Karlsruhe Institute of Technology

Institute for Theoretical Physics

Higgs-Pair Production via Gluon Fusion: Top-Yukawa- and Light-Quark-Induced Electroweak Corrections

with A. Bhattacharya, F. Campanario, J. Chang, J. Mazzitelli, M. M. Mühlleitner, J. Ronca and M. Spira

Sauro Carlotti, ITP - KIT | The 22nd Workshop of the LHC Higgs Working Group, Dec. 04, 2025

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2. Top-Yukawa-Induced Corrections: Computation

3. Results

4. Summary and Conclusions

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Motivation

The Standard Model (SM) has been tested to highest accuracy, but there are open questions that call for new physics.
There are different ways to search for new physics:

- **Direct searches** of new phenomema, e.g. new particles
- **Indirect effects** by computing observables at high precision

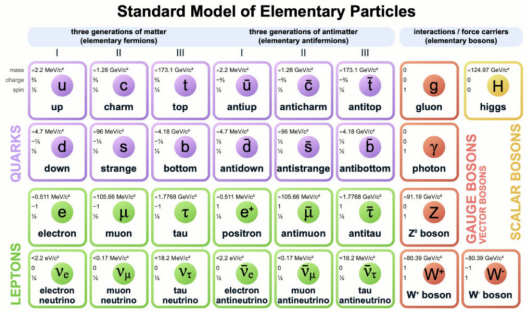

Figure: [*Wikimedia*]

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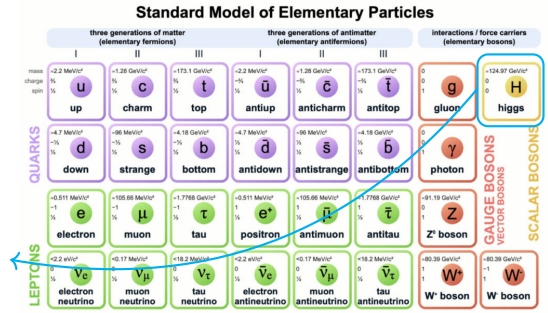

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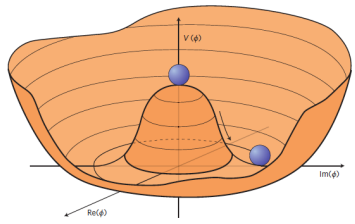
Scalar Higgs Potential in the Standard Model

After the electroweak symmetry breaking (EWSB) the Higgs potential in the SM becomes:

$$V_{\text{SM}} = \frac{1}{2} m_H^2 H^2 + \frac{1}{3!} \lambda_{HHH} H^3 + \frac{1}{4!} \lambda_{HHHH} H^4$$

Figure: *[J.Ellis '13]*



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- Measurement of the Higgs boson mass *[ATLAS, CMS '12]*

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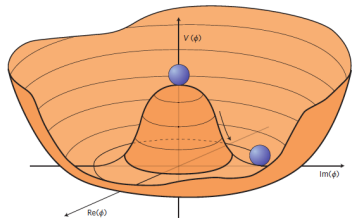
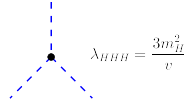


$$m_H = \sqrt{2\lambda} v$$



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- Test the shape of the Higgs potential

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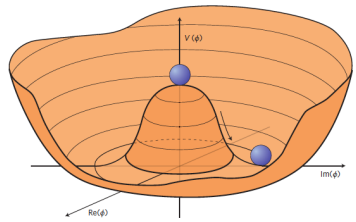

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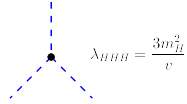

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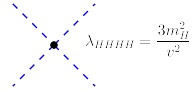

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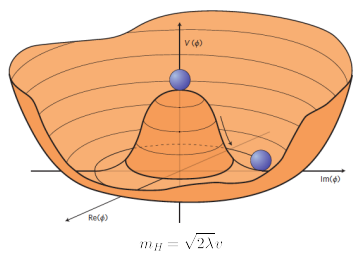

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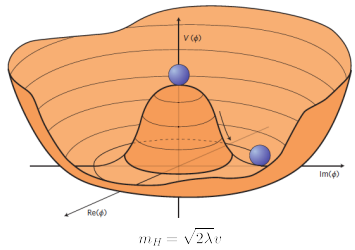
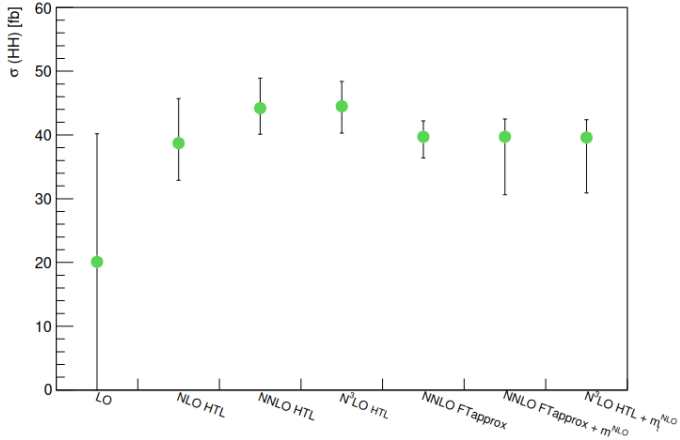
$-1.2 < \frac{\lambda_{obs}}{\lambda} < 7.2, \; -1.4 < \frac{\lambda_{obs}}{\lambda} < 6.4$ *[ATLAS, CMS, '24]*

Figure: *[J.Ellis '13]*

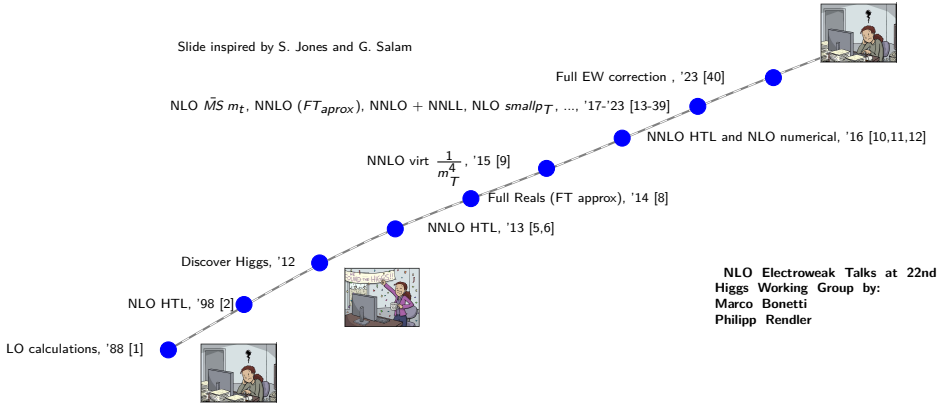


Evolution of the Cross Section over the Years



[M. Spira]

Evolution of Theoretical Calculations for $gg \to HH$



NLO Electroweak Talks at 22nd Workshop of the LHC Higgs Working Group by:
Marco Bonetti
Philipp Rendler

[1] Glover, van der Bij 88; [2] Dawson, Dittmaier, Spira 98; [3] Shao, Li, Li, Wang 13; [4] Grigo, Hoff, Melnikov, Steinhauser 13; [5] de Florian, Mazzitelli 13; [6] Grigo, Melnikov, Steinhauser 14; [7] Grigo, Hoff 14; [8] Maltoni, Vryonidou, Zaro 14; [9] Grigo, Hoff, Steinhauser 15; [10] de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; [11] Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke 16; [12] Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Zirke 16; [13] Ferrera, Pires 16; [14] Heinrich, Jones, Kerner, Luisoni, Vryonidou 17; [15] Jones, Kuttimalai 17; [16] Gröber, Maier, Rauh 17; [17] Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18; [18] Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli 18; [19] de Florian, Mazzitelli 18; [20] Bonciani, Degrassi, Giardino, Gröber 18; [21] Davies, Mishima, Steinhauser, Wellmann 18, 18; [22] Mishima 18; [23] Gröber, Maier, Rauh 19; [24] Davies, Heinrich, Jones, Kerner, Mishima, Steinhauser, David Wellmann 19; [25] Davies, Steinhauser 19; [26] Chen, Li, Shao, Wang 19, 19; [27] Davies, Herren, Mishima, Steinhauser 19, 21; [28] Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 21; [29] Bellafronte, Degrassi, Giardino, Gröber, Vitti 22; [30] Davies, Mishima, Schönwald, Steinhauser, Zhang 22; [31] Davies, Mishima, Schönwald, Steinhauser 23; [32] Davies, Schönwald, Steinhauser 23; [33] Davies, Schönwald, Steinhauser, Zhang 23; [34] Bagnaschi, Degrassi, Gröber 23; [35] Bi, Huang, Huang, Ma Yu 23; [36] Li, Si, Wang, Zhang, Zhao 24; [37] Davies, Schönwald, Steinhauser, Vitti 24; [38] Heinrich, Jones, Kerner, Stone, Vestner: [39] Li, Si, Wang, Zhang, Zhao 24;[40] Bi, Huang, Huang, Ma, Yu.; [41] M.Bonetti, P. Rendler, W. J. Torres., [42] J. Davies, K. Schönwald, M. Steinhauser, D. Stremmer

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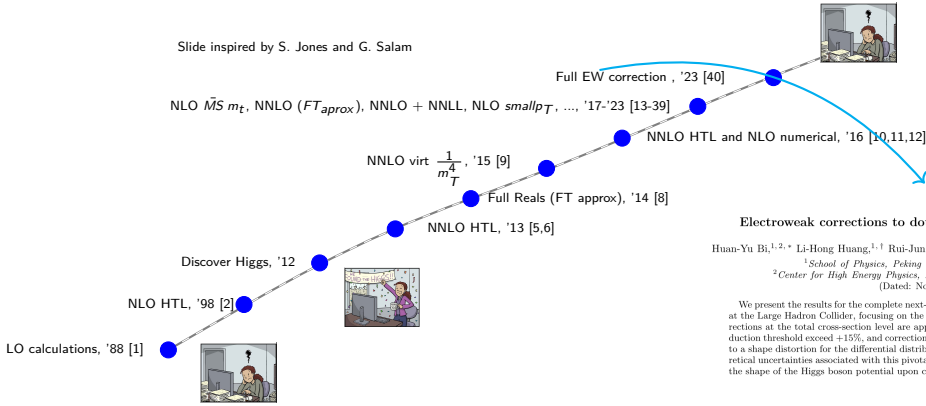

Electroweak corrections to double Higgs production at the LHC

Huan-Yu Bi,[1,2,*] Li-Hong Huang,[1,†] Rui-Jun Huang,[1,‡] Yan-Qing Ma,[1,2,§] and Huai-Min Yu[1,¶]

[1]*School of Physics, Peking University, Beijing 100871, China*
[2]*Center for High Energy Physics, Peking University, Beijing 100871, China*

(Dated: November 29, 2023)

We present the results for the complete next-to-leading order electroweak corrections to $pp \to HH$ at the Large Hadron Collider, focusing on the dominant gluon-gluon fusion process. While the corrections at the total cross-section level are approximately -4%, those near the energy of HH production threshold exceed $+15\%$, and corrections at the high-energy region are around -10%, leading to a shape distortion for the differential distributions. Our findings substantially diminish the theoretical uncertainties associated with this pivotal process, providing valuable input for understanding the shape of the Higgs boson potential upon comparison with experimental measurements.

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$gg \to HH$ is a loop-induced process where we have two types of diagrams

Sizeable contributions of the EW corrections come from the top-Yukawa coupling

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Our framework

- Gaugeless limit $\to$ presence of the Goldstone bosons

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- Massive top and bottom quarks, light quarks massless
- Only top-Yukawa-induced corrections + light quark loops

Top-Yukawa-Induced Corrections

Matrix element split into tensor structures

$$\mathcal{M}^{\mu\nu} = F_1 T_1^{\mu\nu} + F_2 T_2^{\mu\nu}$$

The cross section can be written as

$$\mathrm{d}\sigma \sim \; Re\left[\left(C_\triangle F_{1\triangle} + F_{1\square}\right)^* \left(C_\triangle F_{1\triangle} + F_{1\square}\right) + F_{2\square}^* F_{2\square}\right]$$

Since only virtual corrections are involved, the corrections can be added by shifting the LO terms

$$\begin{aligned} C_\triangle F_{1\triangle} &\to C_\triangle F_{1\triangle}(1 + \delta_1 + \Delta_{HHH}) \\ F_{1\square} &\to F_{1\square}\left(1 + \Delta_{1\square}\right) \\ F_{2\square} &\to F_{2\square}\left(1 + \Delta_{2\square}\right) \end{aligned}$$

$\to$ Expand $\mathrm{d}\sigma$ properly to next-to-leading order (NLO)

$$T_1^{\mu\nu} = g^{\mu\nu} - \frac{p_1^\nu p_2^\mu}{p_1 \cdot p_2} \quad \text{with } p_T = \sqrt{\frac{tu - m_H^4}{s}}$$

$$T_1^{\mu\nu} = g^{\mu\nu} + \frac{m_H^2 p_1^\nu p_2^\mu}{p_T^2 p_1 \cdot p_2} - \frac{2p_1 \cdot p_3 p_3^\nu p_2^\mu}{p_T^2 p_1 \cdot p_2} - \frac{2p_2 \cdot p_3 p_1^\nu p_3^\mu}{p_T^2 p_1 \cdot p_2} + \frac{2p_3^\nu p_3^\mu}{p_T^2}$$

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Contributions computed

$$C_\triangle F_{1\triangle} \to C_\triangle F_{1\triangle}(1 + \delta_1 + \Delta_{HHH})$$
$$F_{1\square} \to F_{1\square}\,(1 + \Delta_{1\square})$$
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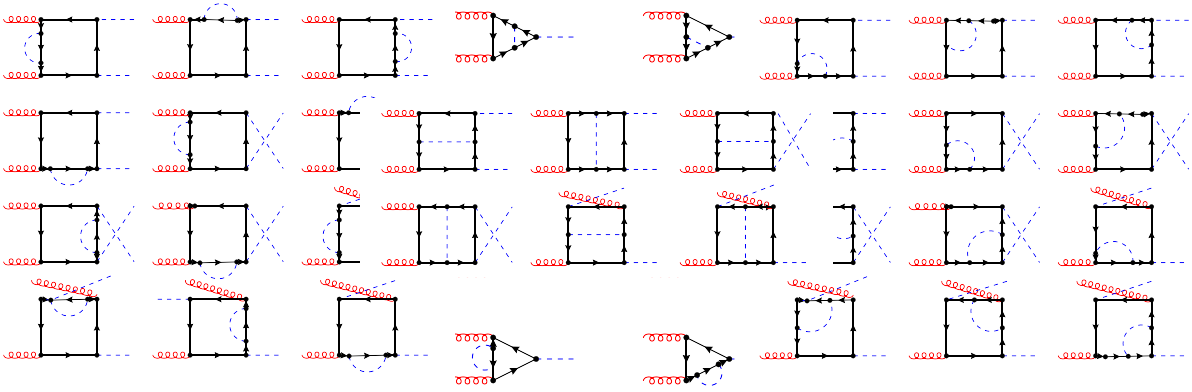

Workflow

- Identify all the Feynman diagrams that contribute to the process for the considered correction

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- Apply Feynman parametrization to all the diagrams

$$\frac{1}{a_1^{\alpha_1} a_2^{\alpha_2} ... a_n^{\alpha_n}} = \frac{\Gamma(\alpha_1 + ... + \alpha_n)}{\Gamma(\alpha_1) ... \Gamma(\alpha_n)} \int_0^1 dx_1 ... \int_0^{1-x_1-...-x_{n-2}} dx_{n-1}$$
$$\frac{(1 - x_1 - ... x_{n-1})^{\alpha_1 - 1} x_1^{\alpha_2 - 1} ... x_{n-1}^{\alpha_n - 1}}{(a_1(1 - x_1 - ... - x_{n-1}) + a_2 x_1 + ... a_n x_{n-1})^{\sum \alpha_i}}$$

$\rightarrow$ up to $(n-1)$-dimensional integral

Workflow

- Identify all the Feynman diagrams that contribute to the process for the considered correction
- Apply Feynman parametrization to all the diagrams
- Small mass regulator for the internal particles + endpoint subtraction → isolate the UV singularities

$$m^2 \to m^2 \left(1 - i\bar{\epsilon}\right)$$

$$\int_0^1 dx \frac{f(x)}{(1-x)^{1-\epsilon}} = \frac{f(1)}{\epsilon} + \int_0^1 dx \frac{f(x) - f(1)}{1-x} + \mathcal{O}(\epsilon)$$

Workflow

- Identify all the Feynman diagrams that contribute to the process for the considered correction
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- Small mass regulator for the internal particles + endpoint subtraction $\rightarrow$ isolate the UV singularities
- Instability? $\rightarrow$ Integration by parts

$$\int_0^1 dx \frac{f(x)}{(a+bx)^3} = \frac{f(0)}{2a^2 b} - \frac{f(1)}{2b\,(a+b)^2} + \int_0^1 \frac{dx}{2b} \frac{f'(x)}{(a+bx)^2}$$

Workflow

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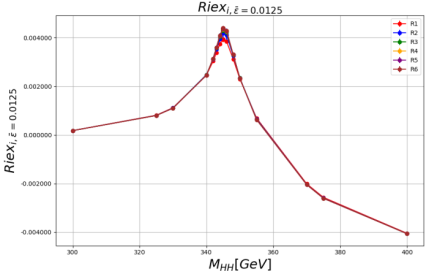
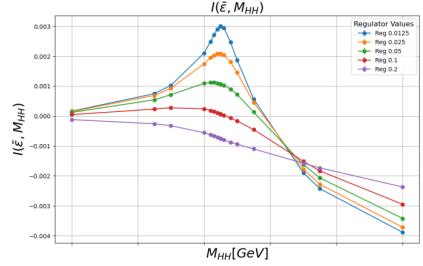
- Instability? $\rightarrow$ Integration by parts
- Richardson extrapolation to do the narrow-width limit $\bar{\epsilon} \rightarrow 0$





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... and if is it not enough? $\rightarrow$ choice of a new parametrization

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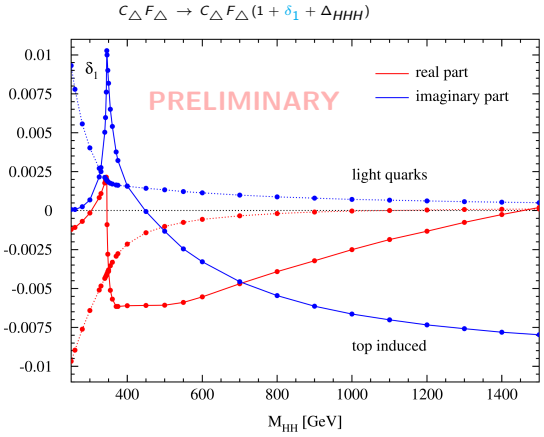
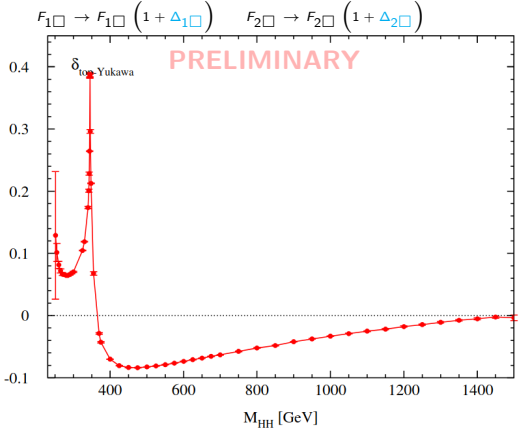
Triangle-like Corrections

- Top-Yukawa-induced corrections to $\mathcal{M}$ as a function of the invariant Higgs-pair mass M_{HH}
- Solid lines: Top-Yukawa-induced corrections
- Dashed lines: light quark loops contributions
- Large contribution around $t\bar{t}$ threshold



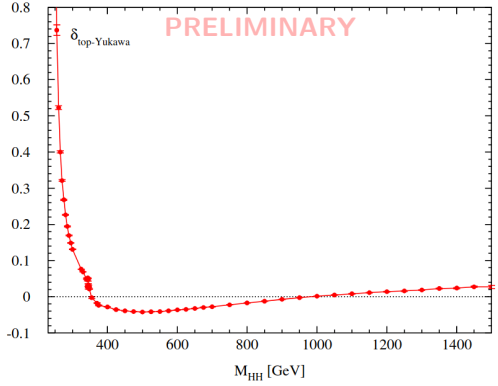
Box-like Corrections

- Pure two loops Top-Yukawa-induced corrections to σ as a function of the invariant Higgs-pair mass M_{HH}
- $\delta_{top-Yukawa}$: corrections proportional to $\Delta_{1\Box}$, $\Delta_{2\Box}$, normalized to σ_{LO}
- Large contributions around the $t\bar{t}$ threshold
- Few percent corrections at the high energy tail



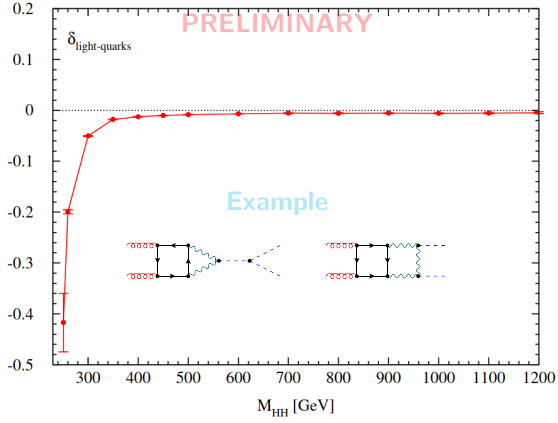
Full Top-Yukawa-induced corrections

- Full Top-Yukawa-induced corrections to σ as a function of the invariant Higgs-pair mass M_{HH}, normalized to σ_{LO}
- Large contributions around the $t\bar{t}$ threshold and below (Δ_{HHH})



Light Quark Loops Corrections

- Light quark loops corrections to σ as a function of the invariant Higgs-pair mass M_{HH}, normalized to σ_{LO}
- Sizeable contributions below $M_{HH} = 400$ GeV
- Small contributions at high energies



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Thank You for your Attention!

BACKUP

Projectors

Matrix element can be written as:

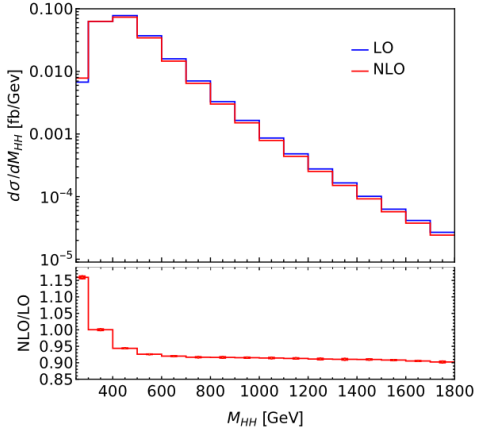
$$M^{\mu\nu} = F_1 T_1^{\mu\nu} + F_2 T_2^{\mu\nu}$$

The Form factors can be obtained via the projectors

$$\begin{aligned} P_i^{\mu\nu} T_{j,\mu\nu} =& \delta_{ij} \\ T_1^{\mu\nu} =& g_{\mu\nu} - \frac{p_1^\nu p_2^\mu}{p_1 \cdot p_2} \\ T_1^{\mu\nu} =& g_{\mu\nu} + \frac{m_H^2 p_1^\nu p_2^\mu}{p_T^2 p_1 \cdot p_2} - \frac{2 p_1 \cdot p_3 p_3^\nu p_2^\mu}{p_T^2 p_1 \cdot p_2} - \frac{2 p_2 \cdot p_3 p_1^\nu p_3^\mu}{p_T^2 p_1 \cdot p_2} + \frac{2 p_3^\nu p_3^\mu}{p_T^2} \end{aligned}$$

with $p_T = \sqrt{\frac{tu - m_H^4}{s}}$

Full EW cross section



Bi, Huang, Huang, Ma, Yu 24.

Integration by parts

$$\int_0^1 dx \frac{f(x)}{N^3(x)} \overset{1'IBP}{=} -\frac{1}{2b}\frac{f(x)}{N^2(x)}|_0^1 + \frac{1}{2b}\int_0^1 dx \frac{f'(x)}{N^2(x)} \overset{2'IBP}{=} \frac{1}{2b}\left[-\frac{f(x)}{N^2(x)}|_0^1 + \frac{1}{b}\left(\frac{f'(x)}{N(x)}|_0^1 + \int_0^1 dx \frac{f''(x)}{N^2(x)}\right)\right]$$

Richardson Extrapolation

Improve accuracy by combining approximations with different step sizes.

1. Compute the approximation S_h for step size h.
2. Compute the approximation S_{2h} for step size $2h$.
3. Extrapolated solution:
$$S_{\text{extr}} = \frac{2S_h - S_{2h}}{1}$$
4. First four polynomials of Richardson extrapolation:
$$S_h = p_0 + p_1 h + p_2 h^2 + p_3 h^3 + \cdots$$
$$S_{2h} = p_0 + p_1(2h) + p_2(2h)^2 + p_3(2h)^3 + \cdots$$
5. Applying Richardson's method, we obtain:
$$S_{\text{extr}} = \frac{2(p_0 + p_1 h + p_2 h^2 + p_3 h^3) - (p_0 + p_1(2h) + p_2(2h)^2 + p_3(2h)^3)}{1}$$

Useful relations for loop integrals

$$\int \frac{d^D k}{(2\pi)^D} \frac{1}{(k^2 - M^2 + i\epsilon)^N} = i \frac{(-1)^N}{(4\pi)^{(\frac{D}{2})}} \frac{\Gamma(N - \frac{D}{2})}{\Gamma(N)} \frac{1}{(M^2 - i\epsilon)^{(N - \frac{D}{2})}}$$

$$\int \frac{d^D k}{(2\pi)^D} \frac{k^2}{(k^2 - M^2 + i\epsilon)^N} = i \frac{1}{2} \frac{(-1)^{N-1}}{(4\pi)^{(\frac{D}{2})}} \frac{\Gamma(N - 1 - \frac{D}{2})}{\Gamma(N)} \frac{1}{(M^2 - i\epsilon)^{(N - 1 - \frac{D}{2})}}$$

$$\int \frac{d^D k}{(2\pi)^D} \frac{k_\mu k_\nu}{(k^2 - M^2 + i\epsilon)^N} = i \frac{1}{2} \frac{(-1)^{N-1}}{(4\pi)^{(\frac{D}{2})}} \frac{\Gamma(N - 1 - \frac{D}{2})}{\Gamma(N)} \frac{g_{\mu\nu}}{(M^2 - i\epsilon)^{(N - 1 - \frac{D}{2})}}$$

Higgs Potential in the Gaugeless Limit

$$\mathcal{V}(\phi) = -\mu|\phi|^2 + \frac{\lambda}{2}|\phi|^4 = -\frac{m_H^2}{8}v^2 + \frac{m_H^2}{2}H^2 + \frac{m_H^2}{v}\left[\frac{H^3}{2} + \frac{H}{2}(G^0)^2 + HG^+G^-\right]$$
$$+\frac{m_H^2}{2v^2}\left[\frac{H^4}{4} + \frac{H^2}{2}(G^0)^2 + H^2G^+G^- + \left(G^+G^-\right)^2 + \left(G^0\right)^2 G^+G^- + \frac{\left(G^0\right)^4}{4}\right]$$

Counter Terms

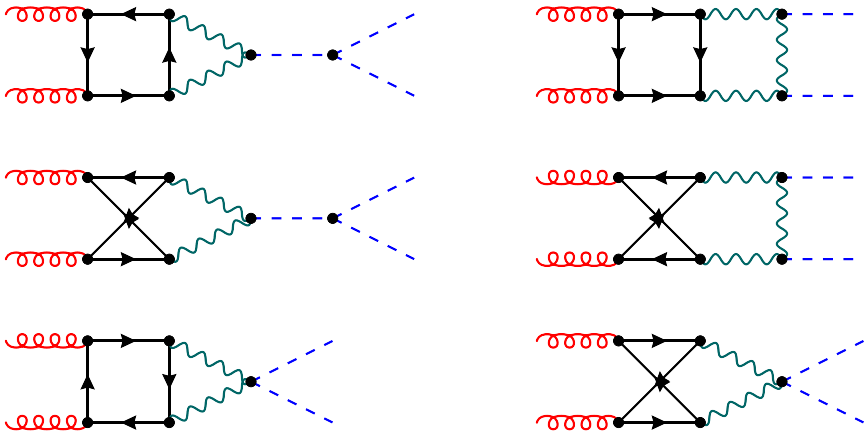
$$CT_1 = \frac{1}{2}\Sigma'_H\left(Q^2\right) = 3x_t\left[\left(4m_t^2 - Q^2\right)B'_0\left(Q^2, m_t, m_t\right) - B_0\left(Q^2, m_t, m_t\right)\right]$$

$$CT_2 = -\frac{\delta v}{v} = -\frac{1}{2}\frac{\delta m_W^2}{m_W^2} = \frac{1}{2}\frac{\Sigma_W(0)}{m_W^2} = \frac{T_1}{v m_H^2} + x_t\left[B_0\left(0, m_t, m_b\right) + 2B_0\left(0, m_t, m_t\right) + m_t^2 B'_0\left(0, m_t, m_b\right)\right]$$

$$CT_3 = -\frac{\delta m_t}{m_t}\left(\frac{m_t \partial}{\partial m_t} F_{LO}^{(n)}\right)\frac{1}{F_{LO}}$$

where $x_t = \frac{G_F m^2}{8\sqrt{2}\pi^2}$

Light quark diagrams



Cross section

The cross section can be decomposed as follows

$$\mathrm{d}\sigma = \mathrm{d}\sigma_{LO} + \mathrm{d}\sigma_{virt}$$

$$\mathrm{d}\sigma_{virt} \sim \; Re\left[\left(C_{\triangle}F_{1\triangle}^{LO} + F_{1\square}^{LO}\right)^{*}\left(C_{\triangle}F_{1\triangle} + F_{1\square}\right) + F_{2\square}^{LO*}F_{2\square}\right]$$

$$\delta_{top-yukawa-\triangle} \; \sim Re\,|\left(C_{\triangle}F_{1\triangle}^{LO}\right)^{*} C_{\triangle}F_{1\triangle}^{LO}|$$

$$\delta_{top-Yukawa} \sim \; Re|\left[\left(C_{\triangle}F_{1\triangle}^{LO} + F_{1\square}^{LO}\right)^{*}\left(C_{\triangle}F_{1\triangle} + F_{1\square}\right) + F_{2\square}^{LO*}F_{2\square}\right]| - \delta_{top-yukawa-\triangle}$$

Heavy Top-quark Limit (HTL)

In the HTL, the top-Yukawa-induced electroweak corrections to the effective *Hgg* and *HHgg* couplings can be obtained as

$$\mathcal{L}_{eff} = C_1 \frac{\alpha_s}{12\pi} G^{a\mu\nu} G^a_{\mu\nu} log\left(1 + C_2 \frac{H}{v}\right)$$

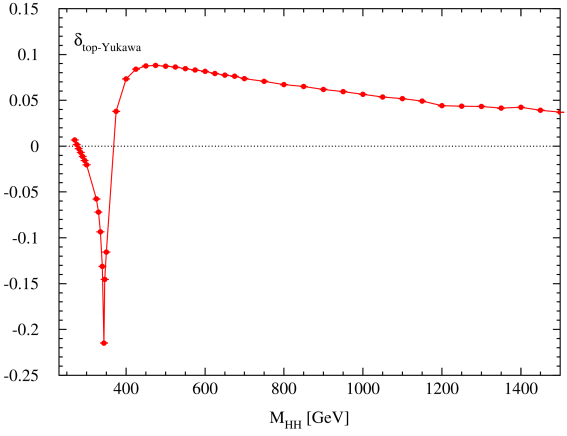
where $G^a_{\mu\nu}$ denotes the gluonic field-strenght tensor, $C_1 = 1 - 3x_t + O(x_t^2)$, and $C_2 = 1 + \frac{7}{2}x_t + O(x_t^2)$. C_1 and C_2 yield the explicit effective *Hgg* and *HHgg* couplings,

$$\mathcal{L}_{eff} = C_1 \frac{\alpha_s}{12\pi} G^{a\mu\nu} G^a_{\mu\nu} log\left[(1+\delta_1)\frac{H}{v} + (1+\eta_1)\frac{H^2}{2v^2} + O(H^3)\right]$$

where

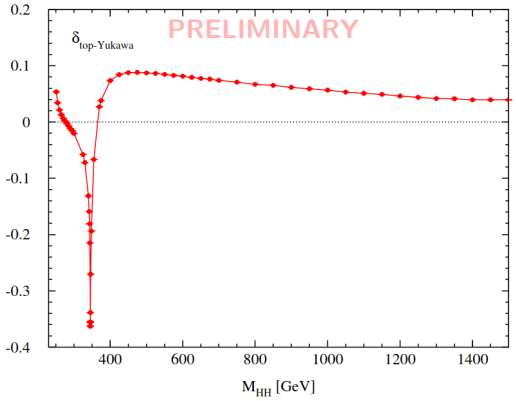
$$\delta_1 = \frac{x_t}{2} + O(x_t^2)$$
$$\eta_1 = 4x_t + O(x_t^2)$$

Counter Terms



Counter Terms Corrections

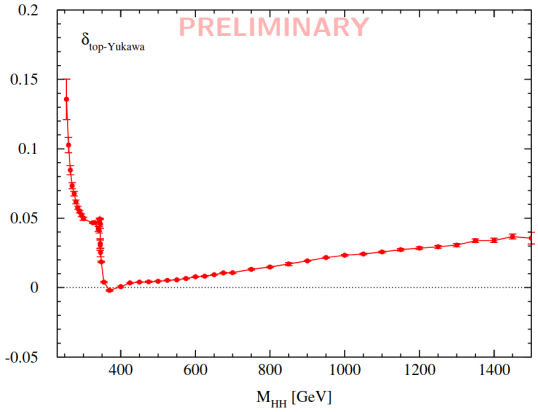
- Counter terms corrections to σ as a function of the invariant Higgs-pair mass M_{HH}
- Large contributions around the $t\bar{t}$ threshold
- Few percent corrections at the high energy tail



Box-like + CT Corrections

- Top-Yukawa-induced corrections to σ as a function of the invariant Higgs-pair mass M_{HH}
- $\delta_{top-Yukawa}$: corrections proportional to $\Delta_{1\Box}$, $\Delta_{2\Box}$, normalized to σ_{LO}
- Large contributions below the $t\bar{t}$ threshold
- Few percent corrections at the high energy tail

$F_{1\Box} \to F_{1\Box}\left(1 + \Delta_{1\Box}\right) \quad F_{2\Box} \to F_{2\Box}\left(1 + \Delta_{2\Box}\right)$



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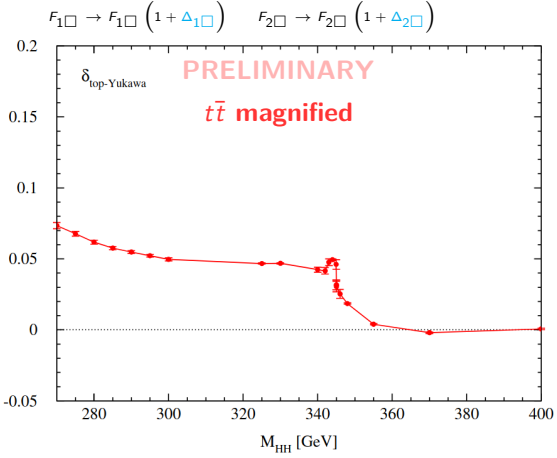
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