NNLO+PS double Higgs boson production with top-quark mass corrections in Geneva

The 22nd workshop of the LHC Higgs WG
CERN, Geneva
04.12.25

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European Research Council
Established by the European Commission

HH at LHC

- Higgs boson sector largely studied

Observation of the Higgs boson by ATLAS and CMS during ***LHC Run 1*** ($H \to \gamma\gamma$ and $H \to ZZ^* \to 4l$ channels)

2012

Ongoing campaign of **measurements** of the Higgs boson **properties**…

2018 **Observation** of the $H \to b\bar{b}$ decay

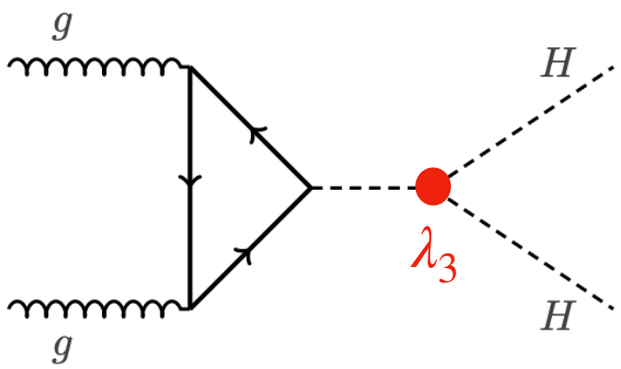
Now

Excellent agreement with the **Standard Model (SM)**

- *Higgs self-coupling* is still largely experimentally *unconstrained* by direct measurements

$$V(\phi^\dagger\phi) = \frac{1}{2}m_H^2H^2 + \lambda_3 vH^3 + \frac{\lambda_4}{4}H^4$$

trilinear Higgs boson coupling accessible through HH



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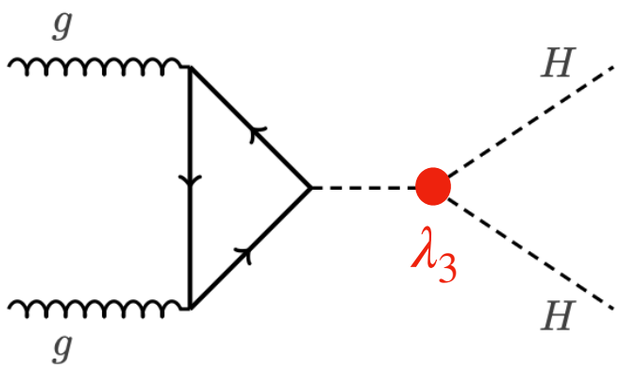
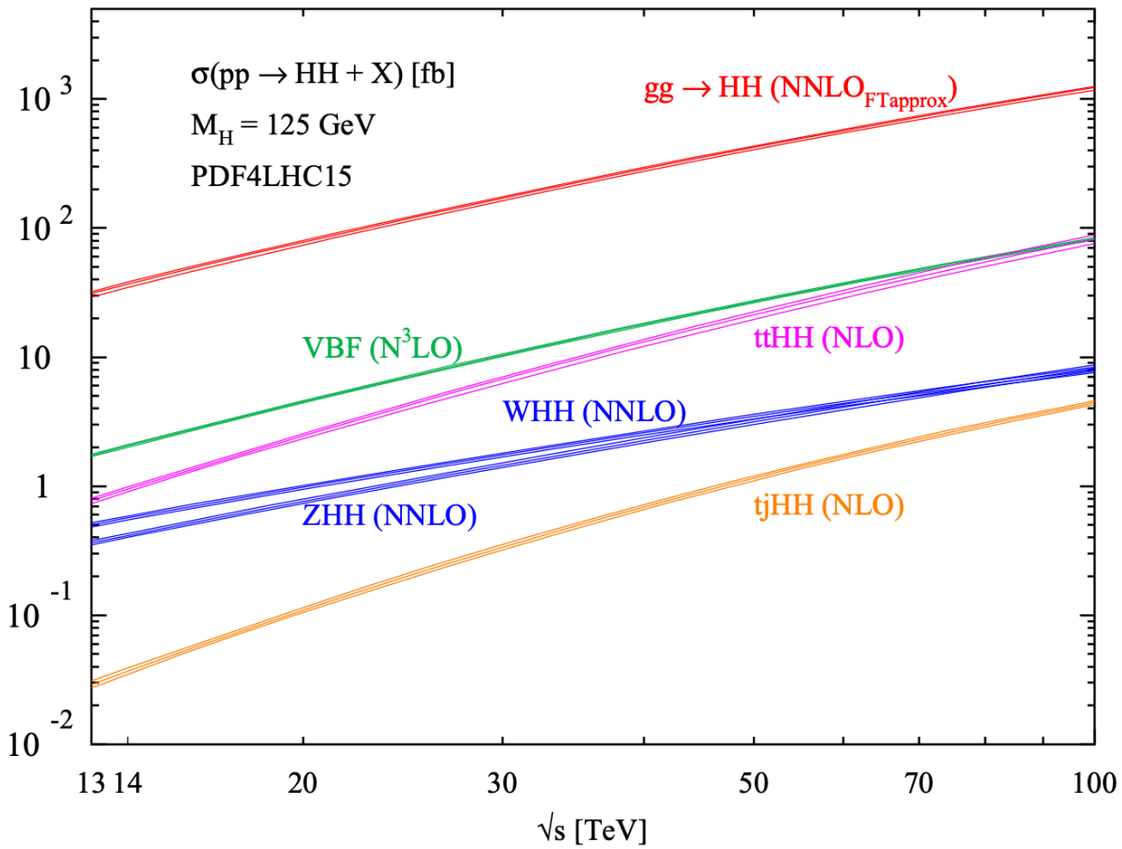


gluon-gluon fusion (ggF) ~ 90%

vector boson fusion (VBF) ~ 5%

For now: no bottom-quark or EW corrections considered in the Geneva implementation



*taken from 10.1016/j.revip.2020.100045

GENEVA method

GENEVA

1. fully differential **NNLO FO** calculations, via jet resolution variable (r_i) subtraction
2. higher-order jet resolution variable resummation up to **N^{3}LL**
3. matching to parton shower plus hadronization and multiparton interactions

For this process: *N*-jettiness as jet resolution variable (Stewart, Tackmann, Waalewijn '09, '10 + Geneva literature)

$$\mathscr{T}_N = \sum_{i=1}^{N} \min_{q_1,\ldots,q_N} \left(q_a \cdot \hat{p}_i, q_b \cdot \hat{p}_i, q_1 \cdot \hat{p}_i, \ldots, q_N \cdot \hat{p}_i\right)$$

$q_{a,b} = (1,0,0,\pm 1)$, $q_1, \ldots q_N$ any jet direction, $\hat{p}_i$ longitudinally boosted momenta from lab to colour singlet with 0 rapidity

- $\mathscr{T}_N \to 0$ in the IR region
- Slicing approach → missing power corrections that vanish as $\mathscr{T}_N^{\text{cut}} \to 0$
- when $\mathscr{T}_N^{\text{cut}} \to 0$, large logs of $\mathscr{T}_N^{\text{cut}}$ and $\mathscr{T}_N$ need to be resummed
  ingredients for the resum. known for each N up to NNLL in SCET (at least)
- IR-finite definition of events based on resolution parameters $\mathscr{T}_0^{\text{cut}}$ and $\mathscr{T}_1^{\text{cut}}$

HH possible approximations

Some approximations that we considered in our framework:

» Infinite top-quark mass limit $m_t \to \infty$

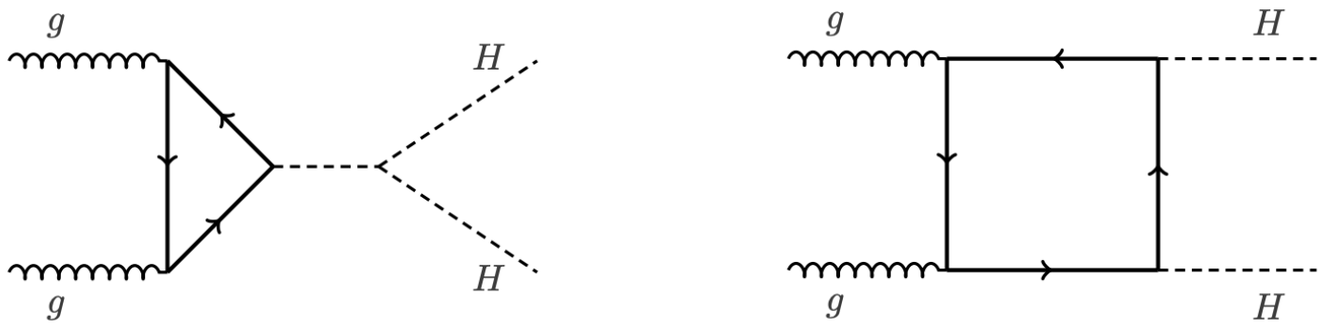
(JHEP 06 (2023) 205)

» B-proj: reweight all squared matrix elements using Born projected matrix elements

$$K\left(\Phi_n\right) = \frac{B_{gg\to HH}\left(\tilde{\Phi}_0(\Phi_n), m_t\right)}{B_{gg\to HH}\left(\tilde{\Phi}_0(\Phi_n), m_t \to \infty\right)} \qquad \begin{aligned} for \quad \Phi_n = gg &\to HH\,, \\ gg &\to HHj\,, \\ gg &\to HHjj \end{aligned}$$

» $\mathrm{FT}_{\mathrm{approx}}$*: include available mass effects, unknown terms are reweighted using the corresponding Born-level contributions with exact top-quark mass dependence

(JHEP 09 (2025) 206)



*original idea Frederix et al '14, Maltoni et al '14, Grazzini et al '18

$\mathrm{FT}_{\mathrm{approx}}$ in Geneva

1 $B_{0,1,2}$ with exact m_t dependence from **Openloops**

Born@LO,NLO,NNLO terms

2 V_0 at NLO with exact m_t dependence from **HHgrid**

Virtual term caveat: using fixed values of m_H, G_F , m_t → m_t uncertainty?
currently playing with ggxy tool
[see Schonwald's talk!]

3 real-virtual term, not known and reweighed

Two-loop massive for 2→3 config.

$$V_1\left(\Phi_1\right) = V_1\left(\Phi_1, m_t \to \infty\right) \frac{B_1\left(\Phi_1, m_t\right)}{B_1\left(\Phi_1, m_t \to \infty\right)}$$

4 W_0 only appears in the hard function:

Double-virtual term

$$H^{(0)} \equiv B_0 \text{ from } \textbf{Openloops},$$

$$H_{\text{fin}}^{(i)}\left(\Phi_0\right) = H_{\text{fin}}^{(i)}\left(\Phi_0, m_t \to \infty\right) \frac{B_0\left(\Phi_0, m_t\right)}{B_0\left(\Phi_0, m_t \to \infty\right)}, i = 1,2$$

ME Instabilities

» **Another caveat**: $gg \to HHgg$ challenging in the double IR limits

Matrix collab.'s approach: for $\alpha_{ij} = p_i \cdot p_j / \hat{s} < 10^{-4}$

use $m_t \to \infty$ ME reweighed by the massive Born ME

projection preserving momenta of Higgs boson

» Care must be taken to avoid spoiling the subtraction mechanism required for the NLO_1 when including top-quark mass!

We chose

$$\mathrm{d}\sigma_{\mathrm{gg}\to\mathrm{HHgg}}(\Phi_2)\Big|_{\min(\alpha_{\mathrm{ij}})<10^{-4}} = \mathrm{d}\sigma_{\mathrm{gg}\to\mathrm{HHgg}}(\tilde{\Phi}_1)\frac{\mathrm{d}\sigma^{\mathrm{m_t}\to\infty}_{\mathrm{gg}\to\mathrm{HHgg}}(\Phi_2)}{\mathrm{d}\sigma^{\mathrm{m_t}\to\infty}_{\mathrm{gg}\to\mathrm{HHgg}}(\tilde{\Phi}_1)}$$

FKS projection mapping to the original Φ_2

IR limit

$$\mathrm{SF}\left(\frac{\mathrm{d}\Phi_2}{\mathrm{d}\tilde{\Phi}_1}\right)$$ used in the subtraction

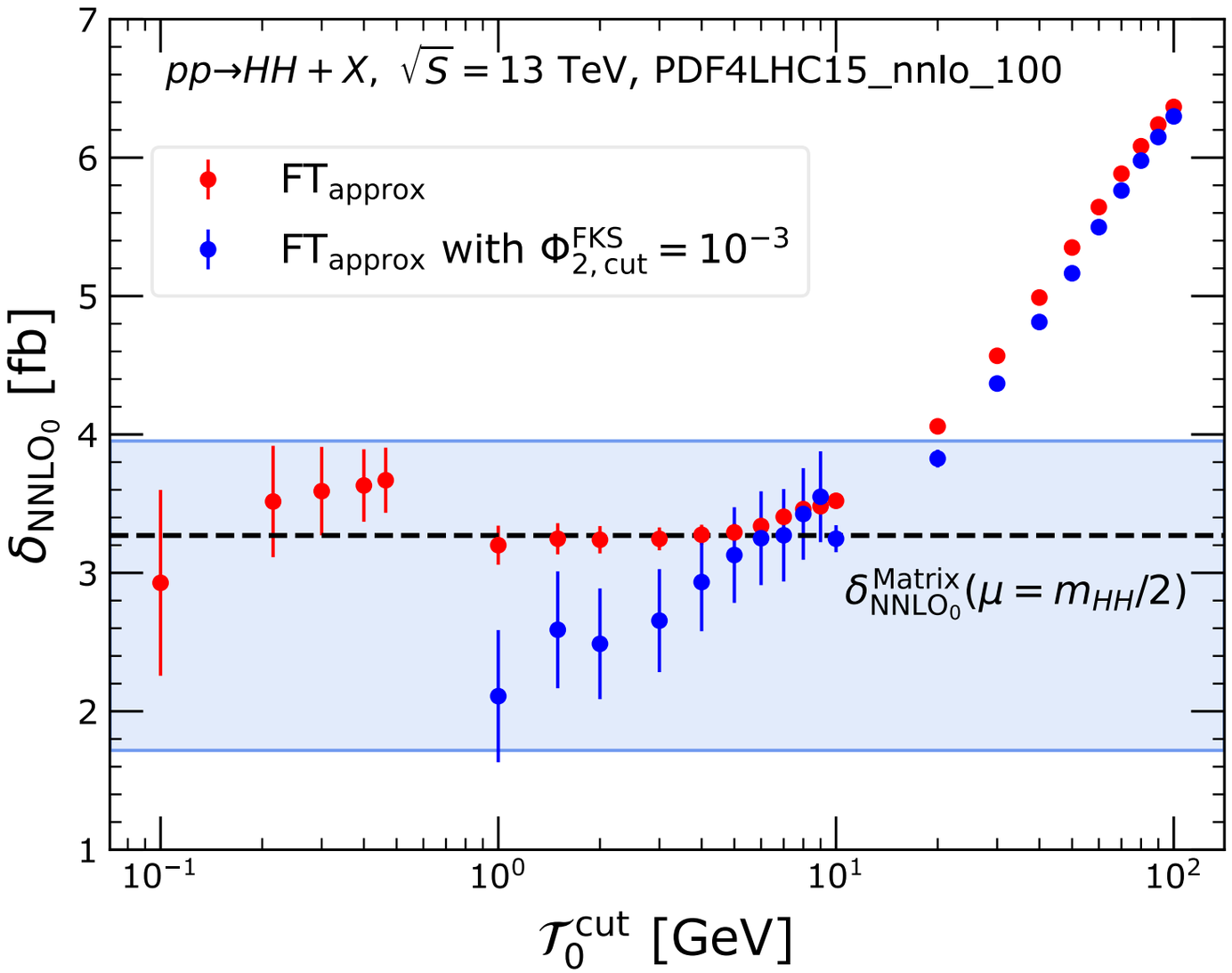
Numerical discrepancies/instabilities in the subtraction for $\mathcal{O}(\alpha_{ij}^{\mathrm{cut}}) \sim \mathcal{O}\left(\frac{\mathcal{T}_0^{\mathrm{cut}}}{Q}\right)$

Validation of NNLO results against Matrix

NNLO correction to the xs in a slicing calculation:

$$\delta_{\mathrm{NNLO}_0} = \Sigma_{\mathcal{O}(\alpha_s^4)}\left(\mathscr{T}_0 < \mathscr{T}_0^{\mathrm{cut}}\right) + \delta_{\mathrm{NLO}_1}\left(\mathscr{T}_0 > \mathscr{T}_0^{\mathrm{cut}}\right) + \mathcal{O}\left(\mathscr{T}_0^{\mathrm{cut}}\right)$$

resummed expanded at fixed order — FO NLO correction to $gg \to HH + j$



» good agreement until $\mathscr{T}_0^{\mathrm{cut}} \lesssim 0.5$ GeV

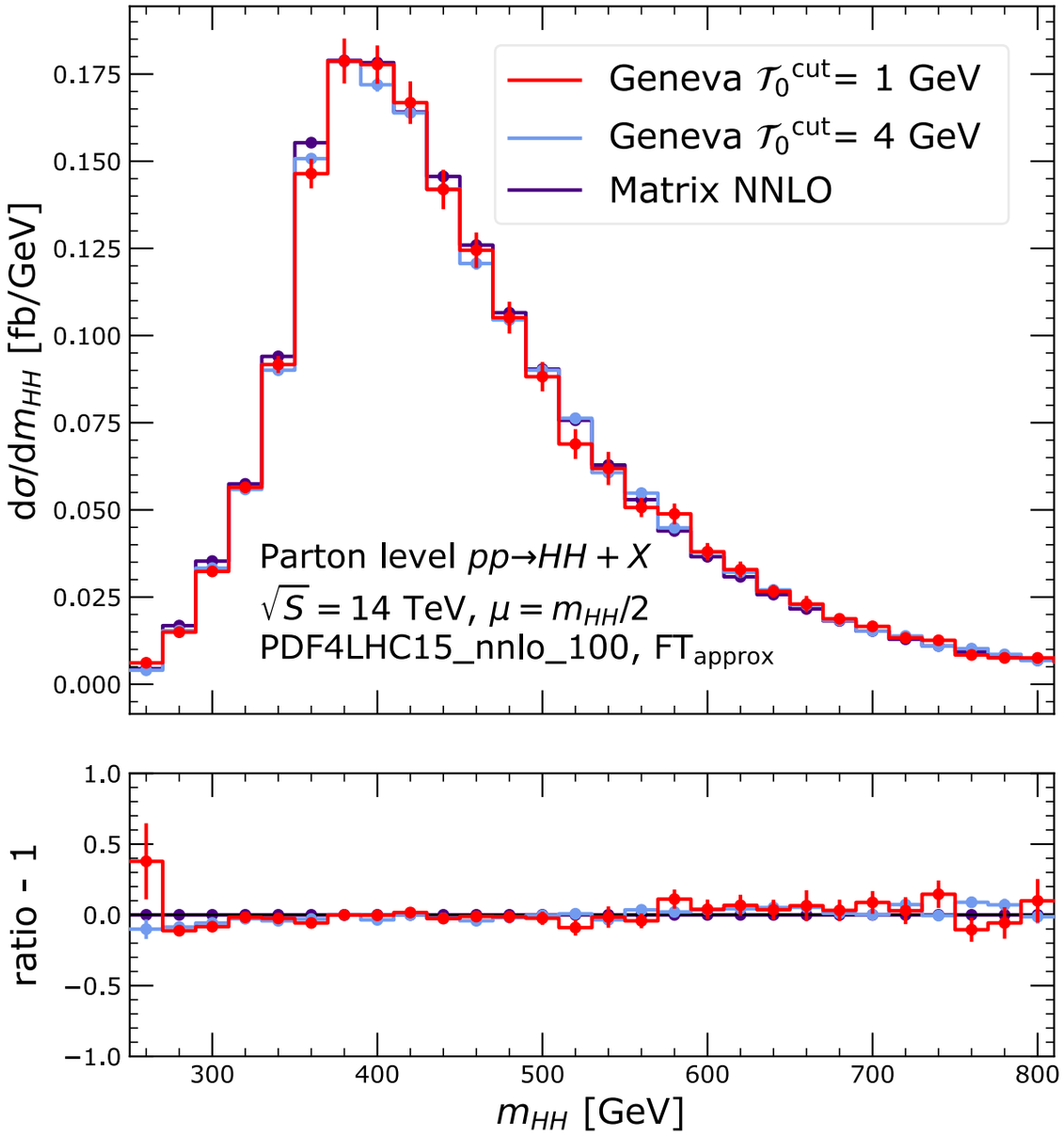
» $gg \to HHgg$ ME with full m_t dependence: cut on ξ and y FKS variables of the extra radiation phase space

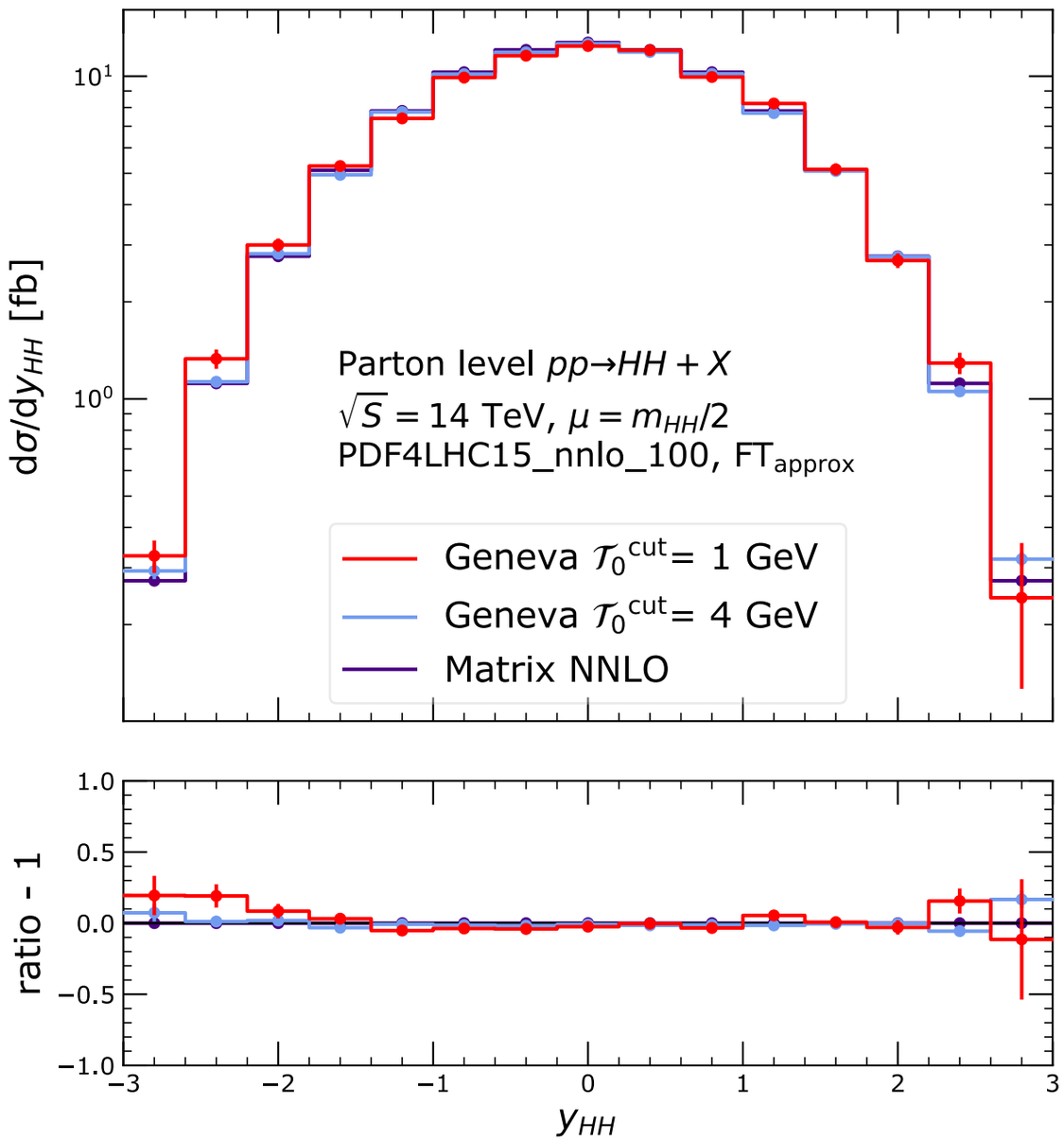
Validation of NNLO results against Matrix

Input parameters: $m_H = 125\,\text{GeV}$ $\quad \mathscr{T}_0^{\text{cut}} = 1, 4\,\text{GeV}$ $\quad$ beam functions from SCETlib

$\mu_F = \mu_R = m_{HH}/2$ $\quad \mathscr{T}_1^{\text{cut}} = 1\,\text{GeV}$ $\quad$ phase space MUNICH

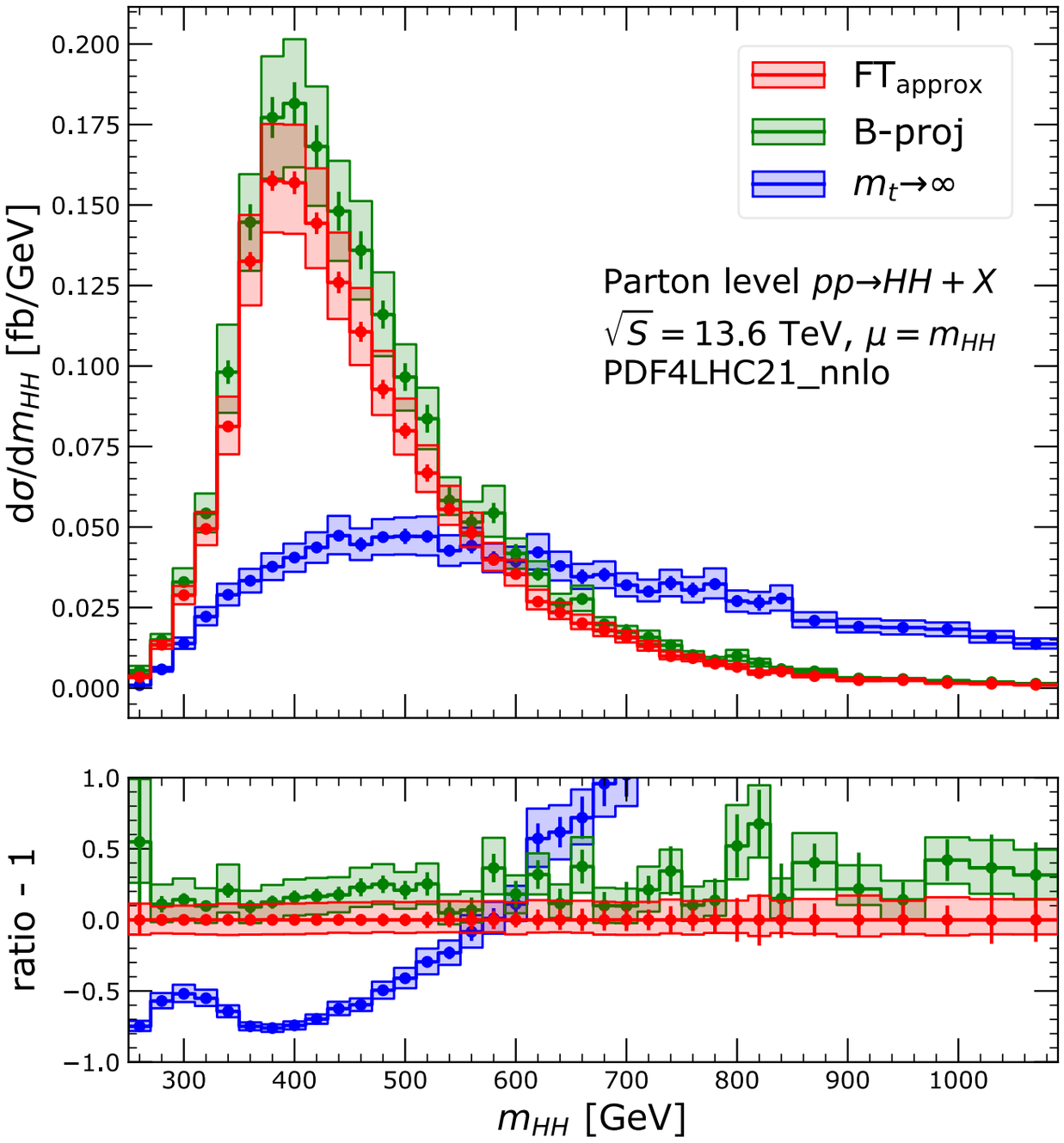
$m_t = 173\,\text{GeV}$ $\quad G_F = 1.1663787\text{e}^{-5}\,\text{GeV}^{-2}$



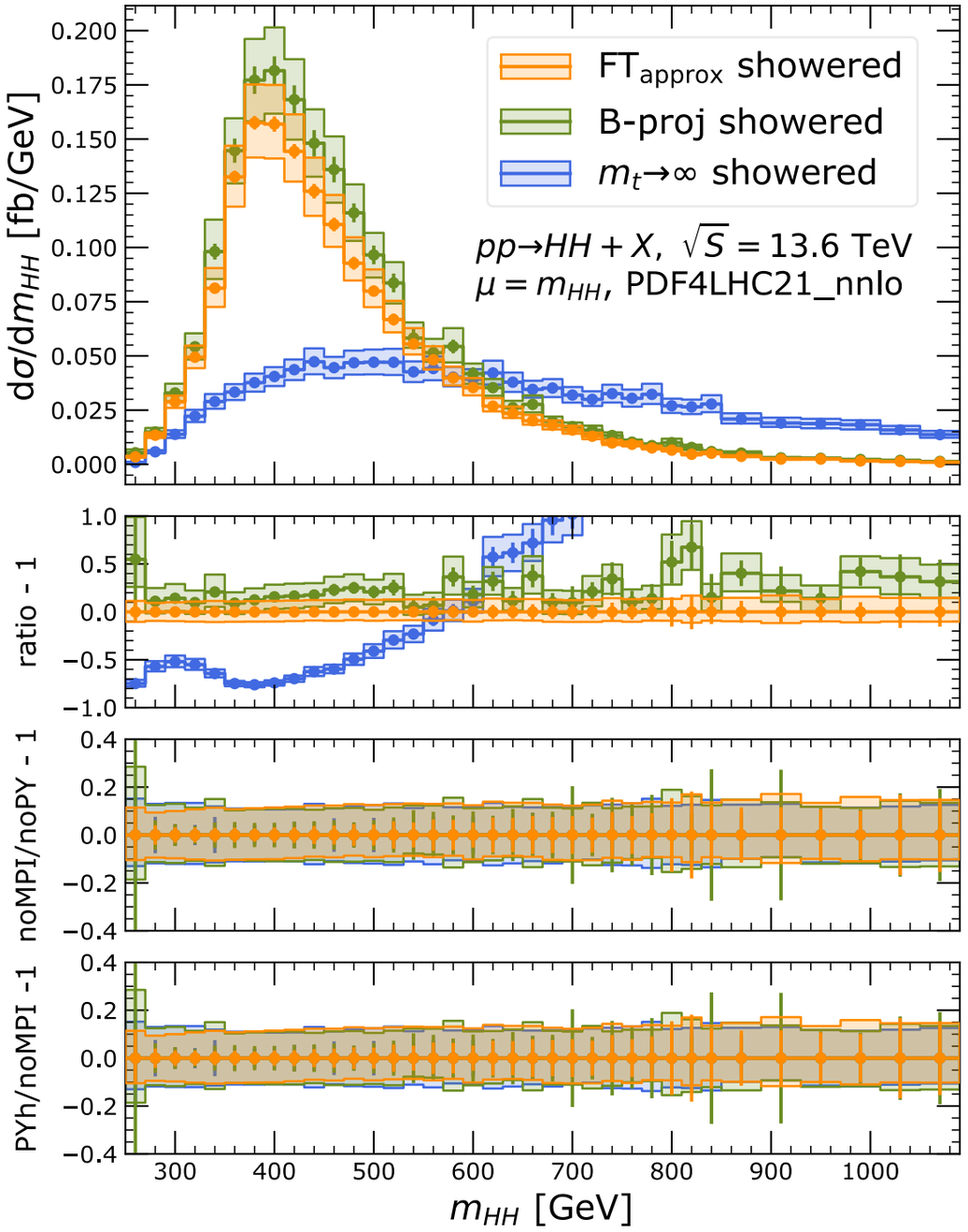


Phenomenological results

Default Geneva options: $\mathscr{T}_0^{\text{cut}} = 1\ \text{GeV}$ $\quad \mu_F = \mu_R = m_{HH}$



Pythia8 shower



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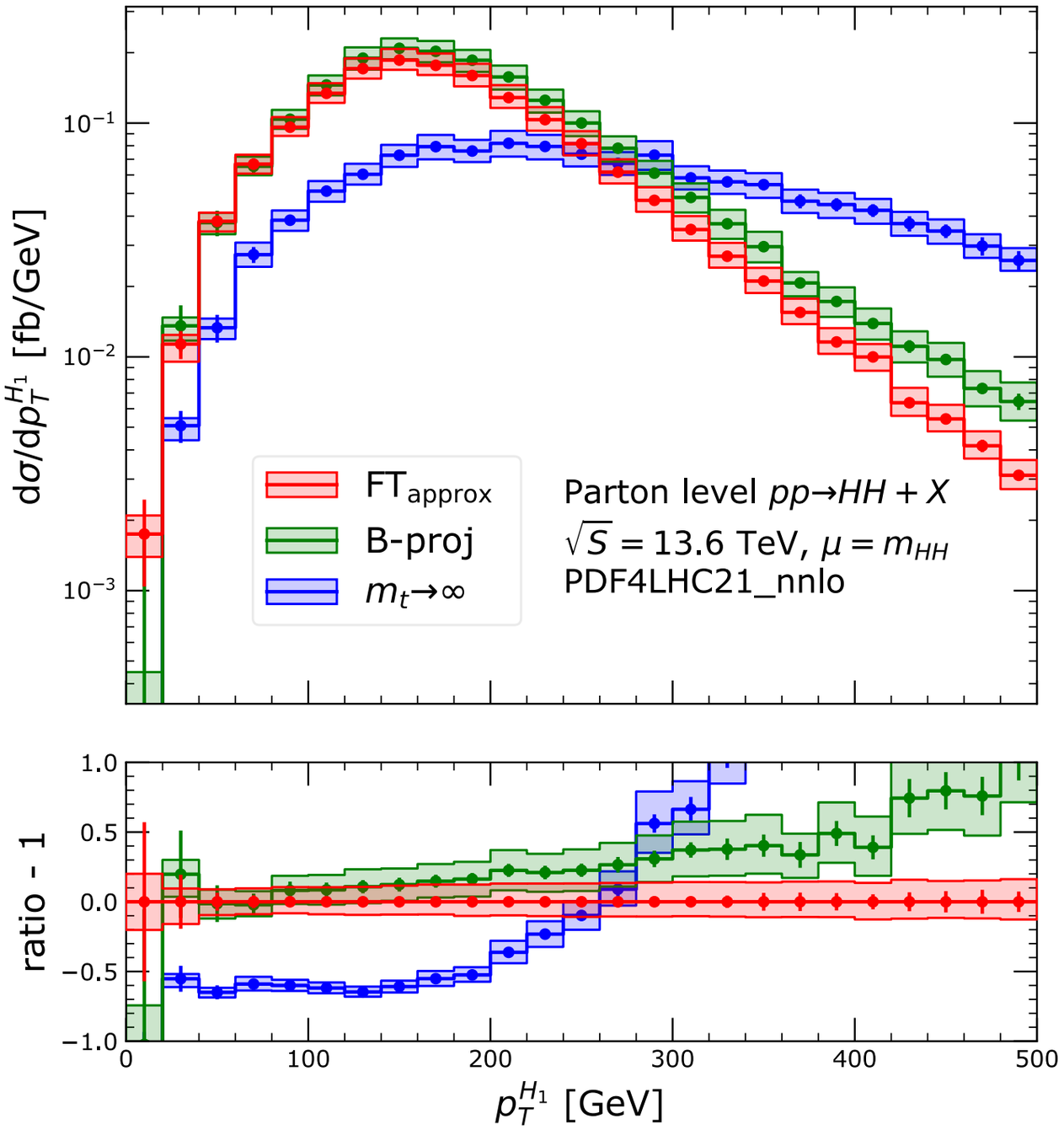
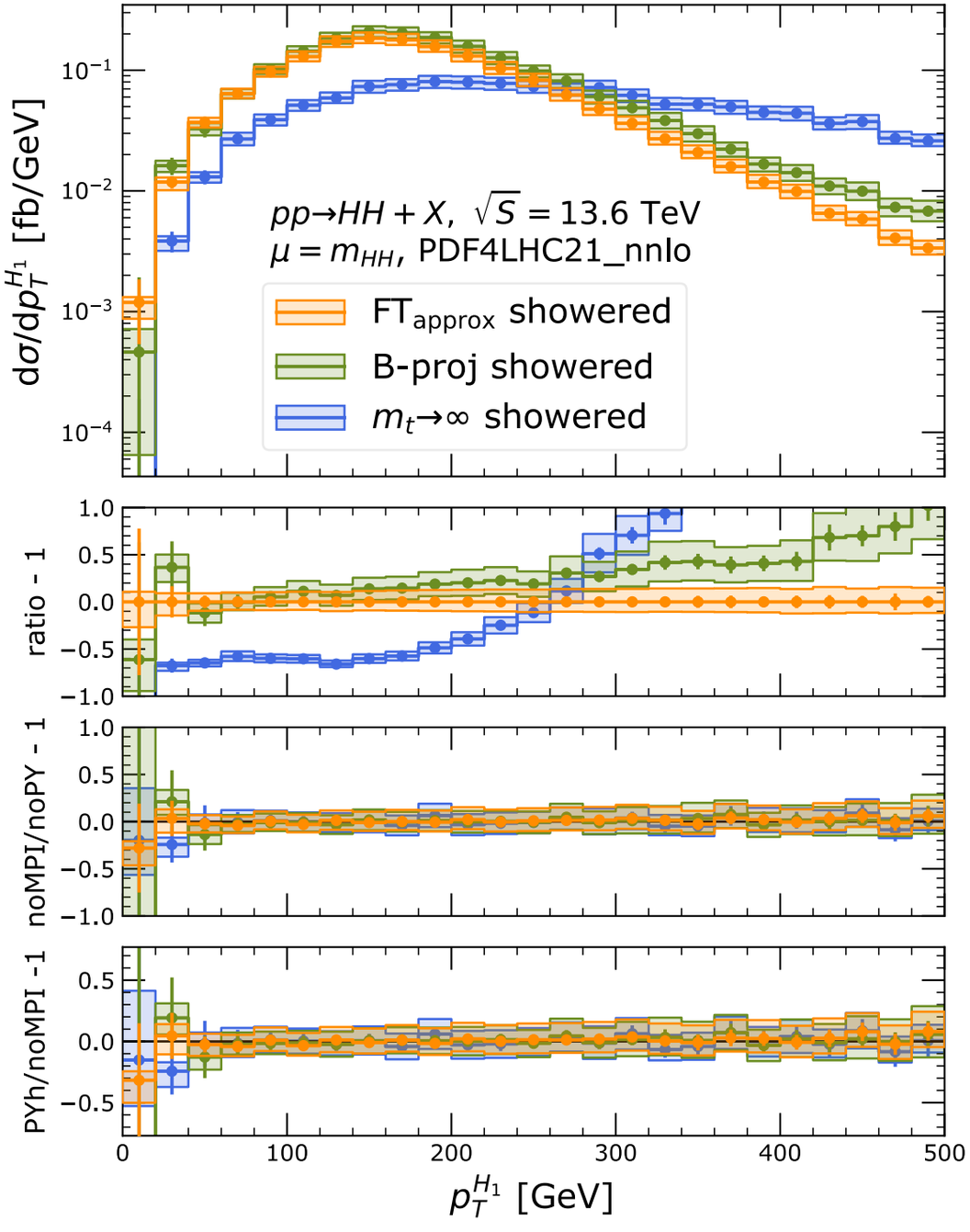




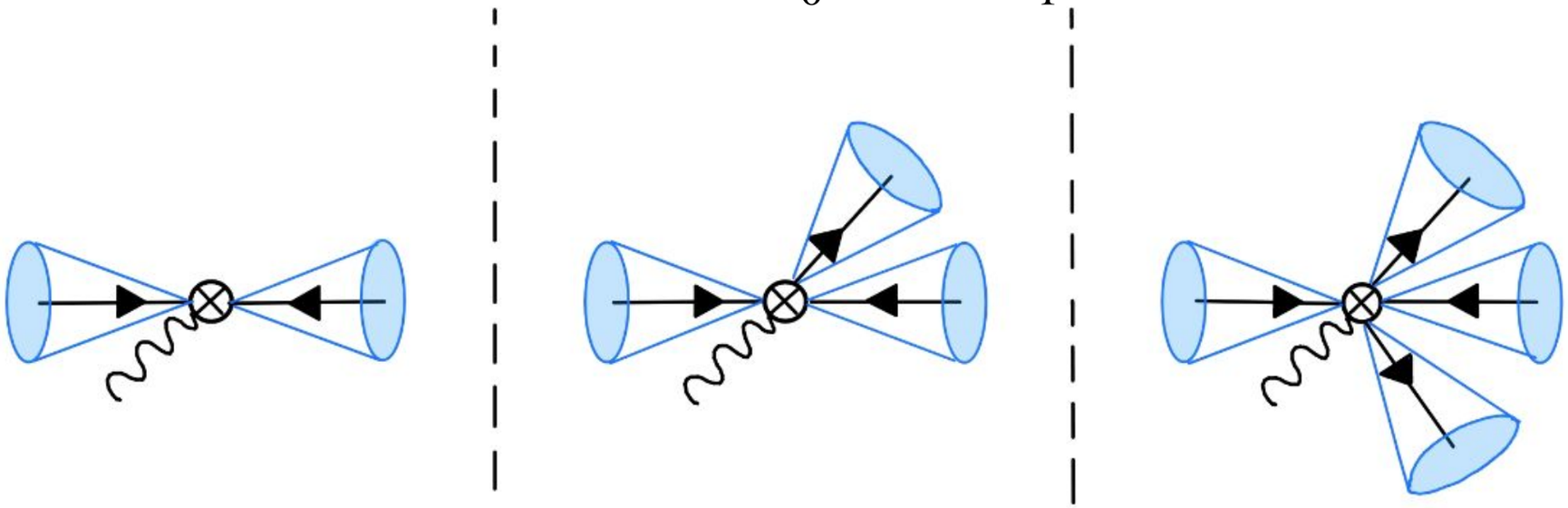
- Parton shower behaves independently of the treatment of m_t effects in the hard interaction
- PS ensures that the accuracy of the observable is *at least* as accurate as the PS is ⟶ **LL**

Thanks!

Backup slides

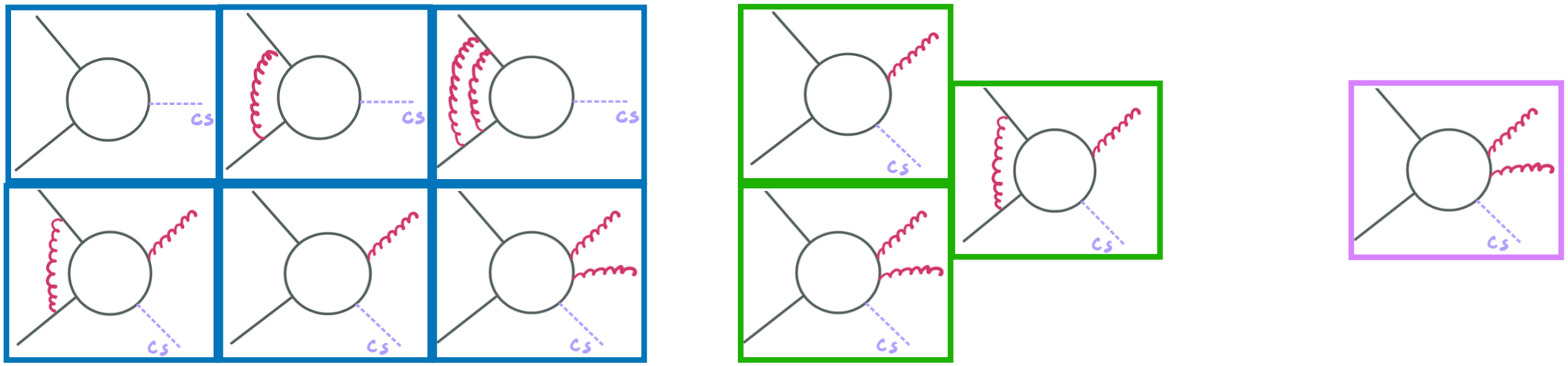
Step 1: choice of the jet resolution variable and slice up the phase space

Phase space divided into jet-bins through $\mathscr{T}_0^{\text{cut}}$ and $\mathscr{T}_1^{\text{cut}}$:



$\mathscr{T}_0 < \mathscr{T}_0^{\text{cut}}$ | $\mathscr{T}_0 > \mathscr{T}_0^{\text{cut}}, \mathscr{T}_1 < \mathscr{T}_1^{\text{cut}}$ | $\mathscr{T}_0 > \mathscr{T}_0^{\text{cut}}, \mathscr{T}_1 > \mathscr{T}_1^{\text{cut}}$

Φ_0 events | $\mathscr{T}_0^{\text{cut}}$ | Φ_1 events | $\mathscr{T}_1^{\text{cut}}$ | Φ_2 events



*taken from Napoletano's talk at QCD@LHC '24

Step 2: combining resummation to fixed-order prediction

$\mathscr{T}_0$ factorization in SCET:

$$\frac{\mathrm{d}\sigma^{\mathrm{SCET}}}{\mathrm{d}\Phi_0\,\mathrm{d}\mathscr{T}_0} = H_\kappa(Q^2,\mu_H)\,U_H(\mu_H,\mu)\int [B_{k_a}(t_a,x_a,\mu_B)\otimes U_B(\mu_B,\mu)]\times[B_{k_b}(t_b,x_b,\mu_B)\otimes U_B(\mu_B,\mu)]$$
$$\times\left[S_\kappa\left(\mathscr{T}_0-\frac{t_a+t_b}{Q},\mu_S\right)\otimes U_S(\mu_S,\mu)\right]\mathrm{d}t_a\,\mathrm{d}t_b$$

$\mathscr{T}_0$ resummation up to NNLL'/N^{3}LL:

Hard function [two-loop virtual corrections calc.]

captures virtual corrections at $\mu_H \sim Q$

Beam functions [two-loop order calc.]

include ISR radiation at $\mu_B \sim \sqrt{\mathscr{T}_0 Q}$ plus PDFs

Soft function [two-loop order calc.]

encodes wide-soft angle radiation at $\mu_S \sim \mathscr{T}_0$

NO LOGS!

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$$\times\left[S_\kappa\left(\mathscr{T}_0-\frac{t_a+t_b}{Q},\mu_S\right)\otimes U_S(\mu_S,\mu)\right]\mathrm{d}t_a\,\mathrm{d}t_b$$

$U_{H,B,S}$ renormalization group evolution factors, resummation performed via RGE to a common scale μ

$$\mu\frac{\mathrm{d}C}{\mathrm{d}\mu}=\gamma_C\otimes C\,,\qquad C=H,B,S$$

To switch-off the resummation and provide a smooth transition to FO:

$\mathscr{T}_0$-dependent profile scales

Step 2: combining resummation to fixed-order prediction

Monte Carlo cross sections:

$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}\Phi_0}(\mathscr{T}_0^{\mathrm{cut}}) = \underbrace{\frac{\mathrm{d}\sigma_0^{\mathrm{NNLO}_0}}{\mathrm{d}\Phi_0}(\mathscr{T}_0^{\mathrm{cut}})}_{\text{fixed-order}} + \underbrace{\frac{\mathrm{d}\sigma_0^{\mathrm{NNLL}'}}{\mathrm{d}\Phi_0}(\mathscr{T}_0^{\mathrm{cut}})}_{\text{resummed}} - \underbrace{\left.\frac{\mathrm{d}\sigma_0^{\mathrm{NNLL}'}}{\mathrm{d}\Phi_0}(\mathscr{T}_0^{\mathrm{cut}})\right|_{\mathrm{NNLO}_0}}_{\text{fo expansion of the resummed cumulant}}$$

$$\frac{\mathrm{d}\sigma_1}{\mathrm{d}\Phi_1}(\mathscr{T}_0 > \mathscr{T}_0^{\mathrm{cut}}, \mathscr{T}_1^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma_1^{\mathrm{NLO}_1}}{\mathrm{d}\Phi_1}(\mathscr{T}_0 > \mathscr{T}_0^{\mathrm{cut}}) + \left[\frac{\mathrm{d}\sigma_0^{\mathrm{NNLL}'}}{\mathrm{d}\mathscr{T}_0\,\mathrm{d}\Phi_0}(\mathscr{T}_0 > \mathscr{T}_0^{\mathrm{cut}}) - \left.\frac{\mathrm{d}\sigma_0^{\mathrm{NNLL}'}}{\mathrm{d}\mathscr{T}_0\,\mathrm{d}\Phi_0}(\mathscr{T}_0 > \mathscr{T}_0^{\mathrm{cut}})\right|_{\mathrm{NLO}_1}\right]\mathscr{P}(\Phi_1)$$

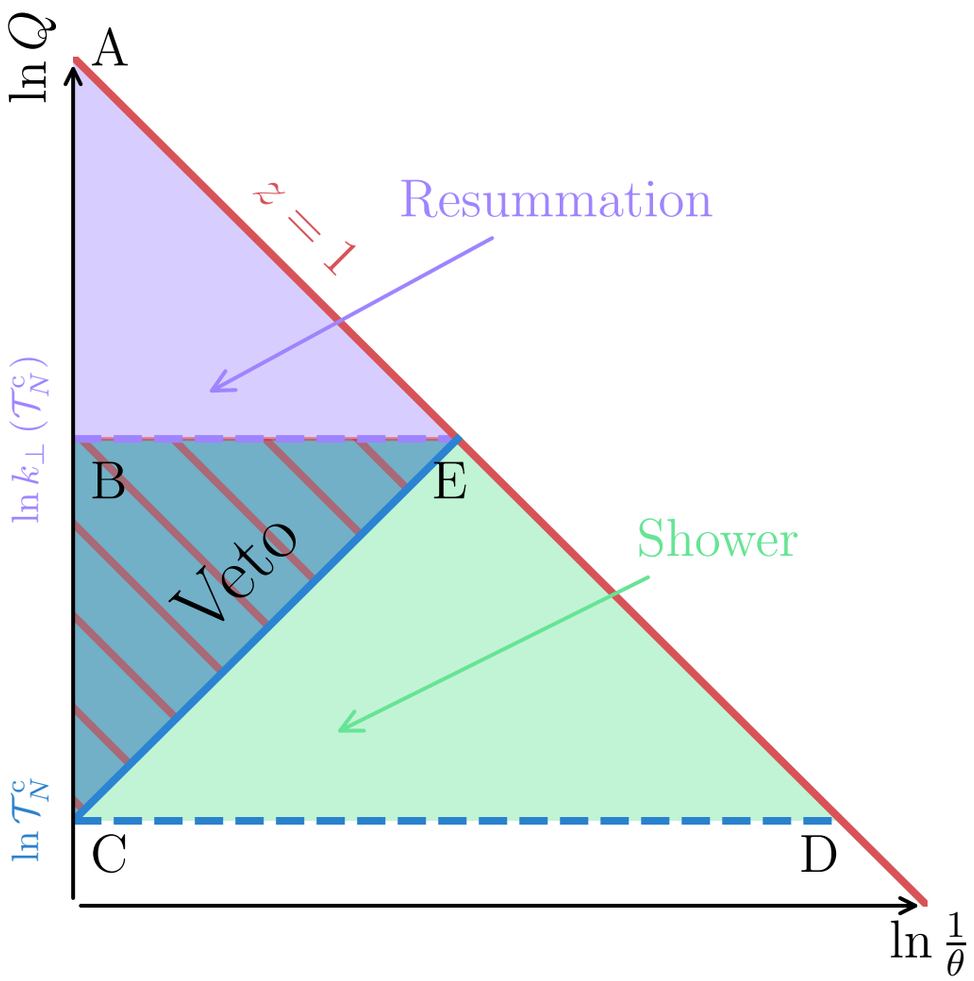
$$\frac{\mathrm{d}\sigma_2}{\mathrm{d}\Phi_2}(\mathscr{T}_0 > \mathscr{T}_0^{\mathrm{cut}}, \mathscr{T}_1 > \mathscr{T}_1^{\mathrm{cut}}) = \frac{\mathrm{d}\sigma_2^{\mathrm{LO}_2}}{\mathrm{d}\Phi_2}(\mathscr{T}_0 > \mathscr{T}_0^{\mathrm{cut}}) + \left[\frac{\mathrm{d}\sigma_0^{\mathrm{NNLL}'}}{\mathrm{d}\mathscr{T}_0\,\mathrm{d}\Phi_0}(\mathscr{T}_0 > \mathscr{T}_0^{\mathrm{cut}}) - \left.\frac{\mathrm{d}\sigma_0^{\mathrm{NNLL}'}}{\mathrm{d}\mathscr{T}_0\,\mathrm{d}\Phi_0}(\mathscr{T}_0 > \mathscr{T}_0^{\mathrm{cut}})\right|_{\mathrm{LO}_2}\right]\mathscr{P}(\tilde{\Phi}_1)\mathscr{P}(\Phi_2)$$

*formulae with $\mathscr{T}_1$ resummation are complicated, see this paper!

Step 3: interface to parton showers (+ hadronization and MPI)

In GENEVA resummation is performed in $\mathscr{T}_N^{\text{cut}} \leq \mathscr{T}_N \leq Q$ (hard scale)

but parton showers have a different resolution variable: **transverse momentum** $k_\perp$



Values of $\mathscr{T}_N^{\text{cut}}$:

- Φ_0 events $\rightarrow \mathscr{T}_N^{\text{cut}} = \mathscr{T}_0^{\text{cut}}$
- Φ_1 events $\rightarrow \mathscr{T}_N^{\text{cut}} = \mathscr{T}_1^{\text{cut}}$
- Φ_2 events $\rightarrow \mathscr{T}_N^{\text{cut}} = \mathscr{T}_1(\Phi_2)$

maximum relative transverse momentum of the emission:

$$k_\perp = \sqrt{\hat{k}_0^2 - \hat{k}_3^2} = \sqrt{\mathscr{T}_N(\hat{k}_0 + \hat{k}_3)} \leq \sqrt{\mathscr{T}_N \sqrt{s}}$$

Veto Step (after M emission):

$$\mathscr{T}_N(\Phi_{N+M}) \leq \mathscr{T}_N$$

Now interface to: Pythia8, Dire, Sherpa

$\mathscr{T}_1$ resummation in HH

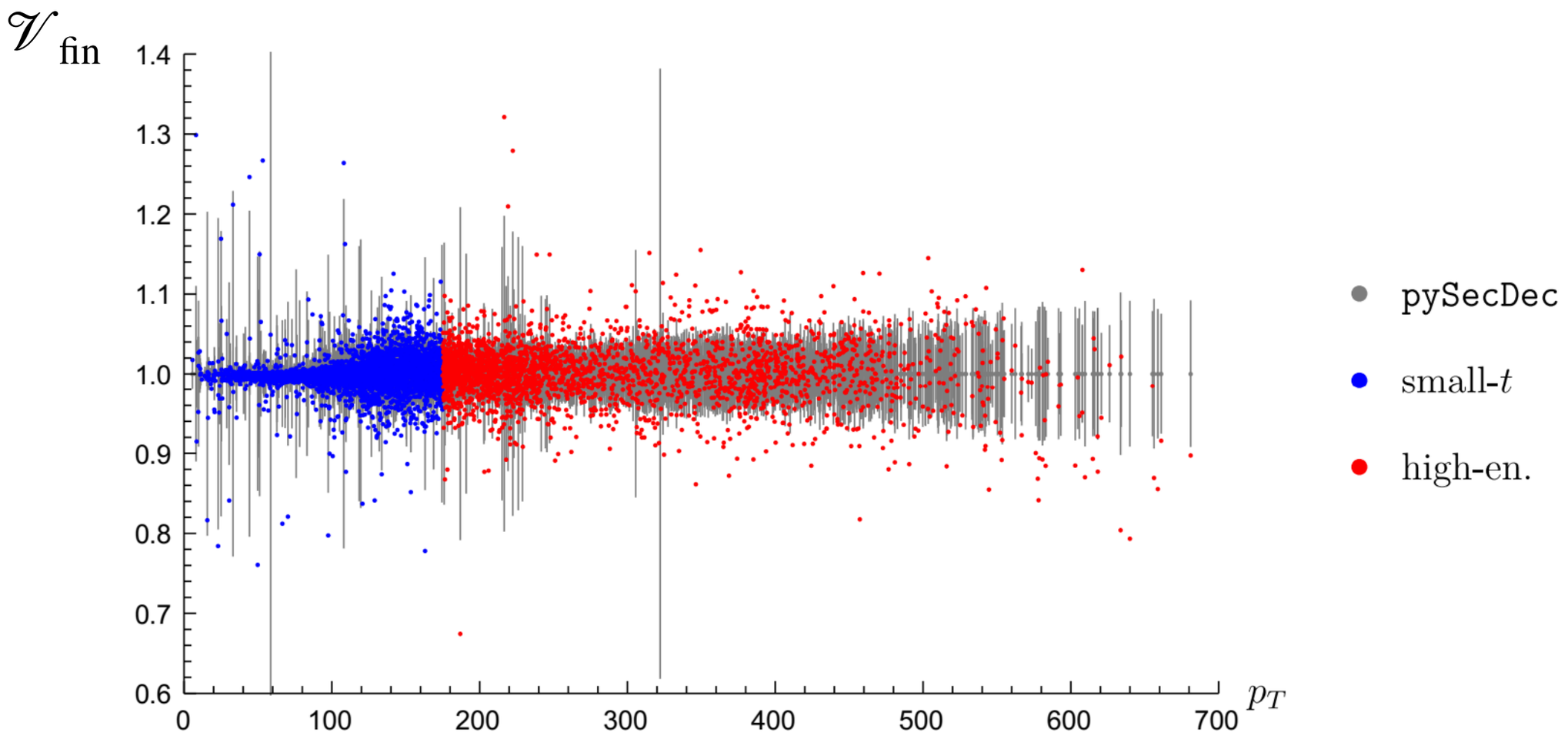
NLL $\mathscr{T}_1$ resummation to obtain a separation between 1-jet exclusive XS and 2-jet inclusive XS:

$$U_1^{ggg}(\Phi_1, \mathscr{T}_1^{\text{cut}}) = \frac{U}{\Gamma\left(1 + 6C_A \eta_{\Gamma_{\text{cusp}}}^{\text{NLL}}(\mu_S, \mu_H)\right)}$$

$$\begin{aligned} U = \exp\Bigg\{ & 6C_A \left[2K_{\Gamma_{\text{cusp}}}^{\text{NLL}}(\mu_J, \mu_H) - K_{\Gamma_{\text{cusp}}}^{\text{NLL}}(\mu_S, \mu_H) \right] \\ & + C_A \left[-\ln\left(\frac{Q_a^2 Q_b^2 Q_J^2}{\mu_H^6}\right) \eta_{\Gamma_{\text{cusp}}}^{\text{NLL}}(\mu_J, \mu_H) + \ln\left(\frac{Q_a^2 Q_b^2 Q_J^2}{stu}\right) \eta_{\Gamma_{\text{cusp}}}^{\text{NLL}}(\mu_S, \mu_H) \right) \Bigg] \\ & - 6\gamma_E C_A \eta_{\Gamma_{\text{cusp}}}^{\text{NLL}}(\mu_S, \mu_J) + 3K_{\gamma_J^g}^{\text{NLL}}(\mu_J, \mu_H) \Bigg\} \end{aligned}$$

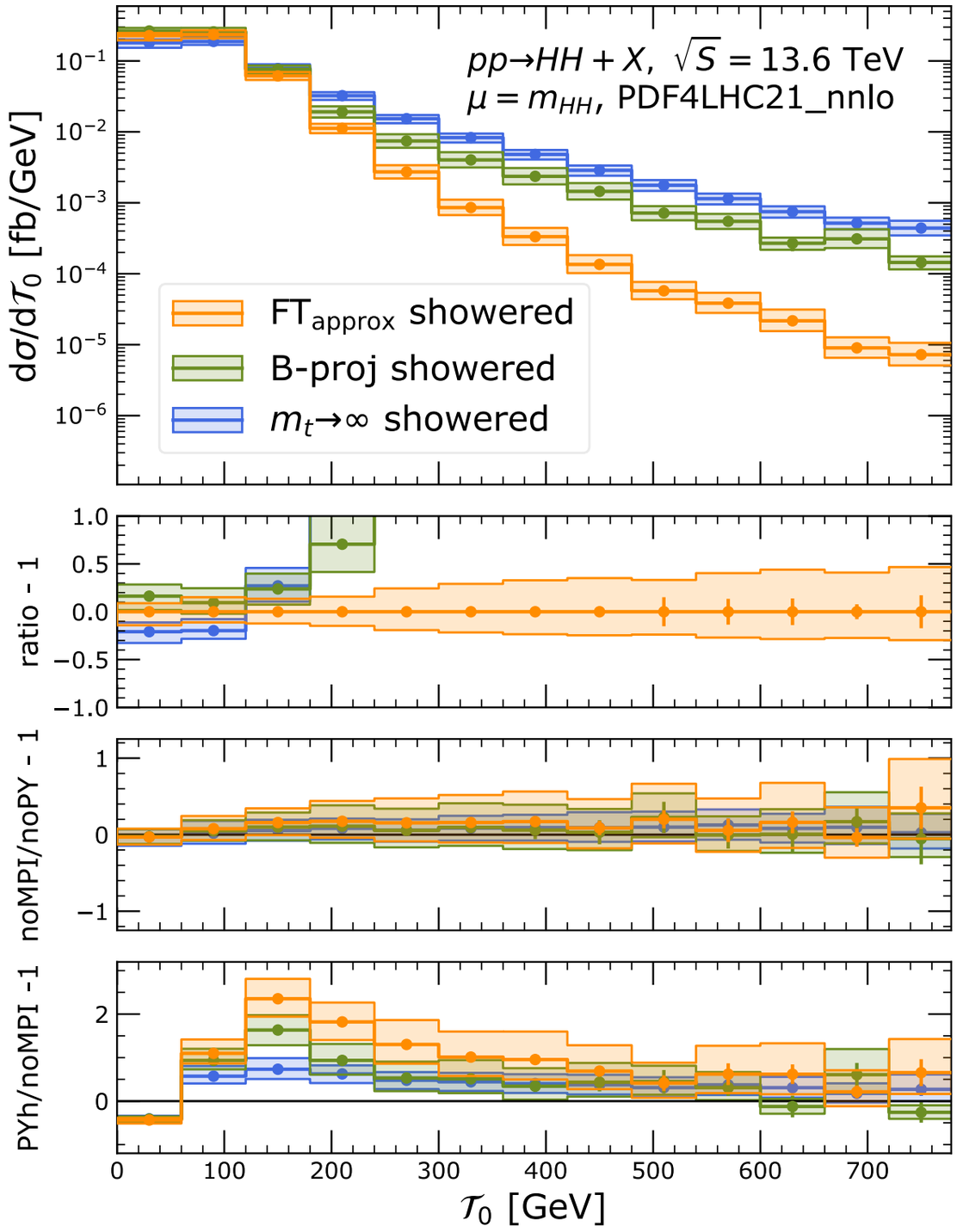
where $\mu_S = \mathscr{T}_1^{\text{cut}}$ $\mu_H = \mathscr{T}_1^{\text{max}}$ $\mu_J = \sqrt{\mu_S \mu_H}$

HHgrid



*taken from this paper!

Phenomenological results

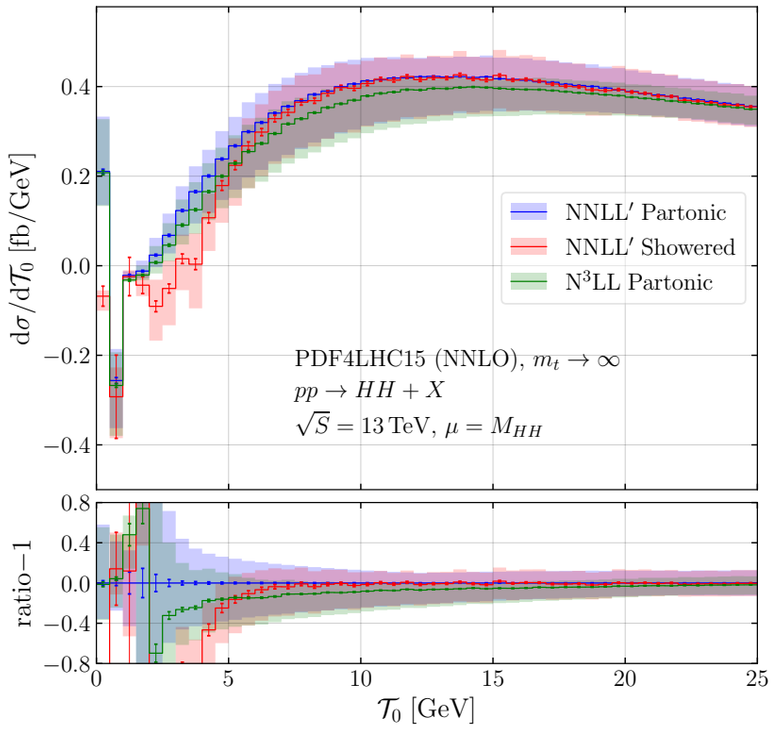


- PS ensures that the accuracy of the observable is *at least* as accurate as the PS is

  ⟶ LL accurate

  No formal claim of NNLL accuracy after PS

- Any numerical mod. should not exceed the typical size of higher-order resummation effects



From the $m_t \to \infty$ dihiggs paper

Acknowledgments

This project has received funding from the European Research Council (ERC)
under the European Union's Horizon 2020
research and innovation programme
(Grant agreement No. 101002090 COLORFREE)

European Research Council

Established by the European Commission