Direct probes of the quartic Higgs self-coupling

Gilberto Tetlalmatzi-Xolocotzi

A. Papaefstathiou, M. Zaro ,GTX: Eur.Phys.J.C 79 (2019) 11, 947 (1909.09166)

A. Papaefstathiou, T. Robens, GTX: JHEP 05 (2021) 193 (2101.00037)

A. Papaefstathiou, GTX: JHEP 06 (2024) 124 (2312.13562)

O. Karkout, A. Papaefstathiou, M. Postma, GTX, J. van de Vis, T du Pree: JHEP 11 (2024) 077 (2404.12425)

A. Papaefstathiou, GTX: (2501.14866)

B. Fuks, A. Papaefstathiou, GTX: Eur.Phys.J.C 85 (2025) 11, 1309 (2509.16364)

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LHC Higgs Working Group

CERN 3rd-5th December

Center for Particle Physics Siegen

TRR 257 - Particle Physics Phenomenology after the Higgs Discovery

Universität Siegen

Higgs Self-Interactions in the SM

$$V\left(\Phi^\dagger \Phi\right) = \mu^2 \Phi^\dagger \Phi + \lambda_{SM}\left(\Phi^\dagger \Phi\right)^2$$

$$\Phi = \left(0, v_0 + h\right)^T / \sqrt{2}$$

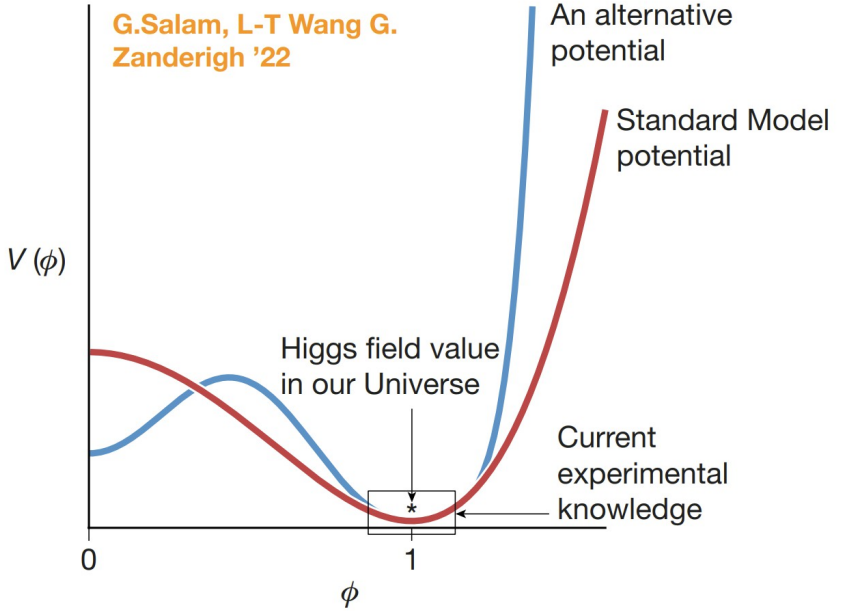
$$V\left(\Phi^\dagger \Phi\right) \supset \frac{1}{2} m_h^2 h^2 + \lambda_{SM}\, v_0\, h^3 + \frac{\lambda_{SM}}{4} h^4$$

In the SM $\quad m_h^2 = \lambda_{SM}\, v_0^2/2 \qquad v_0^2 = -\mu^2/\lambda_{SM}$

Probing the quartic self coupling

Triple Higgs production allows to test the Higgs quartic self couplings directly

We do not know much about the shape of the Higgs potential.

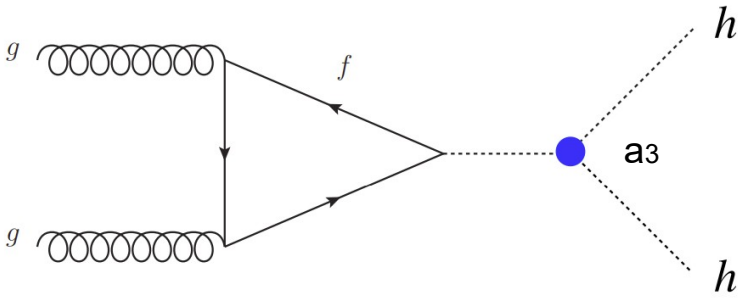


$$V\left(\Phi^{\dagger}\Phi\right) \supset \frac{1}{2} m_h^2 h^2 + a_3 h^3 + a_4 h^4$$

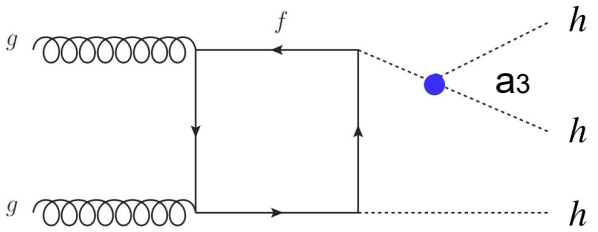
Probing the quartic Higgs self coupling allows to look for Beyond the SM Physics.

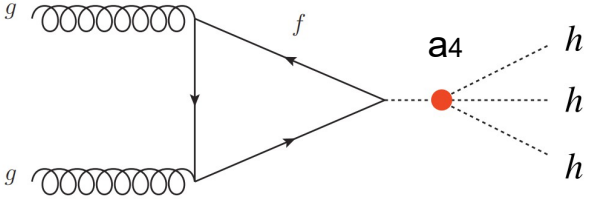
Triple Higgs production

Double Higgs production is the lowest multiplicity to probe for a_3 .



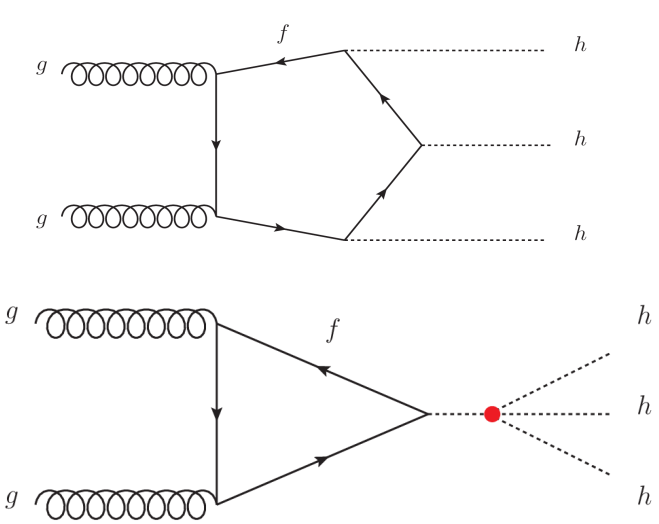
Triple Higgs production is the lowest multiplicity to probe for a_4.

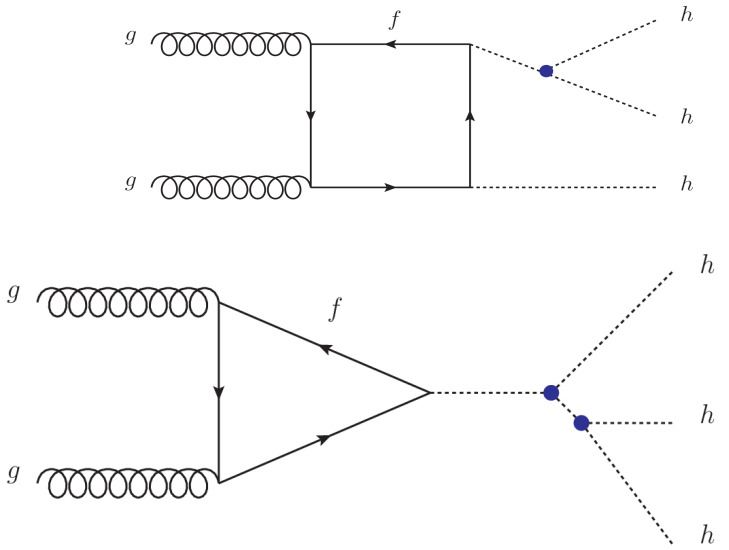




Triple Higgs production at proton-proton colliders

$$g\,g \longrightarrow h\,h\,h$$





At the HL-LHC
(pp @ 14 TeV)

$\sigma = 0.1\ fb$

~*O(100)* events
Very challenging

At the FCC
(pp @ 100 TeV)

$\sigma \sim 5\ fb$

Luminosity=*20 ab*$^{-1}$
~100 000 events.

de Florian, Fabre, Mazzitelli: 1912.02760

Possible Final states

$$h\,h\,h \longrightarrow X$$

X (Final State)	Br(%)
$(b\bar{b})(b\bar{b})(b\bar{b})$	19.21
$(b\bar{b})(b\bar{b})(W\,W_{1l})$	7.20
$(b\bar{b})(b\bar{b})(\tau\,\bar{\tau})$	6.31
$(b\bar{b})(\tau\,\bar{\tau})(W\,W_{1l})$	1.58
$(b\bar{b})(b\bar{b})(W\,W_{2l})$	0.98
$(b\bar{b})(W\,W_{1l})(W\,W_{1l})$	0.90
$(b\bar{b})(\tau\,\bar{\tau})(\tau\,\bar{\tau})$	0.69
$(b\bar{b})(b\bar{b})(\gamma\,\gamma)$	0.23

Papaefstathiou, GTX, Zaro: 1909.09166
A. Papaefstathiou, GTX: (2501.14866)
Fuks, Papaefstathiou, GTX: 2509.16364
Stylianou, Weiglein, 2312.04646

Fuks, Kim, Lee: 1510.07697
1704.04298

Killian et al.: 1702.03554

Stylianou, Weiglein, 2312.04646

Papaefstathiou, Sakurai.: 1508.06524
Chen et al.:1510.04013
Fuks, Kim, Lee: 1510.07697

6-b final state has the largest Branching Fraction

This is the channel we are focusing on in this talk

FCC Study

Backgrounds vs Signal

In the FCC (pp @ 100 TeV)

Signal and Irreducible backgrounds

Process	$\sigma_{\text{GEN}} \times k_{\text{fac}} \times \text{BR} \times \mathcal{P}(6b)$ [fb]
hhh (SM)	4.29×10^{-1}
QCD $(b\bar{b})(b\bar{b})(b\bar{b})$	1.07×10^{4}
$q\bar{q} \to Z(b\bar{b})(b\bar{b})$	3.61×10^{2}
$q\bar{q} \to ZZb\bar{b}$	1.14×10^{1}
$q\bar{q} \to ZZZ$	1.80×10^{-1}
$q\bar{q} \to hZb\bar{b}$	2.04
$q\bar{q} \to hZZ$	1.48×10^{-1}
$q\bar{q} \to hhZ$	8.11×10^{-2}
$q\bar{q} \to hh(b\bar{b})$	1.80×10^{-2}
$q\bar{q} \to h(b\bar{b})(b\bar{b})$	7.25×10^{-1}
$gg \to ZZZ$	5.18×10^{-3}
$gg \to hZZ$	3.59×10^{-2}
$gg \to hhZ$	6.41×10^{-2}

Reducible backgrounds

Process	σ_{GEN} [fb]	$\sigma_{\text{GEN}} \times k_{\text{fac}} \times \mathcal{P}(6b)$ [fb]
$(b\bar{b})(b\bar{b})(c\bar{c})$	73.0×10^{3}	762
$(b\bar{b})(c\bar{c})(c\bar{c})$	72.0×10^{3}	10.4
$(c\bar{c})(c\bar{c})(c\bar{c})$	21.8×10^{3}	4.36×10^{-2}
$(b\bar{b})(b\bar{b})(jj)^*$	3.09×10^{6}	161
$(b\bar{b})(jj)(jj)^*$	1.07×10^{8}	1.54
$(c\bar{c})(c\bar{c})(jj)^*$	1.73×10^{5}	3.46×10^{-3}
$(c\bar{c})(jj)(jj)^*$	6.67×10^{7}	6.67×10^{-3}
$(jj)(jj)(jj)^*$	3.04×10^{9}	6.08×10^{-3}

Assuming a K-factor of 2

Maltoni, Vryonidou, Zaro: 1408.6542

Details on the study of the 6b final state

- Parton level events (signal/background) generated with MadGraph5_aMC@NLO.
- The source of background with the highest XS is QCD-6b-Jets.
- The production of the 6b-final state is challenging, it was generated in the Siegen computer cluster using the gridpack option available in MadGraph5_aMC@NLO.
- Parton shower and non-perturbative effects included with Herwig 7.
- The analysis was performed using HwSim. [*Papaefsathiou*, https://bitbucket.org/andreasp/hwsim].
- Two selection analysis approaches are followed: traditional cuts selection and gradient boosting using XGBoost.

Smearing effects

We introduce smearing effects based on ATLAS and CMS like scenarios

$$E_{\text{smeared}} = E + \Delta E\ , \qquad \Delta E \sim \mathcal{N}(0,\, \sigma_E)\ ,$$

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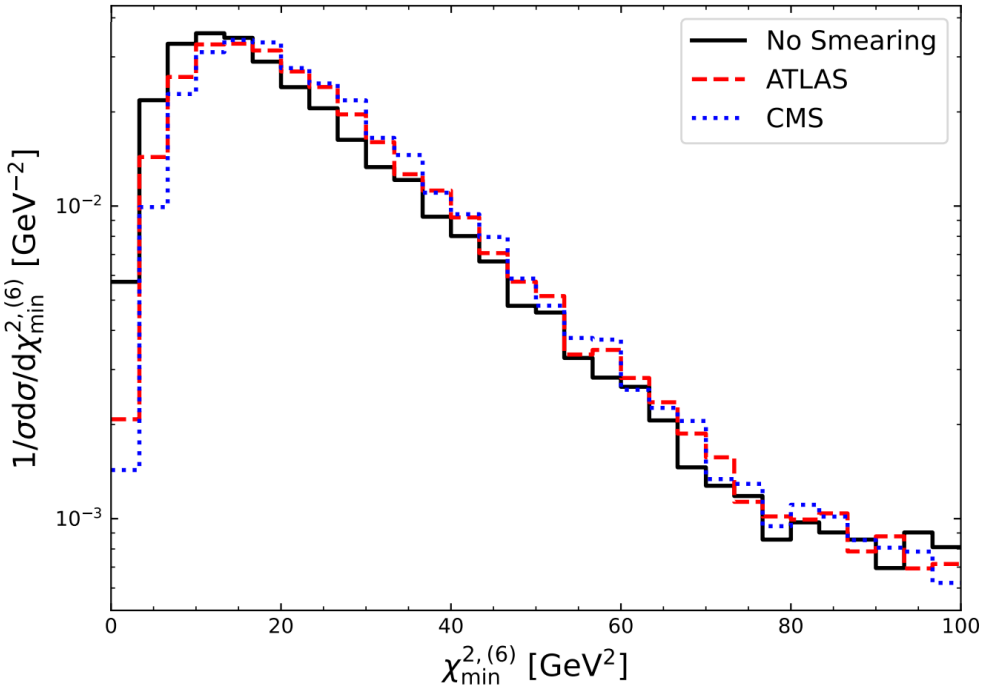
$$E_{\text{smeared}} = E + \Delta E \qquad p_{T,\text{smeared}} = \frac{E + \Delta E}{\cosh \eta}$$



$$\sigma_E^{\text{ATLAS}} = \begin{cases} \sqrt{(0.0302E)^2 + 0.5205^2 E + 1.59^2} & |\eta| \le 1.7 \\ \sqrt{(0.05E)^2 + 0.706^2 E} & |\eta| \in [1.7, 3.2] \\ \sqrt{(0.0942E)^2 + E} & |\eta| \in [3.2, 4.9] \end{cases}\ ,$$

$$\sigma_E^{\text{CMS}} = \begin{cases} \sqrt{(0.05E)^2 + 1.5^2 E} & |\eta| \le 3.0 \\ \sqrt{(0.130E)^2 + 2.7^2 E} & 3.0 < |\eta| \le 5.0 \end{cases}\ ,$$

Selection Analysis

- *Require 6 b-tagged jets*
- *Construct all the possible combinations of 3-pairs of b-jets:* I.
- *For each combination* I calculate the observable

$$\chi^{2,(6)} = \sum_{qr \in I} \left(M_{qr} - m_h \right)^2$$

- *Select the event based on the value of the combination which minimizes* $\chi^{2,(6)}$
- *The combination determining* $\chi^{2,(6)}_{min}$ *defines the best candidates for the set of 3-Higgs bosons in the event.*

Selection Analysis

Set of observables and optimized cuts applied during the *traditional* selection analysis (FCC)

Observable	Threshold
$p_{T,b} >$	35.0 GeV
$\lvert\eta_b\rvert <$	3.0
$\Delta R_{bb} >$	0.3
$p_{T,b_i} >$	[170.0, 135.0, 35.0] GeV
$\chi^{2,(6)} <$	26.0 GeV
$\Delta m_i <$	[8, 8, 8] GeV
$\Delta R_{bb}(h^i) <$	[3.5, 3.5, 3.5]
$\Delta R(h^i, h^j) <$	[3.5, 3.5, 3.5]
$p_T(h^i) >$	[200.0, 190.0, 20.0] GeV

h^i : Higgs boson candidate

$i = 1,2,3$

Sensitivity to self couplings

We consider a generalized version of the SM scalar potential

$$\mathcal{L} \supset -\frac{m_h^2}{2v}\left(1+c_3\right)h^3 - \frac{m_h^2}{8v^2}\left(1+d_4\right)h^4$$

To asses the sensitivity towards the anomalous couplings we construct the following test statistic

$$\chi^2_{\text{tot}}(c_3, d_4) = \chi^2(c_3, d_4) + \left(\frac{c_3}{\delta c_3}\right)^2 \qquad \chi^2(c_3, d_4) = \left[\frac{S_{\text{SM}} - S(c_3, d_4)}{\delta_{\text{SM+B}}}\right]^2$$

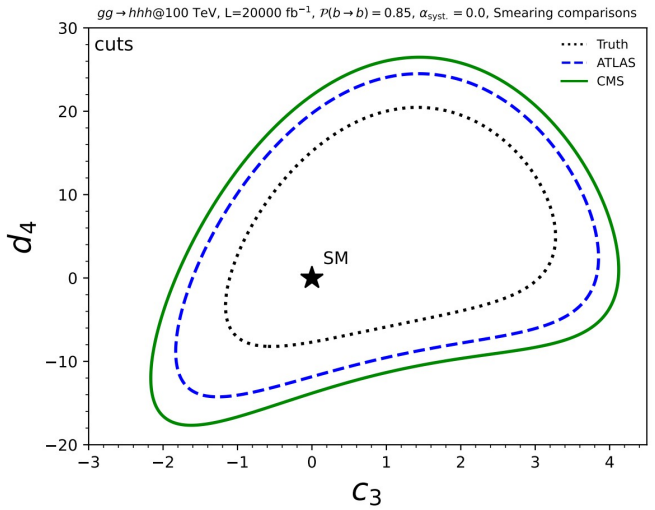
Gaussian prior to introduce information on the cubic coupling

Uncertainty in the number of expected SM events

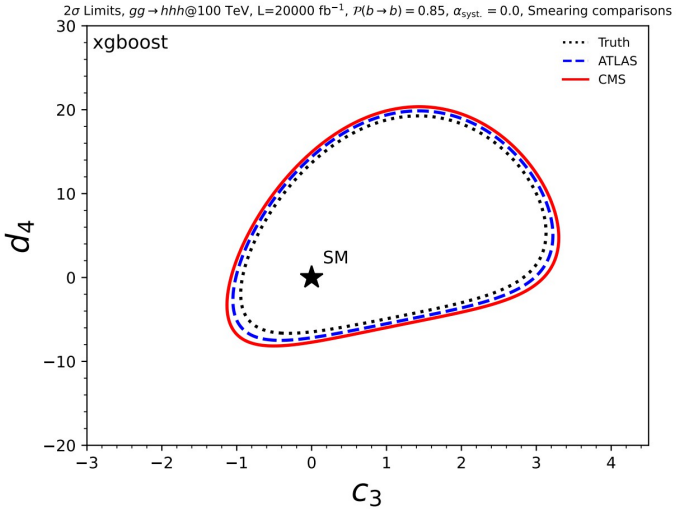
$$\delta_{\text{SM+B}} = \sqrt{S_{\text{SM}} + B + (\alpha B)^2}$$

Systematic uncertainties

Sensitivity to self couplings



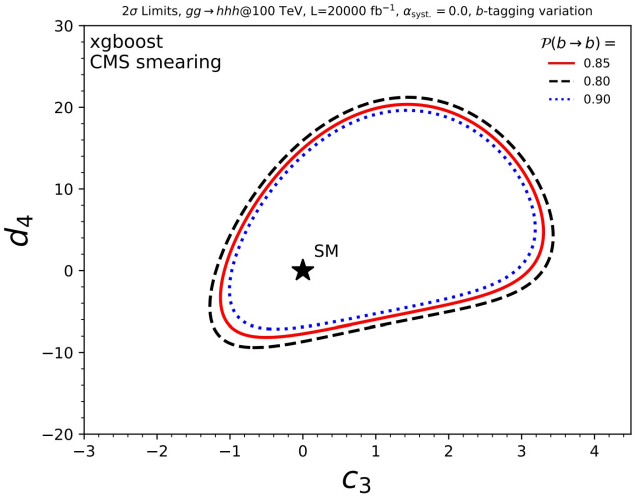
Traditional Cuts



XGBoost

95% C. L. Regions

B. Fuks, A. Papaefstathiou, GTX: 2509.16364



b-Tagging impact

Sensitivity to self couplings

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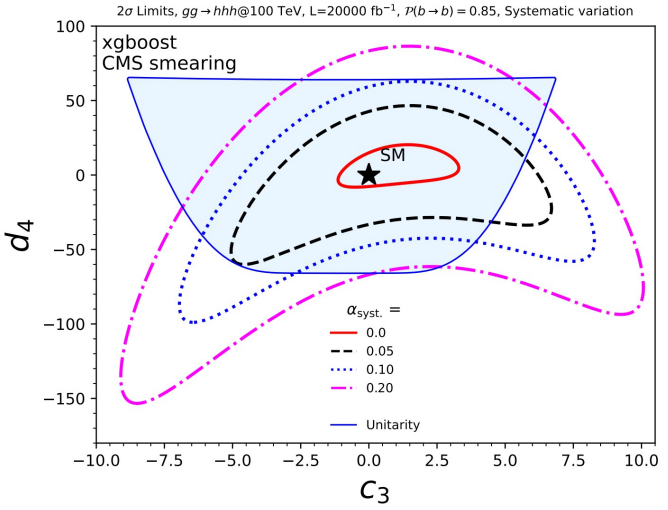
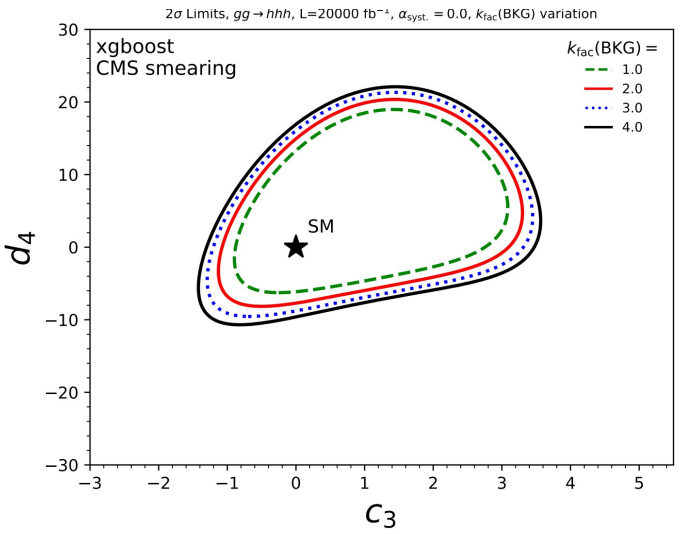
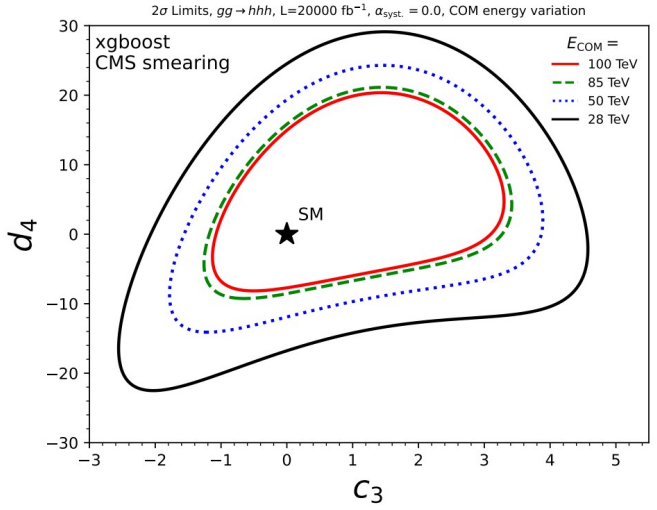


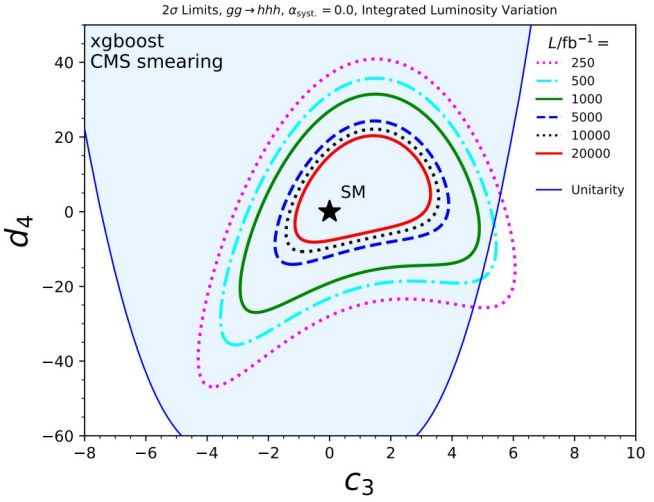
Systematic-error impact



K-factor impact



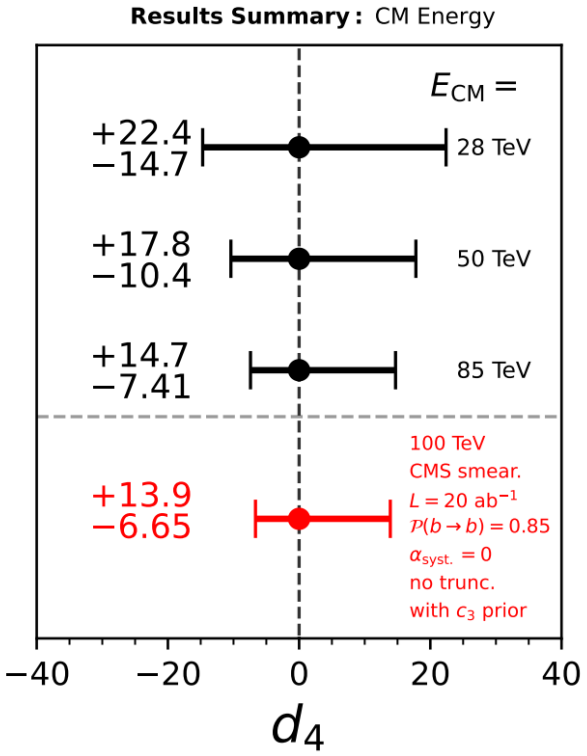
CM-impact

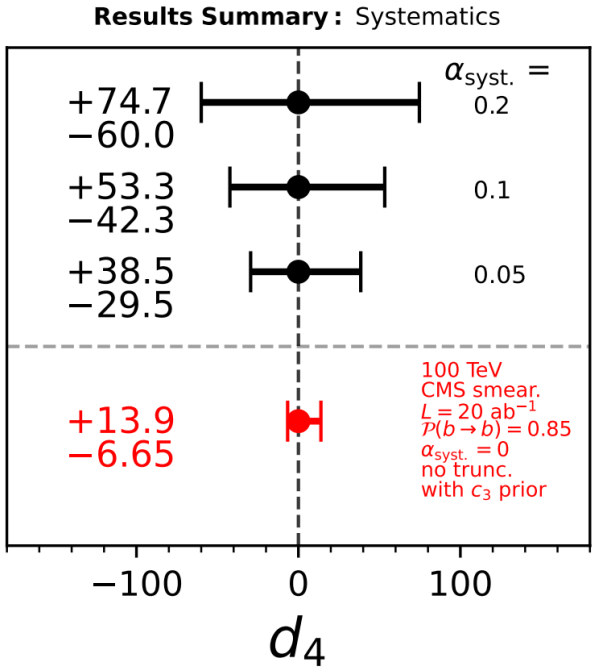


Luminosity impact

Sensitivity to self couplings

Summaries





$$\delta c_3 = 0.05$$

B. Fuks, A. Papaefstathiou, GTX: 2509.16364

LHC
(Extended Scalars)

Two Real Singlet Extension of the SM TRSM

$$V(\Phi,\phi_i)=V_{SM}(\Phi)+V(\Phi,S,X)$$

Reduce the number of parameters by imposing

$$\mathbb{Z}_2^S: S\to -S,\ X\to X$$

$$\mathbb{Z}_2^X: S\to S,\ X\to -X$$

$$V(\Phi,X,S)=\mu_\Phi^2\Phi^\dagger\Phi+\lambda_\Phi(\Phi^\dagger\Phi)^2+\mu_S^2S^2+\lambda_S S^4$$
$$+\mu_X^2X^2+\lambda_X X^4+\lambda_{\Phi S}\Phi^\dagger\Phi X^2+\lambda_{SX}S^2X^2$$

$$S=(\phi_S+v_S)/\sqrt{2}$$

$$X=(\phi_X+v_X)/\sqrt{2}$$

Change to the physical basis

$$\begin{pmatrix}h_1\\h_2\\h_3\end{pmatrix}=R(\theta_X,\theta_S)\begin{pmatrix}\phi_h\\\phi_S\\\phi_X\end{pmatrix}$$

$h_1=h$ *is the SM Higgs boson*

$$M_1=125\,GeV$$

Free independent parameters

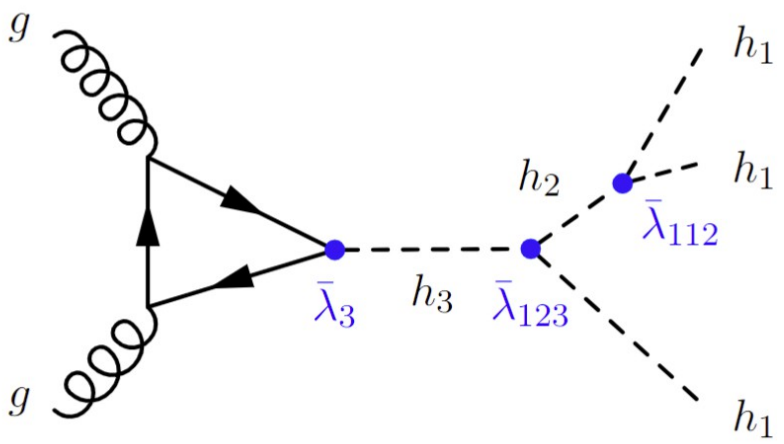
$$M_2, M_3, \theta_{hS}, \theta_{hX}, \theta_{SX}, v_S, v_X$$

Robens, Stefaniak, Wittbrodt: 1908.08554

Old Benchmark Scenario of Study BP3

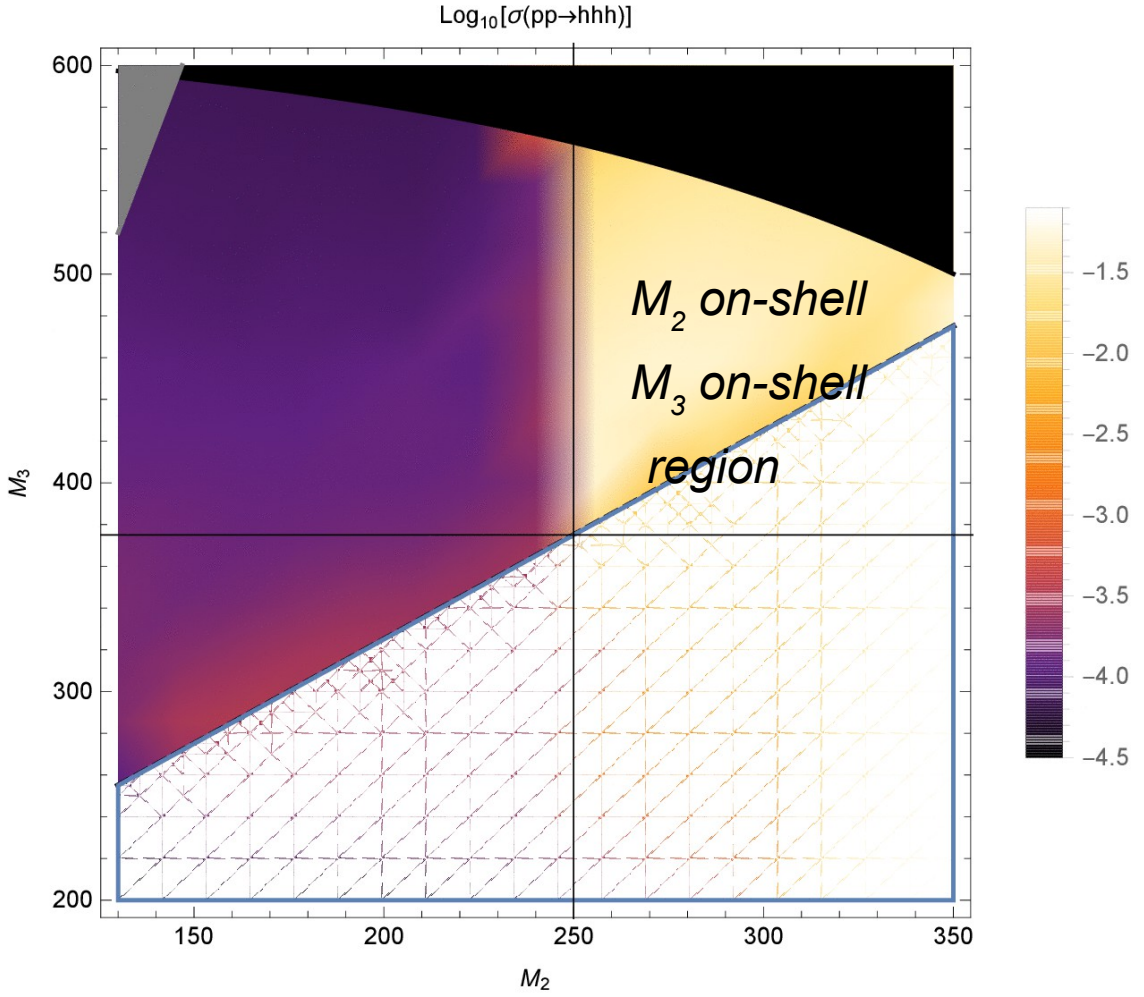
The BP3 Scenario introduced in 1908.08554 which allows for a large $h_1h_1h_1$ production while obeying current theoretical and experimental constraints.

Parameter	Value
M_1	125.09 GeV
M_2	[125, 500] GeV
M_3	[255, 650] GeV
θ_{hS}	−0.129
θ_{hX}	0.226
θ_{SX}	−0.899
v_S	140 GeV
v_X	100 GeV



We consider the mass hierarchy $M_1 < M_2 < M_3$

Production cross section



The X-Section can reach up to 50 fb for M_2~(263,280) GeV and M_3~450 GeV

A. Papaefstathiou, T. Robens, GTX: 2101.00037

Old benchmark points

Label	(M_2, M_3) [GeV]	$\varepsilon_{\text{Sig.}}$	$\text{S}\big\vert_{300\text{fb}^{-1}}$	$\varepsilon_{\text{Bkg.}}$	$\text{B}\big\vert_{300\text{fb}^{-1}}$	$\text{sig}\big\vert_{300\text{fb}^{-1}}$	$\text{sig}\big\vert_{3000\text{fb}^{-1}}$
A	$(255, 504)$	0.025	14.12	8.50×10^{-4}	19.16	2.92	9.23
B	$(263, 455)$	0.019	17.03	3.60×10^{-5}	8.11	4.78	15.11
C	$(287, 502)$	0.030	20.71	9.13×10^{-5}	20.60	4.01	12.68
D	$(290, 454)$	0.044	37.32	1.96×10^{-4}	44.19	5.02	15.86
E	$(320, 503)$	0.051	32.54	2.73×10^{-4}	61.55	3.76	11.88
F	$(264, 504)$	0.028	18.18	9.13×10^{-5}	20.60	3.56	11.27
G	$(280, 455)$	0.044	38.70	1.96×10^{-4}	44.19	5.18	16.39
H	$(300, 475)$	0.054	41.27	2.95×10^{-4}	66.46	4.64	14.68
I	$(310, 500)$	0.063	41.42	3.97×10^{-4}	89.59	4.09	12.94
J	$(280, 500)$	0.029	20.67	9.14×10^{-5}	20.60	4.00	12.65

These points are associated with large couplings which can break perturbativity at the energy scale MZ

A. Papaefstathiou, T. Robens, GTX: 2101.00037

New Benchmark points (LHC)

Determine phase space that enhances triple Higgs production in the TRSM based on

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Perturbative conditions

$$\lambda_{11} < \frac{\pi^2}{3} \approx 3.3, \quad \lambda_{22}, \lambda_{33} < \frac{4\pi^2}{9} \approx 4.4, \quad \lambda_{12}, \lambda_{13}, \lambda_{23} < 2\pi^2 \approx 20$$

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Boundedness from below

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Experimental constraints from HiggsTools (HiggsSignals and HiggsBounds)

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Relevant HiggsBounds Experimental Analyses

Processes	Experiment	Int. Luminosity	arXiv ref.
$gg \to S \to W^+W^-, ZZ$	ATLAS	139 fb^{-1}	2004.14636 [57]
$gg \to S \to ZZ$	ATLAS	139 fb^{-1}	2009.14791 [58]
$gg \to S \to h_1h_1 \to (b\bar{b})(\tau^+\tau^-)$	CMS	137 fb^{-1}	2106.10361 [59]
$(b\bar{b}, \tau^+\tau^-, W^+W^-, ZZ, \gamma\gamma)(b\bar{b})$		35.9 fb^{-1}	1811.09689 [60]
$gg \to S \to h_1h_1 \to (b\bar{b}, \tau^+\tau^-, W^+W^-, \gamma\gamma)^2$	ATLAS	36.1 fb^{-1}	1906.02025 [61]
$gg \to S \to h_1h_1 \to (b\bar{b})(\gamma\gamma)$	ATLAS	36.1 fb^{-1}	1807.04873 [62]
$gg \to S \to W^+W^-, ZZ$	ATLAS	36.1 fb^{-1}	1808.02380 [63]
$pp \to S \to ZZ$ (incl. VBF)	CMS	35.9 fb^{-1}	1804.01939 [64]
$gg \to S \to h_1h_1 \to (b\bar{b})(b\bar{b})$	CMS	35.9 fb^{-1}	1806.03548 [65]
$gg \to S \to h_1h_1 \to (b\bar{b})(b\bar{b})$	ATLAS	36.1 fb^{-1}	1806.03548 [65]

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Boundedness from below

Experimental constraints from HiggsTools (HiggsSignals and HiggsBounds)

We consider the threshold

$$\sigma_{3h_1} > 100\, \sigma_{3h_1}^{\rm SM},$$

Our analysis entailed 530,000 phase space points

Only 130 points fulfilled all the conditions

New Benchmark points (LHC)

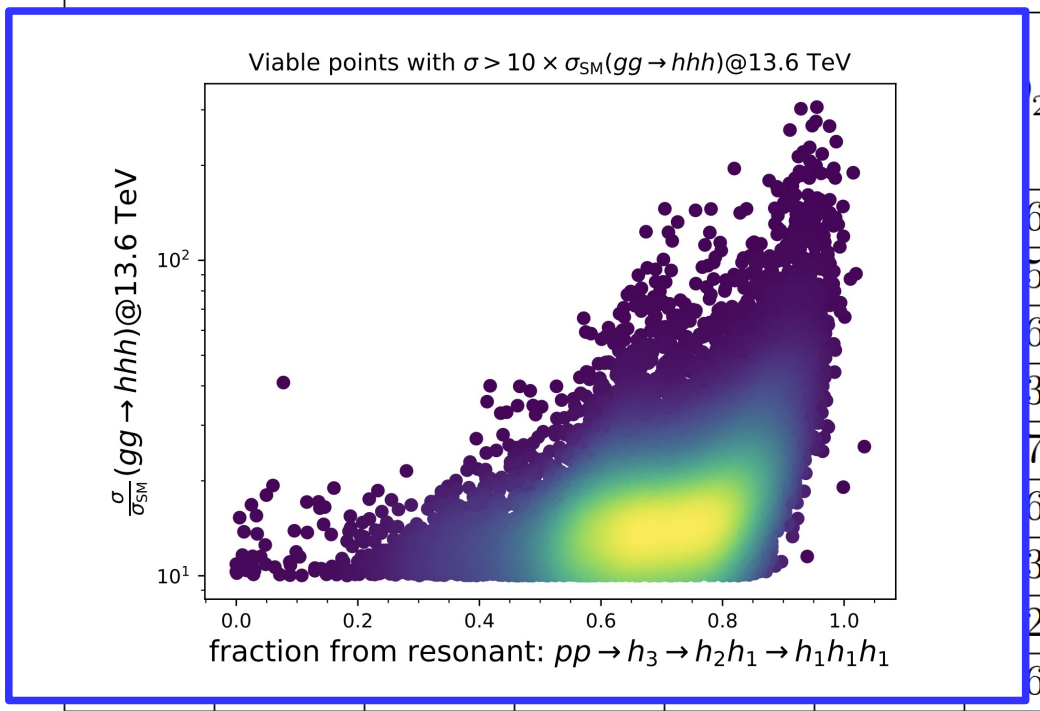
Benchmark points for enhanced triple Higgs production										
M_2	M_3	v_2	v_3	θ_{12}	θ_{13}	θ_{23}	$\frac{\sigma}{\sigma_{SM}}$	Res. Frac.	μ_{pert} GeV	$\frac{\mu_{\text{pert}}}{\mu_{\text{pole}}}$
259.0	495.0	215.8	180.8	6.191	0.163	5.691	306.025	0.955	2.7×10^2	7.3
270.6	444.7	122.4	847.2	0.268	0.030	0.522	302.361	0.929	1.8×10^2	7.4
268.6	452.7	137.8	784.8	0.263	0.023	0.645	275.616	0.954	2.4×10^2	7.3
272.6	480.7	928.3	143.7	3.098	2.9	2.375	267.245	0.948	1.4×10^2	7.3
269.0	409.8	138.0	599.4	0.244	0.004	0.773	266.439	0.976	2.4×10^2	7.4
269.1	486.9	227.5	307.9	0.074	6.149	2.631	157.583	0.956	4.3×10^2	7.3
259.2	577.0	289.0	275.6	0.137	6.148	2.324	145.470	0.781	1.2×10^4	7.3
283.7	575.0	259.4	330.4	0.137	6.152	2.299	122.546	0.779	3.0×10^3	7.3
264.3	469.3	207.3	359.5	0.285	6.277	0.692	119.121	0.999	5.4×10^3	7.3
266.5	461.9	653.1	229.0	2.889	3.046	1.015	112.794	0.863	5.3×10^4	7.4
259.2	399.7	444.5	217.0	2.917	3.046	1.047	103.717	0.973	1.2×10^5	7.4

Update of
A. Papaefstathiou, T. Robens, GTX: 2101.00037/ JHEP 05 (2021), 193

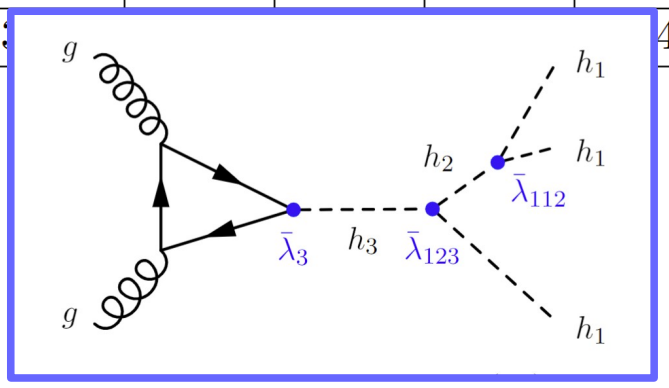
O. Karkout, A. Papaefstathiou, M. Postma, GTX, J. van de Vis, T du Pree: 2404.12425

New Benchmark points (LHC)

Benchmark points for enhanced triple Higgs production



						23	$\frac{\sigma}{\sigma_{SM}}$	Res. Frac.	μ_{pert} GeV	$\frac{\mu_{\text{pert}}}{\mu_{\text{pole}}}$
						691	306.025	0.955	2.7×10^2	7.3
						522	302.361	0.929	1.8×10^2	7.4
						645	275.616	0.954	2.4×10^2	7.3
						375	267.245	0.948	1.4×10^2	7.3
						773	266.439	0.976	2.4×10^2	7.4
						631	157.583	0.956	4.3×10^2	7.3
						324	145.470	0.781	1.2×10^4	7.3
						299	122.546	0.779	3.0×10^3	7.3
						692	119.121	0.999	5.4×10^3	7.3
266.5	461.9	653.1	229.0	2.889	3.046	1.015	112.794	0.863	5.3×10^4	7.4
259.2					46	1.047	103.717	0.973	1.2×10^5	7.4



New Benchmark points (LHC)

Benchmark points for enhanced triple Higgs production

M_2	M_3	v_2	v_3	θ_{12}	θ_{13}	θ_{23}	$\frac{\sigma}{\sigma_{SM}}$	Res. Frac.	$\mu_{\rm pert}$ GeV	$\frac{\mu_{\rm pert}}{\mu_{\rm pole}}$
259.0	495.0	215.8	180.8	6.191	0.163	5.691	306.025	0.955	2.7×10^2	7.3
270.6	444.7	122.4	847.2	0.268	0.030	0.522	302.361	0.929	1.8×10^2	7.4
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$$\lambda_{11} < \frac{\pi^2}{3} \approx 3.3, \quad \lambda_{22}, \lambda_{33} < \frac{4\pi^2}{9} \approx 4.4, \quad \lambda_{12}, \lambda_{13}, \lambda_{23} < 2\pi^2 \approx 20$$

New Benchmark points (LHC)

Benchmark points for enhanced triple Higgs production

M_2	M_3	v_2	v_3	θ_{12}	θ_{13}	θ_{23}	$\frac{\sigma}{\sigma_{SM}}$	Res. Frac.	$\mu_{\rm pert}$ GeV	$\frac{\mu_{\rm pert}}{\mu_{\rm pole}}$
259.0	495.0	215.8	180.8	6.191	0.163	5.691	306.025	0.955	2.7×10^2	7.3
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259.2	399.7	444.5	217.0	2.917	3.046	1.047	103.717	0.973	1.2×10^5	7.4

In practice our points fulfil the following theoretical relationship

$$\ln(\mu_{\rm pole}/\mu_{\rm pert}) = 2 \qquad \mu_{\rm pole} \approx 7.4\mu_{\rm pert}$$

A simplified approach to double resonant triple Higgs production (LHC)

When the double resonant channel is dominant

$$\sigma(m_2, m_3, \Gamma_2, \Gamma_3, \kappa_3, \lambda_{123}, \lambda_{112}) = \hat{\sigma}_u(m_2, m_3) \times \frac{\kappa_3^2 \lambda_{123}^2 \lambda_{112}^2}{\Gamma_2 \Gamma_3} = \hat{\sigma}_u(m_2, m_3) \times \rho^2$$

"Unity" cross section

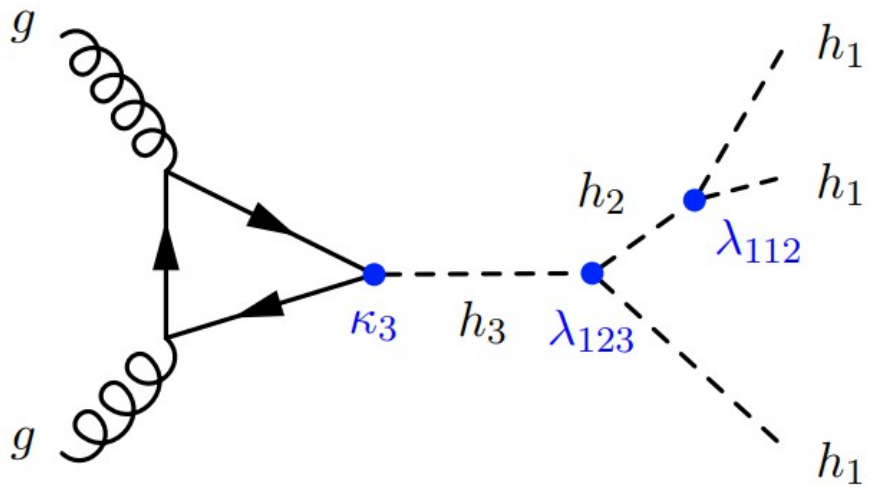
$$\kappa_3 = 1,\ \lambda_{123} = \lambda_{112} = 1\ \text{GeV}$$

$$\Gamma_2 = \Gamma_3 = 1\ \text{GeV}$$

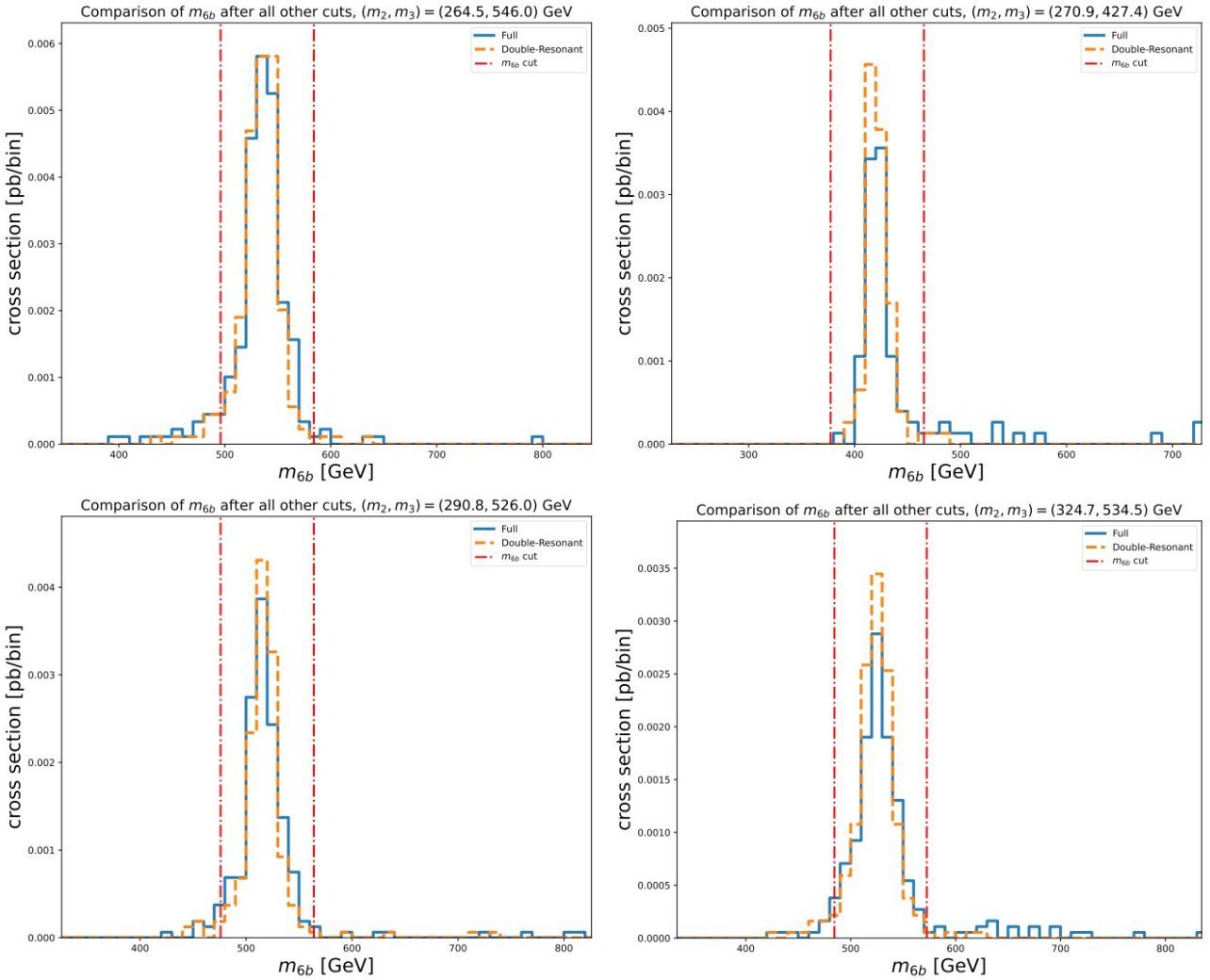
$$\Gamma_i \ll M_i$$

$$\rho^2 \equiv \kappa_3^2 \lambda_{123}^2 \lambda_{112}^2 / (\Gamma_2 \Gamma_3)\ .$$

Model dependent factor

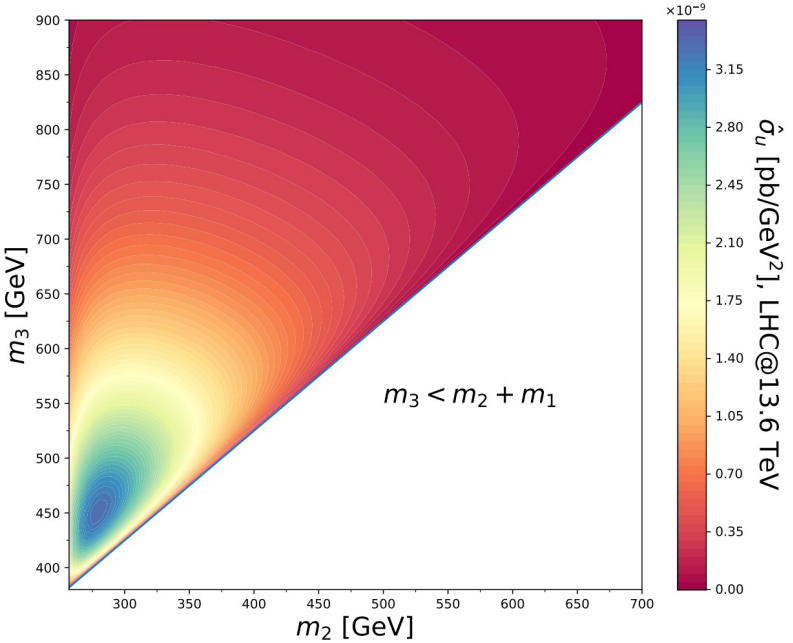


A simplified approach to double resonant triple Higgs production (LHC)



A. Papaefstathiou, GTX: 2501.14866

A simplified approach to double resonant triple Higgs production (LHC)



Define a “universal” set of selection cuts by optimizing

$$\Sigma_\Pi \equiv \prod_{i=1}^{N} \Sigma_i = \prod_{i=1}^{N} \frac{S_i}{\sqrt{B}} \Rightarrow \Sigma_\Pi = \left(\prod_{j=1}^{N} \frac{\sqrt{\mathcal{L}}\sigma_j}{\sqrt{\sigma_B}} \right) \left(\prod_{i=1}^{N} \frac{\varepsilon_i}{\sqrt{\varepsilon_{B,i}}} \right)$$

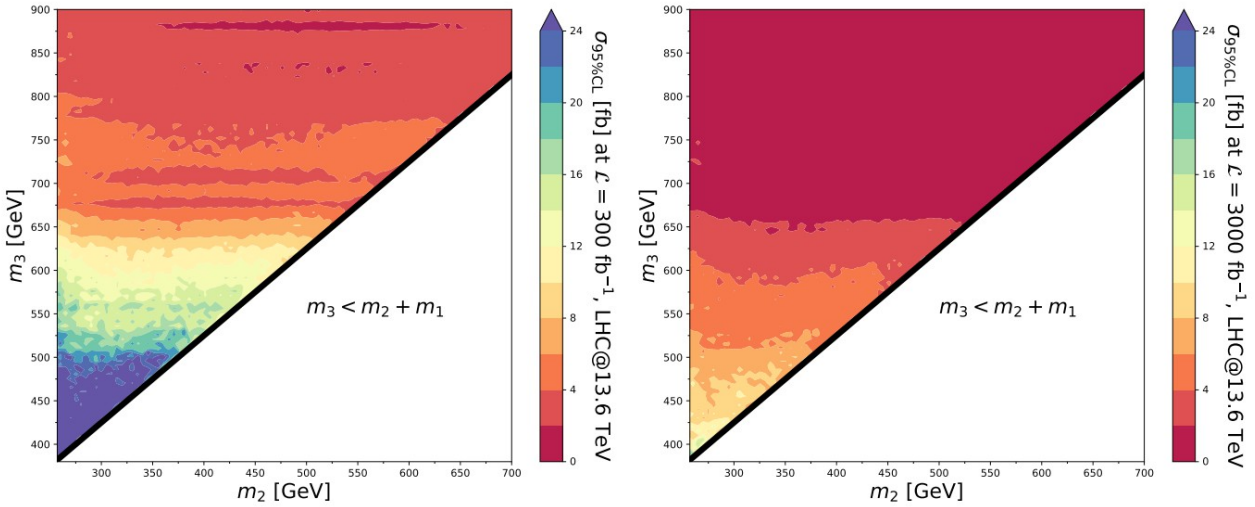
$$\Sigma = \frac{\varepsilon_S \sigma_S \mathcal{L}}{\sqrt{\varepsilon_B \sigma_B \mathcal{L}}} \quad \Sigma = 2 \Rightarrow \quad \sigma_{95\%\mathrm{CL}} = 2\frac{\sqrt{\varepsilon_B \sigma_B}}{\varepsilon_S \sqrt{\mathcal{L}}}$$

A. Papaefstathiou, GTX: 2501.14866

A simplified approach to double resonant triple Higgs production (LHC)

"Universal" set of selection cuts

Observable	Constraint
$p_{Tmin,b}$	37.0
$\lvert\eta_{b,max}\rvert$	2.95
$\chi^{2,(6)}$	12.0
$\chi^{2,(4)}$	34.0
Δm_{6b}^{inv}	$^{+38.0}_{-50.0}$
m_{4b}^{inv}	$\leqslant m_3$
$p_T(h_1^i)$	$\geqslant [50, 50, 0]$
$\Delta m_{\mathrm{min,med,max}}$	$\geqslant [15, 14, 20]$
$\Delta R(h_1^i, h_1^j)$	$\leqslant 3.5$
$\Delta R_{bb}(h_1)$	$\leqslant 3.5$



A. Papaefstathiou, GTX: 2501.14866

Closing Remarks

- The 6-b jets final state is a good candidate to search for $h_1h_1h_1$ *within and beyond the SM*

- *Extended scalar sectors can be probed through* $h_1h_1h_1$ *even in the HL-LHC (consider for instance the TRSM).*

- *We have presented projections on the potential values that future hadron colliders can explore on the cubic and quartic self couplings.*

- *The double resonance dominance allows to find a universal simplified approach which allows to get bounds on potential optimal selection points in a model independent way.*

Acknowledgements

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 945422

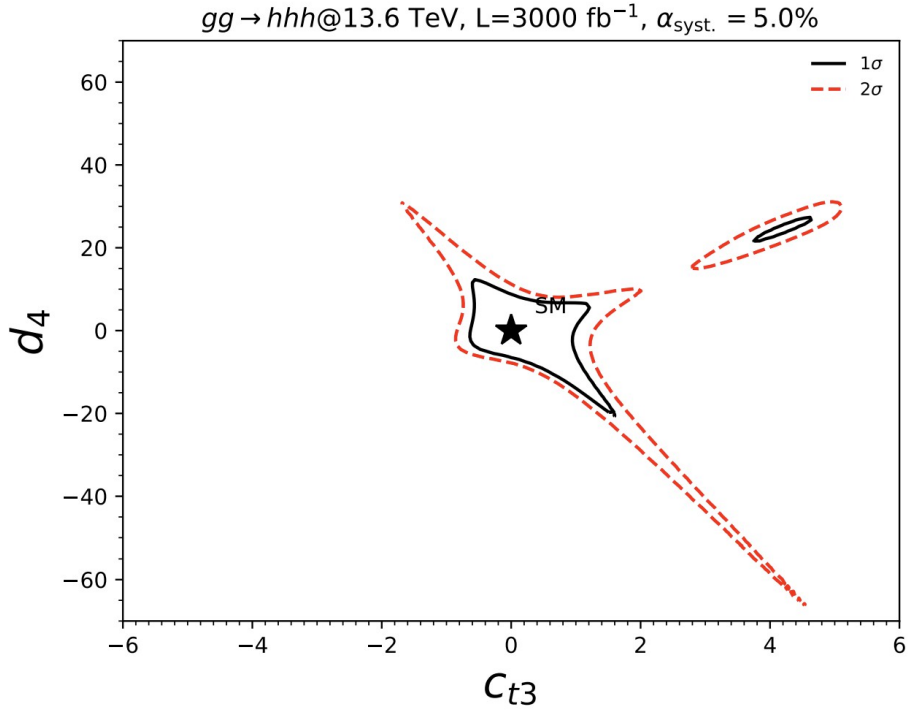
Backup

$$
\begin{aligned}
\mathcal{L}_{h^n} = & -\mu^2|H|^2 - \lambda|H|^4 - \left(y_t \bar{Q}_L H^c t_R + y_b \bar{Q}_L H b_R + \text{h.c.}\right) \\
& + \frac{c_H}{2\Lambda^2}(\partial^\mu |H|^2)^2 - \frac{c_6}{\Lambda^2}\lambda_{\text{SM}}|H|^6 + \frac{\alpha_s c_g}{4\pi\Lambda^2}|H|^2 G^a_{\mu\nu} G_a^{\mu\nu} \\
& - \left(\frac{c_t}{\Lambda^2} y_t |H|^2 \bar{Q}_L H^c t_R + \frac{c_b}{\Lambda^2} y_b |H|^2 \bar{Q}_L H b_R + \text{h.c.}\right),
\end{aligned}
$$

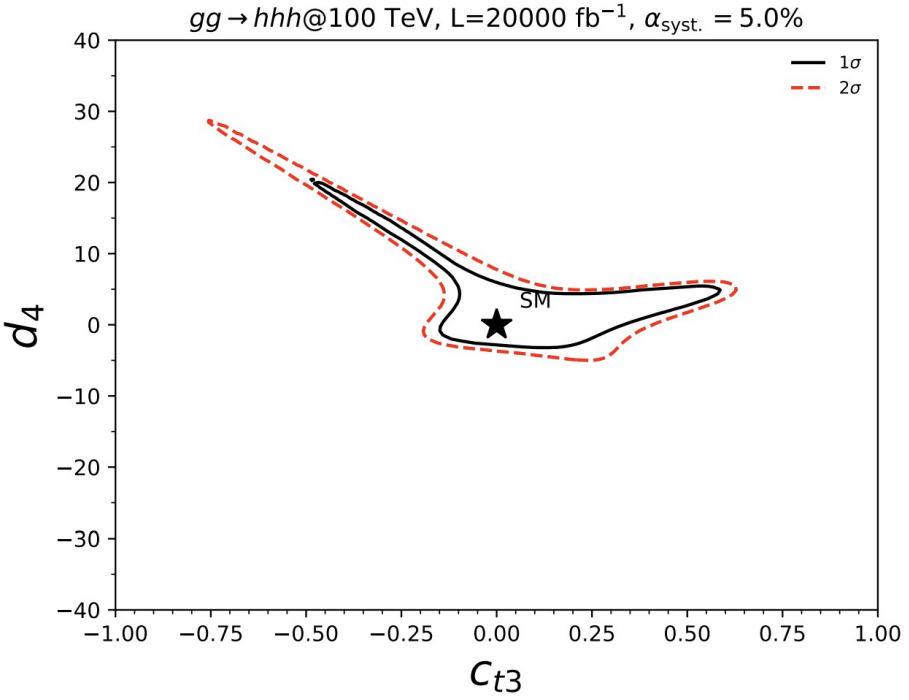
Anomalous couplings

Confidence regions on the anomalous couplings at proton-proton colliders

HL-LHC



FCC



In this plot it is assumed that the SM is the underlying theory

Adding an Extra-Scalar Singlet

The x-SM potential

$$V(\Phi, S) = \mu_\Phi^2 \Phi^\dagger \Phi + \lambda_\Phi (\Phi^\dagger \Phi)^2 + \left(\frac{a_1}{2}\right)(\Phi^\dagger \Phi) S$$

$$+ \left(\frac{a_2}{2}\right)(\Phi^\dagger \Phi) S^2 + \left(\frac{b_2}{2}\right) S^2 + \left(\frac{b_3}{3}\right) S^3 + \left(\frac{b_4}{4}\right) S^4$$

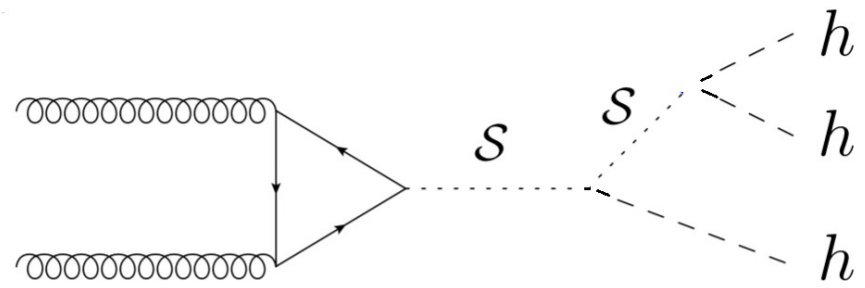
Kotwal et al. 1605.06123

Mass Eigenstates

$$h_1 = h \cos\theta + \phi_s \sin\theta$$

$$h_2 = -h \sin\theta + \phi_s \cos\theta$$

$$S = (\phi_S + v_S)/\sqrt{2}$$



Triple Higgs production in the presence of an extra-scalar

Analysis results

Benchmark points which lead to a Strong-First Order EW Phase Transition

Benchmark	$\cos\theta$	$\sin\theta$	m_2 (GeV)	Γ_{h_2} (GeV)	x_0 (GeV)	λ	a_1 (GeV)	a_2	b_3 (GeV)	b_4	$\frac{\sigma(h_1h_1)}{\sigma(hh)_{\rm SM}}$	$\frac{\sigma(h_1h_1h_1)}{\sigma(hhh)_{\rm SM}}$
B1max	0.976	0.220	341	2.42	257	0.92	-377	0.392	-403	0.77	22.44	60.55
B2max	0.982	0.188	353	2.17	265	0.99	-400	0.446	-378	0.69	22.43	56.69
B3max	0.983	0.181	415	1.59	54.6	0.17	-642	3.80	-214	0.16	6.43	3.01
B4max	0.984	0.176	455	2.08	47.4	0.18	-707	4.63	-607	0.85	5.19	3.37
B5max	0.986	0.164	511	2.44	40.7	0.18	-744	5.17	-618	0.82	3.49	2.94
B6max	0.988	0.153	563	2.92	40.5	0.19	-844	5.85	-151	0.083	2.79	3.60
B7max	0.992	0.129	604	2.82	36.4	0.18	-898	7.36	-424	0.28	2.51	4.70
B8max	0.994	0.113	662	2.97	32.9	0.17	-976	8.98	-542	0.53	2.28	4.91
B9max	0.993	0.115	714	3.27	29.2	0.18	-941	8.28	497	0.38	1.98	2.68
B10max	0.996	0.094	767	2.83	24.5	0.17	-920	9.87	575	0.41	1.95	2.35
B11max	0.994	0.105	840	4.03	21.7	0.19	-988	9.22	356	0.83	1.76	1.03

Identification of the Extra-scalar at *100 TeV*

Benchmark	Significance
B1max	46.6
B2max	42.9
B3max	2.9
B4max	3.7
B5max	3.0
B6max	3.8
B7max	5.3
B8max	7.8
B9max	5.9
B10max	4.9
B11max	2.3