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ggxy - A library library for loop induced $gg \to XY$ processes

Higg Working Group Meeting (CERN), 2025

Kay Schönwald — in collaboration with Joshua Davies, Matthias Steinhauser, and Daniel Stremmer
CERN (Geneva), December 4, 2025

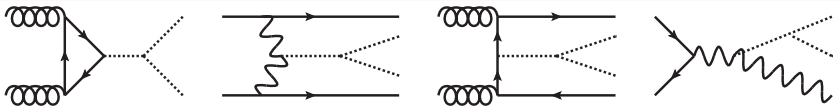
CERN
Based on: [Comput. Phys. Commun. 320 (2026) 109933]

Higgs Self Coupling

- Standard Model Higgs potential:

$$V(H) = \frac{1}{2} m_H^2 H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4, \quad \text{where } \lambda = m_H^2/(2v^2) \approx 0.13.$$

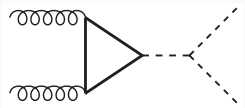
- Want to measure λ, to determine if $V(H)$ is consistent with nature.
  - Challenging! Cross-section $\approx 10^{-3} \times H$ prod.
  - $-1.24 < \lambda/\lambda_{SM} < 6.49$ [CMS '22] ; $-0.6 < \lambda/\lambda_{SM} < 6.6$ [Atlas '22]
- λ appears in various production channels:

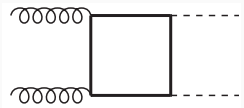


- Gluon fusion – dominant, 10x
- VBF
- $t\bar{t}$ associated production
- H-strahlung

Gluon Fusion

- Leading order (1 loop) partonic amplitude:





$$\mathcal{M}^{\mu\nu} \sim \mathcal{A}_1^{\mu\nu}(\mathcal{F}_{tri} + \mathcal{F}_{box1}) + \mathcal{A}_2^{\mu\nu}(\mathcal{F}_{box2})$$

  - $\mathcal{F}_{tri}$ contains the dependence on λ at LO

- Form factors:
  - LO: known exactly [Glover, van der Bij '88]
  - Beyond LO... no fully-exact (analytic) results to date
    - **QCD:** numerical evaluation, expansion in various kinematic limits
    - **EW:** heavy top expansion, high-energy expansion (subsets) [Davies, Mishima, Schönwald, Steinhauser, Zhang '22-'25]
    - see also Yuakwa corrections in (partial) HTL [Mühlleitner,Schlenk,Spira '22]
    - full (numerical) EW corrections [Bi, Huang, Huang, Ma, Yu '23]
    - numerical Yukawa- and Higgs self-coupling corrections [Heinrich, Jones, Kerner, Stone, Vestner '24]
    - light quark corrections [Bonetti, Rendler, Bobadilla '25]

$gg \to HH$ Beyond LO

NLO QCD:

- large-m_t [Dawson, Dittmaier, Spira '98] [Grigo, Hoff, Melnikov, Steinhauser '13]
- numeric [Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke '16] [Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher '19]
- large-m_t + threshold exp. Padé [Gröber, Maier, Rauh '17]
- high-energy expansion [Davies, Mishima, Steinhauser, Wellmann '18,'19]
- small-p_T expansion + high-energy expansion [Bonciani, Degrassi, Giardino, Gröber '18] [Bagnaschi, Degrassi, Gröber '23]

NNLO QCD:

- large-m_t virtuals [de Florian, Mazzitelli '13] [Grigo, Hoff, Steinhauser '15, Davies; Steinhauser '19]
- HTL+numeric real ("FTapprox") [Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli '18]
- large-m_t reals [Davies, Herren, Mishima, Steinhauser '19 '21]
- $p_T = 0$ (n_l and leading color n_h contributions) [Davies, Schönwald, Steinhauser '23-'25]

N3LO QCD:

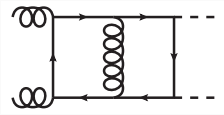
- Wilson coefficient C_{HH} [Spira '16; Gerlach, Herren, Steinhauser '18]
- HTL [Chen, Li, Shao, Wang '19]

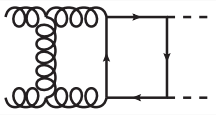
Small t expansion

Consider the hierarchy of scales: $\boxed{s, m_t^2 \gg |t|, m_H^2}$

- Form factors written in term of Feynman integrals: $I(m_H, m_t, s, t, \epsilon)$
- Taylor expand in $m_H^2 \to 0$: $I(m_H, ...) = I(0, ...) + m_H^2 I'(0, ...)$
- IBP reduction to master integrals: $J(m_t, s, t, \epsilon)$
- Expand master integrals in $t \to 0$:

$$\boxed{J(m_t, s, t, \epsilon) = \sum_{i,j} c_{ij}(s, m_t^2) \epsilon^i t^j}$$





High energy expansion

Consider the hierarchy of scales: $s, |t| \gg m_t^2 \gg m_H^2$

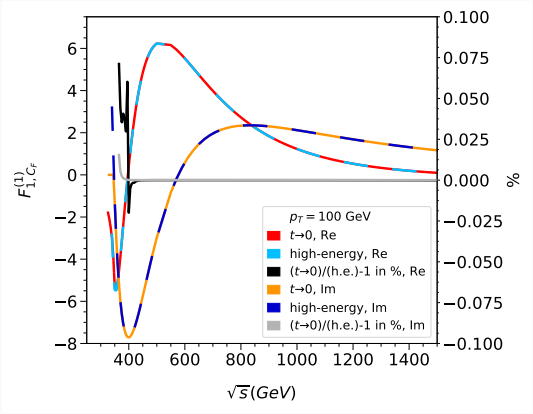
- Form factors written in term of Feynman integrals: $I(m_H, m_t, s, t, \epsilon)$
- Taylor expand in $m_H^2 \to 0$: $I(m_H, ...) = I(0, ...) + m_H^2 I'(0, ...)$
- IBP reduction to master integrals: $J(m_t, s, t, \epsilon)$
- Expand master integrals in $m_t \to 0$:

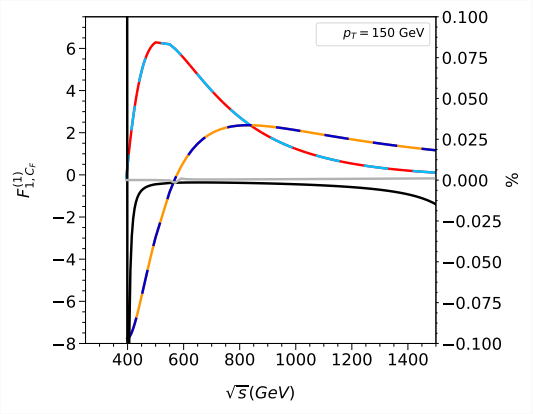
$$J(m_t, s, t, \epsilon) = \sum_{i,j,k} c_{ijk}(s, t)\epsilon^i (m_t)^j \ln^k(m_t)$$

- Improve convergence with Padé approximation:

$$\sum_{k=0}^{N} c_k (m_t^2)^k = \frac{a_0 + a_1(m_t^2) + ... + a_r(m_t^2)^r}{1 + b_1(m_t^2) + ... + b_s(m_t^2)^r} = [r/s](m_t^2) \ , \quad r + s = N$$

Combination of expansions





- Convergence over the whole phase space with **small-t** and **high-energy expansion** with Padé approximation.

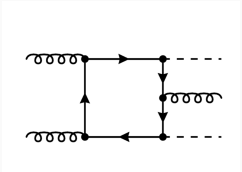
Real corrections

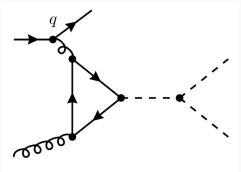
- New partonic processes: $gg \to gHH$, $qg \to qHH$, $q\bar{q} \to gHH$
- IR singularities handled with **Catani-Seymour** subtraction scheme. [Catani, Seymour '97]
- Calculation of one-loop matrix elements with:

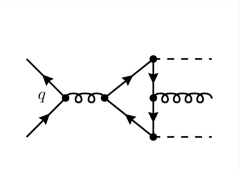
  `Recola` [Actis, Denner, Hofer, Lang, Scharf, Uccirati '17]

  `Collier` [Denner, Dittmaier, Hofer '17]

  - Implemented alternative reduction to scalar integrals for exceptional phase-space points with `CutTools` [Ossola, Papadopoulos, Pittau '09] and `OneLOop` [van Hameren '11] .







ggxy: [https://gitlab.com/ggxy/ggxy-release]

- Public c++ library for the computation of LO and NLO amplitudes and cross sections for gluon initiated top-quark loop induced processes.
- Contains all necessary functions for $gg \to HH$ at NLO QCD.
- Analytic expansions allow to **modify** m_t, m_H and top-quark mass renormalization scheme (OS,$\overline{\text{MS}}$).
- Supports variation of $\boxed{\kappa_\lambda = \lambda_3/\lambda_3^{\text{SM}}}$.

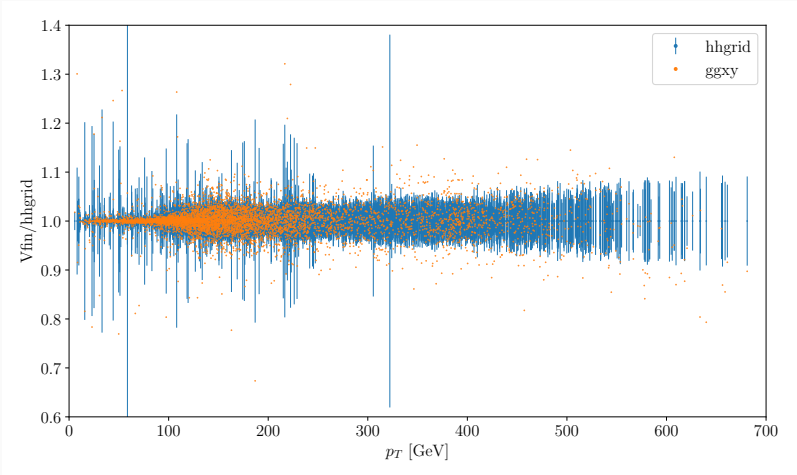
Installation:

- External dependencies: `Boost`, `CMake`, `Eigen`
- Path to `LHAPDF` installation needed for hadronic cross sections.
- The following libraries are directly installed and linked to ggxy:

- `Collier` [Denner, Dittmaier, Hofer '17]
- `CutTools` [Ossola, Papadopoulos, Pittau '09]
- `OneLOop` [van Hameren '11]
- `Recola` [Actis, Denner, Hofer, Lang, Scharf, Uccirati '17]
- `avhlib/Kaleu` [van Hameren '10]
- `CRunDec` [Herren, Steinhauser '18]
- `lievaluate` [Frellesvig, Tommasini, Wever '16]

Example: check-hhgrid.cpp

- Compare to 2-loop **finite remainder** $\mathcal{V}_{\text{fin}}$ with hhgrid [https://github.com/mppmu/hhgrid]
  - Interpolation grid of 6320 phase-space points, combination of numerical evaluations with pySecDec and (lower-order) high-energy expansion
- **ggxy** reproduces results in less than 10 seconds on a single core
  - Sufficiently fast for a direct use in MC tools (with increased flexibility)



Example: nlo-gghh.cpp

[Browka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke '16]

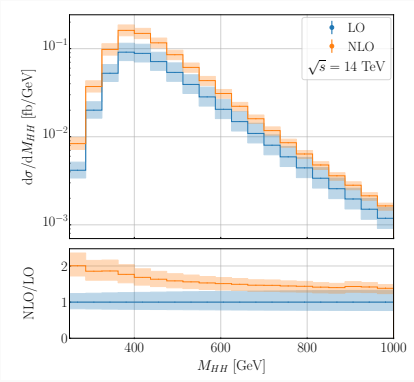
$\sqrt{s}$		ggxy	Ref.
14 TeV	σ^{LO} [fb]	$19.848(4)^{+27.6\%}_{-20.5\%}$	$19.85^{+27.6\%}_{-20.5\%}$
	σ^{NLO} [fb]	$32.92(2)^{+13.6\%}_{-12.6\%}$	$32.91^{+13.6\%}_{-12.6\%}$
100 TeV	σ^{LO} [fb]	$731.2(2)^{+20.9\%}_{-15.9\%}$	$731.3^{+20.9\%}_{-15.9\%}$
	σ^{NLO} [fb]	$1150(1)^{+10.8\%}_{-10.0\%}$	$1149^{+10.8\%}_{-10.0\%}$

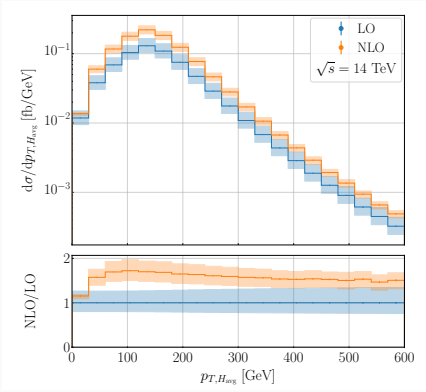
[Baglio Campanario, Glaus, Mühlleitner, Spira, Streicher '20]

$\sqrt{s}$		ggxy	Ref.
13 TeV	σ^{NLO} [fb]	27.72(2)	27.73(7)
14 TeV	σ^{NLO} [fb]	32.79(2)	32.81(7)

- Reproduces LO/NLO hadronic cross sections from the literature together with scale uncertainties from 7-point scale variation.
- Fast runtime: 30 minutes on a single core for a precision of $0.1\% - 0.2\%$

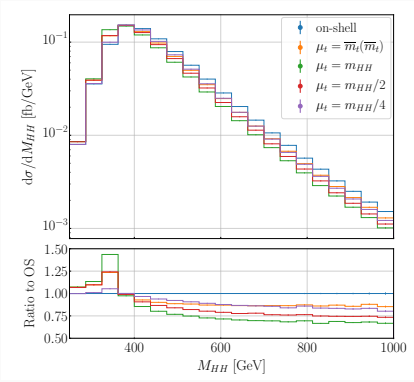
Example: nlo-gghh.cpp

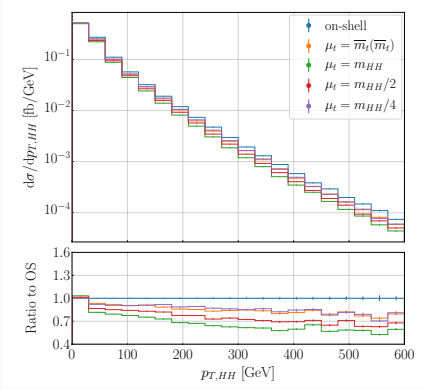




- Differential cross sections with scale uncertainties also produced by example program.
- Renormalization scheme of the top flexible.
- Runtime depends highly on the requested precision.

Example: nlo-gghh.cpp

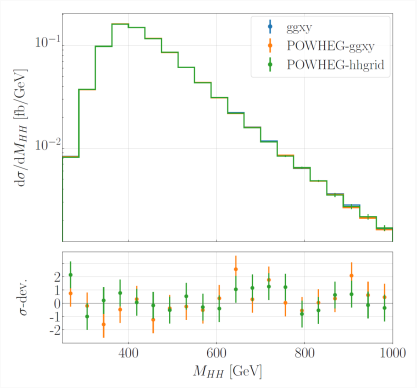


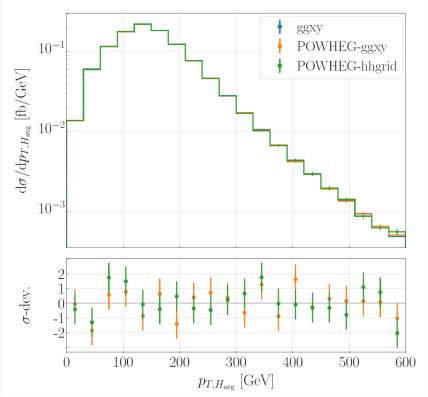


- Differential cross sections with scale uncertainties also produced by example program.
- Renormalization scheme of the top flexible.
- Runtime depends highly on the requested precision.

Interfaces

- $gg \to HH$ implementation in POWHEG-BOX V2 [Aliolo, Nason, Leari, Re '10] with **ggxy**.
  [https://gitlab.com/POWHEG-BOX/V2/User-Processes/ggxy_ggHH]
  - Supports same features as **ggxy**: Variation of m_t, m_H and κ_λ
  - Fast evaluation (faster by a factor **4-5** compared to previous implementation of $gg \to HH$).
- We can also provide an interface to `Herwig`.



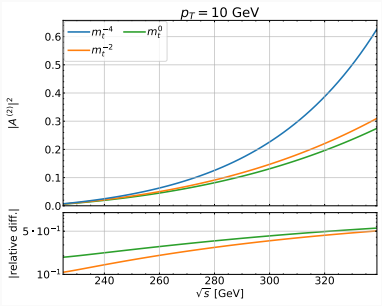


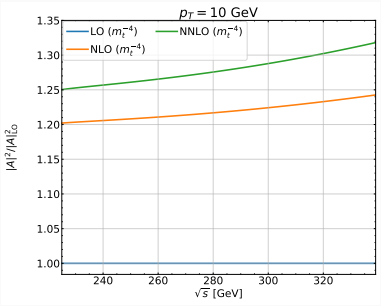
Next processes in ggxy

Already implemented:

- large-m_t expansion of $gg \to ZH$ (virtual corrections) up to **3-loop**.

[Davies, Grau, Schönwald, Steinhauser, Stremmer '25]





Next steps:

- 2-loop top-quark mediated corrections to **(off-shell)** $\boxed{gg \to ZZ}$ and $\boxed{gg \to ZH}$.

[Davies, Grau, Schönwald, Steinhauser, Stremmer, Vitti '25]

$\Rightarrow$ **See talk by Dominik Grau on Friday!**

Conclusions

- New public c++ library **ggxy**:
  - All relevant functions for $gg \to HH$ at NLO QCD provided.
  - 2-loop amplitude calculated with analytic expansions.
  - Input parameters (m_t,m_H,κ_λ) and top-quark renormalization scheme (OS,$\overline{\text{MS}}$) can be chosen freely.
- Several cross-checks with previous results in the literature.
- Can be used directly to compute hadronic cross sections or matrix element provider for MC programs like Powheg, Herwig,... .
- Fast runtime: $\sim$30 minutes for per mille precision

Outlook

- Implementation fo new processes into **ggxy**:
  - $gg \to ZH$
  - $gg \to ZZ$ ($gg \to Z\gamma$ and $gg \to \gamma\gamma$)
- Include electroweak corrections.