# Hadron Mass Effects

Iain Stewart
MIT

Strong Coupling from Thrust at Lepton Colliders
CERN workshop
November 27, 2025

Based on:

Mateu, IS, Thaler, arXiv:1209.3781 (JHEP)
Salam, Wicke, hep-ph/0102343 (JHEP); Lee, Sterman hep-ph/0611061 (PRD); ...
Benitez, Hoang, Mateu, IS, Vita, arXiv:2412.15164 (PRD)

Massachusetts Institute of Technology

- For $e^+e^- \to$ N-jets, $d\sigma_N$ $(e \ll 1)$ dominant power corrections $\sim \frac{\Lambda_{\rm QCD}}{Qe}$

- Induced by soft hadronization effects, $Qe =$ pert. soft scale

  2-jets, eg. $e =$ thrust $\qquad d\sigma_2 = H_2 \times J_q^2 \otimes S_2 \otimes F_2$

  3-jets, eg. $e =$ 3-jettiness $\qquad d\sigma_3 = H_3 \times J_q^2 J_g \otimes S_3 \otimes F_3$

- Hadron masses are crucial: $\qquad m_{\rm H} \sim \Lambda_{\rm QCD}$

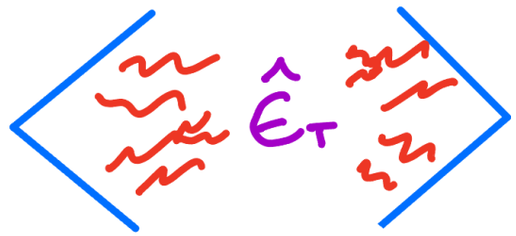
  for $\Lambda_{\rm QCD}$ power corrections: can't neglect $m_H$ in any limit !!

- For $e^+e^- \to$ N-jets, $d\sigma_N$ $(e \ll 1)$ dominant power corrections $\sim \dfrac{\Lambda_{\rm QCD}}{Qe}$

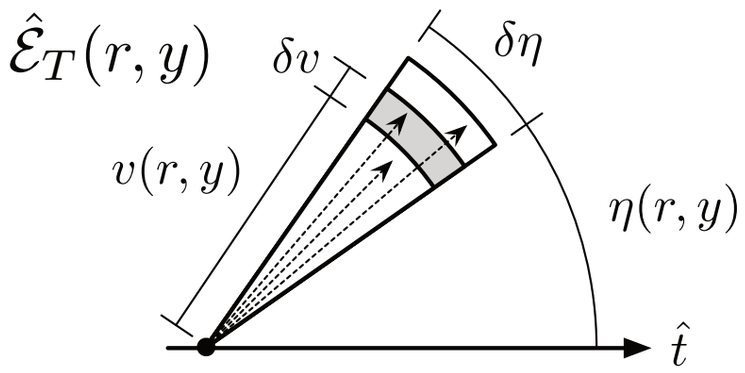
- For $d\sigma_2$ with $\Lambda_{\rm QCD} \ll Qe$ ("OPE" or "tail" region)
  the leading hadron mass effects in QCD are encoded in:

$$\Omega_1(r) \equiv \langle\, 0 \,|\, \overline{Y}^\dagger_{\bar n} Y^\dagger_n \hat{\mathcal{E}}_T(r,0) Y_n \overline{Y}_{\bar n} \,|\, 0 \,\rangle$$

finite velocity energy-flow operator




use boost symmetry $y \to 0$




$$r = \frac{p_\perp}{\sqrt{p_\perp^2 + m_{\rm H}^2|}}$$ transverse velocity

$$\hat{\mathcal{E}}_T(r,y) = \frac{r\,\mathrm{sech}^4 y}{\sqrt{r^2 + \sinh^2 y}} \lim_{R\to\infty} R^3 \int_0^{2\pi} \mathrm{d}\phi\, \hat{n}_i\, T^{0i}(R, R\,v\,\hat{n})$$

energy mom. tensor

Mateu, IS, Thaler, arXiv:1209.3781 (PRD)

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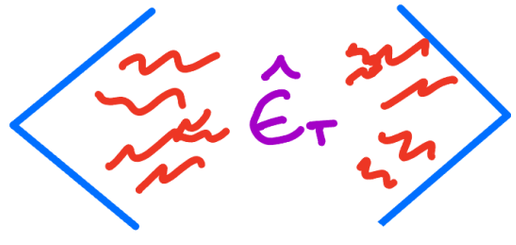


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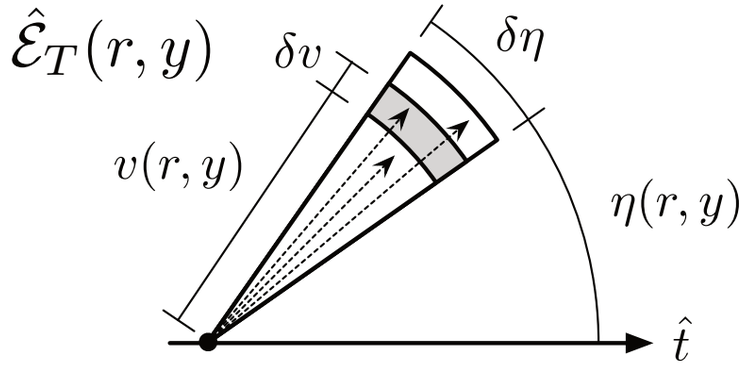


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energy mom. tensor

- For $d\sigma_2$ with $\Lambda_{\rm QCD} \sim Qe$ ("peak" region) QCD predicts higher moments $\Omega_k \sim \Lambda^k_{\rm QCD}$ are equally important ($\to$ full $F_2$ shape fn.)

→ fits for $\alpha_s$ exclude "peak" to avoid model bias from form of $F_2$

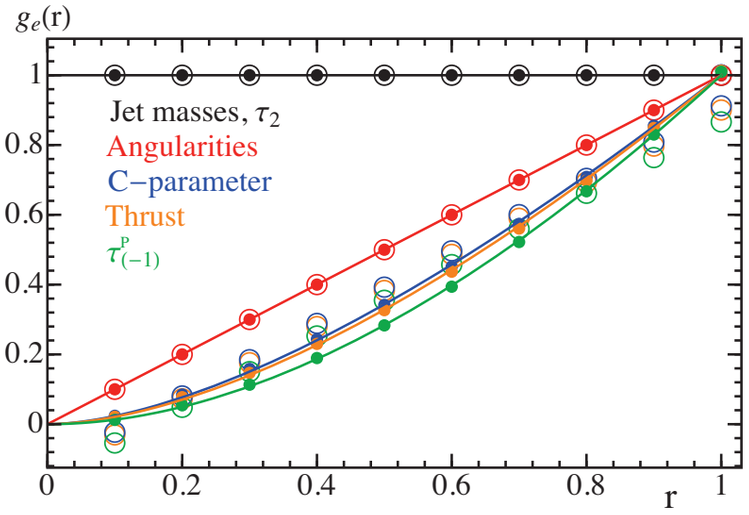
- $\Omega_1(r) \equiv \langle\, 0 \,|\, \overline{Y}_{\bar{n}}^\dagger Y_n^\dagger \hat{\mathcal{E}}_T(r,0) Y_n \overline{Y}_{\bar{n}} \,|\, 0 \,\rangle$ universal 2-jet hadronization function

- Dijet event shapes $e = \sum_i \frac{m_i^\perp}{Q} f_e(r_i, y_i) \ll 1$

power correction: $\Omega_1^e = \int_0^1 \mathrm{d}r \left[ \int_{-\infty}^{+\infty} \mathrm{d}y\, f_e(r,y) \right] \Omega_1(r) = c_e \int_0^1 \mathrm{d}r\, g_e(r)\, \Omega_1(r)$

naive "massless" universality
$c_\tau = 2,\ c_\rho = 1,\ \ldots$

hadron mass universality classes!

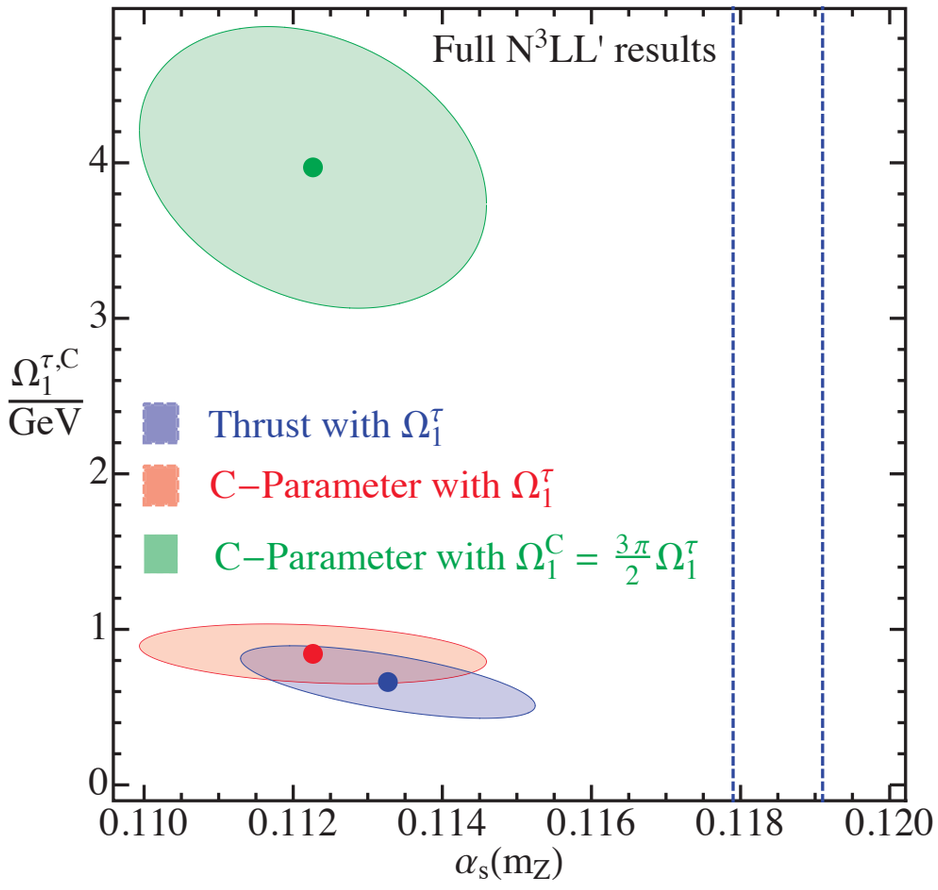



$\{\tau, C\} \neq \{\rho, \ldots\} \neq \{\mathrm{EEC}, \ldots\}$

describes differences between E-scheme, p-scheme, etc.

- Thrust and C-parameter universality is obeyed by global fits

Hoang, Kolodrubetz, Mateu, IS, arXiv:150.04111 (PRD)



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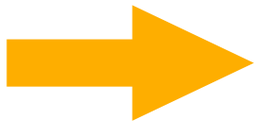
  power correction



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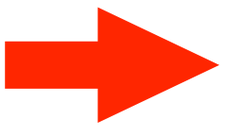
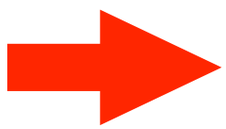


For fit to <u>single</u> event shape (eg. thrust), if we assume that all expts. treat $m_H$ effects with same method, then fit <u>one parameter</u> $\Omega_1^e$



Fits using <u>multiple</u> event shapes (in different classes) should <u>not</u> be done with only a single hadronic parameter!

# Time Permitting …

- $\Omega_1$ anomalous dimension
- Validation with Monte Carlo
- Impact of Renormalon subtractions and Schemes

- $\Omega_1(r) \equiv \langle\, 0 \,|\, \overline{Y}_{\bar{n}}^{\dagger} Y_n^{\dagger} \hat{\mathcal{E}}_T(r,0) Y_n \overline{Y}_{\bar{n}} \,|\, 0 \,\rangle$ universal 2-jet hadronization function

- Dijet event shapes $e = \sum_i \frac{m_i^{\perp}}{Q} f_e(r_i, y_i) \ll 1$

$$\Omega_1^e = c_e \int_0^1 \mathrm{d}r \, g_e(r) \, \Omega_1(r)$$

power correction

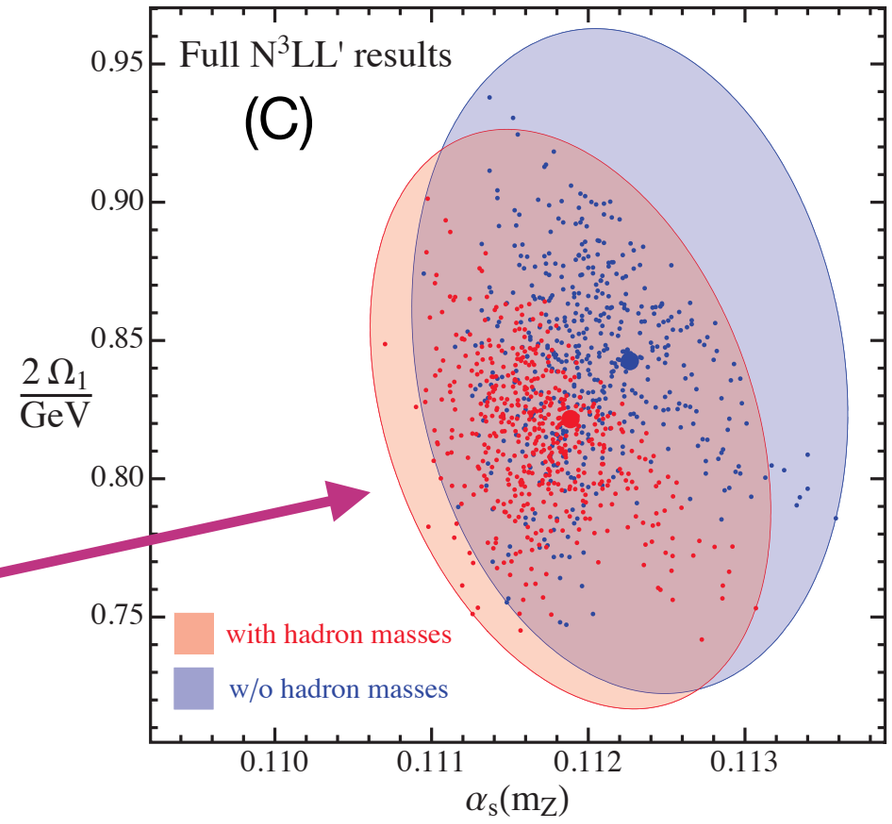
$$\{\tau, C\} \neq \{\rho, \ldots\} \neq \{\mathrm{EEC}, \ldots\}$$

- $\Omega_1(r)$ has an anomalous dimension

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \Omega_1(r,\mu) = \Big[ -\frac{\alpha_s C_A}{\pi} \ln(1-r^2) \Big] \, \Omega_1(r,\mu) + \mathcal{O}(\alpha_s^2)$$

$\mu$ dependence is in 1-to-1 correspondence with that of OPE matching coefficient

this running does not seem to cause large changes



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power correction

$$\{\tau, C\} \neq \{\rho, \ldots\} \neq \{\mathrm{EEC}, \ldots\}$$

- Monte Carlo agrees with QCD predictions for leading hadron mass effects

$$\Omega_1^0(\mu_Q) - \Omega_1^n(\mu_Q) = \frac{Q}{c_a}\left(\left\langle \tau_{(0,a)} \right\rangle - \left\langle \tau_{(n,a)} \right\rangle\right)$$

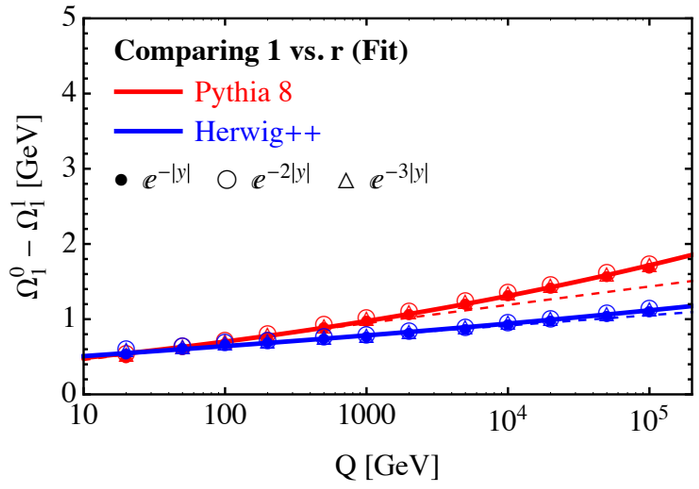
1st moments of R-scheme angularities $a$
in different $(r^n)$ hadron mass schemes
(difference → pert. moment drops out)

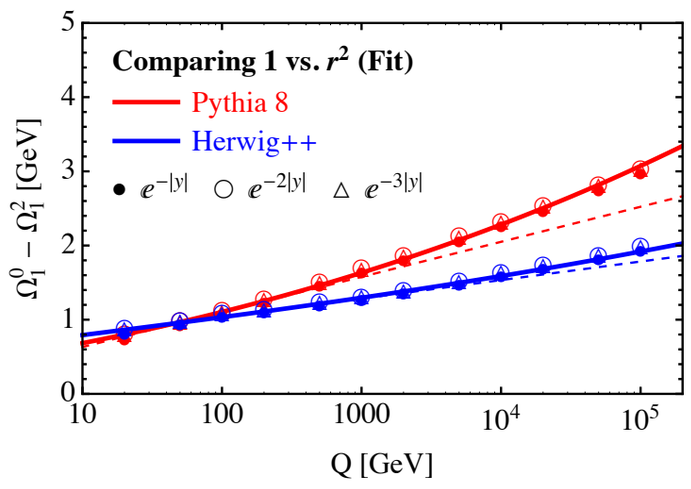
y dependence ✓

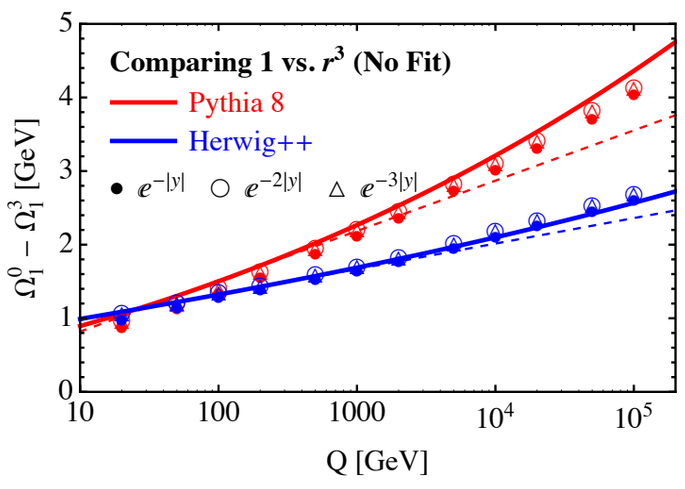
$\Omega_1(r)$ RGE ✓

different hadronic parameters in
Pythia vs. Herwig,
but both agree with expectations ✓

Mateu, IS, Thaler, arXiv:1209.3781 (PRD)



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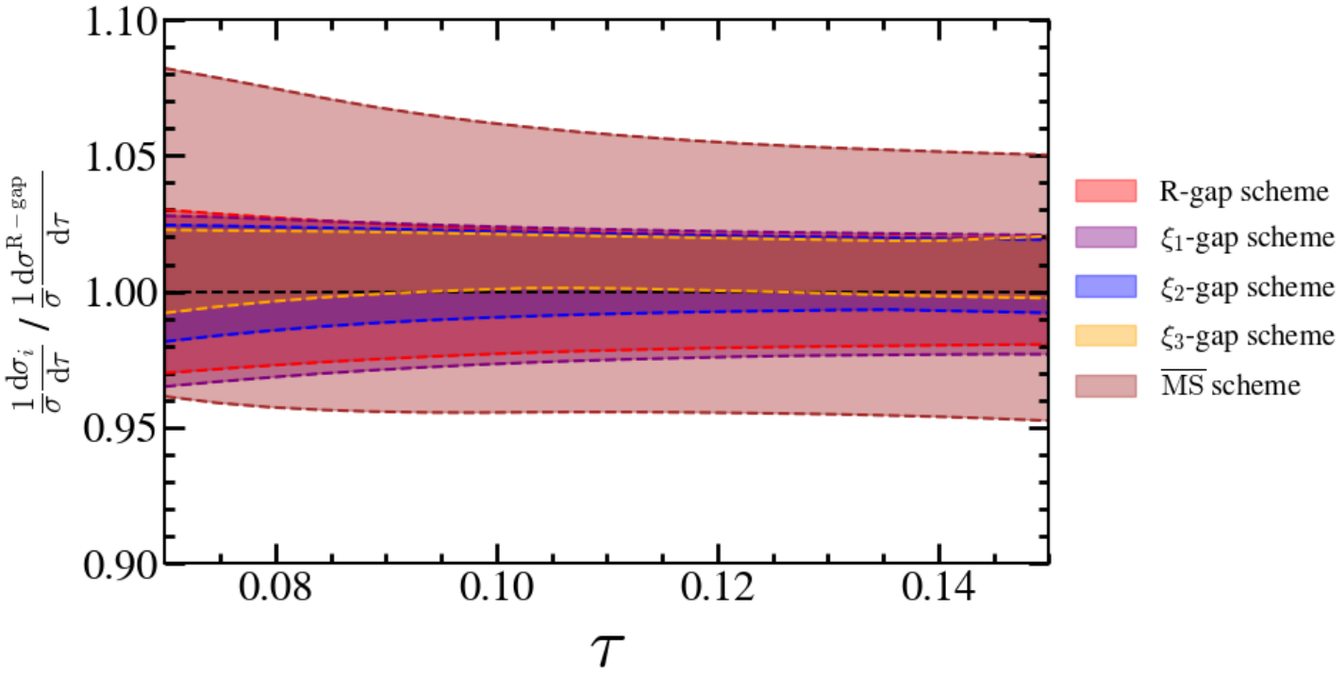
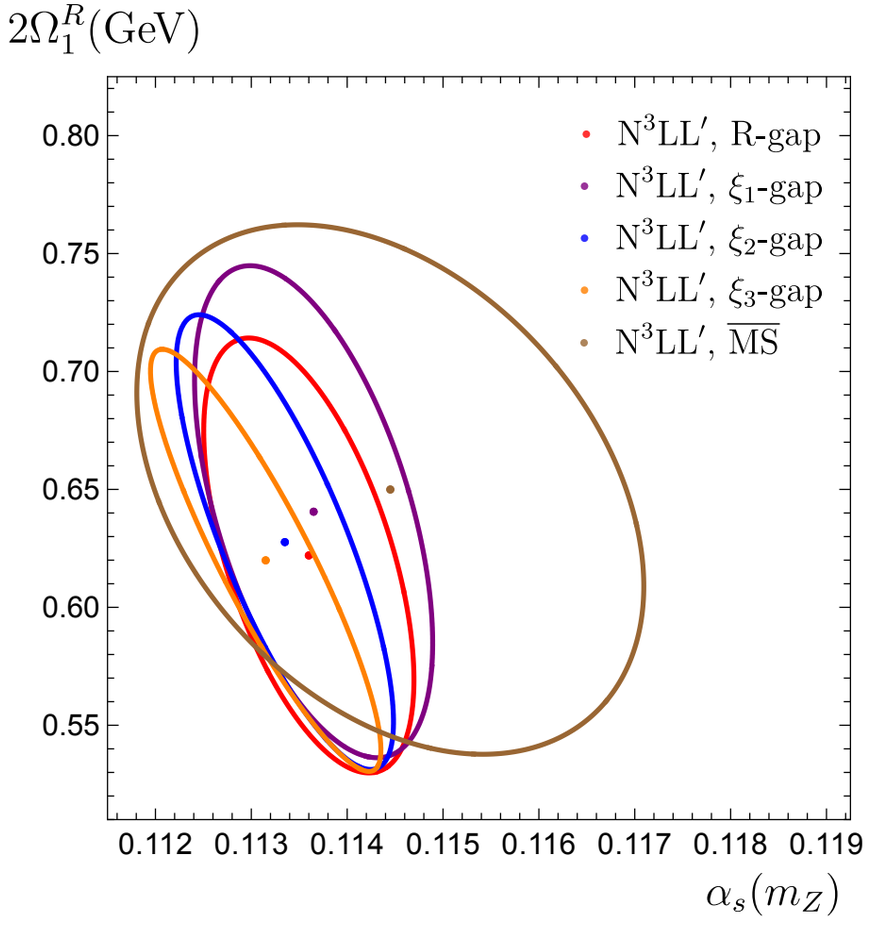
  power correction

$\Rightarrow \quad \{\tau, C\} \neq \{\rho, \ldots\} \neq \{\mathrm{EEC}, \ldots\}$

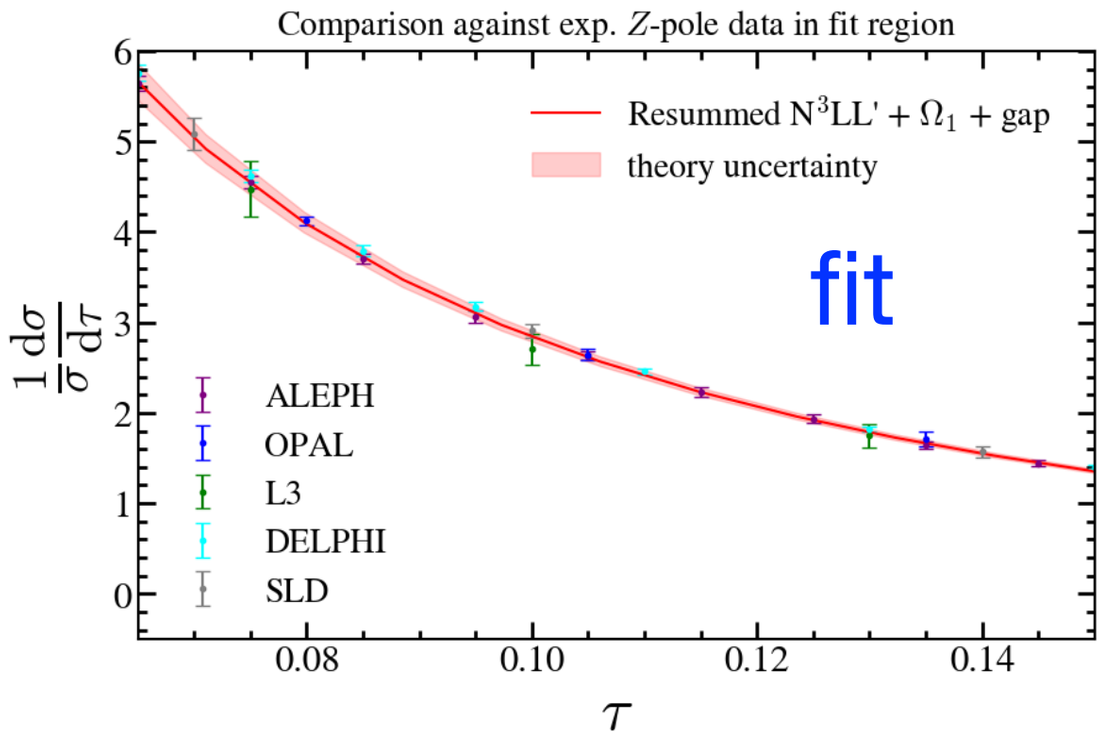
- $\Omega_1$ in $\overline{\mathrm{MS}}$ scheme has leading $\Lambda_{\mathrm{QCD}}$ renormalon, which can be removed using other subtraction schemes



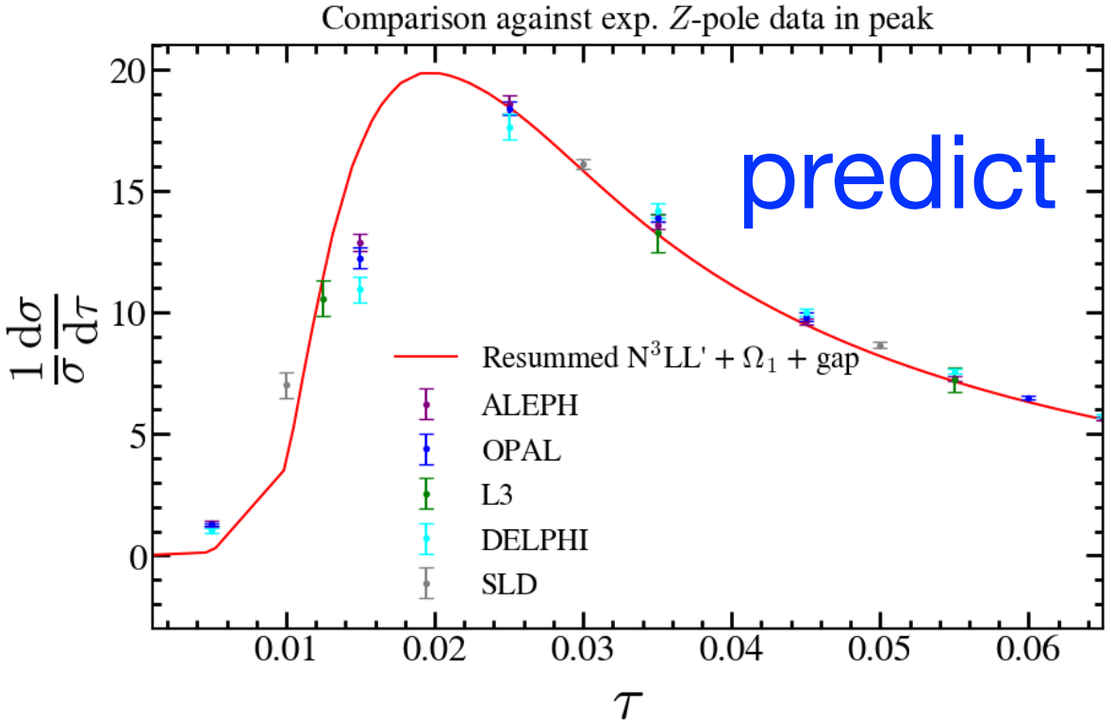

Benitez, Hoang, Mateu, IS, Vita, arXiv:2412.15164 (JHEP)



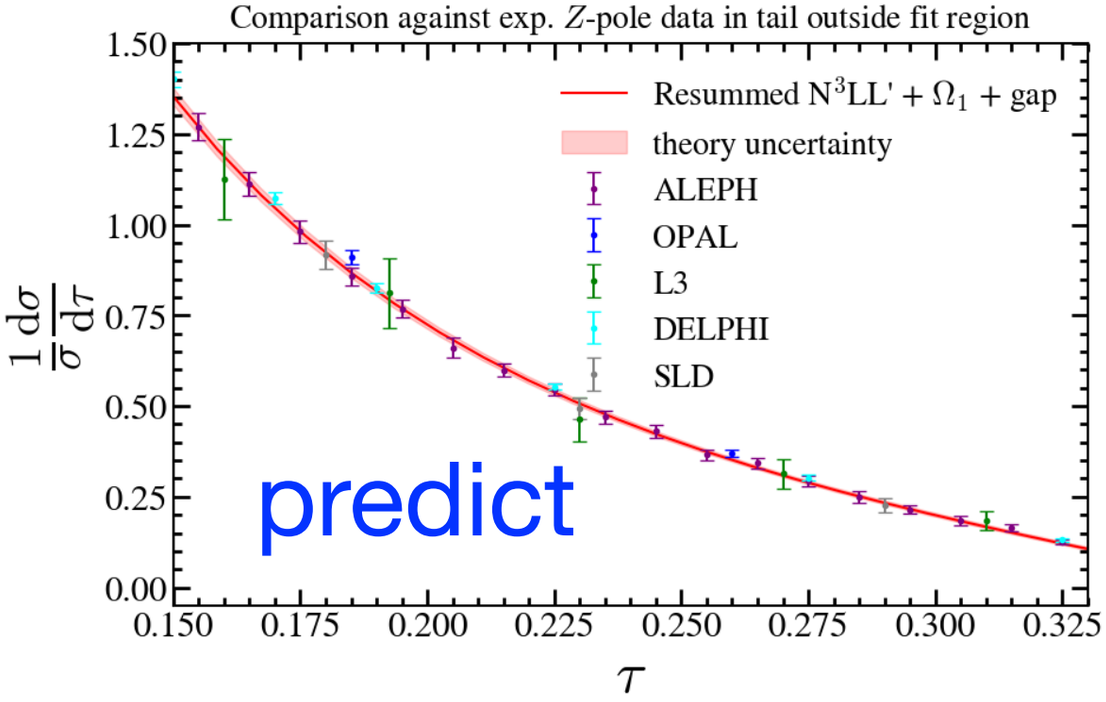
# Backup


Comparison against exp. $Z$-pole data in fit region
$\frac{1}{\sigma}\frac{d\sigma}{d\tau}$
$\tau$
Resummed N$^3$LL' + $\Omega_1$ + gap
theory uncertainty
fit
ALEPH
OPAL
L3
DELPHI
SLD

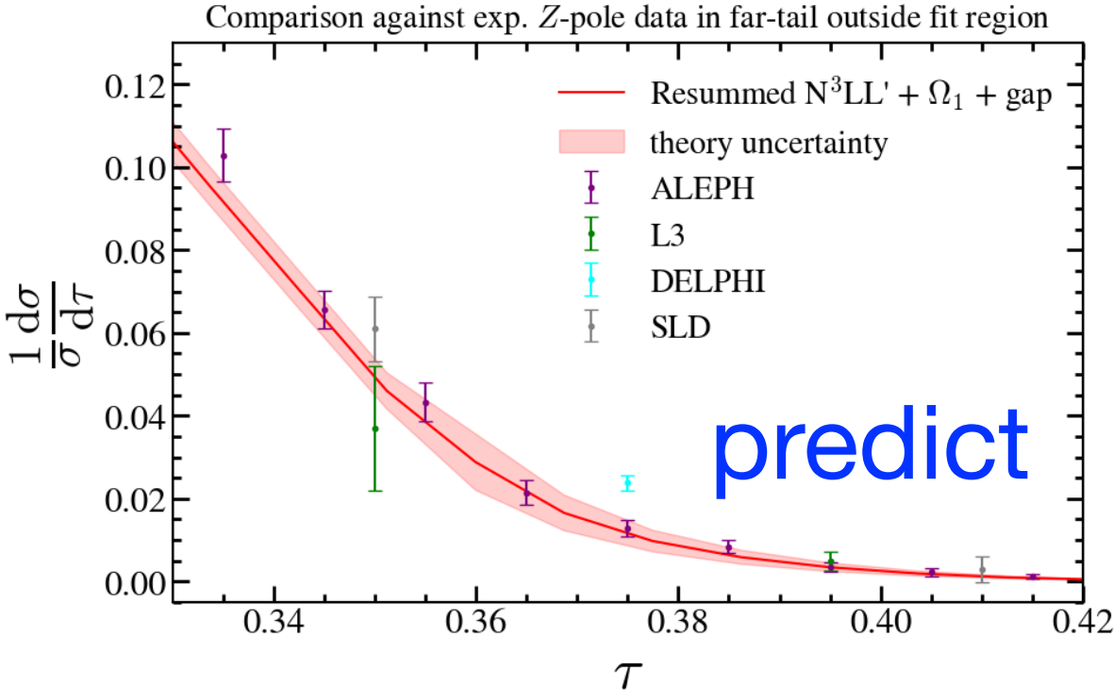
(a)


Comparison against exp. $Z$-pole data in peak
$\frac{1}{\sigma}\frac{d\sigma}{d\tau}$
$\tau$
Resummed N$^3$LL' + $\Omega_1$ + gap
predict
ALEPH
OPAL
L3
DELPHI
SLD

(b)


Comparison against exp. $Z$-pole data in tail outside fit region
$\frac{1}{\sigma}\frac{d\sigma}{d\tau}$
$\tau$
Resummed N$^3$LL' + $\Omega_1$ + gap
theory uncertainty
ALEPH
OPAL
L3
DELPHI
SLD
predict

(c)


Comparison against exp. $Z$-pole data in far-tail outside fit region
$\frac{1}{\sigma}\frac{d\sigma}{d\tau}$
$\tau$
Resummed N$^3$LL' + $\Omega_1$ + gap
theory uncertainty
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(d)