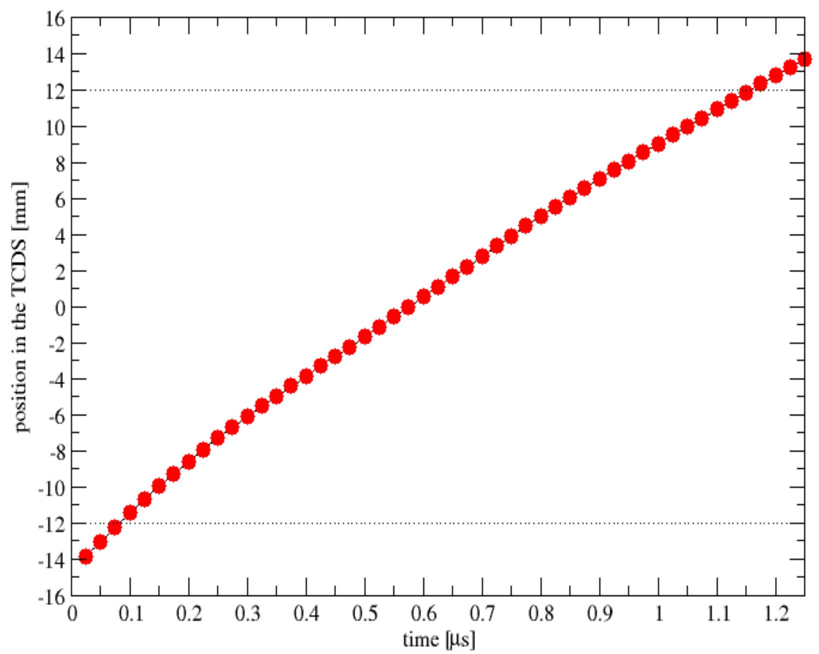
Target Collimator Dump Septum

- In the first design the TCDS was 3.0 long and had the following material composition: 1m of graphite, 2m of a carbon composite, 1.5m of graphite again, 1m of aluminum nitride and 0.5m of a titanium alloy
- The core had a wedge shape determined by the extreme orbit trajectories and is realized by a set of parallelepiped blocks (80mm high, ~24mm thick and 25mm long)
- In the revised design the core consists of 24 blocks, each 250mm long with the following materials: 0.5m of graphite, 0.5m of high density carbon composite, 2m of low density carbon composite, 0.5m of high density CC, 1.75m of graphite again, followed by 0.5 of a titanium alloy and 0.25m of a nickel alloy



TCDS beam load

- The proton beam is composed by 2808 bunches
- Each bunch contains 1.7 10^{11} protons with an energy of 7TeV
- The time step between the bunches is 25ns
- The beam is swept across the TCDS section in 1μs
- The energy of almost 40 bunches is deposited on the blocks



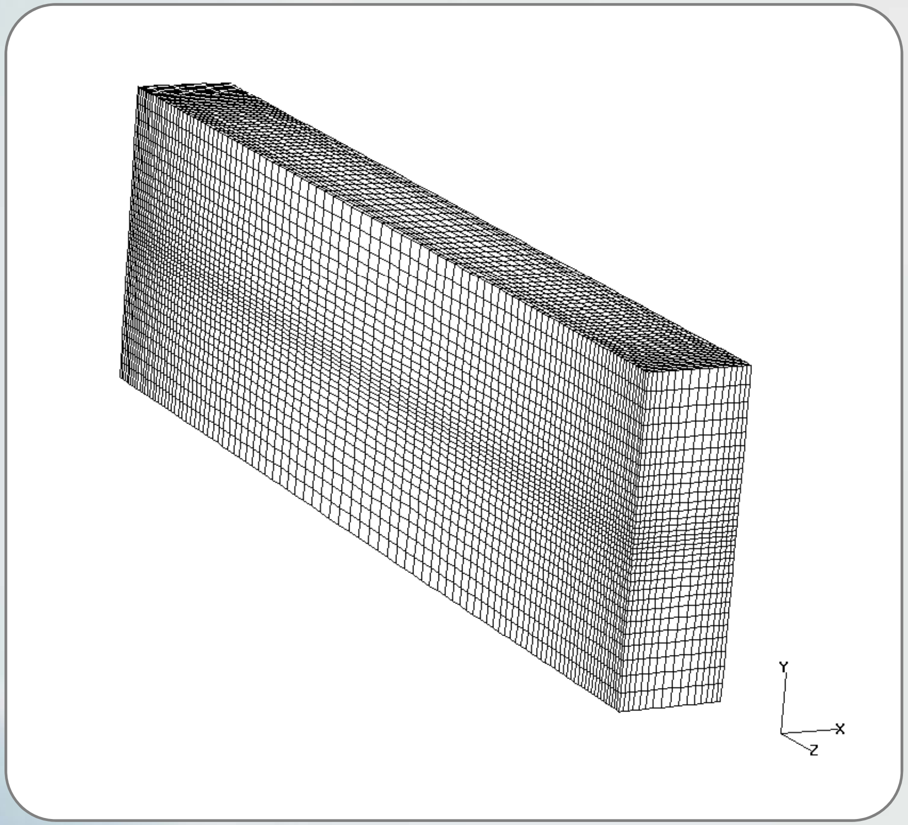
TCDS material properties RT

Material	Graphite	Titanium	Steel	CC 1.4 X	CC 1.4 YZ
Name	C2020	Ti 6Al 4V	316L	SG1.4	SG1.4
Density (kg m^{-3})	1760	4420	7990	1400	1400
Specific heat (J kg^{-1} K^{-1})	685	586	500	685	685
Thermal exp. (10^{-6}K^{-1})	3.6	9.0	16.0	1.0 - 5.0	-1.0 - 2.0
Young's mod. (GPa)	9.5	107	193	2.8	10
Tensile strength (MPa)	29.1	1036	290	46	61
Compres. strength (MPa)	29.1	1036	290	69.6	82.4

- The properties of SG1.75 are equal to those of SG1.4 except for the density
- The thermal expansion coefficient along the three direction for the Carbon Composite has been measured from 20°C up to 1000°C

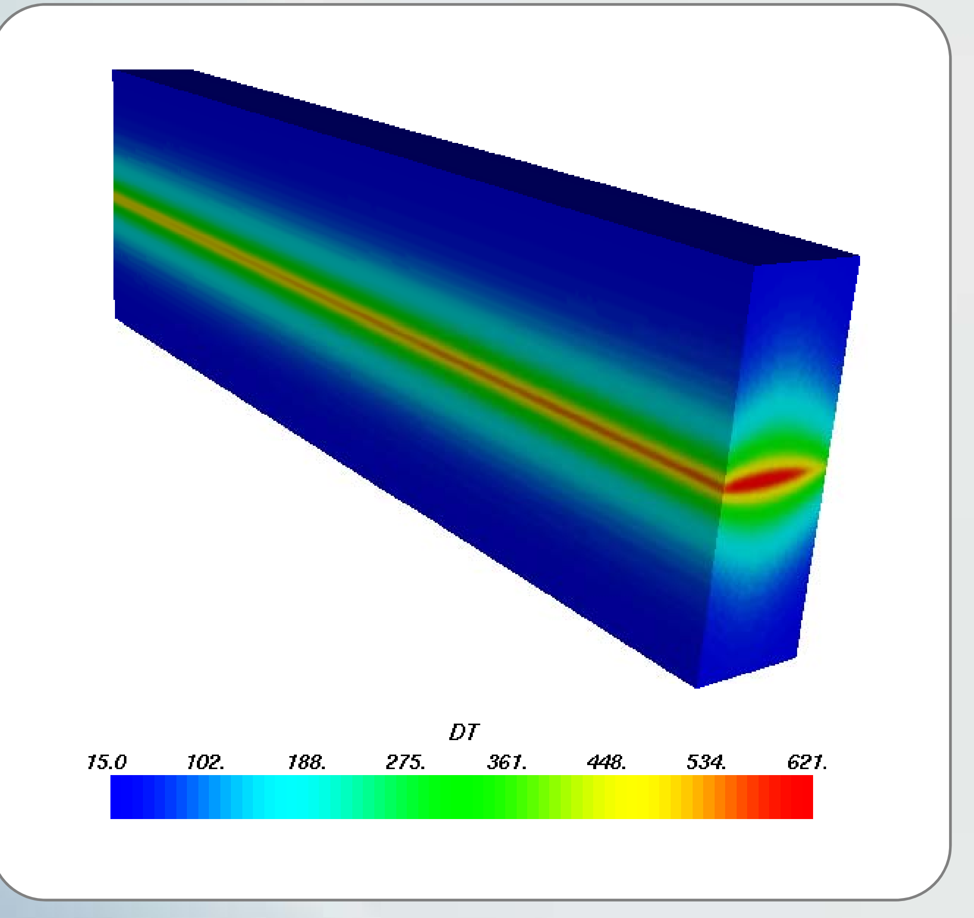
TCDS phase 2 model

- Each block has been simulated separately
- The mesh dimensions are 24x72x250mm
- The mesh consists of 51200 spectral elements
- A minimum spectral degree of 3 was adopted
- Internodal distance is in the range 0.2-2.0mm
- Total number of DOF ≈ 6 millions



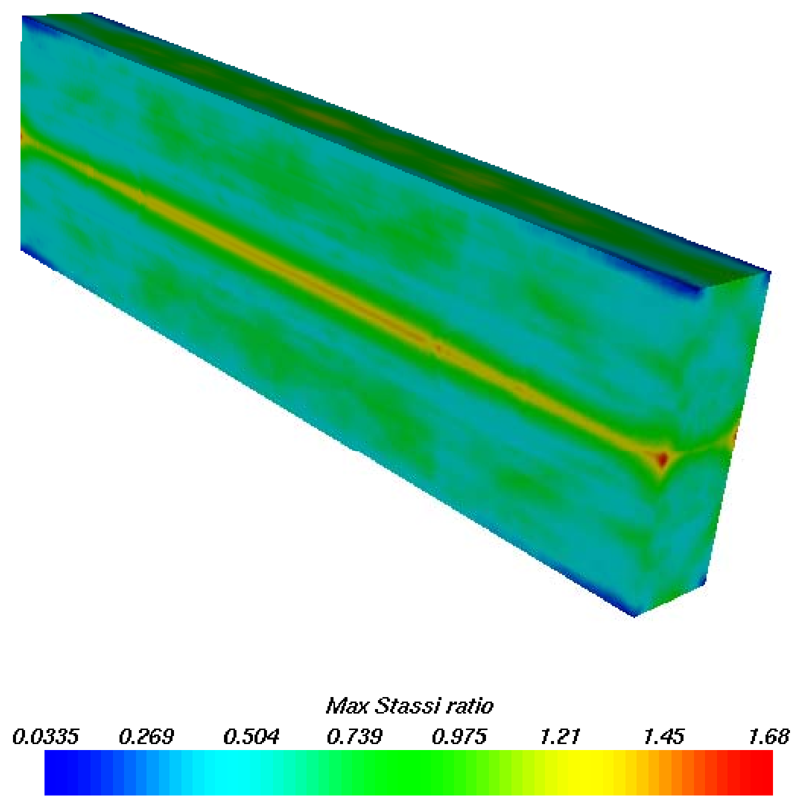
TCDS phase 2 results: graphite block 15 temperature increase

- ΔT in the block 15, made from graphite from 3.5m
- ΔT is reducing along the axis and the mean radius of the energy deposition widens
- Sweep velocity has an effect on the ΔT along the sweep direction

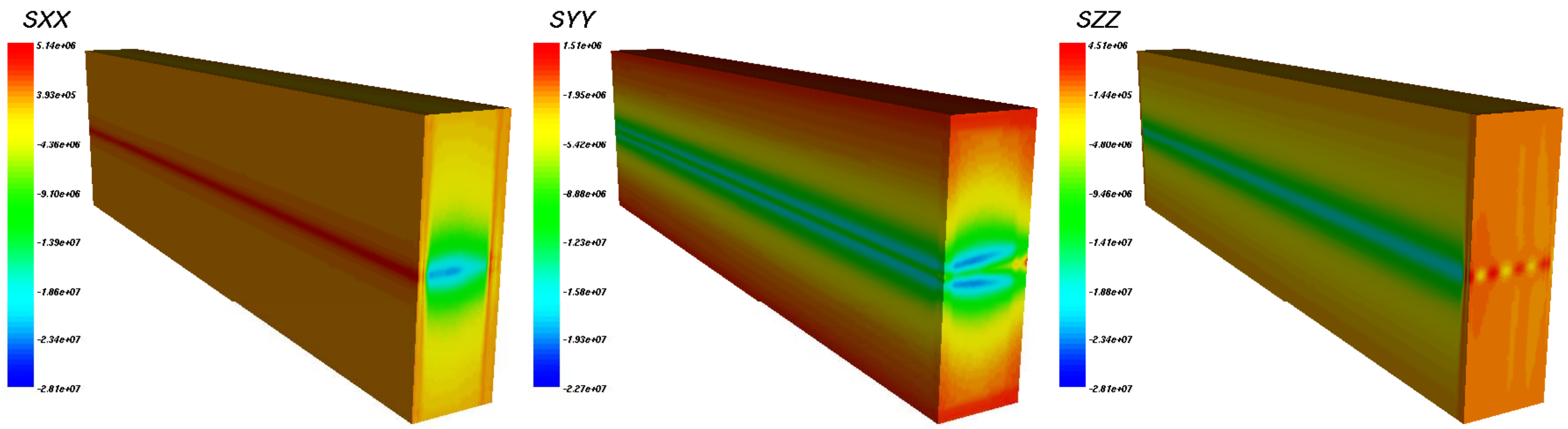


TCDS phase 2 results: graphite block 15 max Stassi ratio

- Max Stassi ratio in block 15, for a 200μs simulation
- High stress peaks on the lateral surfaces and on the vertexes
- The stresses are higher than the failure limit

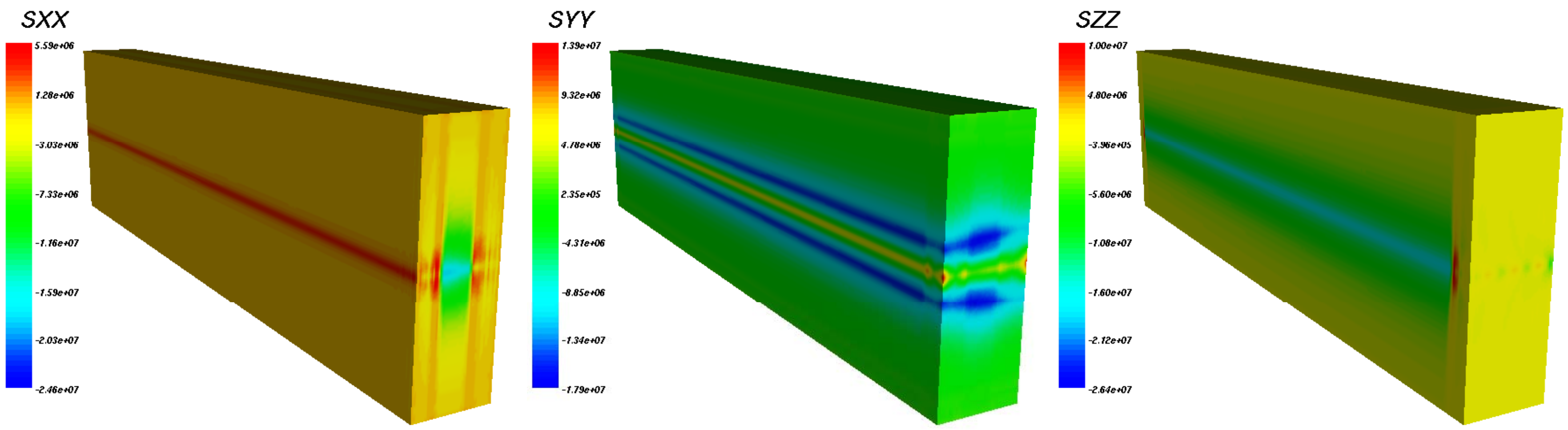


TCDS phase 2 results: stress waves in graphite block 15



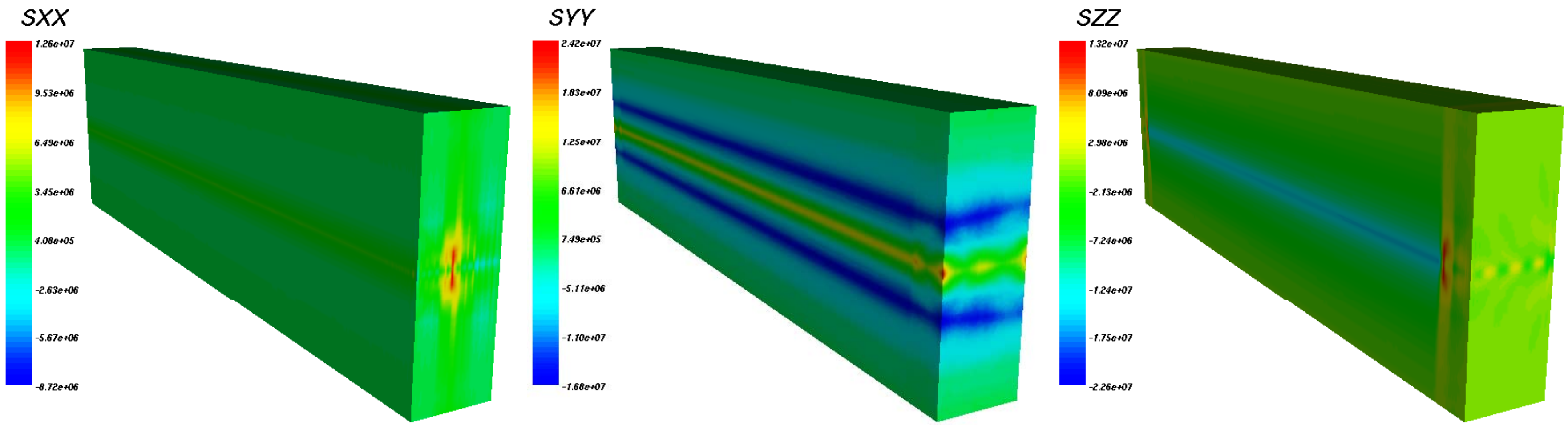
2μs

TCDS phase 2 results: stress waves in graphite block 15



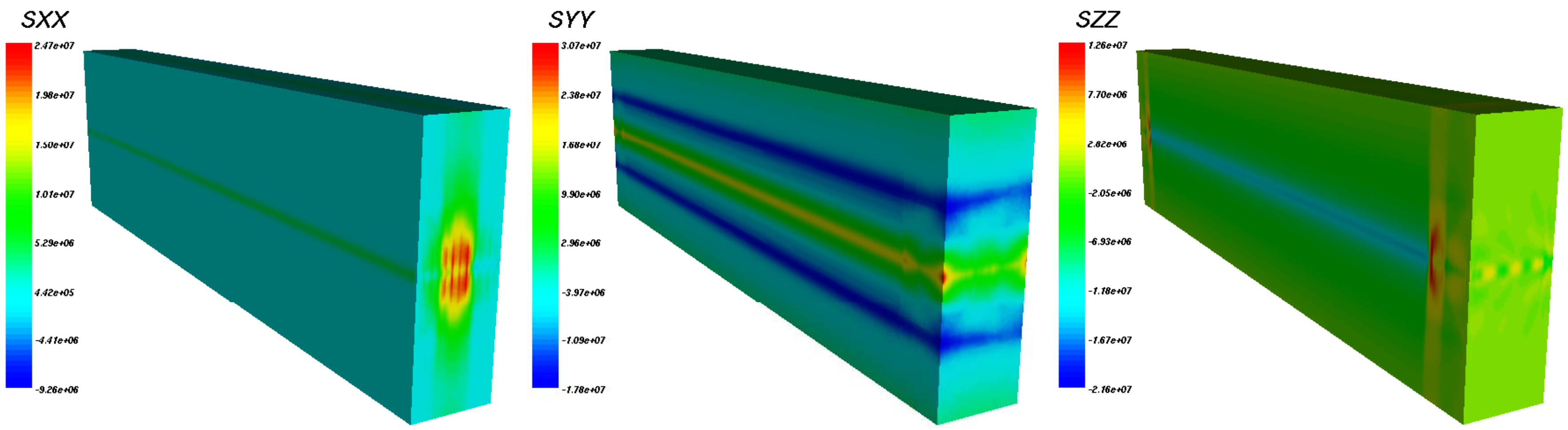
4μs

TCDS phase 2 results: stress waves in graphite block 15



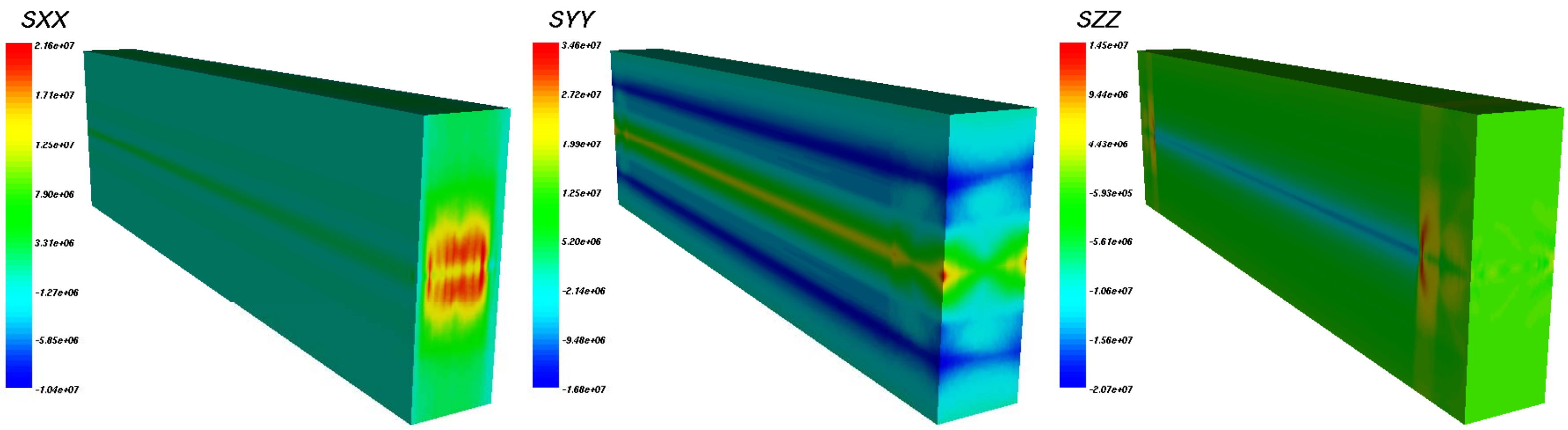
6μs

TCDS phase 2 results: stress waves in graphite block 15



8μs

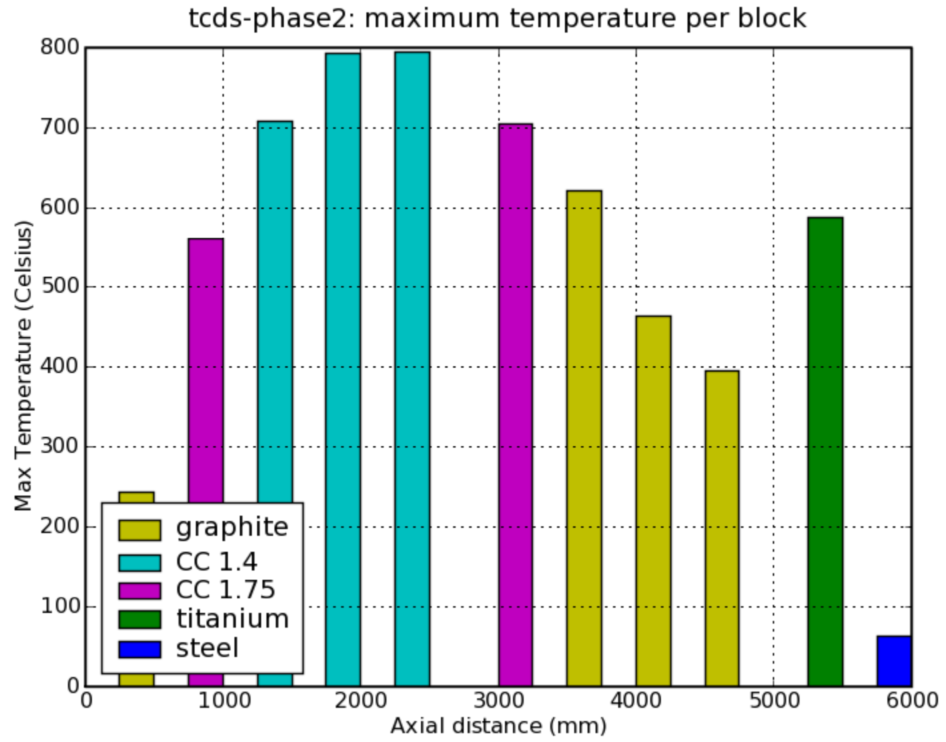
TCDS phase 2 results: stress waves in graphite block 15



10μs

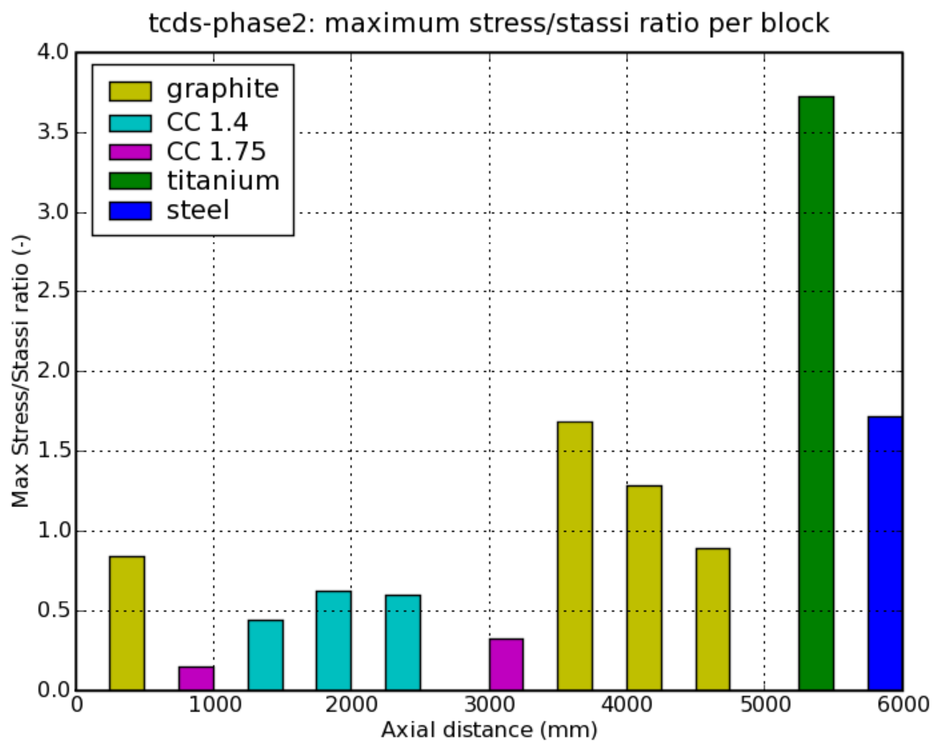
TCDS phase 2 results: temperature increase

- The temperature increase is high but not critic for material integrity
- The maximum values are reached in the low density carbon composite, but there are relatively high values also in the second graphite and in the titanium part



TCDS phase 2 results: max stress ratio

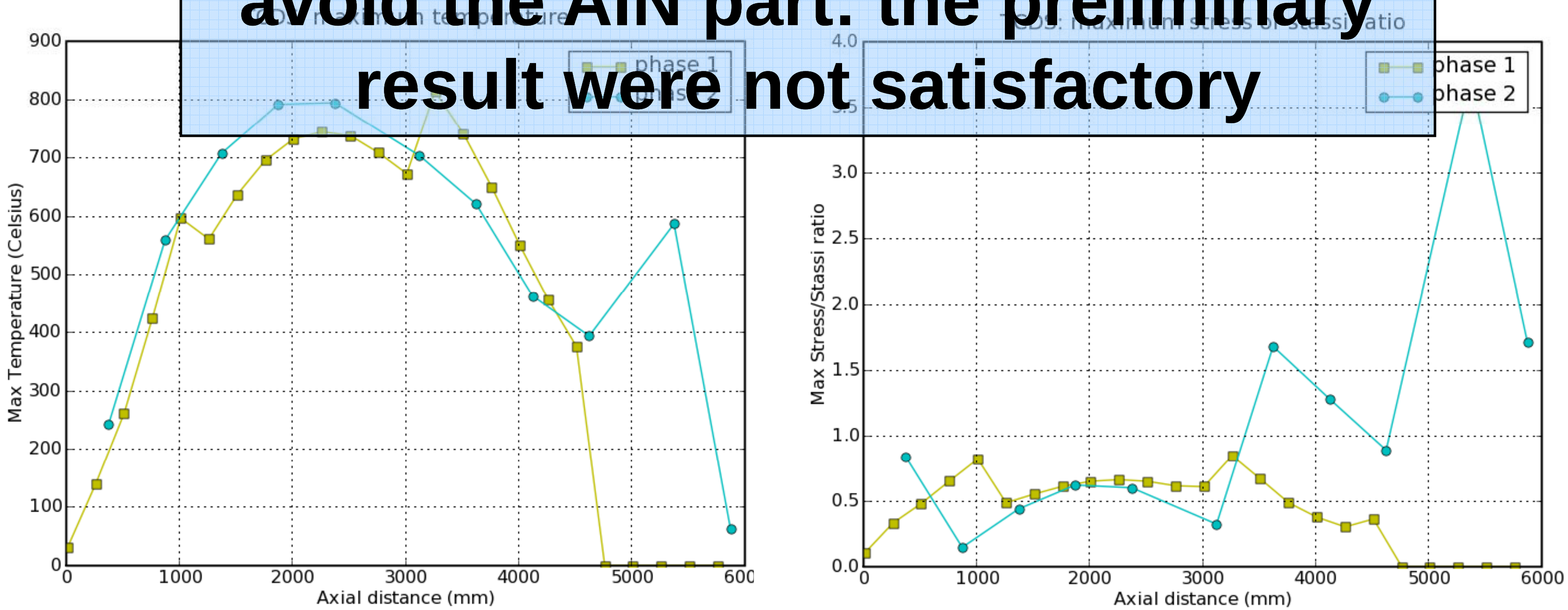
- High stresses are found on the second part of graphite blocks and on the titanium and steel blocks
- Values higher than unity imply a failure or a yielding



TCDS results comparison

It was necessary to extend the graphite portion of the target to avoid the AlN part: the preliminary result were not satisfactory

The results for phase 1 appear to be better than phase 2!



TCDS conclusions

- The temperature increase and the stress wave propagation in the blocks of the TCDS have been analyzed under the beam sweeping conditions
- The carbon composite seems to have excellent material properties, hence the relatively low values of the max stress ratio
- The most stressed part of each block are located on the heated plane, and on the lateral surfaces and vertexes in particular. Stresses may be reduced by an offset of the heating plane and a rounding of the block vertexes

TCDS conclusions: phase 3

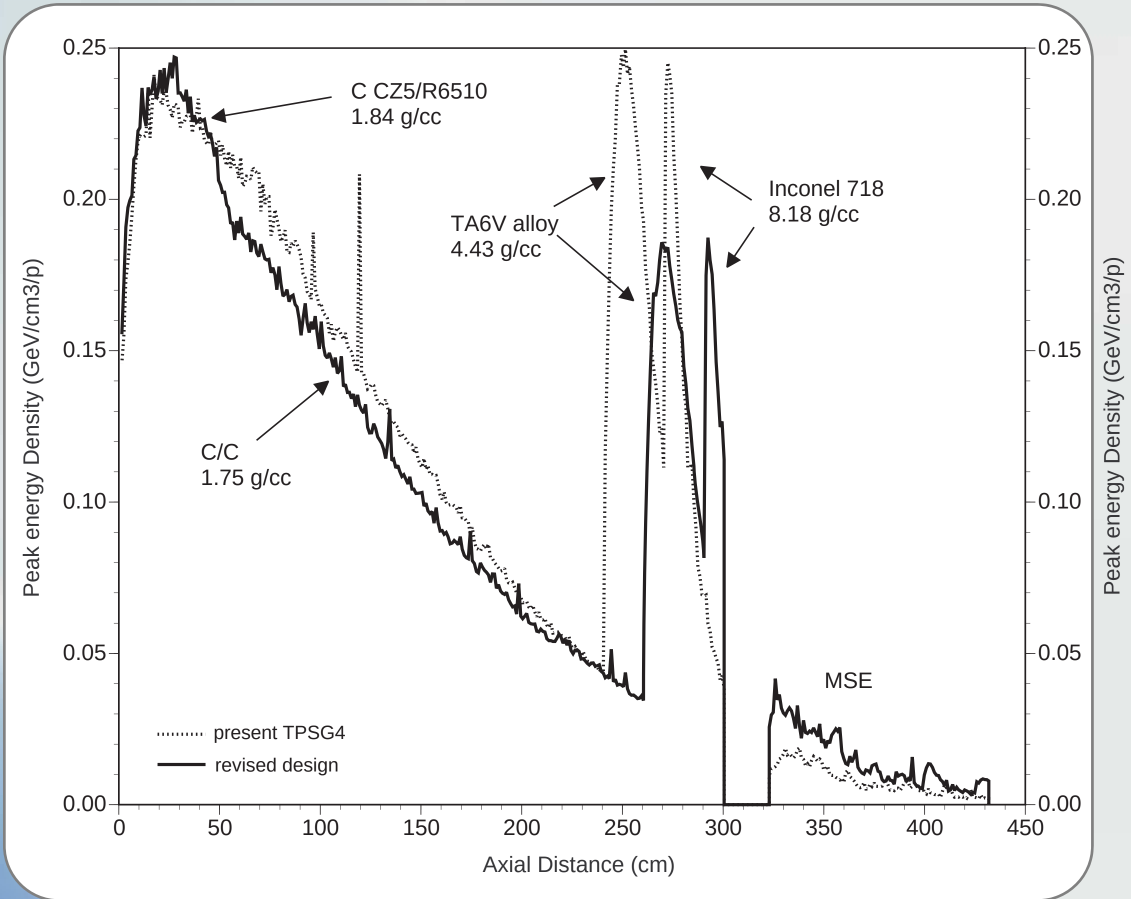
- A new design has then been adopted by CERN that appears as a good compromise
- Some graphite blocks are substituted by high density carbon composite, the steel block is no longer present and the two titanium blocks are moved at the end of the target; the following materials are adopted: 0.5m of graphite, 0.5m of high density carbon composite, 2m of low density carbon composite, 1.5m of high density CC, 1.0m of graphite again and 0.5m of a titanium alloy
- The results were satisfactory throughout the whole target. The highest stresses are found in the 23rd block, made from titanium, in which a temperature increase of 401°C and a maximum Stassi ratio of 2,08 are reached. This value reduces to 1,65 when an offset beam is considered

TPSG4 beam diluter

- In the first design the TPSG4 was 3.0 long and had the following material composition: 2.4m of graphite, 0.3m of a titanium alloy, and 0.3m of a Nickel based alloy
- The design has then been modified by substituting several graphite blocks with a CC composite and by adding another 10cm long graphite block
- The three section were composed of several blocks each having a cross section of 30 x 19.25 mm, the block length is 240-300mm

TPSG4: phase 1 vs. phase 2

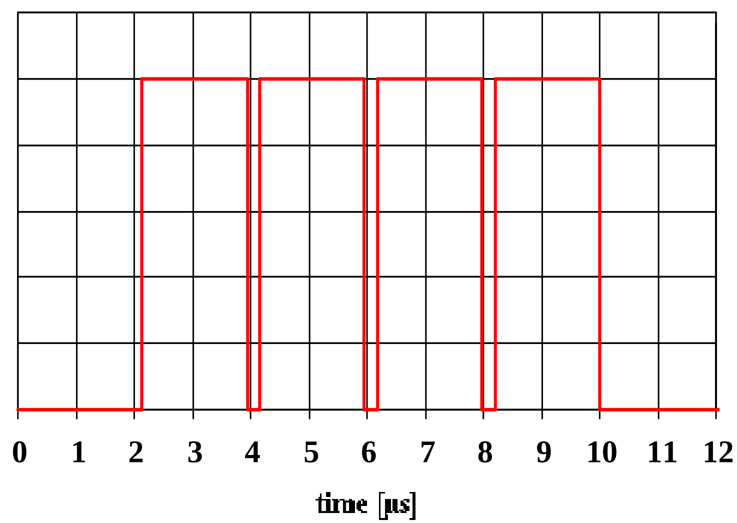
- The carbon composite has better mechanical properties than graphite
- but also a lower density which implies a downstream shift of the energy deposition
- To compensate for this effect the graphite/carbon section has been extended



TPSG4 beam load

- For the purpose of the analysis, the LHC ultimate beam intensity is considered as the worst case

Momentum	450	GeV/c
Time structure	25ns x 72 x 4	
Bunch intensity	$1.7\ 10^{11}$	protons
Total intensity	$3.2\ 10^{11}$	protons
Beam size H	0.97	mm
Beam size V	0.40	mm

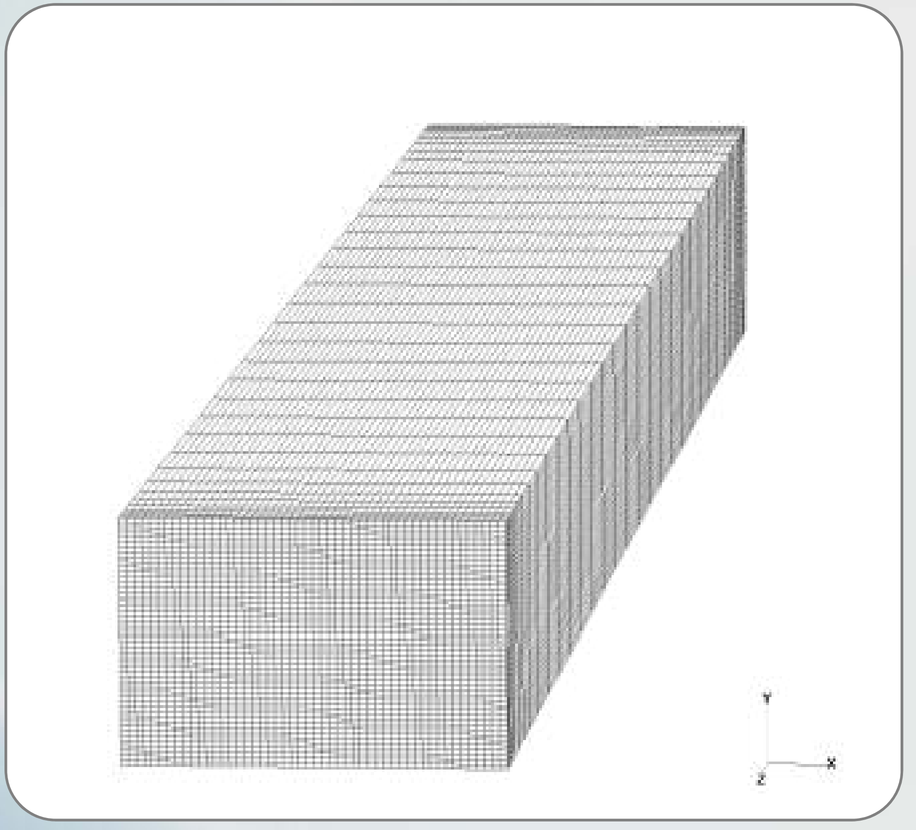


TPSG material properties RT

Material	Graphite	CC 1.4 X	CC 1.4 YZ	Titanium	Nickel alloy
Name	CZ5	SG1.4	SG1.4	Ti 6Al 4V	INCONEL 718
Density (kg m^{-3})	1840	1400	1400	4420	8190
Specific heat (J kg^{-1} K^{-1})	685	685	685	562	437
Thermal exp. ($10^{-6}K^{-1}$)	3.92	1.0 - 5.0	-1.0 - 2.0	8.80	12.80
Young's mod. (GPa)	11.4	2.8	10	111.9	207.7
Tensile strength (MPa)	33	46	61	1036	1408
Compres. strength (MPa)	125	69.6	82.4	1036	1408

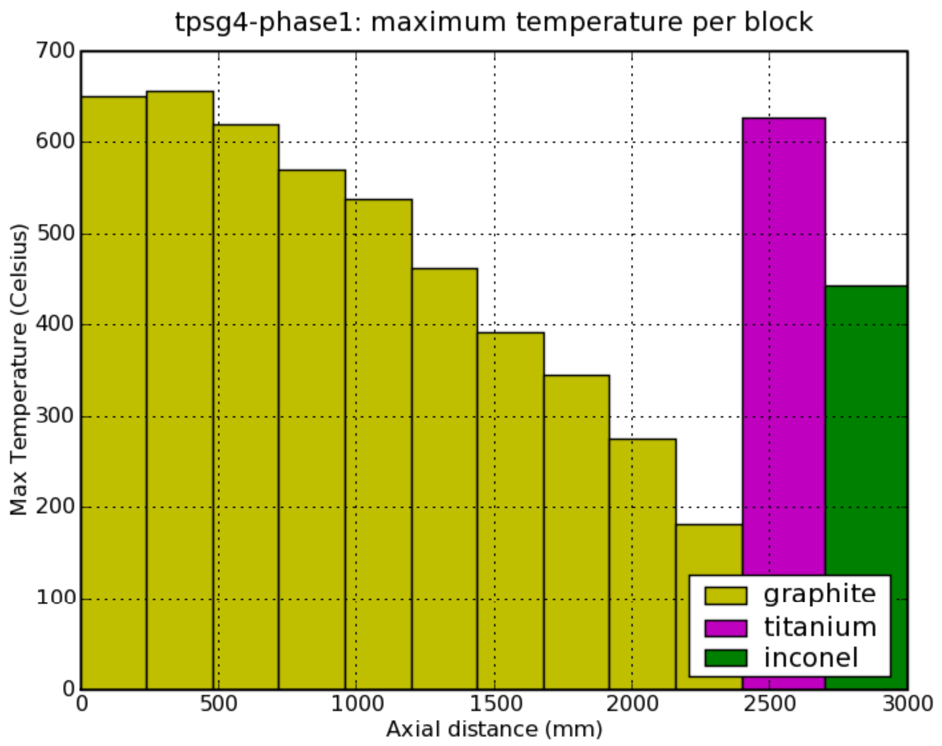
TPSG4 model

- Each block has been simulated separately
- The mesh dimensions are 19x30mm for the section, the length of the block is 240mm for Graphite and 300mm for Titanium and Inconel
- The mesh consists of 85000 spectral elements and a spectral degree of 2 was adopted
- Internodal distance is in the range 0.2-5.0mm
- Total number of DOF ≈ 3 millions



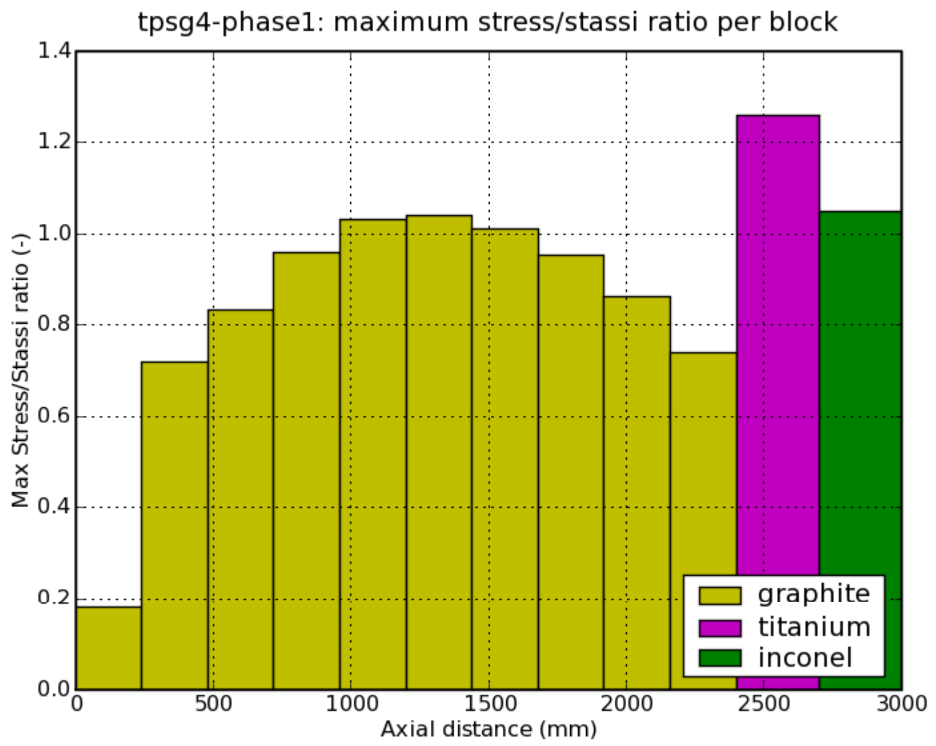
TPSG4 phase 1 results: temperature increase

- The temperature increase is high but not critic for the material integrity, the maximum values are reached in the 2nd graphite block, high values are also reached in the Titanium and Inconel blocks



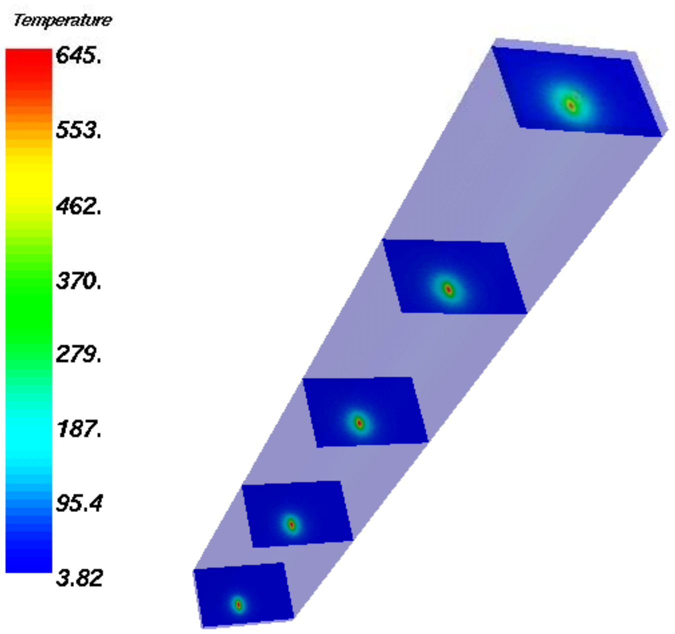
TPSG4 phase 1 results: max Stassi ratio

- The maximum value of the Stassi stress ratio are found on the 6th graphite block, in which a wider area is heated.
- High values are found on the Titanium and Inconel blocks as well
- Values higher than unity imply a failure

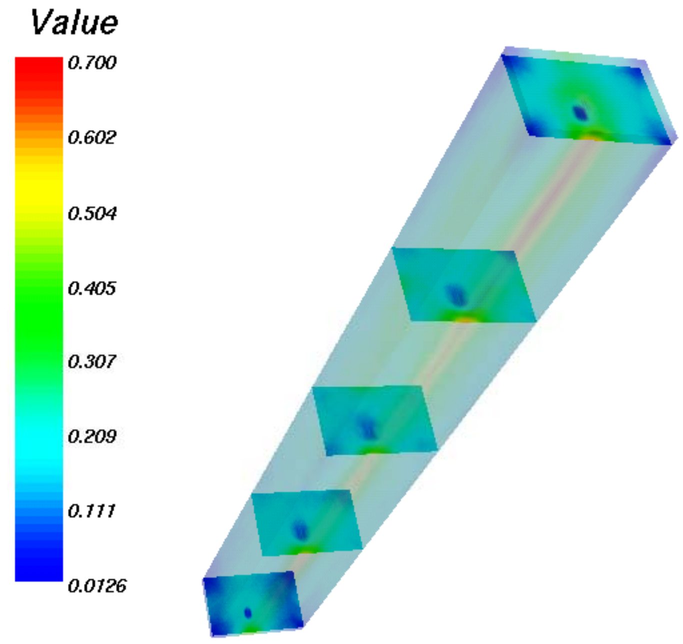


TPSG4 phase 1 results: graphite block 2

Temperature increase

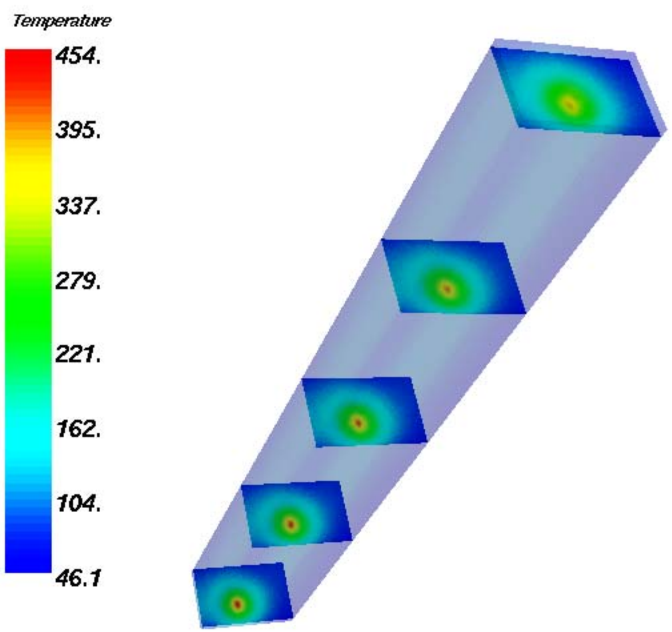


Max Stassi ratio

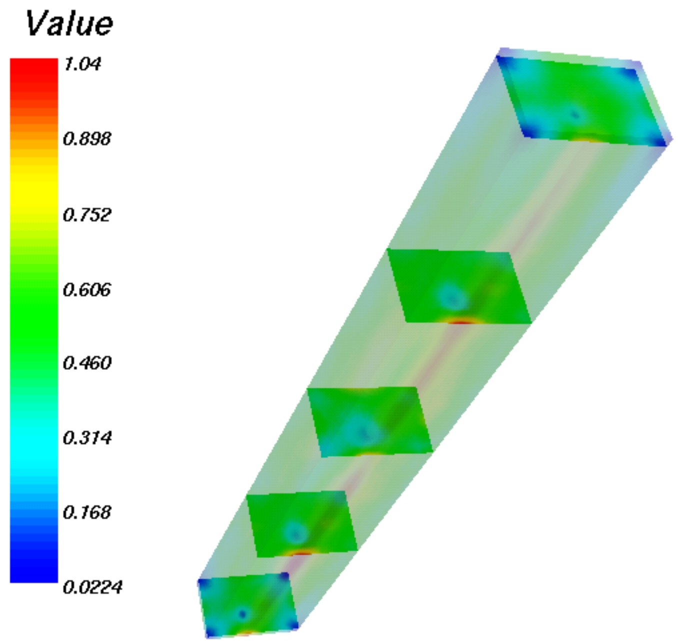


TPSG4 phase 1 results: graphite block 6

Temperature increase

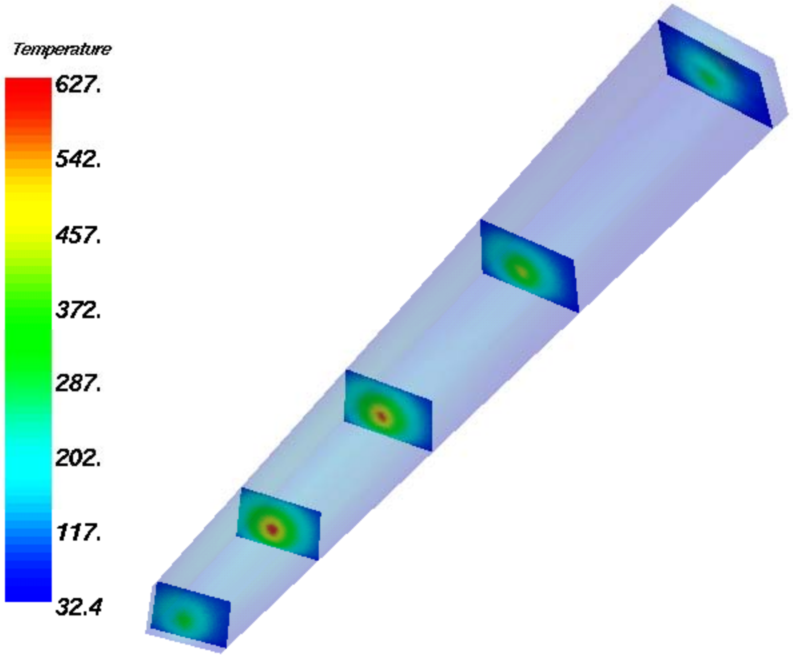


Max Stassi ratio

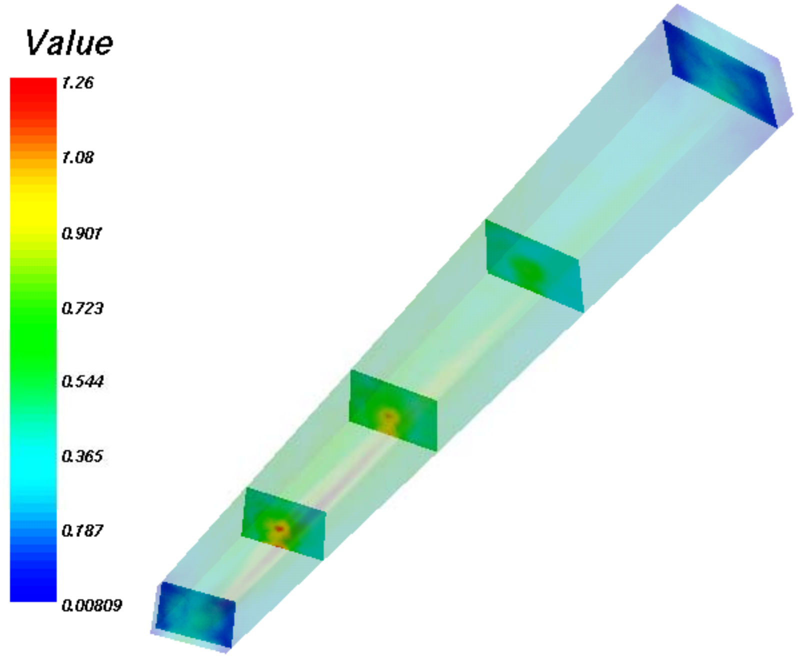


TPSG4 phase 1 results: titanium block

Temperature increase

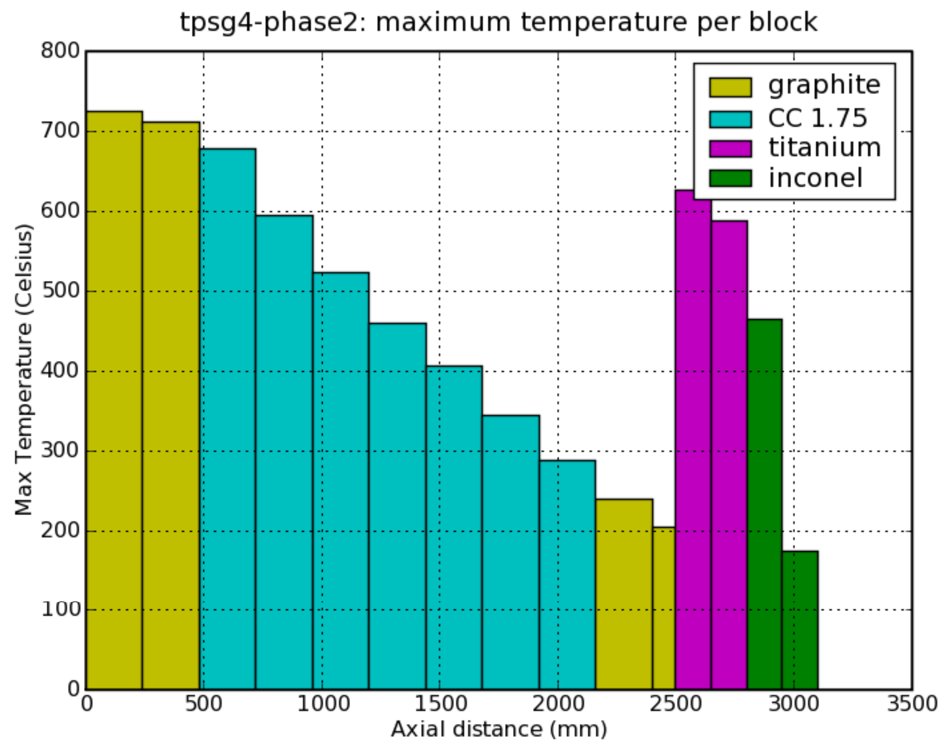


Max Stassi ratio



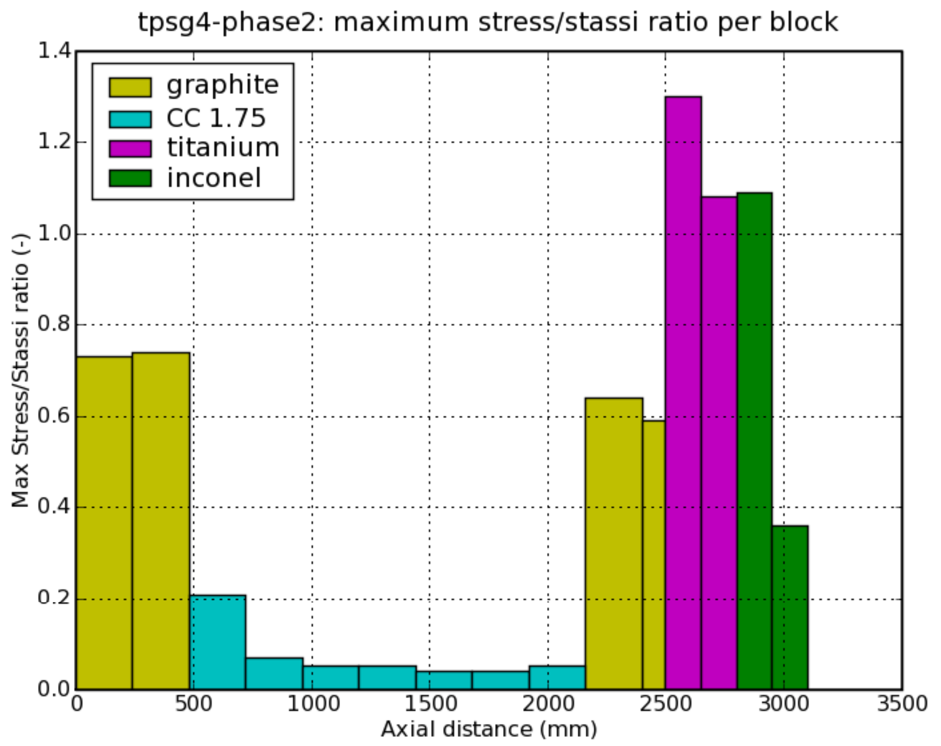
TPSG4 phase 2 results: temperature increase

- The temperature increase is similar to the results of phase 1
- The max ΔT is found in the 1st and 2nd blocks, the beam is also highly focalized
- The presence of the additional graphite block greatly reduces the effect of the lower density of CC

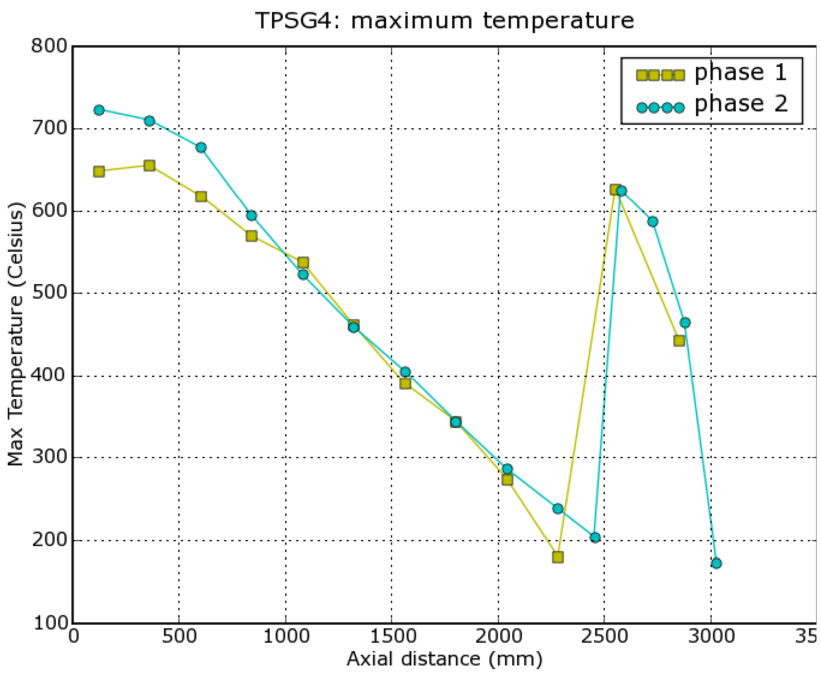


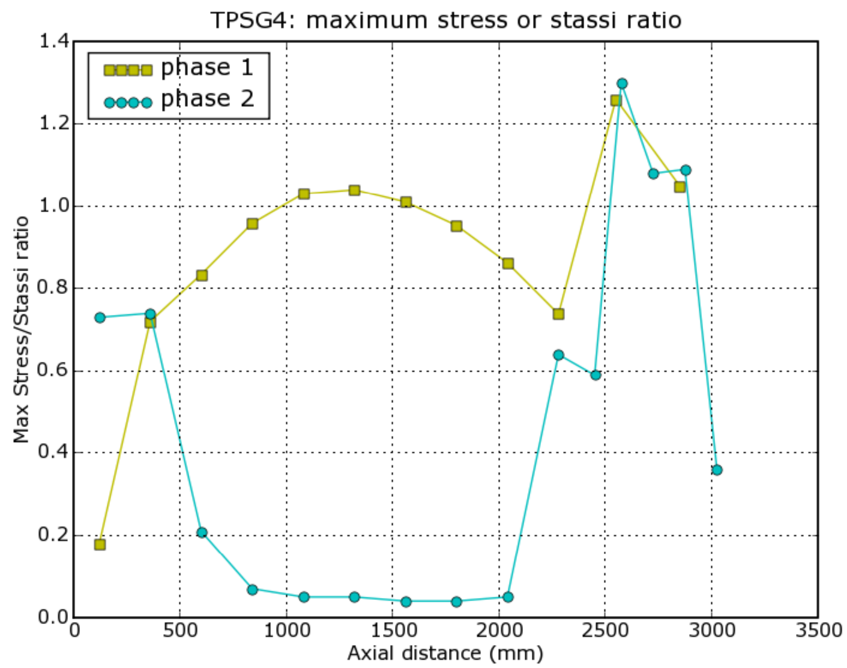
TPSG4 phase 2 results: max Stassi ratio

- The CC greatly reduces the resulting equivalent stresses
- The results are acceptable for the graphite block, are well below he failure limit for the CC
- The stress ratio is high for the Ti and Ni blocks but these alloys have a ductile behavior



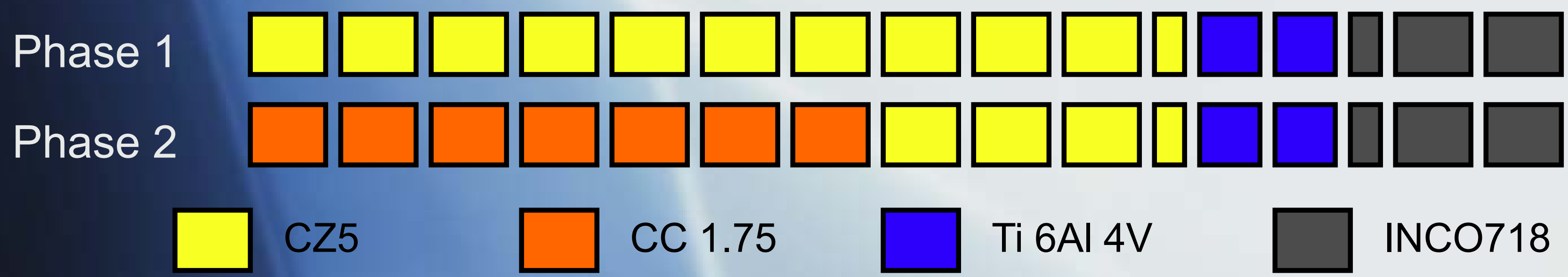
TPSG4 results comparison





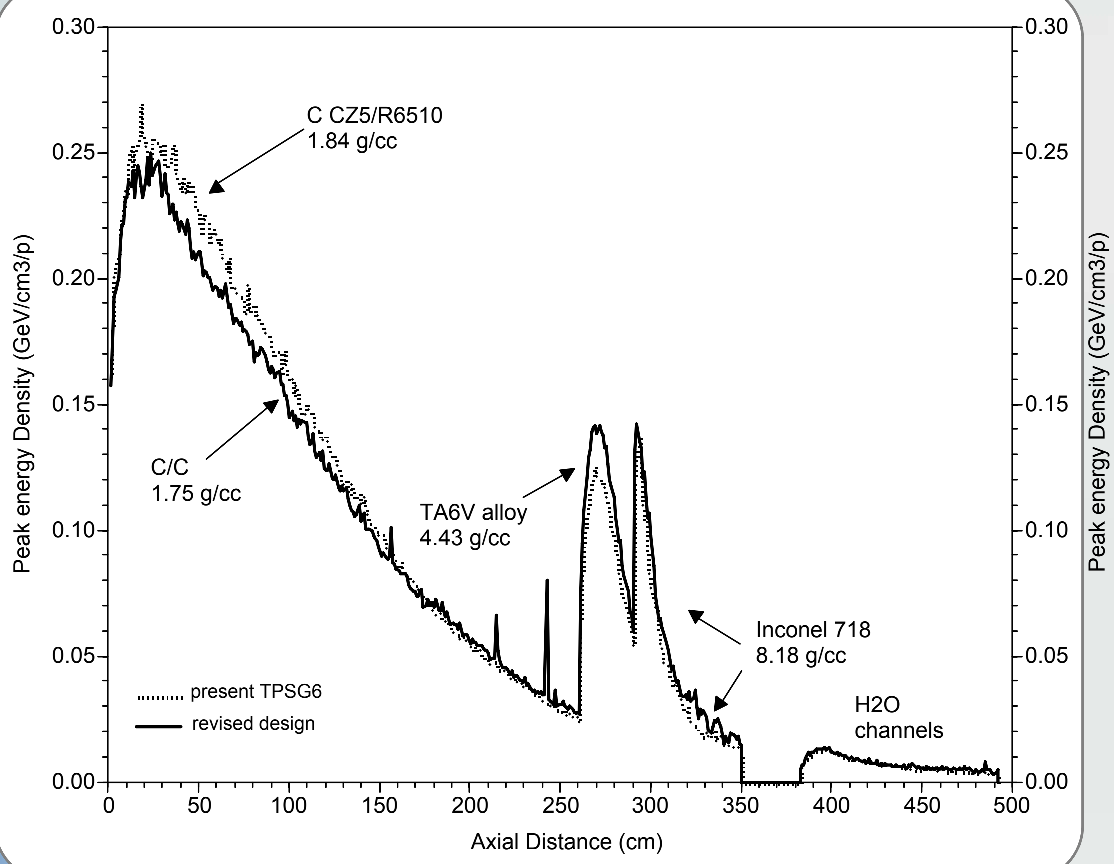
TPSG6 beam diluter

- In the first design the TPSG6 was 3.5m long, and was formed by several blocks of different materials: ten 250mm long graphite blocks were followed by a 100mm block; the final part was formed by two 150mm long titanium blocks followed by a 100mm long block and two 250mm long all made from Inconel
- The cross section is almost constant along the diluter axis and is 6x30mm
- In the new design the first 7 graphite blocks are substituted by high density carbon composite ones



TPSG6: phase 1 vs. phase 2

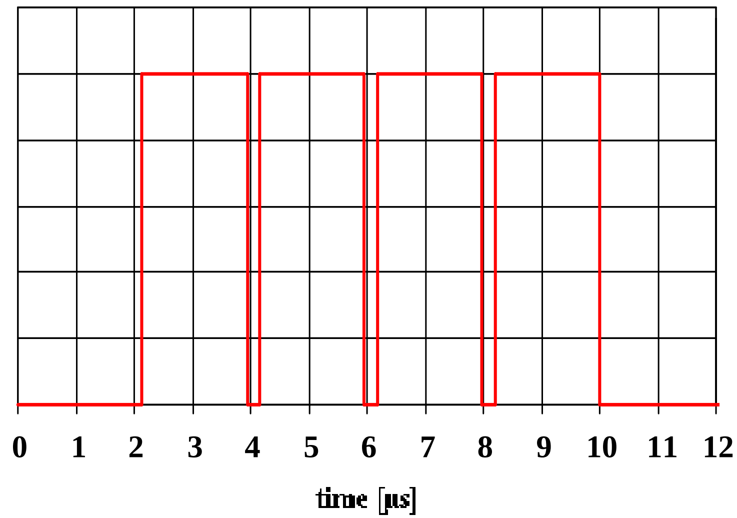
- Graphite blocks are replaced with carbon composite blocks in the first section
- The consequent shift of the energy deposition profile is not as strong as in TPSG4 due to the different beam parameters
- The diluter geometry was not changed



TPSG6 beam load

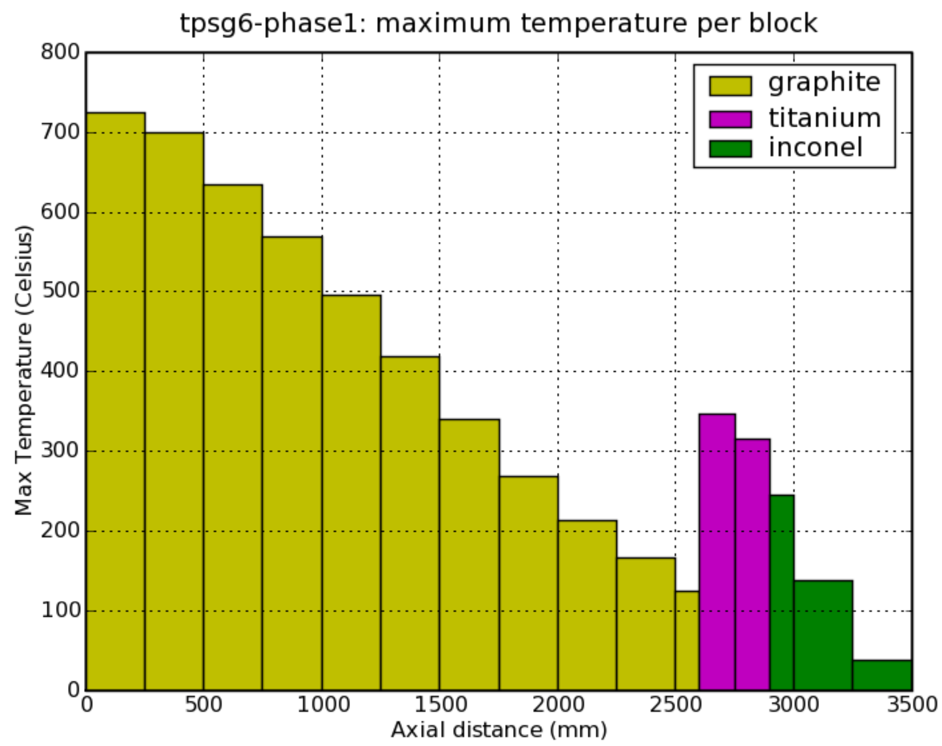
- For the purpose of the analysis, the LHC ultimate beam intensity is considered as the worst case

Momentum	450	GeV/c
Time structure	25ns x 72 x 4	
Bunch intensity	1.7 10^{11}	protons
Total intensity	3.2 10^{11}	protons
Beam size H	0.63	mm
Beam size V	0.58	mm



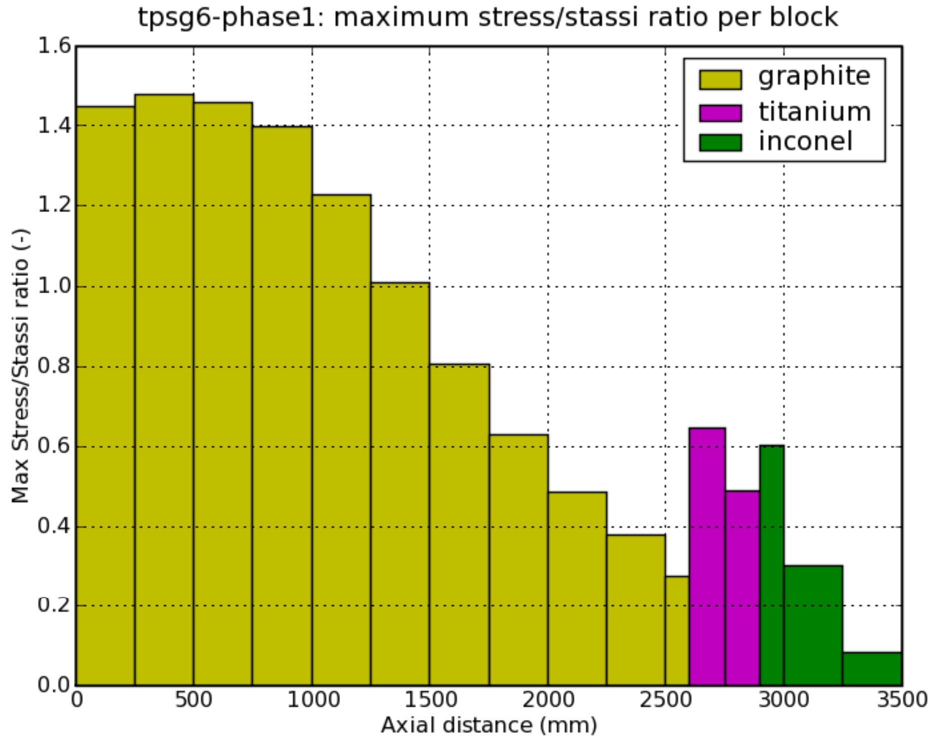
TPSG6 phase 1 results: temperature increase

- The temperature increase is similar to the TPSG4 but is concentrated on the first blocks
- The values found on the Titanium and Inconel parts are lower than the TPSG4 results



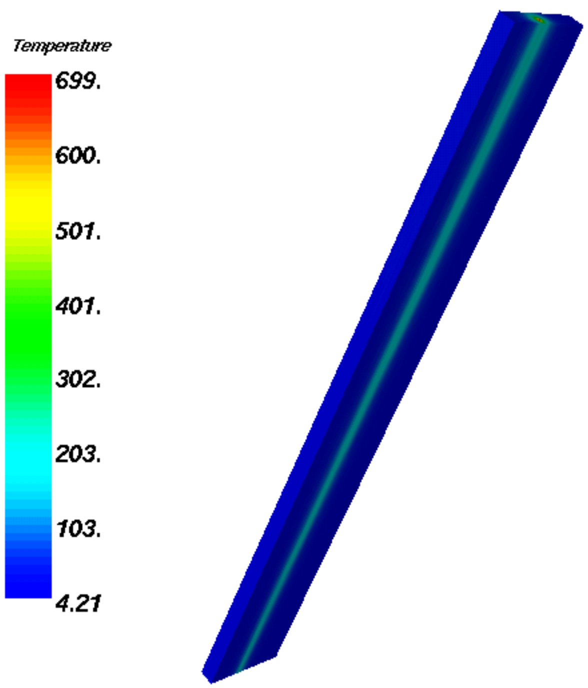
TPSG6 phase 1 results: max Stassi ratio

- The maximum value of the Stassi stress ratio are almost constant on the first graphite blocks.
- The values are much higher than unity and a failure is probable for the the first graphite blocks
- Titanium and Inconel parts are less stressed

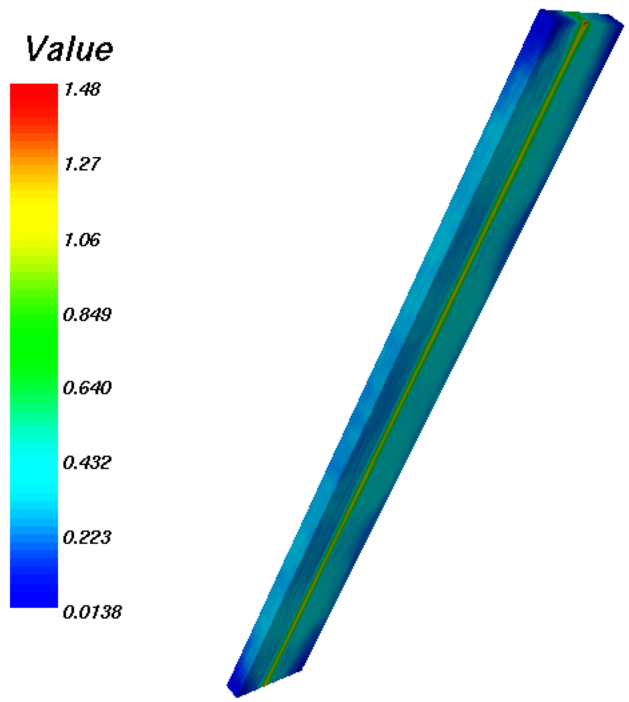


TPSG6 phase 1 results: graphite block 2

Temperature increase

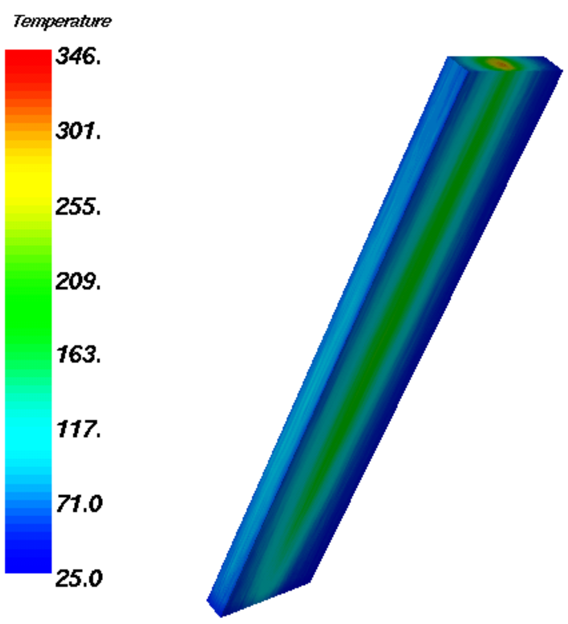


Max Stassi ratio

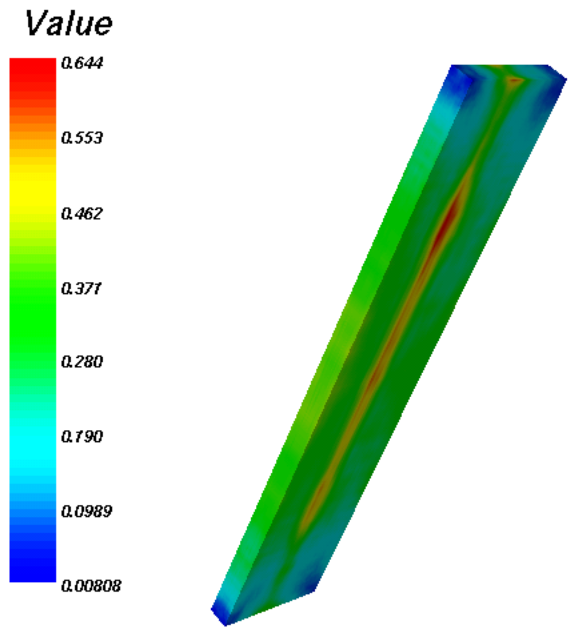


TPSG6 phase 1 results: 1st titanium block

Temperature increase

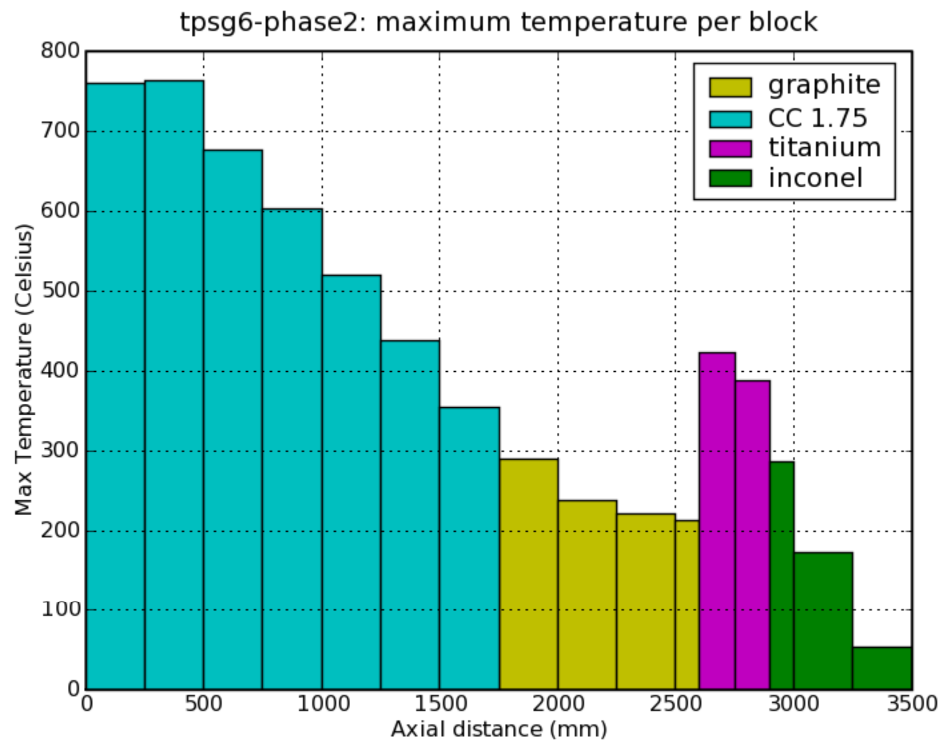


Max Stassi ratio



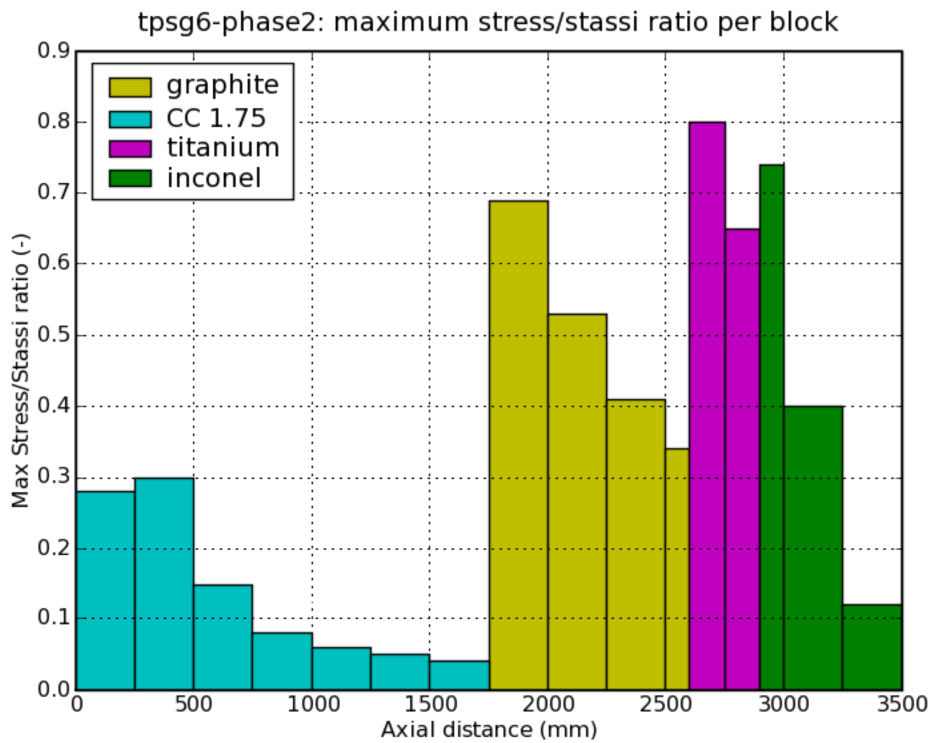
TPSG6 phase 2 results: temperature increase

- The temperature increase is similar to the results of phase 1
- The max ΔT is found in the 1st and 2nd blocks, the beam is also highly focalized
- The slightly lower density of the first blocks determines a higher power deposition on the final part

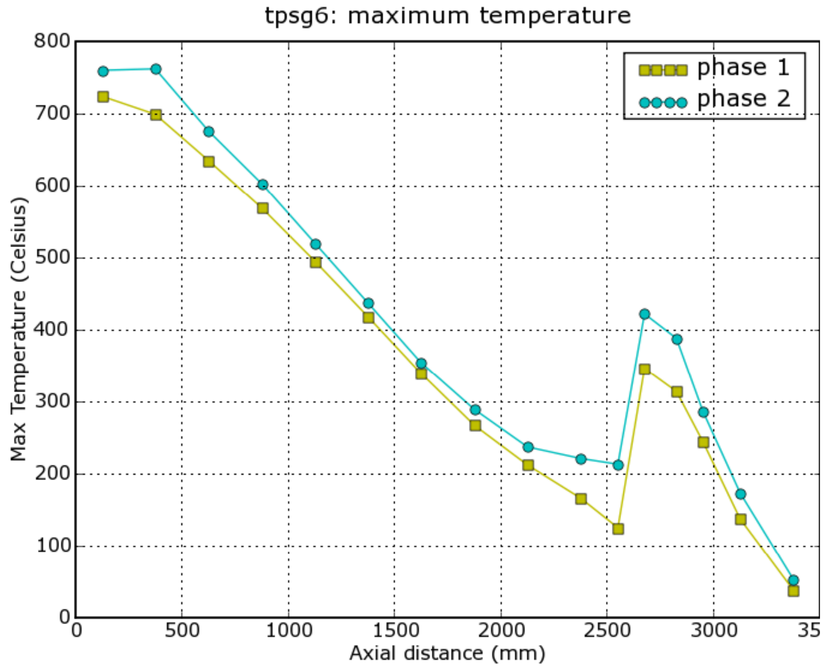


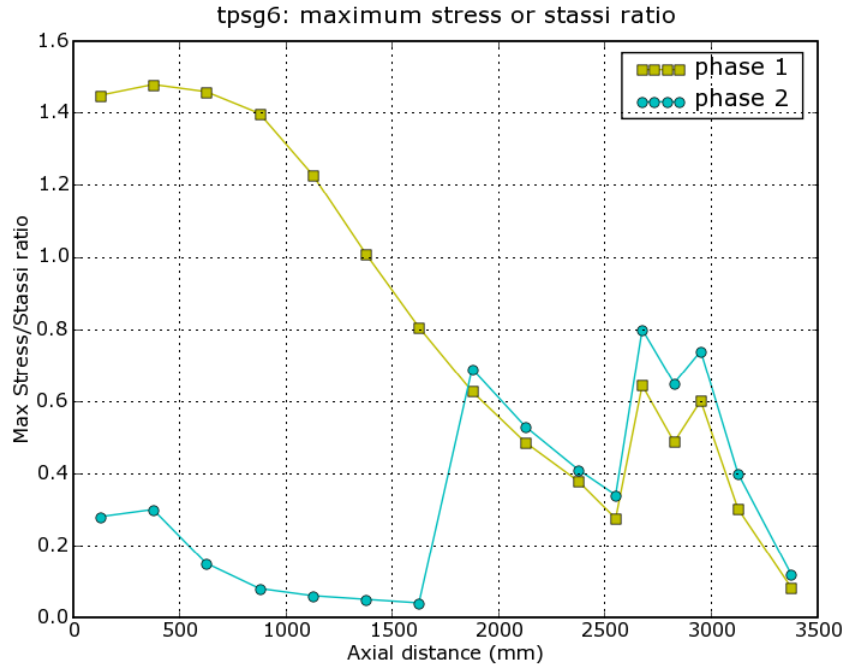
TPSG6 phase 2 results: max Stassi ratio

- The CC greatly reduces the resulting equivalent stresses
- The stress ratio is lower than unity on all the target and in particular for the CC blocks



TPSG6 results comparison





TPSG4/6 conclusions

- The numerical simulations of the TPSG4 and TPSG6 have allowed to calculate the temperature increase in the blocks, and to simulate the propagation of elastic waves and dynamic stresses in the structures
- In the first design of these targets the blocks made from graphite were subject to an high level of stress, higher than the safety limit; the Stassi equivalent stress at some time of the simulation resulted to be higher than the failure limit of the material. High values of the equivalent stresses were found on the final blocks of the TPSG4, but these did not appear as dangerous due to the ductile properties of the metallic alloys adopted
- The design was modified substituting the most stressed graphite blocks with a new high performance Carbon Composite and increasing the length of the TPSG4 to compensate the lower density of the new blocks
- The results appear satisfactory, with stress levels lower than the failure limit, with the only exception of the final part of the TPSG4 target, in the ductile alloy blocks

Some thoughts on materials and numerical simulations

- The accuracy on material properties is essential
  - The results of a numerical simulation are deeply influenced by material properties values
- Material data for the newest materials such as carbon composite are not yet well established and difficult to measure
  - An orthotropic material has a lot more properties than an isotropic material, some of them are hard to measure and can only be established as an “educated guess”
- The strength criteria are important.
  - There is a good experience on isotropic materials, and on the relative strength criteria, anisotropic materials are less common, moreover are often used with simple stress patterns.
  - The beam dump application is rather unique, some specific theoretical and experimental study may be important

Some thoughts on materials and numerical simulations

- The continuum formulation may be at its limits, detailed formulation of fiber, matrix and interface may be required and modeled
- Something more can be done on solid targets apart from the use of new materials, for instance by working on the target geometry
- Finally a fault tolerant design may be of interest
  - The mechanical failure does not necessarily cause the target to be out of order
  - The mechanical damage sometimes cannot be avoided but may be controlled

Thanks for your attention