Impact of resummation space choice and perturbative uncertainty estimation

Miguel Benitez

VNiVERSiDAD Đ SALAMANCA

IUFFyM
Instituto Universitario
de Física Fundamental y Matemáticas

MB, André H. Hoang, Vicent Mateu, Iain W. Stewart and Gherardo Vita — JHEP 07 (2025) 249, WIP

MB, Vicent Mateu, Matthew Schwartz and Xiaoyuan Zhang — WIP

Dijet Factorization Formula for e^+e^- Thrust

- Theoretical prediction based on factorization formula derived within SCET

[Abbate, Fickinger, Hoang, Mateu, Stewart 2010] [Fleming, Hoang, Mantry, Stewart 2007] [Becher, Schwartz 2008] [Bauer, Lee, Fleming, Sterman 2008]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \int \mathrm{d}k \left(\frac{\mathrm{d}\hat{\sigma}_{\mathrm{s}}}{\mathrm{d}\tau} + \frac{\mathrm{d}\hat{\sigma}_{\mathrm{ns}}}{\mathrm{d}\tau} \right) \left(\tau - \frac{k}{Q} \right) F_\tau(k)$$

Includes Resummation

$$\begin{aligned}
\frac{\mathrm{d}\hat{\sigma}_s^{\mathrm{QCD}}}{\mathrm{d}\tau} &= Q \sum_I \sigma_o^I H_Q^I(Q, \mu_H) U_H(Q, \mu_H, \mu) \\
&\times \int \mathrm{d}s \mathrm{d}s' \; J_\tau(s', \mu_J) U_J^\tau(s - s', \mu, \mu_J) \\
&\times \int \mathrm{d}k' U_S^\tau(k', \mu, \mu_s) e^{\frac{-2\delta(R,\mu_s)}{Q} \frac{\partial}{\partial \tau}} S_\tau^{\mathrm{part}} \left(Q\tau - \frac{s}{Q} - k', \mu_s \right) \\
&\approx \sum_n \alpha_s^n \delta(\tau) + \sum_{n,l} \alpha_s^n \left[\frac{\ln^l(\tau)}{\tau} \right]_+ + \text{subtractions}
\end{aligned}$$

Singular partonic for massless quarks

QCD final states

Dijet Factorization Formula for e^+e^- Thrust

- Theoretical prediction based on factorization formula derived within SCET

[Abbate, Fickinger, Hoang, Mateu, Stewart 2010] [Fleming, Hoang, Mantry, Stewart 2007] [Becher, Schwartz 2008] [Bauer, Lee, Fleming, Sterman 2008]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \int \mathrm{d}k \left(\frac{\mathrm{d}\hat{\sigma}_{\mathrm{s}}}{\mathrm{d}\tau} + \frac{\mathrm{d}\hat{\sigma}_{\mathrm{ns}}}{\mathrm{d}\tau} \right) \left(\tau - \frac{k}{Q} \right) F_\tau(k)$$

Includes Resummation

$$\frac{\mathrm{d}\hat{\sigma}_s^{\mathrm{QCD}}}{\mathrm{d}\tau} = Q \sum_I \sigma_o^I H_Q^I(Q,\mu_H) U_H(Q,\mu_H,\mu)$$
$$\times \int \mathrm{d}s\mathrm{d}s'\; J_\tau(s',\mu_J) U_J^\tau(s-s',\mu,\mu_J)$$
$$\times \int \mathrm{d}k' U_S^\tau(k',\mu,\mu_s) e^{\frac{-2\delta(R,\mu_s)}{Q}\frac{\partial}{\partial\tau}} S_\tau^{\mathrm{part}}\left(Q\tau - \frac{s}{Q} - k',\right.$$
$$\approx \sum_n \alpha_s^n \delta(\tau) + \sum_{n,l} \alpha_s^n \left[\frac{\ln^l(\tau)}{\tau}\right]_+ + \text{subtractions}$$

Singular partonic for massless quarl

QCD final states

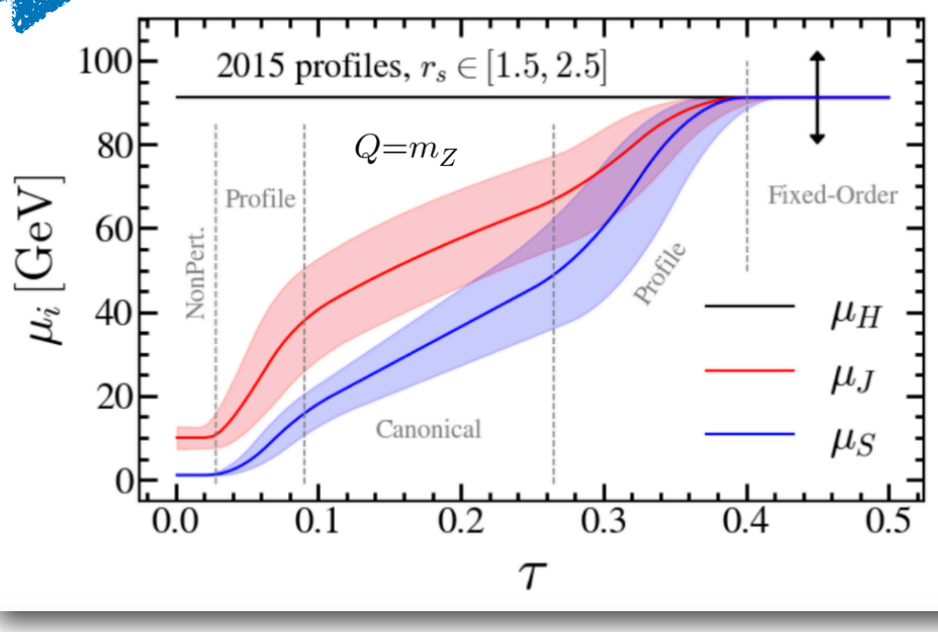
Profile functions [Momentum space scale setting]

- Factorization formula for the singular partonic thrust distribution governed by three renormalization scales

Hard scale: μ_H Jet scale: μ_J Soft scale: μ_S

- To avoid large logs while not spoiling singular vs non-singular cancellation in the far tail, the above scales must satisfy

peak:	$\mu_H \sim Q$,	$\mu_J \sim \sqrt{\Lambda_{\mathrm{QCD}} Q}$,	$\mu_S \gtrsim \Lambda_{\mathrm{QCD}}$
tail:	$\mu_H \sim Q$,	$\mu_J \sim Q\sqrt{\tau}$,	$\mu_S \sim Q\tau$
far-tail:	$\mu_H = \mu_J = \mu_S \sim Q$		



provides order-by-order convergence

gives conservative estimate for perturbative uncertainties

Singular distribution in Fourier Space

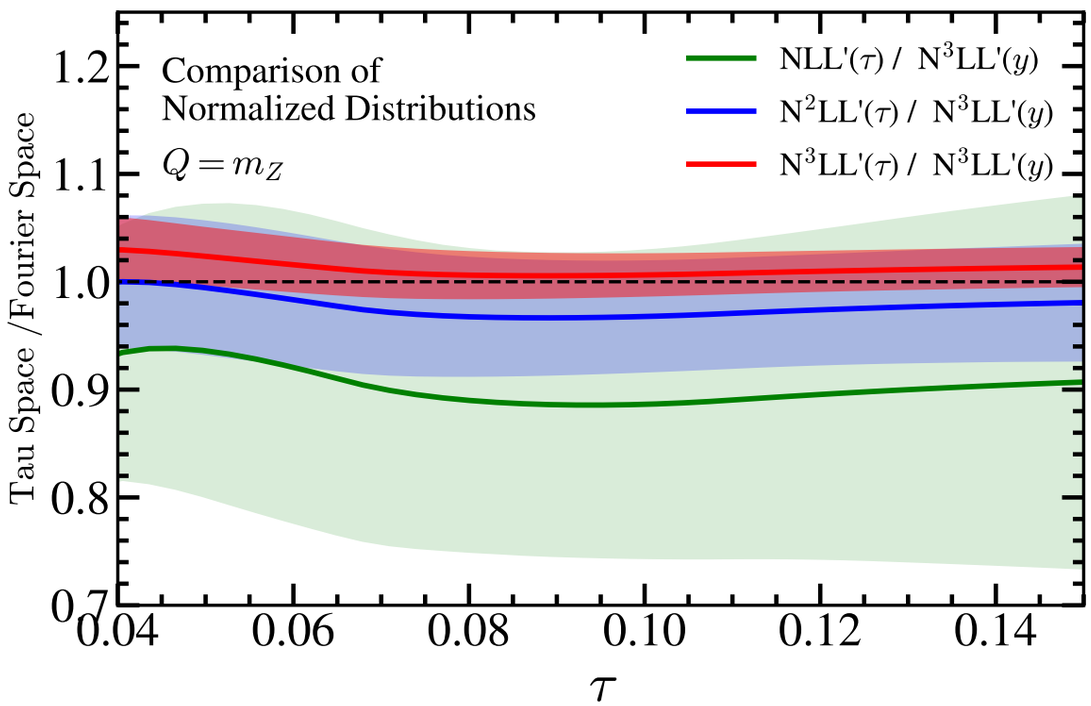
- Singular distribution in Fourier space takes the form

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \frac{H_Q(Q,\mu_H)}{p}\left(\frac{e^{\gamma_E}\mu_S}{p}\right)^{\tilde{\omega}} R(Q,\mu_H,\mu_S,\mu_J,\mu)\int\frac{\mathrm{d}y}{2\pi}\,e^{iy}(iy)^{\tilde{\omega}}\,\tilde{J}_\tau\left(\frac{y}{Qp},\mu_J\right)\tilde{S}_\tau\left(\frac{y}{p},\mu_S\right)$$

$$R(Q,\mu_H,\mu_S,\mu_J,\mu) = Q\left(\frac{\mu_H}{Q}\right)^{-2\tilde{\omega}_H} e^{\tilde{k}}\left(\frac{\mu_J^2}{Q\mu_S}\right)^{\tilde{\omega}_J} \qquad Q\tau \equiv p$$

- Fourier space scale setting:

$$\mu_H = \xi_H Q,$$
$$\mu_s = \xi_s \mu_s^c \equiv \xi_s (iye^{\gamma_E})^{-1},$$
$$\mu_J = \xi_J\sqrt{\mu_H}(iye^{\gamma_E})^{-1/2} = \xi_J\sqrt{\frac{\mu_H\mu_s}{\xi_s}}$$



Analytic Expectations

- At N^{3}LL$'$ + $\mathcal{O}(\alpha_s^3)$ differences between space of resummation are of $\mathcal{O}(\alpha_s^4)$
  - $\sim \alpha_s^4/\tau,\ \alpha_s^5\ln(\tau)/\tau$ (N^{4}LL resummation terms)
  - $\sim \alpha_s^4\tau\ln^n(\tau)$ ($\mathcal{O}(\alpha_s^4)$ matching terms)
- Discrepancy should decrease order-by-order
- If perturbative uncertainty estimate is reliable, compatibility between resummation in momentum and Laplace space must be given at any order in perturbation theory

→ Precisely follows analytic expectations

→ Both predictions converge to same result

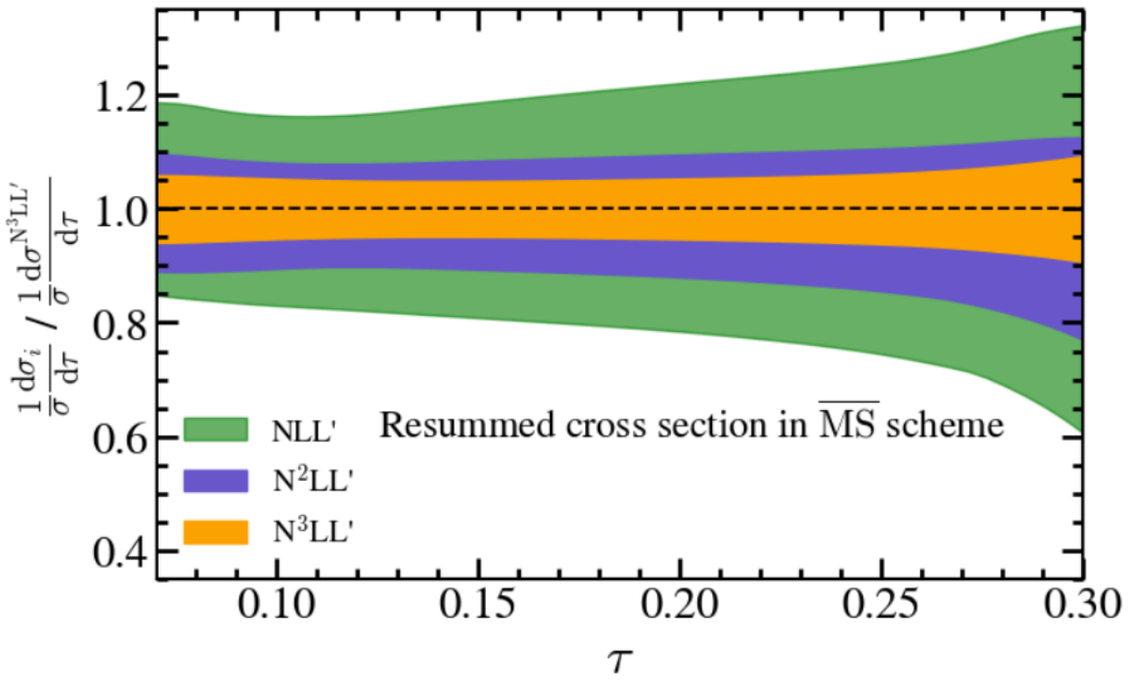
Convergence

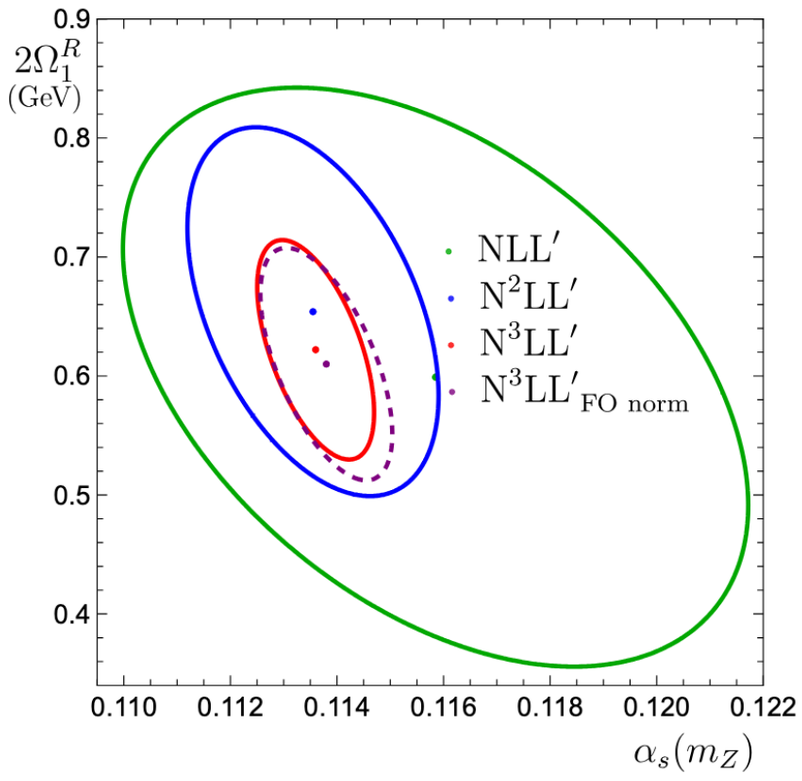
Resummed + FO Prediction

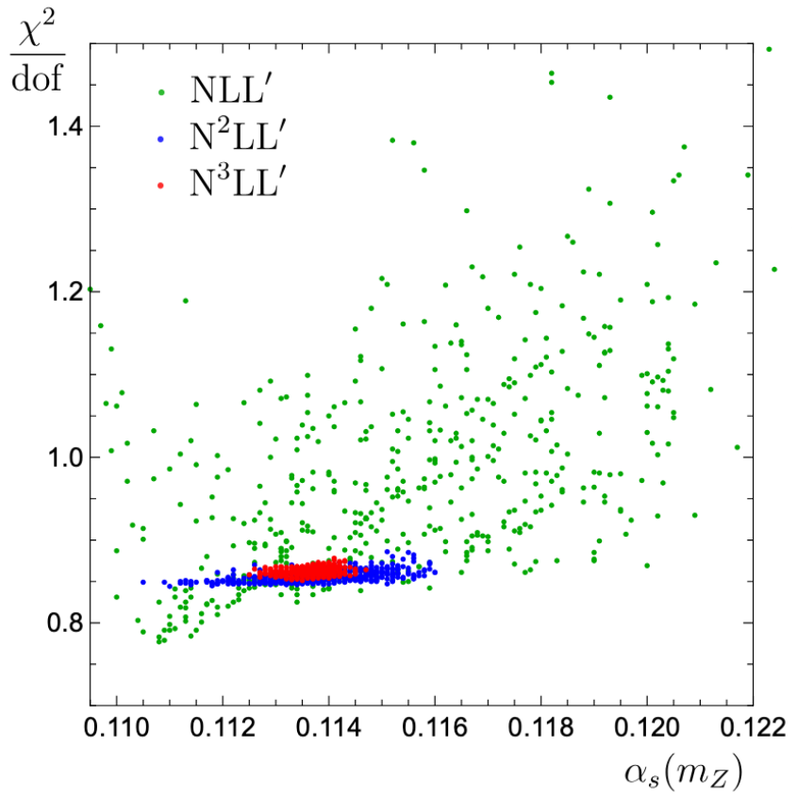
Order-by Order convergence

Gives reliable fit results for event shapes w.r.t. variation of the fit range

Order-by Order convergence confirms reasonable uncertainty estimation from profile parameter variation







Alternative resummation approaches

Comparison of:

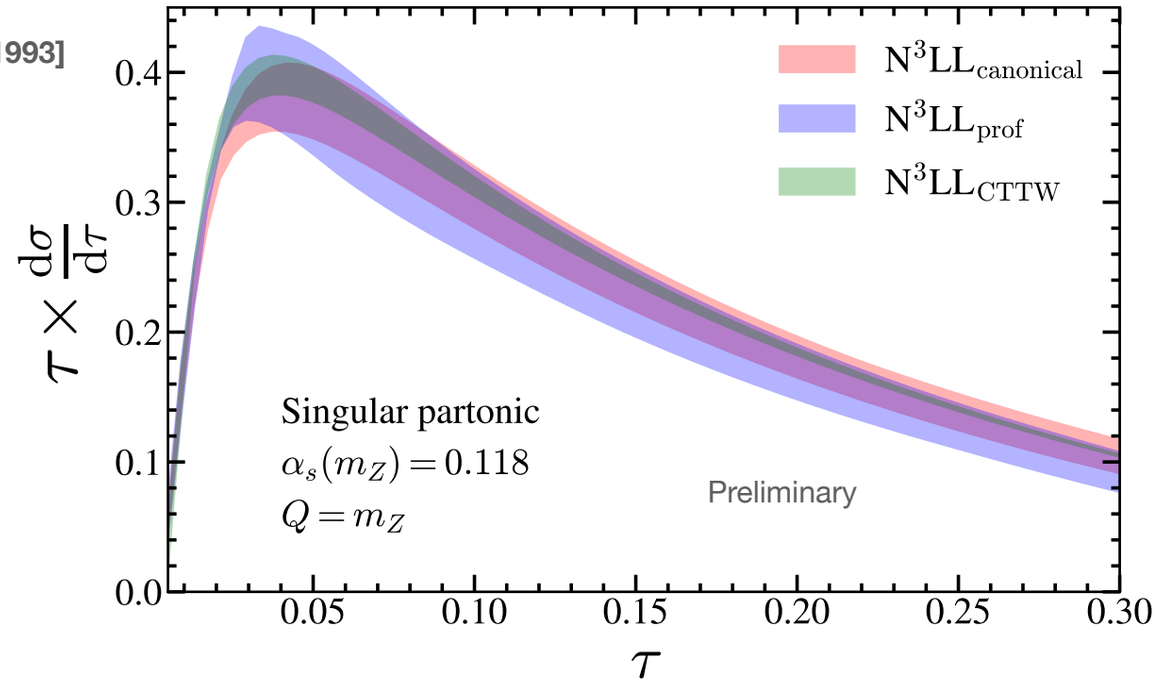
- CTTW radiator given by [Catani, Trentadue, Turnock, Webber 1991, 1993]

$$R(\tau) = C(\alpha_s(\mu))\Sigma(\tau, \alpha_s(\mu)) + D(\tau, \alpha_s(\mu))$$

$$\Sigma(\tau, \alpha_s) = \exp\left[\sum_{n=1}^{\infty}\sum_{m=1}^{n+1}\left(\frac{\alpha_s}{4\pi}\right)^n G_{nm}\log^m\frac{1}{\tau}\right] \quad C(\alpha_s) = 1 + \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{4\pi}\right)^n C_n$$

$D(\tau, \alpha_s(\mu))$ represents the non-singular contribution

- SCET factorization theorem with profiles
- SCET factorization theorem with canonical scales (no profile functions)



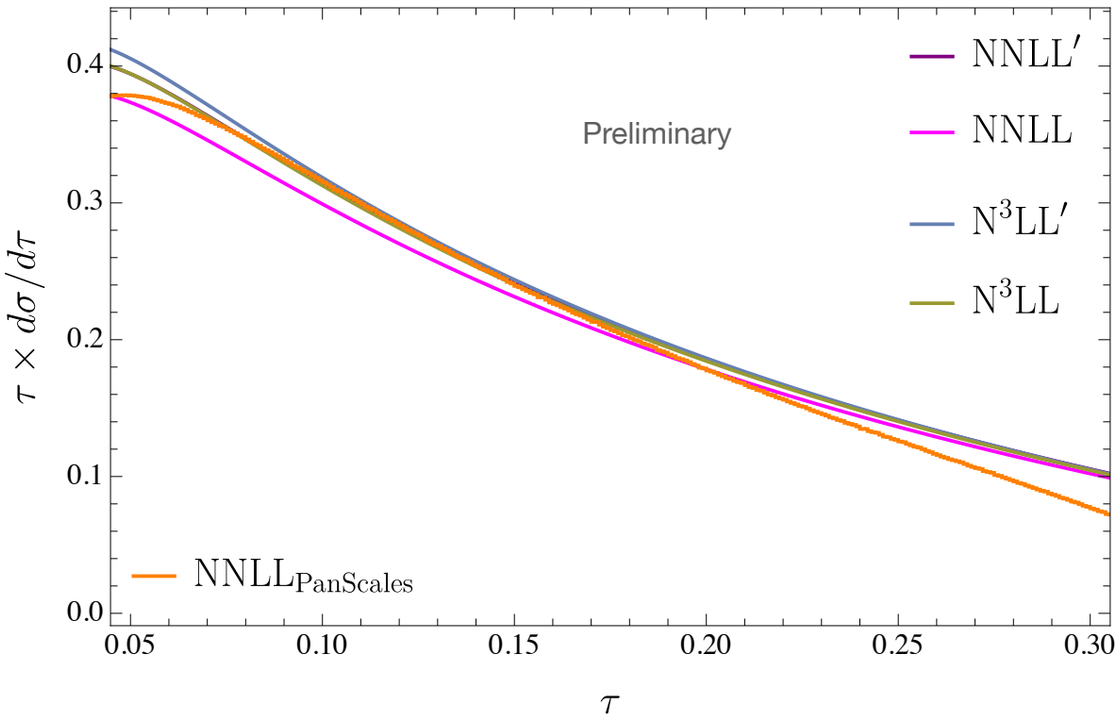
Conclusion:

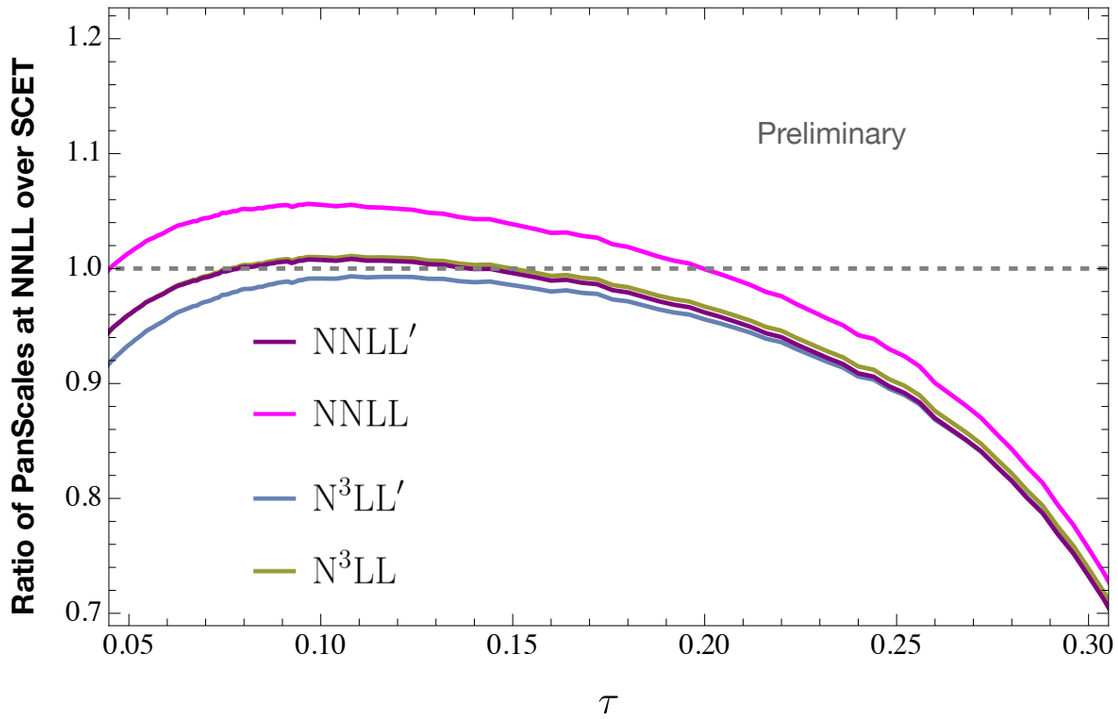
- Mutually compatible
- Using standard convention within different approaches current implementation of profile functions actually most conservative in direct comparison

Alternative resummation approaches

Comparison of:

- PanScales framework to determine resummed (no matching) cross section up to NNLL [PanScales Collaboration 2025]
- SCET factorization theorem with canonical scales





Conclusion:

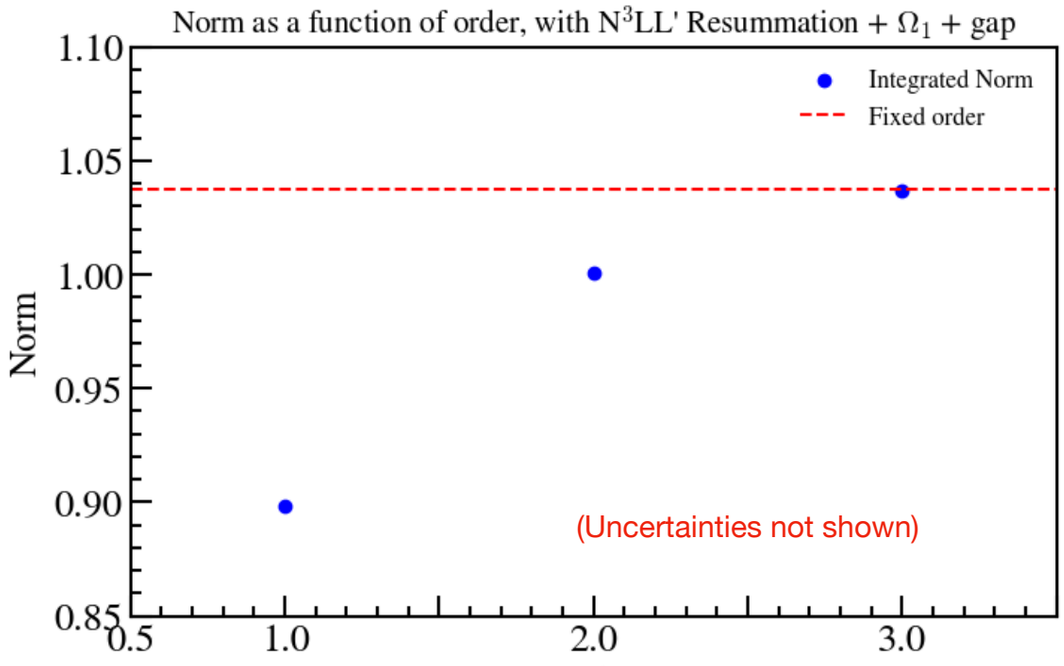
- Mutually compatible
- At NNLL PanScales result lies within uncertainty band of analytic SCET prediction with profile functions

Back-Up

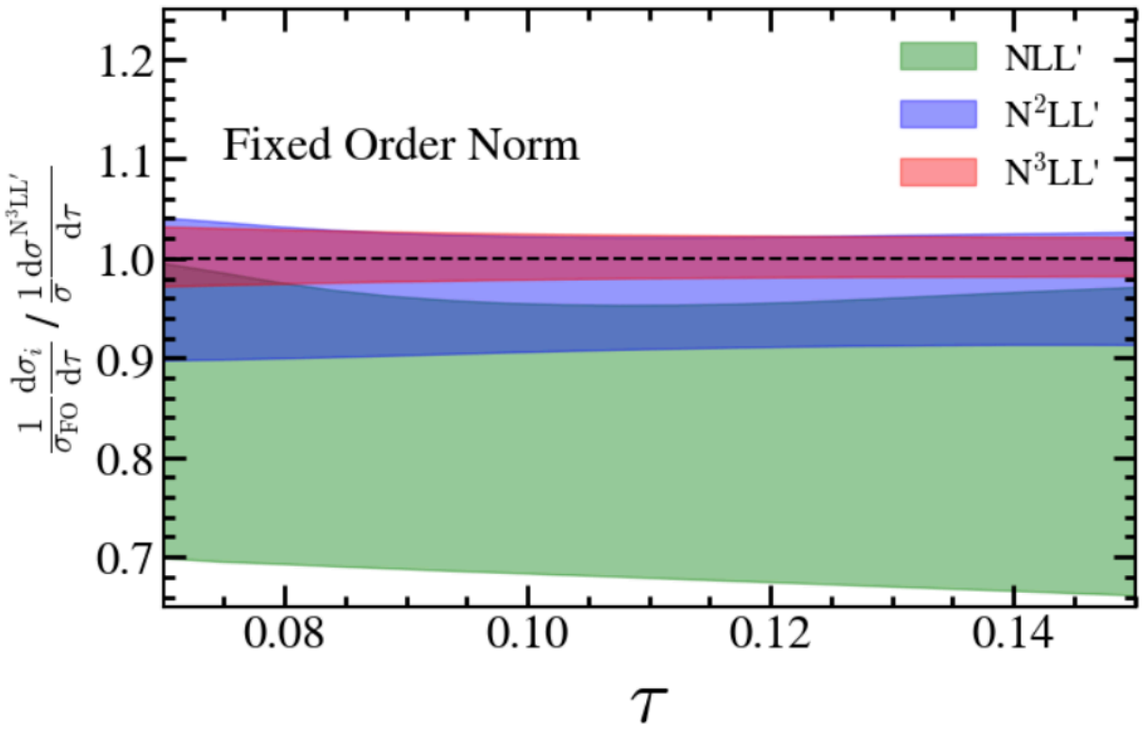
Normalization of Cross Section

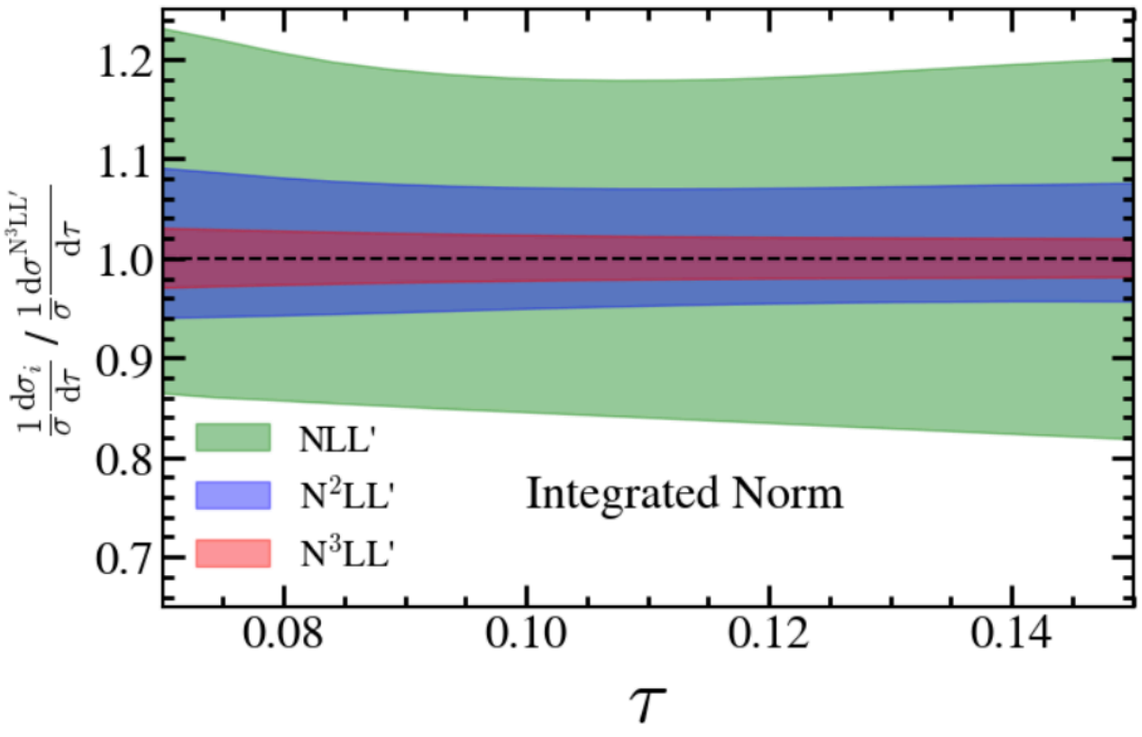
Self normalization of Cross Section

- Experimental data normalized to total number of events
- Need to normalize distribution to the total Cross section
- Can use available fixed-order result for total hadronic Cross Section or integral of $\mathrm{d}\sigma/\mathrm{d}\tau$ distribution
- Using the integrated norm suppresses τ independent uncertainty present when using fixed-order norm



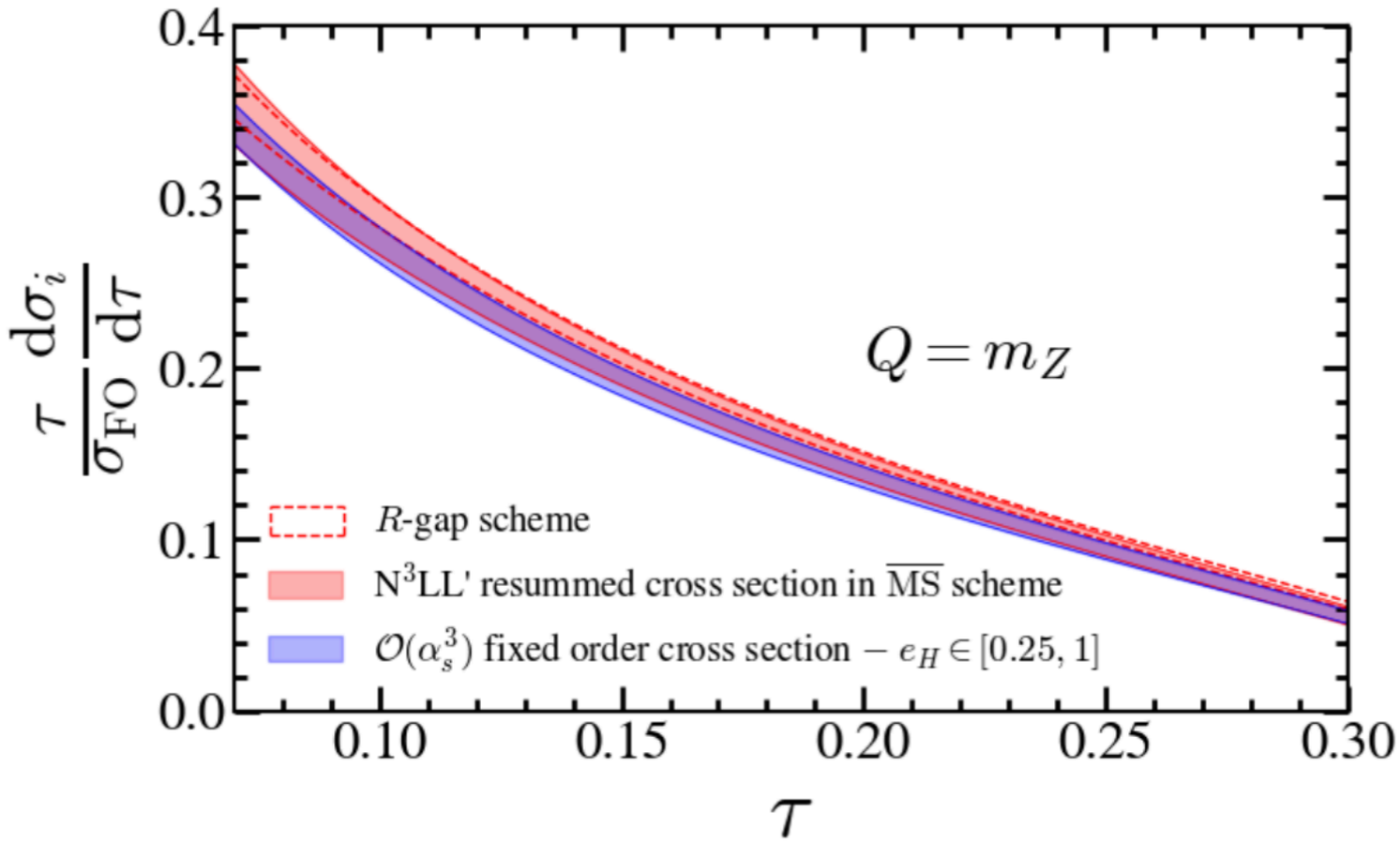
Order-by-Order convergence of resummed Cross Section





Matching uncertainty

- Resummed prediction approaches fixed-order in far tail



Fit range variation

- Resummed prediction gives stable fit result w.r.t. variation of fit range