Hadron mass effects

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- First considered in Salam,Wicke 2001, hep-ph/0102343 (SW from now on)
- Shape variables can have different definitions that all agree when hadron masses are neglected.
- Measurements with different definitions are not always possible. They require particle identification, so that the 4-momentum of the final state particles can be reconstructed from momentum measurements.
- The impact of these definition ambiguities can however be assessed using Monte Carlo generators.
- The only requirement on the MC generator is that it gives a fair representation of the data, i.e. it fits reasonably well the shape variables and hadron fragmentation functions for all species. The result does not rely upon the theoretical accuracy of the MC generator.

SW suggested three alternative schemes, all leading to consider massless final state momenta.
Call p_i the 4-momenta of the final state particles.

- p-scheme: the time component of the momenta p_i^0 are set to $|\vec{p}_i|$ before the computation of the shape variable.
- E-scheme: the space components are rescaled $\vec{p}_i \to p^0 \vec{p}_i / |\vec{p}_i|$.
- D-scheme: all particles are decayed isotropically into a pair of massless pseudoparticles. The shape variables are evaluated using these pseudoparticles.

In the following we denote by N-scheme the procedure that follows the standard definition of the shape variable.

To estimate the effect of the scheme change, we ran a large sample of $e^+e^- \to \mathrm{hadrons}$ events. For each event we

- Compute a given shape variable in the N, p, E and D scheme, in the bins adopted by each experiment. Fill two dimensional matrices $T_{ij}^{(s)}$ where s is either p, E or D. i is the bin where the shape variable computed in the N scheme falls, and j is the bin where the shape variable computed in the s scheme falls.
- Use the $T_{ij}^{(s)}$ migration matrices to correct the measured data to the s scheme, using the formula

$$n_j^{(s)} = \sum_i n_i^{(\mathrm{data})} \frac{T_{ij}}{\sum_k T_{ik}}.$$

  such that if we replace n_i^{data} with the N-scheme result for the MC sample, we get exactly the s-scheme result for the MC sample.

Variations: fit T

Variation	$\alpha_s(M_Z)$	α_0	χ^2	χ^2/dof	dof
Central	0.1169	0.6245	651.7492	1.4419	452
High scale	0.1208	0.5253	687.2753	1.5205	452
Low scale	0.1159	0.7082	639.8972	1.5236	420
Std scheme	0.1152	0.5876	534.0098	1.1841	451
p scheme	0.1152	0.5876	534.0098	1.1841	451
D scheme	0.1205	0.7232	372.8273	0.8248	452
high low-lim	0.1180	0.5896	596.8003	1.5303	390
low low-lim	0.1154	0.6672	713.4200	1.4771	483
non-pert scheme 2	0.1185	0.6557	660.7301	1.4618	452
non-pert scheme 3	0.1192	0.5888	672.7682	1.4884	452
non-pert scheme 4	0.1160	0.6758	647.1783	1.4318	452
minus non-pert error	0.1165	0.6506	648.7400	1.4353	452
plus non-pert error	0.1172	0.6038	662.0738	1.4648	452

Variations: global fit CTy3

Variation	$\alpha_s(M_Z)$	α_0	χ^2	χ^2/dof	dof
Central	0.1180	0.5892	1125.6976	1.2578	895
High scale	0.1167	0.5846	1465.9393	1.6021	915
Low scale	0.1167	0.6683	1940.0007	2.3775	816
Std scheme	0.1173	0.5347	1090.8732	1.2202	894
p scheme	0.1160	0.5624	1051.1005	1.1757	894
D scheme	0.1199	0.7252	747.3571	0.8350	895
high low-lim	0.1177	0.5673	947.3134	1.2498	758
low low-lim	0.1165	0.6260	1579.9496	1.6073	983
non-pert scheme 2	0.1193	0.5923	1249.1436	1.3957	895
non-pert scheme 3	0.1189	0.5825	1232.5919	1.3772	895
non-pert scheme 4	0.1185	0.5927	1158.0191	1.2939	895
minus non-pert error	0.1187	0.5865	1122.1407	1.2538	895
plus non-pert error	0.1189	0.5649	1228.4413	1.3726	895

Summary

- We find a strong sensitivity of the extracted value of α_s to the adopted hadron mass scheme.
- The sensitivity is even more pronounced if we consider thrust alone.