Differential Observables for Higgs-strahlung process to all orders in EFT

Sourav Bera

Tata Institute of Fundamental Research (TIFR), Mumbai

In collaboration with : Debsubhra Chakraborty, Susobhan Chattopadhyay, Rick S Gupta

(Based on : arXiv:2512.xxxxx)

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Motivation

- After the discovery of Higgs boson, no new particles observed in LHC.
  $\Longrightarrow$ New Physics scale (Λ) $\gg$ EW scale.

- Motivates us to study New Physics effects at low energies through the use of EFTs.

- So far, given the present experimental sensitivities, most studies consider lowest order corrections in EFT, e.g. dimension-6 SMEFT or NLO operators at HEFT.

- As we enter the era of higher luminosities (HL-LHC) and higher experimental sensitivities :

  Experimental aim : Measurement of differential (both energy and angular) distributions

  multivariate observables.

- Theoretical need :
  - Go beyond lower order corrections in EFT expansion, e.g : dimension-8 SMEFT operators.
  - A systematic approach to study the effect of higher dimensional operators on differential observables — in particular, the angular distributions.

Higgs-strahlung process to all EFT orders

- We want to study differential (especially, angular) distributions of $pp \to Z(l^+l^-)\,h(\bar{b}b)$ to all EFT orders.

- The most natural formalism for this is the partial-wave expansion.

  For $ff \to Z(l^+l^-)h$ process :

$ff \to Zh$

$$\mathscr{A}_{\lambda_1\lambda_2}(s,\Theta,\theta,\phi) = \frac{-ig_l^Z}{\Gamma_Z} \sum_{\lambda_Z=0,\pm} \left[\sum_{J=J_{min}}^{\infty} (2J+1)\, d^J_{\sigma,\lambda_Z}(\Theta)\, a^J_{\lambda_1\lambda_2;\lambda_Z}(s) \right] d^{J=1}_{1,\lambda_Z}(\theta)\, e^{i\lambda_Z\phi}$$

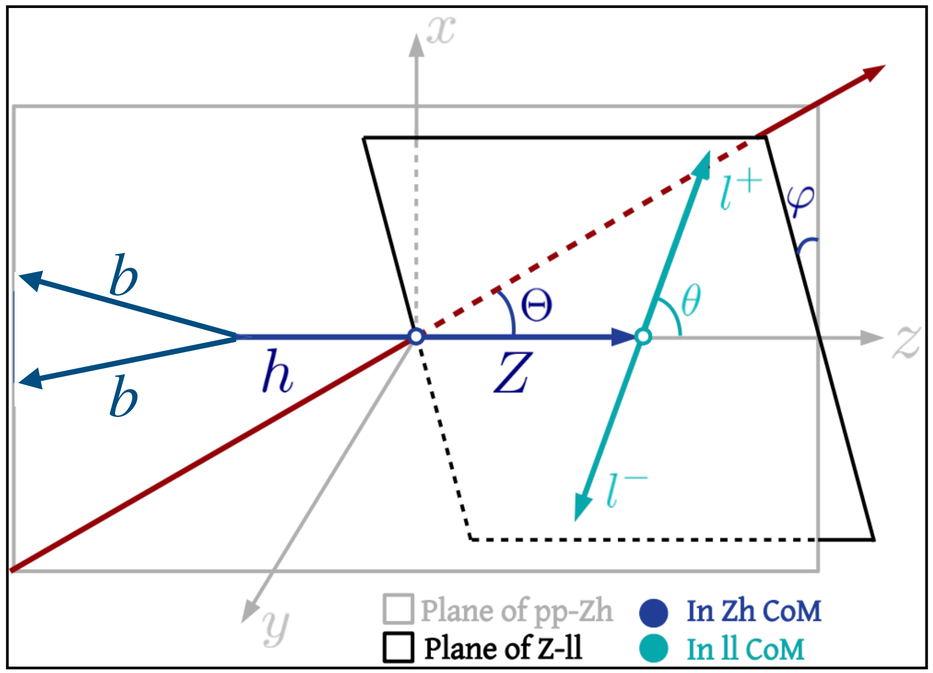
$Z-$decay amplitude

$\lambda_1, \lambda_2 \leftarrow$ Helicities of initial fermions and $\sigma = \lambda_1 - \lambda_2$
$\lambda_Z \leftarrow$ Helicity of $Z-$boson

- This expansion has infinite number of parameters. No information about dimensional analysis.

- Impossible to truncate the process to finite number of parameters. Need EFT input for truncation.

- At a given EFT order, how many partial wave coefficients should be considered?



Adapted from : S. Banerjee et. al., Phys.Rev.D 100(2019)11

EFT-informed partial-wave expansion

EFT expansion

$$\mathscr{L}_{EFT} = \sum_i \frac{C_i}{\Lambda^{d_i-4}} \mathcal{O}_i^{d_i>4}$$

Partial-wave expansion

$$\{a^J_{\lambda_a\lambda_b;\lambda_c\lambda_d}\}$$

- We will find the mapping between EFT expansion and partial-wave expansion to all orders.
- We will organize the EFT expansion following : Chang, Chen, Liu and Luty, JHEP07(2023)030 with some modifications.

Amplitudes : Mandelstam Principals & Descendants

- An convenient EFT parametrization for Higgs/electroweak processes using amplitude approach.

- The tree level on-shell amplitude that captures beyond SM correction to Higgs-strahlung process $ff \longrightarrow Zh$:

$$\delta\mathscr{M} = \frac{(\bar{v}_2 \not{\epsilon}_3^* u_1)}{\Lambda}\left[C^1_{(0,0)} + C^1_{(1,0)}\frac{s}{M^2} + C^1_{(0,1)}\frac{t-u}{M^2} + \mathcal{O}(E^4/\Lambda^4)\right]$$
$$+ \frac{i}{\Lambda}\epsilon^{\mu\nu\rho\sigma}(\bar{v}_2\gamma^\mu u_1)(p_1-p_2)_\nu(p_{3\rho}\epsilon^*_{3\sigma})\left[C^2_{(0,0)} + C^2_{(1,0)}\frac{s}{M^2} + C^2_{(0,1)}\frac{t-u}{M^2} + \mathcal{O}(E^4/M^4)\right] + \sum_{k=3}^{12}\sum_{m,n=0}^{\infty} C^k_{(m,n)}\frac{P_k}{\Lambda^{d_k-4}}\frac{s^m(t-u)^n}{(\Lambda^2)^{m+n}}$$

Principal : $hZ_\mu\bar{\psi}\gamma^\mu\psi$

Principal : $ih\widetilde{Z}_{\mu\nu}\bar{\psi}\gamma^\mu\overleftrightarrow{\partial^\nu}\psi$

- The series in Mandelstam variables are called descendants.

- For a given process, there are a finite number of principal amplitudes.

Chang, Chen, Liu and Luty, JHEP07(2023)030

Effective Lagrangian

- The effective Lagrangian corresponding to the EFT expansion is :

$$\mathscr{L}_{CCLL} = \mathscr{L}_{SM} + \sum_i \frac{C_i^{3p}}{\Lambda^{d_i-4}} O_i^{3-point} + \sum_{k=1}^{N_p} \sum_{m,n=0}^{\infty} \frac{C_{(m,n)}^k}{\Lambda^{d_k-4+2(m+n)}} O_{m,n}^k$$

$O_i^{3-point}$: correction to 3-point vertices

$O_{(m,n)}^k$: operator corresponding to amplitude $s^m(t-u)^n P_k$

Example :

$$O_{(m,0)}^2 = hZ_\mu \Box_f^m (\bar{\psi}_L \gamma^\mu \psi_L)$$

$$O_{(0,n)}^2 = -i^n \bar{\psi}_L \gamma^\mu \Box^n (\psi_L h) Z_\mu + h.c.$$

i	$\mathcal{O}_{hZZ/\bar{f}ZZ}^{3-point}$	CP	$d_\mathcal{O}$
1	$hZ_\mu Z^\mu$	+	3
2	$Z_\mu \bar{\psi}_R \gamma^\mu \psi_R$	+	4
3	$Z_\mu \bar{\psi}_L \gamma^\mu \psi_L$	+	
4	$hZ_{\mu\nu} Z^{\mu\nu}$	+	5
5	$hZ_{\mu\nu} \tilde{Z}^{\mu\nu}$	−	
6	$Z_{\mu\nu} \bar{\psi}_L \sigma^{\mu\nu} \psi_R + \text{h.c.}$	+	
7	$iZ_{\mu\nu} \bar{\psi}_L \sigma^{\mu\nu} \psi_R + \text{h.c.}$	−	

i	$\mathcal{O}_{(0,0)}^k$	CP	$d_\mathcal{O}$
$J=1$			
1	$hZ^\mu \bar{\psi}_R \gamma_\mu \psi_R$	+	5
2	$hZ^\mu \bar{\psi}_L \gamma_\mu \psi_L$	+	
3	$ih\widetilde{Z}_{\mu\nu}(\bar{\psi}_R \gamma^\mu \overleftrightarrow{\partial}_\nu \psi_R)$	+	7
4	$ih\widetilde{Z}_{\mu\nu}(\bar{\psi}_L \gamma^\mu \overleftrightarrow{\partial}_\nu \psi_L)$	+	
5	$h\widetilde{Z}_{\mu\nu} \partial^\mu (\bar{\psi}_R \gamma^\nu \psi_R)$	−	
6	$h\widetilde{Z}_{\mu\nu} \partial^\mu (\bar{\psi}_L \gamma^\nu \psi_L)$	−	
7	$hZ^{\mu\nu} \bar{\psi}_L \sigma_{\mu\nu} \psi_R + \text{h.c}$	+	6
8	$ihZ^{\mu\nu} \bar{\psi}_L \sigma_{\mu\nu} \psi_R + \text{h.c}$	−	
9	$ihZ^\mu (\bar{\psi}_L \overleftrightarrow{\partial}_\mu \psi_R) + \text{h.c.}$	+	
10	$hZ^\mu (\bar{\psi}_L \overleftrightarrow{\partial}_\mu \psi_R) + \text{h.c.}$	−	
$J=0$			
11	$hZ^\mu \partial_\mu (\bar{\psi}_L \psi_R) + \text{h.c.}$	−	6
12	$ihZ^\mu \partial_\mu (\bar{\psi}_L \psi_R) + \text{h.c.}$	+	

Principal operators for Higgs-strahlung process

Chang, Chen, Liu and Luty, JHEP07(2023)030

Mapping CCLL Lagrangian to partial-waves?

- The CCLL Lagrangian systematically parametrizes EFT to all orders.
- For Higgs-strahlung process, the principals are fixed$-J$ operators $\Longrightarrow$ they contribute to partial-wave coefficients with single J value.
- Amplitudes with 3-point corrections also have fixed$-J$ as they are mediated by spin$-J$ (here, $J = 1$) particles.
- $s-$descendants will also correspond to fixed$-J$ operators.
- $(t-u)^n \propto (\cos\Theta)^n \Longrightarrow$ The operator $O^k_{(0,n)}$ corresponding to amplitude $(t-u)^n P_k$ will contribute to partial-wave coefficients with different J values from $J = J_k, J_k+1, \ldots, J_k+n$.
- For CCLL EFT basis, partial-wave amplitudes at a fixed$-J$ will receive contributions from infinite number of operators

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- For Higgs-strahlung process, the principals are fixed$-J$ operators $\Longrightarrow$ they contribute to partial-wave coefficients with single J value.
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- For CCLL EFT basis, partial-wave amplitudes at a fixed$-J$ will receive contributions from infinite number of operators

Not the most convenient basis for the mapping

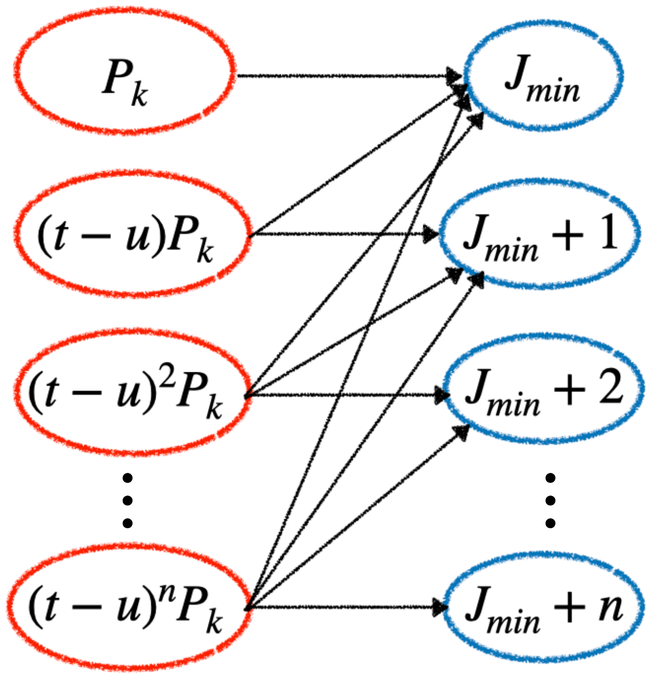
Definition of EFT basis

CCLL Basis

- Any operator can be written as a sum of fixed$-J$ operators :

$$O^k_{(m,n)} = \sum_J \alpha_{(k|J)} O^k_{(m,n|J)}$$

$(t-u)-$ descendants

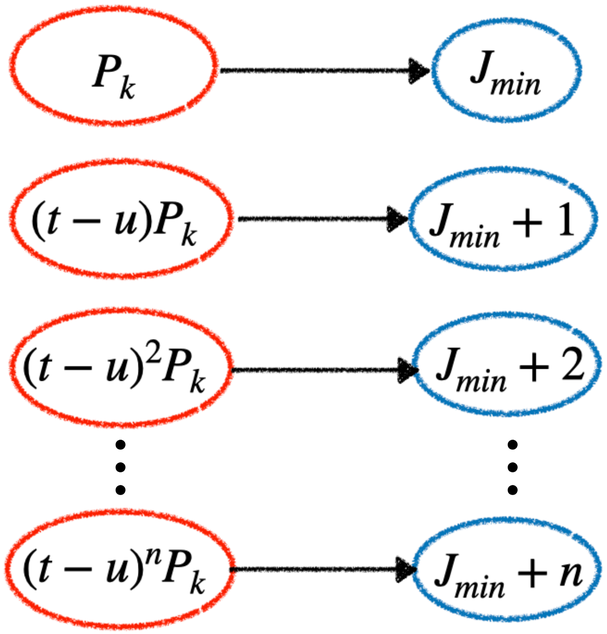


Our basis

- All operators are traded for their maximum$-J$ component :

$$O^k_{(m,n)} \longrightarrow O^k_{(m,n|J_{n,k})} \quad \text{where } J_{n,k} = J_k + n$$

$J-$descendants



Definition of EFT basis

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Our basis

- All operators are traded for their maximum$-J$ component :

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$J-$descendants

- The principal operators and their $s-$descendants remain same in both the basis.

- Partial-waves with minimum allowed J ($J_{min} = \max\{|\sigma|, |\lambda|\}$) value get contributions from N_P operators at a given order in s/Λ^2.

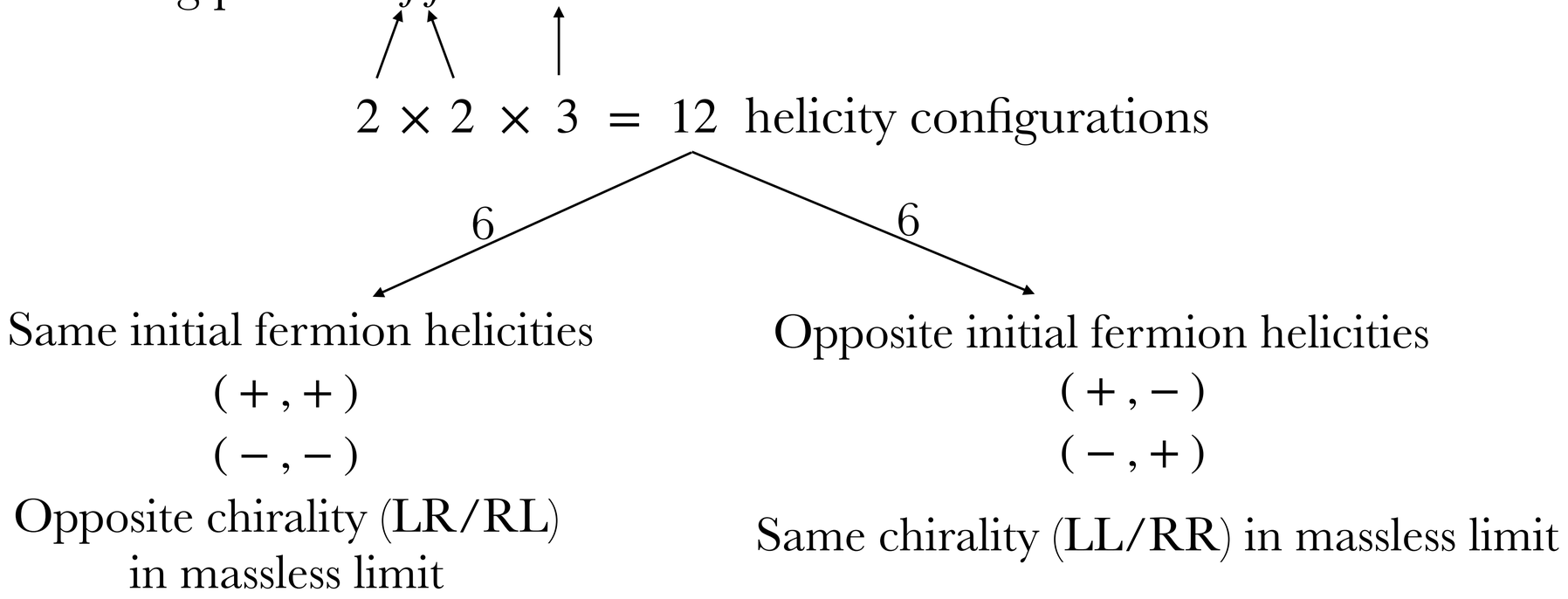
- Partial-waves with a given $J > J_{min}$ receive contributions from N_P $J-$descendants at a given order in s.

- The Lagrangian in our basis : $\mathscr{L} = \mathscr{L}_{SM} + \sum_i \frac{\mathscr{C}^{3p}_i}{\Lambda^{d_i-4}} O^{3-point}_i + \sum_{k=1}^{N_p} \sum_{m,n=0}^{\infty} \frac{\mathscr{C}^k_{(m,J_{n,k})}}{\Lambda^{d_k-4+2(m+n)}} O^k_{(m,n|J_{k,n})}$

$$\mathscr{C}^k_{(m,n)} = v^2\Lambda^2 \left(\frac{1}{\Lambda}\right)^{n_\partial} \left(\frac{1}{v}\right)^{n_h} \left(\frac{g_Z}{\Lambda}\right)^{n_Z} \left(\frac{g_\psi}{\Lambda^{3/2}}\right)^{n_\psi} c^k_{(m,n)}$$

Number of helicity configurations

- For Higgs-strahlung process : $ff \longrightarrow Zh$



- A given helicity configuration corresponds to a complex partial-wave amplitude.

- At a given $-J$, there are 24 partial-wave amplitudes for this process.

Number of independent partial-wave coefficients

- Unitarity of $\mathcal{S}$ - matrix implies :

$$\mathcal{S}\mathcal{S}^\dagger = I$$

$\Longrightarrow \; \mathcal{T} - \mathcal{T}^\dagger = i\mathcal{T}\mathcal{T}^\dagger$, where $\mathcal{S} = I + i\mathcal{T}$

0 (for effective operators below new-physics cut-off, $s \ll \Lambda^2$)

CPT

Constraints on the partial-wave amplitudes :

$$a^J_{\lambda_1,\lambda_2;\,\lambda_Z}(s) = (-1)^{J-\sigma}\, a^{J*}_{-\lambda_2,-\lambda_1;\,-\lambda_Z}(s)$$

- For a fixed$-J$, the number of possible partial-wave amplitudes : $N_h = \prod_i (2s_i + 1)$
  (For Higgs-strahlung process, $N_h = 12$)

- Note : $N_P = N_h = 12$. Do we understand this?

Mapping principals to minimum–J partial waves

EFT WCs	Partial wave Amplitudes					
	$(+-;0)$	$(+-;+)$	$(+-;-)$	$(-+;0)$	$(-+;+)$	$(-+;-)$
c_1^{3p}	α_0	α_0	α_0	α_0	α_0	α_0
c_2^{3p}	$-\frac{m_Z^2}{\Lambda^2}\alpha_0$	$-\frac{m_Z^2}{\Lambda^2}\alpha_0$	$-\frac{m_Z^2}{\Lambda^2}\alpha_0$	0	0	0
c_3^{3p}	0	0	0	$-\frac{m_Z^2}{\Lambda^2}\alpha_0$	$-\frac{m_Z^2}{\Lambda^2}\alpha_0$	$-\frac{m_Z^2}{\Lambda^2}\alpha_0$
c_4^{3p}	$\frac{s}{\Lambda^2}\left(\frac{4\varepsilon}{\alpha_1}\alpha_0\right)$	$\frac{s}{\Lambda^2}\alpha_1\alpha_0$	$\frac{s}{\Lambda^2}\alpha_1\alpha_0$	$\frac{s}{\Lambda^2}\left(\frac{4\varepsilon}{\alpha_1}\alpha_0\right)$	$\frac{s}{\Lambda^2}\alpha_1\alpha_0$	$\frac{s}{\Lambda^2}\alpha_1\alpha_0$
c_5^{3p}	0	$-i\frac{s}{\Lambda^2}\alpha_2\alpha_0$	$i\frac{s}{\Lambda^2}\alpha_2\alpha_0$	0	$-i\frac{s}{\Lambda^2}\alpha_2\alpha_0$	$i\frac{s}{\Lambda^2}\alpha_2\alpha_0$
$c_{(0,0)}^{1}$	$-2\frac{s}{\Lambda^2}$	$-2\frac{s}{\Lambda^2}$	$-2\frac{s}{\Lambda^2}$	0	0	0
$c_{(0,0)}^{2}$	0	0	0	$-2\frac{s}{\Lambda^2}$	$-2\frac{s}{\Lambda^2}$	$-2\frac{s}{\Lambda^2}$
$c_{(1,0)}^{1}$	$-2\frac{s^2}{\Lambda^4}$	$-2\frac{s^2}{\Lambda^4}$	$-2\frac{s^2}{\Lambda^4}$	0	0	0
$c_{(1,0)}^{2}$	0	0	0	$-2\frac{s^2}{\Lambda^4}$	$-2\frac{s^2}{\Lambda^4}$	$-2\frac{s^2}{\Lambda^4}$
$c_{(0,0)}^{3}$	$-\frac{s^2}{\Lambda^4}\left(\frac{4\varepsilon}{\alpha_1}\right)$	$-\frac{s^2}{\Lambda^4}\alpha_1$	$-\frac{s^2}{\Lambda^4}\alpha_1$	0	0	0
$c_{(0,0)}^{4}$	0	0	0	$\frac{s^2}{\Lambda^4}\left(\frac{4\varepsilon}{\alpha_1}\right)$	$\frac{s^2}{\Lambda^4}\alpha_1$	$\frac{s^2}{\Lambda^4}\alpha_1$
$c_{(0,0)}^{5}$	0	$i\frac{s^2}{\Lambda^4}\alpha_2$	$-i\frac{s^2}{\Lambda^4}\alpha_2$	0	0	0
$c_{(0,0)}^{6}$	0	0	0	0	$i\frac{s^2}{\Lambda^4}\alpha_2$	$-i\frac{s^2}{\Lambda^4}\alpha_2$
$c_{(m,0)}^{k}$	$(s/\Lambda^2)^m\ \vec{V}_{(0,0)}^{k}$					

- We work in the limit of massless fermions

  ⇓

  Operators with fermion chiralities LL or RR and LR/RL contribute to disjoint set of helicity configurations.

- Here, $\varepsilon = \dfrac{m_Z^2}{s}$, $\alpha_0 = \dfrac{1}{1-\varepsilon}$ and α_1, α_2 are kinematic factors arising from energy and momentum of Z–boson.

- Mapping of $J = 1$ partial-waves to all orders in EFT.

Contribution of all operators to $J = 1$ partial waves up to arbitrarily higher dimensions can be determined.

Mapping principals to minimum−J partial waves

EFT WCs	Partial wave Amplitudes					
	$(+-;0)$	$(+-;+)$	$(+-;-)$	$(-+;0)$	$(-+;+)$	$(-+;-)$
c_1^{3p}	α_0	α_0	α_0	α_0	α_0	α_0
c_2^{3p}	$-\frac{m_Z^2}{\Lambda^2}\alpha_0$	$-\frac{m_Z^2}{\Lambda^2}\alpha_0$	$-\frac{m_Z^2}{\Lambda^2}\alpha_0$	0	0	0
c_3^{3p}	0	0	0	$-\frac{m_Z^2}{\Lambda^2}\alpha_0$	$-\frac{m_Z^2}{\Lambda^2}\alpha_0$	$-\frac{m_Z^2}{\Lambda^2}\alpha_0$
c_4^{3p}	$\frac{s}{\Lambda^2}\left(\frac{4\varepsilon}{\alpha_1}\alpha_0\right)$	$\frac{s}{\Lambda^2}\alpha_1\alpha_0$	$\frac{s}{\Lambda^2}\alpha_1\alpha_0$	$\frac{s}{\Lambda^2}\left(\frac{4\varepsilon}{\alpha_1}\alpha_0\right)$	$\frac{s}{\Lambda^2}\alpha_1\alpha_0$	$\frac{s}{\Lambda^2}\alpha_1\alpha_0$
c_5^{3p}	0	$-i\frac{s}{\Lambda^2}\alpha_2\alpha_0$	$i\frac{s}{\Lambda^2}\alpha_2\alpha_0$	0	$-i\frac{s}{\Lambda^2}\alpha_2\alpha_0$	$i\frac{s}{\Lambda^2}\alpha_2\alpha_0$
$c_{(0,0)}^1$	$-2\frac{s}{\Lambda^2}$	$-2\frac{s}{\Lambda^2}$	$-2\frac{s}{\Lambda^2}$	0	0	0
$c_{(0,0)}^2$	0	0	0	$-2\frac{s}{\Lambda^2}$	$-2\frac{s}{\Lambda^2}$	$-2\frac{s}{\Lambda^2}$
$c_{(1,0)}^1$	$-2\frac{s^2}{\Lambda^4}$	$-2\frac{s^2}{\Lambda^4}$	$-2\frac{s^2}{\Lambda^4}$	0	0	0
$c_{(1,0)}^2$	0	0	0	$-2\frac{s^2}{\Lambda^4}$	$-2\frac{s^2}{\Lambda^4}$	$-2\frac{s^2}{\Lambda^4}$
$c_{(0,0)}^3$	$-\frac{s^2}{\Lambda^4}\left(\frac{4\varepsilon}{\alpha_1}\right)$	$-\frac{s^2}{\Lambda^4}\alpha_1$	$-\frac{s^2}{\Lambda^4}\alpha_1$	0	0	0
$c_{(0,0)}^4$	0	0	0	$\frac{s^2}{\Lambda^4}\left(\frac{4\varepsilon}{\alpha_1}\right)$	$\frac{s^2}{\Lambda^4}\alpha_1$	$\frac{s^2}{\Lambda^4}\alpha_1$
$c_{(0,0)}^5$	0	$i\frac{s^2}{\Lambda^4}\alpha_2$	$-i\frac{s^2}{\Lambda^4}\alpha_2$	0	0	0
$c_{(0,0)}^6$	0	0	0	0	$i\frac{s^2}{\Lambda^4}\alpha_2$	$-i\frac{s^2}{\Lambda^4}\alpha_2$
$c_{(m,0)}^k$	$(s/\Lambda^2)^m\ \vec{V}_{(0,0)}^k$					

- In high-energy limit ($s \gg m_{h,W,Z}^2$), $\varepsilon \to 0$ and $\alpha_i \to 1$.
- In this limit, each row, $\vec{V}_i = \vec{r}_i \left(s/\Lambda^2\right)^n$, where $\vec{r}_i$ is an array of $\mathcal{O}(1)$ numbers of length $N_h^{LL/RR}$.
- In a non-redundant EFT basis, all these vectors $\vec{r}_i$ must be linearly independent.
  $\Longrightarrow$ $N_P = N_h$ in high energy-limit.
- 4-point principal operators become $s-$descendants of the 3-point couplings.

Mapping principals to minimum–J partial waves

EFT WCs	Partial wave Amplitudes					
	$(+-;0)$	$(+-;+)$	$(+-;-)$	$(-+;0)$	$(-+;+)$	$(-+;-)$
c_1^{3p}	α_0	α_0	α_0	α_0	α_0	α_0
c_2^{3p}	$-\frac{m_Z^2}{\Lambda^2}\alpha_0$	$-\frac{m_Z^2}{\Lambda^2}\alpha_0$	$-\frac{m_Z^2}{\Lambda^2}\alpha_0$	0	0	0
c_3^{3p}	0	0	0	$-\frac{m_Z^2}{\Lambda^2}\alpha_0$	$-\frac{m_Z^2}{\Lambda^2}\alpha_0$	$-\frac{m_Z^2}{\Lambda^2}\alpha_0$
c_4^{3p}	$\frac{s}{\Lambda^2}\left(\frac{4\varepsilon}{\alpha_1}\alpha_0\right)$	$\frac{s}{\Lambda^2}\alpha_1\alpha_0$	$\frac{s}{\Lambda^2}\alpha_1\alpha_0$	$\frac{s}{\Lambda^2}\left(\frac{4\varepsilon}{\alpha_1}\alpha_0\right)$	$\frac{s}{\Lambda^2}\alpha_1\alpha_0$	$\frac{s}{\Lambda^2}\alpha_1\alpha_0$
c_5^{3p}	0	$-i\frac{s}{\Lambda^2}\alpha_2\alpha_0$	$i\frac{s}{\Lambda^2}\alpha_2\alpha_0$	0	$-i\frac{s}{\Lambda^2}\alpha_2\alpha_0$	$i\frac{s}{\Lambda^2}\alpha_2\alpha_0$
$c_{(0,0)}^1$	$-2\frac{s}{\Lambda^2}$	$-2\frac{s}{\Lambda^2}$	$-2\frac{s}{\Lambda^2}$	0	0	0
$c_{(0,0)}^2$	0	0	0	$-2\frac{s}{\Lambda^2}$	$-2\frac{s}{\Lambda^2}$	$-2\frac{s}{\Lambda^2}$
$c_{(1,0)}^1$	$-2\frac{s^2}{\Lambda^4}$	$-2\frac{s^2}{\Lambda^4}$	$-2\frac{s^2}{\Lambda^4}$	0	0	0
$c_{(1,0)}^2$	0	0	0	$-2\frac{s^2}{\Lambda^4}$	$-2\frac{s^2}{\Lambda^4}$	$-2\frac{s^2}{\Lambda^4}$
$c_{(0,0)}^3$	$-\frac{s^2}{\Lambda^4}\left(\frac{4\varepsilon}{\alpha_1}\right)$	$-\frac{s^2}{\Lambda^4}\alpha_1$	$-\frac{s^2}{\Lambda^4}\alpha_1$	0	0	0
$c_{(0,0)}^4$	0	0	0	$\frac{s^2}{\Lambda^4}\left(\frac{4\varepsilon}{\alpha_1}\right)$	$\frac{s^2}{\Lambda^4}\alpha_1$	$\frac{s^2}{\Lambda^4}\alpha_1$
$c_{(0,0)}^5$	0	$i\frac{s^2}{\Lambda^4}\alpha_2$	$-i\frac{s^2}{\Lambda^4}\alpha_2$	0	0	0
$c_{(0,0)}^6$	0	0	0	0	$i\frac{s^2}{\Lambda^4}\alpha_2$	$-i\frac{s^2}{\Lambda^4}\alpha_2$
$c_{(m,0)}^k$	$(s/\Lambda^2)^m\ \vec{V}_{(0,0)}^k$					

$\mathcal{O}(s^0)$: Two operators contribute to 6 independent partial-wave amplitudes $\Longrightarrow$ correlations among partial waves.

$$a^1_{+-+} - a^1_{+-0} = 0\,, \quad a^1_{+--} - a^1_{+-0} = 0$$
$$a^1_{-++} - a^1_{-+0} = 0\,, \quad a^1_{-+-} - a^1_{-+0} = 0$$

Mapping principals to minimum–J partial waves

EFT WCs	Partial wave Amplitudes					
	$(+-;\,0)$	$(+-;\,+)$	$(+-;\,-)$	$(-+;\,0)$	$(-+;\,+)$	$(-+;\,-)$
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c_4^{3p}	$\frac{s}{\Lambda^2}\left(\frac{4\varepsilon}{\alpha_1}\alpha_0\right)$	$\frac{s}{\Lambda^2}\alpha_1\alpha_0$	$\frac{s}{\Lambda^2}\alpha_1\alpha_0$	$\frac{s}{\Lambda^2}\left(\frac{4\varepsilon}{\alpha_1}\alpha_0\right)$	$\frac{s}{\Lambda^2}\alpha_1\alpha_0$	$\frac{s}{\Lambda^2}\alpha_1\alpha_0$
c_5^{3p}	0	$-i\frac{s}{\Lambda^2}\alpha_2\alpha_0$	$i\frac{s}{\Lambda^2}\alpha_2\alpha_0$	0	$-i\frac{s}{\Lambda^2}\alpha_2\alpha_0$	$i\frac{s}{\Lambda^2}\alpha_2\alpha_0$
$c_{(0,0)}^{1}$	$-2\frac{s}{\Lambda^2}$	$-2\frac{s}{\Lambda^2}$	$-2\frac{s}{\Lambda^2}$	0	0	0
$c_{(0,0)}^{2}$	0	0	0	$-2\frac{s}{\Lambda^2}$	$-2\frac{s}{\Lambda^2}$	$-2\frac{s}{\Lambda^2}$
$c_{(1,0)}^{1}$	$-2\frac{s^2}{\Lambda^4}$	$-2\frac{s^2}{\Lambda^4}$	$-2\frac{s^2}{\Lambda^4}$	0	0	0
$c_{(1,0)}^{2}$	0	0	0	$-2\frac{s^2}{\Lambda^4}$	$-2\frac{s^2}{\Lambda^4}$	$-2\frac{s^2}{\Lambda^4}$
$c_{(0,0)}^{3}$	$-\frac{s^2}{\Lambda^4}\left(\frac{4\varepsilon}{\alpha_1}\right)$	$-\frac{s^2}{\Lambda^4}\alpha_1$	$-\frac{s^2}{\Lambda^4}\alpha_1$	0	0	0
$c_{(0,0)}^{4}$	0	0	0	$\frac{s^2}{\Lambda^4}\left(\frac{4\varepsilon}{\alpha_1}\right)$	$\frac{s^2}{\Lambda^4}\alpha_1$	$\frac{s^2}{\Lambda^4}\alpha_1$
$c_{(0,0)}^{5}$	0	$i\frac{s^2}{\Lambda^4}\alpha_2$	$-i\frac{s^2}{\Lambda^4}\alpha_2$	0	0	0
$c_{(0,0)}^{6}$	0	0	0	0	$i\frac{s^2}{\Lambda^4}\alpha_2$	$-i\frac{s^2}{\Lambda^4}\alpha_2$
$c_{(m,0)}^{k}$	$(s/\Lambda^2)^m\ \vec{V}_{(0,0)}^{k}$					

$\mathcal{O}(s^0)$: Two operators contribute to 6 independent partial-wave amplitudes $\Longrightarrow$ correlations among partial waves.

$$a^1_{+-+} - a^1_{+-0} = 0\,, \quad a^1_{+--} - a^1_{+-0} = 0$$
$$a^1_{-++} - a^1_{-+0} = 0\,, \quad a^1_{-+-} - a^1_{-+0} = 0$$

$\mathcal{O}(s^1)$: Two of the combinations receive non-zero contributions, but two correlations remain.

$$a^1_{+-+} - a^1_{+-0} - a^1_{-++} + a^1_{-+0} = 0$$
$$a^1_{+--} - a^1_{+-0} - a^1_{-+-} + a^1_{-+0} = 0$$

Mapping principals to minimum–J partial waves

EFT WCs	Partial wave Amplitudes					
	$(+-;0)$	$(+-;+)$	$(+-;-)$	$(-+;0)$	$(-+;+)$	$(-+;-)$
c_1^{3p}	α_0	α_0	α_0	α_0	α_0	α_0
c_2^{3p}	$-\frac{m_Z^2}{\Lambda^2}\alpha_0$	$-\frac{m_Z^2}{\Lambda^2}\alpha_0$	$-\frac{m_Z^2}{\Lambda^2}\alpha_0$	0	0	0
c_3^{3p}	0	0	0	$-\frac{m_Z^2}{\Lambda^2}\alpha_0$	$-\frac{m_Z^2}{\Lambda^2}\alpha_0$	$-\frac{m_Z^2}{\Lambda^2}\alpha_0$
c_4^{3p}	$\frac{s}{\Lambda^2}\left(\frac{4\varepsilon}{\alpha_1}\alpha_0\right)$	$\frac{s}{\Lambda^2}\alpha_1\alpha_0$	$\frac{s}{\Lambda^2}\alpha_1\alpha_0$	$\frac{s}{\Lambda^2}\left(\frac{4\varepsilon}{\alpha_1}\alpha_0\right)$	$\frac{s}{\Lambda^2}\alpha_1\alpha_0$	$\frac{s}{\Lambda^2}\alpha_1\alpha_0$
c_5^{3p}	0	$-i\frac{s}{\Lambda^2}\alpha_2\alpha_0$	$i\frac{s}{\Lambda^2}\alpha_2\alpha_0$	0	$-i\frac{s}{\Lambda^2}\alpha_2\alpha_0$	$i\frac{s}{\Lambda^2}\alpha_2\alpha_0$
$c_{(0,0)}^1$	$-2\frac{s}{\Lambda^2}$	$-2\frac{s}{\Lambda^2}$	$-2\frac{s}{\Lambda^2}$	0	0	0
$c_{(0,0)}^2$	0	0	0	$-2\frac{s}{\Lambda^2}$	$-2\frac{s}{\Lambda^2}$	$-2\frac{s}{\Lambda^2}$
$c_{(1,0)}^1$	$-2\frac{s^2}{\Lambda^4}$	$-2\frac{s^2}{\Lambda^4}$	$-2\frac{s^2}{\Lambda^4}$	0	0	0
$c_{(1,0)}^2$	0	0	0	$-2\frac{s^2}{\Lambda^4}$	$-2\frac{s^2}{\Lambda^4}$	$-2\frac{s^2}{\Lambda^4}$
$c_{(0,0)}^3$	$-\frac{s^2}{\Lambda^4}\left(\frac{4\varepsilon}{\alpha_1}\right)$	$-\frac{s^2}{\Lambda^4}\alpha_1$	$-\frac{s^2}{\Lambda^4}\alpha_1$	0	0	0
$c_{(0,0)}^4$	0	0	0	$\frac{s^2}{\Lambda^4}\left(\frac{4\varepsilon}{\alpha_1}\right)$	$\frac{s^2}{\Lambda^4}\alpha_1$	$\frac{s^2}{\Lambda^4}\alpha_1$
$c_{(0,0)}^5$	0	$i\frac{s^2}{\Lambda^4}\alpha_2$	$-i\frac{s^2}{\Lambda^4}\alpha_2$	0	0	0
$c_{(0,0)}^6$	0	0	0	0	$i\frac{s^2}{\Lambda^4}\alpha_2$	$-i\frac{s^2}{\Lambda^4}\alpha_2$
$c_{(m,0)}^k$	$(s/\Lambda^2)^m\ \vec{V}_{(0,0)}^k$					

$\mathcal{O}(s^0)$: Two operators contribute to 6 independent partial-wave amplitudes $\Longrightarrow$ correlations among partial waves.

$$a^1_{+-+} - a^1_{+-0} = 0\,, \quad a^1_{+--} - a^1_{+-0} = 0$$
$$a^1_{-++} - a^1_{-+0} = 0\,, \quad a^1_{-+-} - a^1_{-+0} = 0$$

$\mathcal{O}(s^1)$: Two of the combinations receive non-zero contributions, but two correlations remain.

$$a^1_{+-+} - a^1_{+-0} - a^1_{-++} + a^1_{-+0} = 0$$
$$a^1_{+--} - a^1_{+-0} - a^1_{-+-} + a^1_{-+0} = 0$$

$\mathcal{O}(s^2)$: Two more contributing operators at this order, and all partial wave coefficients become independent.

Principal Directions

$$\mathbf{a}_{1,1} = (1+\varepsilon_2)a^1_{+-0} + (1+\varepsilon_2)a^1_{-+0} - \frac{\varepsilon_2}{2}(a^1_{+-+} + a^1_{+--} + a^1_{-++} + a^1_{-+-})$$

$$\mathbf{a}_{1,2} = (1+\varepsilon_2)a^1_{+-0} - (1+\varepsilon_2)a^1_{-+0} - \frac{\varepsilon_2}{2}(a^1_{+-+} + a^1_{+--} - a^1_{-++} - a^1_{-+-})$$

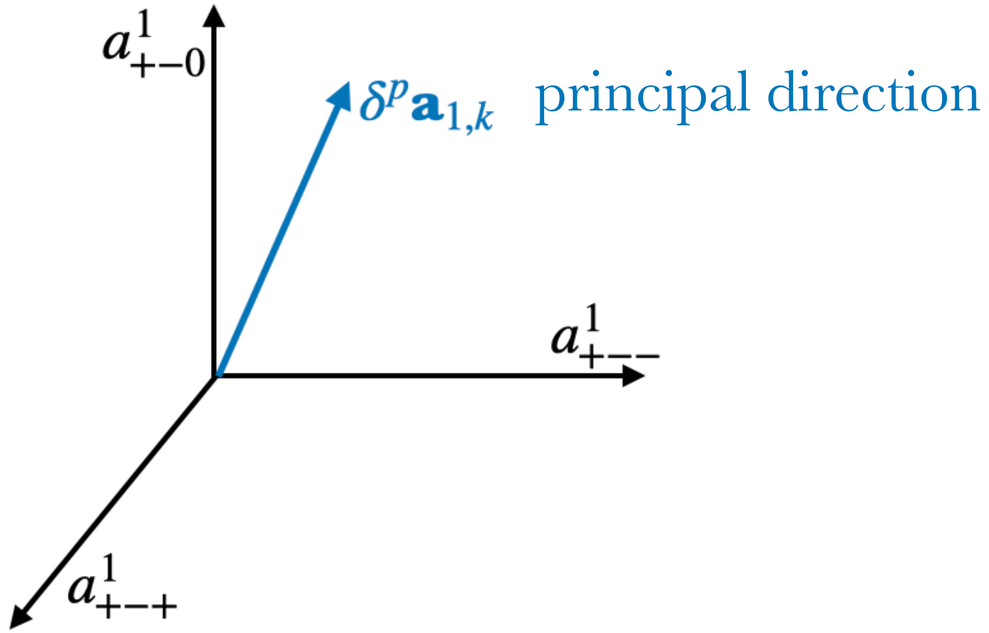
$$\delta\mathbf{a}_{1,3} = \frac{1}{2\alpha_3}(a^1_{+-+} + a^1_{+--} - 2a^1_{+-0})$$

$$\delta\mathbf{a}_{1,4} = \frac{i}{2\alpha_2}(a^1_{+-+} - a^1_{+--})$$

$$\delta^2\mathbf{a}_{1,5} = \frac{1}{2\alpha_3}(a^1_{+-+} + a^1_{+--} - 2a^1_{+-0} - a^1_{-++} - a^1_{-+-} + 2a^1_{-+0})$$

$$\delta^2\mathbf{a}_{1,6} = -\frac{i}{2\alpha_2}(a^1_{+-+} + a^1_{-+-} - a^1_{+--} - a^1_{-++})$$

Linear combinations of partial-wave amplitudes which receive non-vanishing contributions at different orders of $\left(\frac{s}{\Lambda^2}\right)$



Principal Directions

$$\mathbf{a}_{1,1} = (1+\varepsilon_2)a^1_{+-0} + (1+\varepsilon_2)a^1_{-+0} - \frac{\varepsilon_2}{2}(a^1_{+-+} + a^1_{+--} + a^1_{-++} + a^1_{-+-})$$

$$\mathbf{a}_{1,2} = (1+\varepsilon_2)a^1_{+-0} - (1+\varepsilon_2)a^1_{-+0} - \frac{\varepsilon_2}{2}(a^1_{+-+} + a^1_{+--} - a^1_{-++} - a^1_{-+-})$$

$$\delta\mathbf{a}_{1,3} = \frac{1}{2\alpha_3}(a^1_{+-+} + a^1_{+--} - 2a^1_{+-0})$$

$$\delta\mathbf{a}_{1,4} = \frac{i}{2\alpha_2}(a^1_{+-+} - a^1_{+--})$$

$$\delta^2\mathbf{a}_{1,5} = \frac{1}{2\alpha_3}(a^1_{+-+} + a^1_{+--} - 2a^1_{+-0} - a^1_{-++} - a^1_{-+-} + 2a^1_{-+0})$$

$$\delta^2\mathbf{a}_{1,6} = -\frac{i}{2\alpha_2}(a^1_{+-+} + a^1_{-+-} - a^1_{+--} - a^1_{-++})$$

Linear combinations of partial-wave amplitudes which receive non-vanishing contributions at different orders of $\left(\frac{s}{\Lambda^2}\right)$

Relation to principals

$$\mathbf{a}_{1,1} = \left[2c_1^{3p} - \left(c_2^{3p} + c_3^{3p}\right)\frac{m_Z^2}{\Lambda^2}\right]\alpha_0 - 2\sum_{m=0}^{\infty}\left(\frac{s}{\Lambda^2}\right)^{m+1}(c^1_{(m,0)} + c^2_{(m,0)})$$

$$\mathbf{a}_{1,2} = (c_3^{3p} - c_2^{3p})\alpha_0\frac{m_Z^2}{\Lambda^2} - 2\sum_{m=0}^{\infty}\left(\frac{s}{\Lambda^2}\right)^{m+1}(c^1_{(m,0)} - c^2_{(m,0)})$$

$$\delta\mathbf{a}_{1,3} = \frac{s}{\Lambda^2}c_4^{3p}\,\alpha_0 - \sum_{m=0}^{\infty}\left(\frac{s}{\Lambda^2}\right)^{m+2}c^3_{(m,0)}$$

$$\delta\mathbf{a}_{1,4} = \frac{s}{\Lambda^2}c_5^{3p}\,\alpha_0 - \sum_{m=0}^{\infty}\left(\frac{s}{\Lambda^2}\right)^{m+2}c^5_{(m,0)}$$

$$\delta^2\mathbf{a}_{1,5} = -\sum_{m=0}^{\infty}\left(\frac{s}{\Lambda^2}\right)^{m+2}(c^3_{(m,0)} + c^4_{(m,0)})$$

$$\delta^2\mathbf{a}_{1,6} = \sum_{m=0}^{\infty}\left(\frac{s}{\Lambda^2}\right)^{m+2}(c^5_{(m,0)} - c^6_{(m,0)})$$

Directions in vector space of $J = J_{min}$ partial-wave coefficients that correspond to principal operators (and their s-descendants)

Higher–J partial wave amplitudes

EFT Couplings	Helicity Amplitudes					
	$(+-;\,0)$	$(+-;\,+)$	$(+-;\,-)$	$(-+;\,0)$	$(-+;\,+)$	$(-+;\,-)$
$c^1_{(0,1)}$	$i\frac{s^2}{\Lambda^4}\frac{2\alpha_2}{\sqrt{6}}$	$-i\frac{s^2}{\Lambda^4}\frac{\alpha_2}{\sqrt{2}}$	$-i\frac{s^2}{\Lambda^4}\frac{\alpha_2}{\sqrt{2}}$	0	0	0
$c^2_{(0,1)}$	0	0	0	$-i\frac{s^2}{\Lambda^4}\frac{2\alpha_2}{\sqrt{6}}$	$i\frac{s^2}{\Lambda^4}\frac{\alpha_2}{\sqrt{2}}$	$i\frac{s^2}{\Lambda^4}\frac{\alpha_2}{\sqrt{2}}$
$c^1_{(1,1)}$	$i\frac{s^3}{\Lambda^6}\frac{2\alpha_2}{\sqrt{6}}$	$-i\frac{s^3}{\Lambda^6}\frac{\alpha_2}{\sqrt{2}}$	$-i\frac{s^3}{\Lambda^6}\frac{\alpha_2}{\sqrt{2}}$	0	0	0
$c^2_{(1,1)}$	0	0	0	$-i\frac{s^3}{\Lambda^6}\frac{2\alpha_2}{\sqrt{6}}$	$i\frac{s^3}{\Lambda^6}\frac{\alpha_2}{\sqrt{2}}$	$i\frac{s^3}{\Lambda^6}\frac{\alpha_2}{\sqrt{2}}$
$c^3_{(0,1)}$	$i\frac{s^3}{\Lambda^6}\left(\frac{4\,\alpha_2\,\varepsilon}{\sqrt{6}\,\alpha_1}\right)$	$-i\frac{s^3}{\Lambda^6}\frac{\alpha_1\alpha_2}{2\sqrt{2}}$	$-i\frac{s^3}{\Lambda^6}\frac{\alpha_1\alpha_2}{2\sqrt{2}}$	0	0	0
$c^4_{(0,1)}$	0	0	0	$i\frac{s^3}{\Lambda^6}\left(\frac{4\,\alpha_2\,\varepsilon}{\sqrt{6}\,\alpha_1}\right)$	$-i\frac{s^3}{\Lambda^6}\frac{\alpha_1\alpha_2}{2\sqrt{2}}$	$-i\frac{s^3}{\Lambda^6}\frac{\alpha_1\alpha_2}{2\sqrt{2}}$
$c^5_{(0,1)}$	0	$-\frac{s^3}{\Lambda^6}\frac{\alpha_2^2}{2\sqrt{2}}$	$\frac{s^3}{\Lambda^6}\frac{\alpha_2^2}{2\sqrt{2}}$	0	0	0
$c^6_{(0,1)}$	0	0	0	0	$\frac{s^3}{\Lambda^6}\frac{\alpha_2^2}{2\sqrt{2}}$	$-\frac{s^3}{\Lambda^6}\frac{\alpha_2^2}{2\sqrt{2}}$
$c^k_{(m,1)}$	$(s/\Lambda^2)^m\vec{V}^k_{(0,1)}$					

$J = 2$ Directions

EFT Couplings	Helicity Amplitudes		
	$(+-;0)$	$(+-;+)$	$(+-;-)$
$c^6_{(0,1)}$	0	0	0
$c^k_{(m,1)}$	$(s/\Lambda^2)^m \vec{V}^k_{(0,1)}$		

$$\delta^2 \mathbf{a}_{2,1} = -\frac{i}{\alpha_2}[(1+\varepsilon_2)a^2_{+-0} + \frac{\varepsilon_2}{\sqrt{3}}(a^2_{+--} + a^2_{+-+})]$$

$$\delta^2 \mathbf{a}_{2,2} = \frac{i}{\alpha_2}[(1+\varepsilon_2)a^2_{-+0} + \frac{\varepsilon_2}{\sqrt{3}}(a^2_{-+-} + a^2_{-++})]$$

$$\delta^3 \mathbf{a}_{2,3} = \frac{i}{\alpha_2\alpha_3}(a^2_{+--} + a^2_{+-+} + \sqrt{3}\, a^2_{+-0})$$

$$\delta^3 \mathbf{a}_{2,4} = \frac{1}{\alpha_2^2}(a^2_{+--} - a^2_{+-+})$$

$$\delta^3 \mathbf{a}_{2,5} = \frac{i}{\alpha_2\alpha_3}(a^2_{-+-} + a^2_{-++} + \sqrt{3}\, a^2_{-+0})$$

$$\delta^3 \mathbf{a}_{2,6} = \frac{1}{\alpha_2^2}(a^2_{-++} - a^2_{-+-})$$

Relation to principals and descendants

$$\delta^2 \mathbf{a}_{2,1} = \sqrt{\frac{2}{3}} \sum_{m=0}^{\infty} \left(\frac{s}{\Lambda^2}\right)^{m+2} c^1_{(m,1)}$$

$$\delta^2 \mathbf{a}_{2,2} = \sqrt{\frac{2}{3}} \sum_{m=0}^{\infty} \left(\frac{s}{\Lambda^2}\right)^{m+2} c^2_{(m,1)}$$

$$\delta^3 \mathbf{a}_{2,3} = \frac{1}{\sqrt{2}} \sum_{m=0}^{\infty} \left(\frac{s}{\Lambda^2}\right)^{m+3} c^3_{(m,1)}$$

$$\delta^3 \mathbf{a}_{2,4} = \frac{1}{\sqrt{2}} \sum_{m=0}^{\infty} \left(\frac{s}{\Lambda^2}\right)^{m+3} c^5_{(m,1)}$$

$$\delta^3 \mathbf{a}_{2,5} = \frac{1}{\sqrt{2}} \sum_{m=0}^{\infty} \left(\frac{s}{\Lambda^2}\right)^{m+3} c^4_{(m,1)}$$

$$\delta^3 \mathbf{a}_{2,6} = \frac{1}{\sqrt{2}} \sum_{m=0}^{\infty} \left(\frac{s}{\Lambda^2}\right)^{m+3} c^6_{(m,1)}$$

Mapping EFT operators to differential observables

- Aim : Construct angular observables for $pp \longrightarrow Z(ll)h(\bar{b}b)$ and map them to principal directions ($\delta^p \mathbf{a}_{J,k}$).

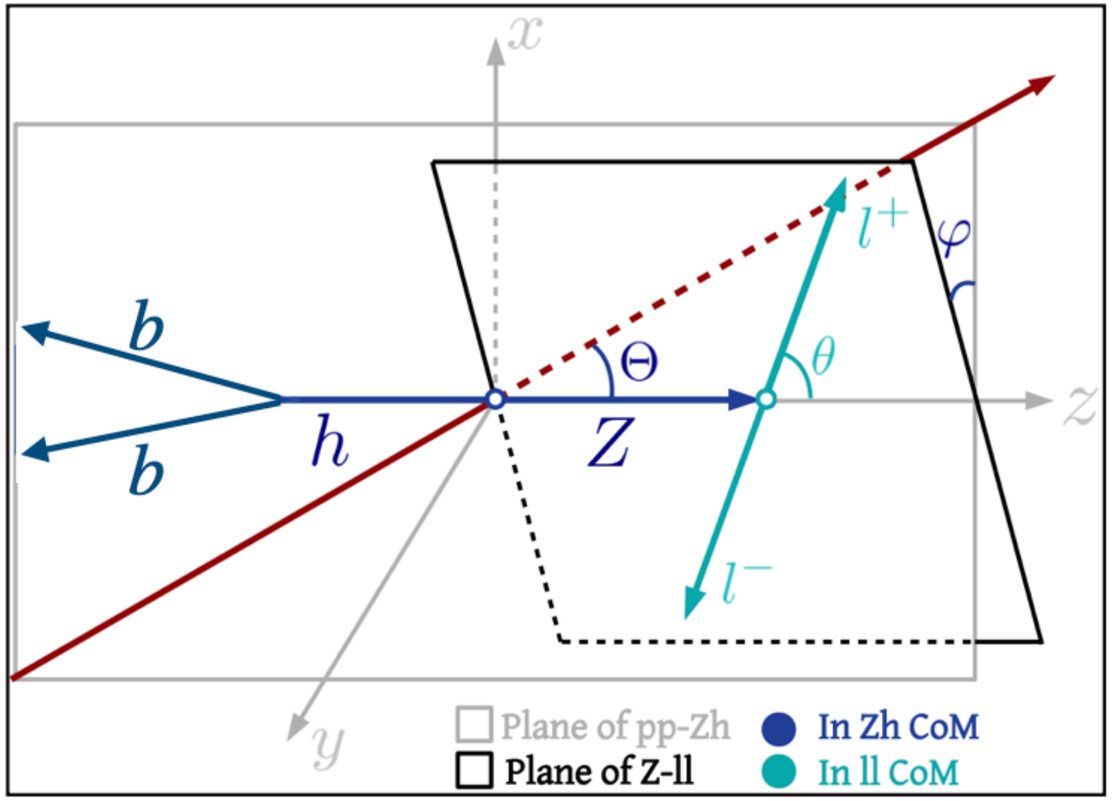
- To probe different helicity amplitudes for this process, require full differential analysis of angular distribution of leptons from $Z-$decay products.

- The amplitude for $pp \longrightarrow Z(ll)h(\bar{b}b)$ is :

$$\mathscr{A}_{\lambda_1\lambda_2}(s,\Theta,\theta,\phi) = \frac{-ig_l^Z}{\Gamma_Z} \sum_{\lambda_Z=0,\pm} \underbrace{\mathscr{M}_{\lambda_1\lambda_2;\lambda_Z}(s,\Theta)}_{ff\to Zh} \underbrace{d^{J=1}_{\lambda_Z,1}(\theta)\, e^{i\lambda_Z\phi}}_{Z-\text{decay}}$$

- Unitarity + CPT :

$$\mathscr{A}^*_{\lambda_1,\lambda_2}(s,\Theta,\theta,\phi) = -\,\mathscr{A}_{-\lambda_2,-\lambda_1}(s,\pi-\Theta,\pi-\theta,\phi)$$



Adapted from : S. Banerjee et. al., Phys.Rev.D 100(2019)11

Angular Moments

- To connect to experiments : Compute the squared amplitude with sum over the initial and final state lepton polarizations.

$$\overline{|\mathscr{A}|^2} = \sum_{\lambda_1,\lambda_2} \left(\alpha_L \, |\mathscr{A}_{\lambda_1,\lambda_2}(s,\Theta,\theta,\phi)|^2 + \alpha_R \, |\mathscr{A}_{\lambda_1,\lambda_2}(s,\Theta,\pi-\theta,\pi-\phi)|^2 \right)$$

- Unitarity + CPT $\Longrightarrow$ the squared amplitude is manifestly even under : $\Theta \longrightarrow \pi - \Theta$, $\theta \longrightarrow \pi - \theta$

- The full angular dependence of squared amplitude is given as :

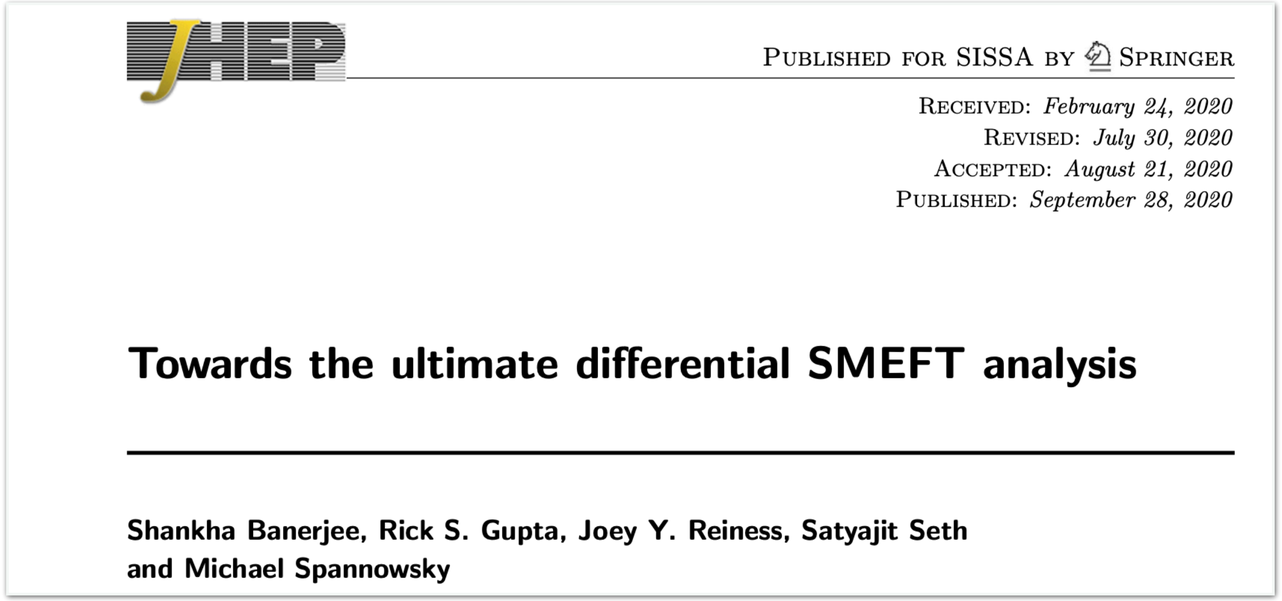
$$\begin{aligned}
\overline{|\mathcal{A}|^2} = {} & [a^1_{LL}(s,\Theta)\, s^2_\Theta + a^2_{LL}(s,\Theta)] s^2_\theta + a^1_{TT}(s,\Theta)\, c_\Theta\, c_\theta \\
& + [a^2_{TT}(s,\Theta)(1+c^2_\Theta) + a^3_{TT}(s,\Theta) s^2_\Theta](1+c^2_\theta) \\
& + [a^1_{LT}(s,\Theta) + a^2_{LT}(s,\Theta)\, c_\Theta\, c_\theta]\, s_\Theta\, s_\theta\, c_\phi \\
& + [\tilde{a}^1_{LT}(s,\Theta) + \tilde{a}^2_{LT}(s,\Theta)\, c_\Theta\, c_\theta]\, s_\Theta\, s_\theta\, s_\phi \\
& + a_{TT'}(s,\Theta)\, c_{2\phi}\, s^2_\Theta\, s^2_\theta + \tilde{a}_{TT'}(s,\Theta)\, s_{2\phi}\, s^2_\Theta\, s^2_\theta
\end{aligned}$$

To all orders in EFT

$$a_i(s,\Theta) = \sum_n a^n_i(s) \underbrace{(\cos\Theta)^{2n}}_{J-\text{descendants}}$$

- Additional constraints from CP symmetry : $\overline{|\mathscr{A}|^2}$ must be even under the transformation $\phi \longrightarrow 2\pi - \phi$ $\Longrightarrow$ CP violation is encoded in the moments with $\sin\phi$ or $\sin 2\phi$

Measurements of Angular coefficients



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Towards the ultimate differential SMEFT analysis

Shankha Banerjee, Rick S. Gupta, Joey Y. Reiness, Satyajit Seth and Michael Spannowsky

Detailed collider analysis to measure the angular moments using "Method of Moments"

Fourier analysis technique

Measurements of Angular coefficients

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Towards the ultimate differential SMEFT analysis

Constraints on standard model effective field theory for a Higgs boson produced in association with W or Z bosons in the $H \to b\bar{b}$ decay channel in proton-proton collisions at $\sqrt{s} = 13$ TeV

The CMS collaboration

E-mail: cms-publication-committee-chair@cern.ch

JHEP03(2025

Detailed collider analysis to measure the angular moments using "Method of Moments"

Fourier analysis technique

The squared amplitude of $V(\to \ell_1\ell_2)H(\to b\bar{b})$ production, when summing over the lepton helicities, has the form

$$|\mathcal{M}(\hat{s}, \Theta, \theta, \varphi)|^2 = \sum_i a_i(\hat{s})\, f_i(\Theta, \theta, \varphi), \tag{2.2}$$

where index i represents the lepton helicity, $a_i(\hat{s})$ are functions of the Wilson coefficients and the partonic center-of-mass energy $\sqrt{\hat{s}}$ involved in the process.

The functions f_i depend on the three angles in figure 2 and are summarized in eq. (2.3) following ref. [38]:

$$\begin{aligned}
f_1 &= f_{LL} = \sin^2\Theta \sin^2\theta \\
f_2 &= f^1_{TT} = \cos\Theta\cos\theta \\
f_3 &= f^2_{TT} = (1+\cos^2\Theta)(1+\cos^2\theta) \\
f_4 &= f^1_{LT} = \cos\varphi \sin\Theta \sin\theta \\
f_5 &= f^2_{LT} = \cos\varphi \sin\Theta \sin\theta \cos\Theta\cos\theta \\
f_6 &= \tilde{f}^1_{LT} = \sin\varphi \sin\Theta \sin\theta \\
f_7 &= \tilde{f}^2_{LT} = \sin\varphi \sin\Theta \sin\theta \cos\Theta\cos\theta \\
f_8 &= f_{TT'} = \cos^2\varphi \sin^2\Theta \sin^2\theta \\
f_9 &= \tilde{f}_{TT'} = \sin^2\varphi \sin^2\Theta \sin^2\theta,
\end{aligned} \tag{2.3}$$

CMS has also started using these angular observables.

Angular Moments in terms of Principal Directions

a_i	Relation to correlators
a_{LL}	$\frac{\alpha_0}{4}\left[\lvert a^{SM}_{+,0}\rvert^2(\mathbf{a}^0_{1,1}+\mathbf{a}^0_{1,2})(4+\alpha_0(\mathbf{a}^0_{1,1}+\mathbf{a}^0_{1,2}))+\lvert a^{SM}_{-,0}\rvert^2(\mathbf{a}^0_{1,1}-\mathbf{a}^0_{1,2})(4+\alpha_0(\mathbf{a}^0_{1,1}-\mathbf{a}^0_{1,2}))\right]+$
	$\frac{1}{2}\Big[\lvert a^{SM}_{+,0}\rvert^2(\mathbf{a}^1_{1,1}+\mathbf{a}^1_{1,2}+2\,\varepsilon_1\,\delta\mathbf{a}^1_{1,3})(2+\alpha_0(\mathbf{a}^0_{1,1}+\mathbf{a}^0_{1,2}))$
	$+\lvert a^{SM}_{-,0}\rvert^2(\mathbf{a}^1_{1,1}-\mathbf{a}^1_{1,2}+2\,\varepsilon_1\,\delta\mathbf{a}^1_{1,3})(2+\alpha_0(\mathbf{a}^0_{1,1}-\mathbf{a}^0_{1,2}))\Big]\frac{s}{\Lambda^2}$
	$+\frac{1}{4}\Big[\lvert a^{SM}_{+,0}\rvert^2\{(\mathbf{a}^1_{1,1}+\mathbf{a}^1_{1,2})^2+2(2+\alpha_0(\mathbf{a}^0_{1,1}+\mathbf{a}^0_{1,2}))(\mathbf{a}^2_{1,1}+\mathbf{a}^2_{1,2})$
	$+4\varepsilon_1[\delta\mathbf{a}^1_{1,3}(\mathbf{a}^1_{1,1}+\mathbf{a}^1_{1,2})+(2+\alpha_0(\mathbf{a}^0_{1,1}+\mathbf{a}^0_{1,2}))\delta\mathbf{a}^2_{1,3}]+4\varepsilon_1^2(\delta\mathbf{a}^1_{1,3})^2\}$
	$+\lvert a^{SM}_{-,0}\rvert^2\{(\mathbf{a}^1_{1,1}-\mathbf{a}^1_{1,2})^2+2(2+\alpha_0(\mathbf{a}^0_{1,1}-\mathbf{a}^0_{1,2}))(\mathbf{a}^2_{1,1}-\mathbf{a}^2_{1,2}\}$
	$+4\varepsilon_1[\delta\mathbf{a}^1_{1,3}(\mathbf{a}^1_{1,1}-\mathbf{a}^1_{1,2})+(2+\alpha_0(\mathbf{a}^0_{1,1}-\mathbf{a}^0_{1,2}))(\delta\mathbf{a}^2_{1,3}-\delta^2\mathbf{a}^2_{1,5})]+4\varepsilon_1^2(\delta\mathbf{a}^1_{1,3})^2\}\Big]\frac{s^2}{\Lambda^4}$
	0
a^1_{LT}	$\frac{\alpha_0}{2}\epsilon_{RL}\left[a^{SM}_{+,0}a^{SM}_{+}(\mathbf{a}^0_{1,1}+\mathbf{a}^0_{1,2})(4+\alpha_0(\mathbf{a}^0_{1,1}+\mathbf{a}^0_{1,2}))-a^{SM}_{-,0}a^{SM}_{-}(\mathbf{a}^0_{1,1}-\mathbf{a}^0_{1,2})(4+\alpha_0(\mathbf{a}^0_{1,1}-\mathbf{a}^0_{1,2}))\right]$
	$+\epsilon_{RL}\Big[a^{SM}_{+,0}a^{SM}_{+}(\mathbf{a}^1_{1,1}+\mathbf{a}^1_{1,2}+(\alpha_1+\varepsilon_1)\delta\mathbf{a}^1_{1,3})(2+\alpha_0(\mathbf{a}^0_{1,1}+\mathbf{a}^0_{1,2}))$
	$-a^{SM}_{-,0}a^{SM}_{-}(\mathbf{a}^1_{1,1}-\mathbf{a}^1_{1,2}+(\alpha_1+\varepsilon_1)\delta\mathbf{a}^1_{1,3})(2+\alpha_0(\mathbf{a}^0_{1,1}-\mathbf{a}^0_{1,2}))\Big]\frac{s}{\Lambda^2}$
	$+\epsilon_{RL}\frac{1}{2}\Big[a^{SM}_{+,0}a^{SM}_{+}\{(\mathbf{a}^1_{1,1}+\mathbf{a}^1_{1,2})(\mathbf{a}^1_{1,1}+\mathbf{a}^1_{1,2}+2(\alpha_1+\varepsilon_1)\delta\mathbf{a}^1_{1,3})+4\varepsilon_0(\delta\mathbf{a}^1_{1,3})^2$
	$+2(2+\alpha_0(\mathbf{a}^0_{1,1}+\mathbf{a}^0_{1,2}))(\mathbf{a}^2_{1,1}+\mathbf{a}^2_{1,2}+2(\varepsilon_1+\alpha_1)\delta\mathbf{a}^2_{1,3})\}$
	$-a^{SM}_{-,0}a^{SM}_{-}\{(\mathbf{a}^1_{1,1}-\mathbf{a}^1_{1,2})(\mathbf{a}^1_{1,1}-\mathbf{a}^1_{1,2}+2(\varepsilon_1+\alpha_1)\delta\mathbf{a}^1_{1,3})+4\varepsilon_0(\delta\mathbf{a}^1_{1,3})^2$
	$+2(2+\alpha_0(\mathbf{a}^0_{1,1}-\mathbf{a}^0_{1,2}))(\mathbf{a}^2_{1,1}-\mathbf{a}^2_{1,2}+2(\varepsilon_1+\alpha_1)(\delta\mathbf{a}^2_{1,3}-\delta^2\mathbf{a}^2_{1,5}))\}\Big]\frac{s^2}{\Lambda^4}$
	0
$\tilde{a}^1_{LT}$	0
	$+\epsilon_{RL}\,\alpha_2\left[a^{SM}_{+,0}a^{SM}_{+}(2+\alpha_0(\mathbf{a}^0_{1,1}+\mathbf{a}^0_{1,2}))-a^{SM}_{-,0}a^{SM}_{-}(2+\alpha_0(\mathbf{a}^0_{1,1}-\mathbf{a}^0_{1,2}))\right]\delta\mathbf{a}^1_{1,4}\frac{s}{\Lambda^2}$
	$+\epsilon_{RL}\,\frac{\alpha_2}{4}\Big[a^{SM}_{+,0}a^{SM}_{+}\left(4\delta\mathbf{a}^1_{1,4}(\mathbf{a}^1_{1,1}+\mathbf{a}^1_{1,2}+2\varepsilon_1\delta\mathbf{a}^1_{1,3})+(4\delta\mathbf{a}^2_{1,4}-\sqrt{6}\,\delta^2\mathbf{a}^2_{2,1})[2+\alpha_0(\mathbf{a}^0_{1,1}+\mathbf{a}^0_{1,2})]\right)$
	$-a^{SM}_{-,0}a^{SM}_{-}\left(4\delta\mathbf{a}^1_{1,4}(\mathbf{a}^1_{1,1}-\mathbf{a}^1_{1,2}+2\varepsilon_1\delta\mathbf{a}^1_{1,3})+(4\delta\mathbf{a}^2_{1,4}+4\delta^2\mathbf{a}^2_{1,6}+\sqrt{6}\,\delta^2\mathbf{a}^2_{2,2})[2+\alpha_0(\mathbf{a}^0_{1,1}-\mathbf{a}^0_{1,2})]\right)\Big]\frac{s^2}{\Lambda^4}$
	0

Preliminary

Angular Moments in terms of Principal Directions

a_i	Relation to correlators	
a_{LL}	$\frac{\alpha_0}{4}\left[\lvert a^{SM}_{+,0}\rvert^2(\mathbf{a}^0_{1,1}+\mathbf{a}^0_{1,2})(4+\alpha_0(\mathbf{a}^0_{1,1}+\mathbf{a}^0_{1,2}))+\lvert a^{SM}_{-,0}\rvert^2(\mathbf{a}^0_{1,1}-\mathbf{a}^0_{1,2})(4+\alpha_0(\mathbf{a}^0_{1,1}-\mathbf{a}^0_{1,2}))\right]+$	$\mathcal{O}(1)$
	$\frac{1}{2}\Big[\lvert a^{SM}_{+,0}\rvert^2(\mathbf{a}^1_{1,1}+\mathbf{a}^1_{1,2}+2\,\varepsilon_1\,\delta\mathbf{a}^1_{1,3})(2+\alpha_0(\mathbf{a}^0_{1,1}+\mathbf{a}^0_{1,2}))$	
	$+\lvert a^{SM}_{-,0}\rvert^2(\mathbf{a}^1_{1,1}-\mathbf{a}^1_{1,2}+2\,\varepsilon_1\,\delta\mathbf{a}^1_{1,3})(2+\alpha_0(\mathbf{a}^0_{1,1}-\mathbf{a}^0_{1,2}))\Big]\frac{s}{\Lambda^2}$	$\frac{s}{\Lambda^2}$
	$+\frac{1}{4}\Big[\lvert a^{SM}_{+,0}\rvert^2\{(\mathbf{a}^1_{1,1}+\mathbf{a}^1_{1,2})^2+2(2+\alpha_0(\mathbf{a}^0_{1,1}+\mathbf{a}^0_{1,2}))(\mathbf{a}^2_{1,1}+\mathbf{a}^2_{1,2})$	
	$+4\varepsilon_1[\delta\mathbf{a}^1_{1,3}(\mathbf{a}^1_{1,1}+\mathbf{a}^1_{1,2})+(2+\alpha_0(\mathbf{a}^0_{1,1}+\mathbf{a}^0_{1,2}))\delta\mathbf{a}^2_{1,3}]+4\varepsilon_1^2(\delta\mathbf{a}^1_{1,3})^2\}$	
	$+\lvert a^{SM}_{-,0}\rvert^2\{(\mathbf{a}^1_{1,1}-\mathbf{a}^1_{1,2})^2+2(2+\alpha_0(\mathbf{a}^0_{1,1}-\mathbf{a}^0_{1,2}))(\mathbf{a}^2_{1,1}-\mathbf{a}^2_{1,2}\}$	$\frac{s^2}{\Lambda^4}$
	$+4\varepsilon_1[\delta\mathbf{a}^1_{1,3}(\mathbf{a}^1_{1,1}-\mathbf{a}^1_{1,2})+(2+\alpha_0(\mathbf{a}^0_{1,1}-\mathbf{a}^0_{1,2}))(\delta\mathbf{a}^2_{1,3}-\delta^2\mathbf{a}^2_{1,5})]+4\varepsilon_1^2(\delta\mathbf{a}^1_{1,3})^2\}\Big]\frac{s^2}{\Lambda^4}$	
a^1_{LT}	0	$\cos\Theta\frac{s^2}{\Lambda^4}$

A priori, all the partial-wave amplitudes should contribute to every order in s.

EFT input

Restricts the number of linear combinations of partial-wave amplitudes that should contribute at a given order in s and $\cos\Theta$.

Measurements of differential (both energy and angular) distributions of these angular coefficients will help to probe EFT to all orders.

Conclusion

- We constructed differential angular observables that probes EFT to all orders in the Higgstrahlung process.
- Partial-wave expansion ⟷ EFT expansion : Derived an explicit mapping between the partial-waves and EFT expansion valid to all orders.
- At lower orders, EFT predicts correlations between the different partial-wave amplitudes ⟶ allows us to construct linear combinations of partial-wave amplitudes, called “principal directions” that get leading contribution from higher dimensional EFT operators.
- We introduce experimental observables, the so-called angular moments, that probe these linear combinations of partial waves.
- Measurements of differential distributions of these observables allows us to systematically probe all higher dimension EFT operators contributing to this process.

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Thank you

Supplementary Slides

Why $(t-u)^n$?

$$s,\ t,\ u \longrightarrow \begin{cases} t+u = \sum_i m_i^2 - s \\ t-u \end{cases}$$

$$\mathscr{M}_{\lambda_1,\lambda_2;\lambda_Z}(s,\Theta) = \sum_{i=1}^{N_P} P^i_{\lambda_1,\lambda_2;\lambda_Z}(s,\Theta)\left(1 + \sum_{m,n>0} c^i_{(m,n)} s^m (t-u)^n\right) \qquad \Longrightarrow \quad c^i_{(m,n)} = (-1)^n \left(c^i_{(m,n)}\right)^*$$

Unitarity + CPT : $\mathscr{M}_{\lambda_1,\lambda_2;\lambda_Z}(s,\Theta) = \left(\mathscr{M}_{-\lambda_2,-\lambda_1;-\lambda_Z}(s,\pi-\Theta)\right)^*$

Projecting on to fixed-J operators

- For the process : $ab \longrightarrow cd$, where all particles are distinct, any operator contributing to the amplitude is given by :

$$\mathcal{O}_i = [ab]_{\mu\nu\rho\ldots}[cd]^{\mu\nu\rho\ldots}$$

- Therefore, $\mathcal{O}_i = [ab]_{\mu\nu\rho\ldots}\left(\sum_J \mathscr{P}^{\mu\nu\rho\ldots}_{J,\alpha\beta\kappa\ldots}\mathscr{P}^{\alpha\beta\kappa\ldots}_{J,\mu'\nu'\rho'\ldots}\right)[cd]^{\mu'\nu'\rho'\ldots}$

$$= \sum_J \alpha_{(i|J)}\mathcal{O}_{(i|J)}$$

Projection operators that act on a rank-n tensor, $T^{\mu_1\mu_2\ldots\mu_n}$,

$$\mathscr{T}^{\nu_1\nu_2..\nu_n}_J = \mathscr{P}^{\nu_1\nu_2..\nu_n}_{J,\mu_1\mu_2..\mu_n} T^{\mu_1\mu_2..\mu_n}$$

such that resulting tensor $\mathscr{T}^{\nu_1\nu_2..\nu_n}_J$ transforms under the Lorentz group with fixed-J

SMEFT Operators

Principal Operators	SMEFT Operators	
$hZ^\mu\bar{\psi}_L\gamma_\mu\psi_L$	$i\left(H^\dagger\overleftrightarrow{D}_\mu H\right)\bar{Q}_L\gamma^\mu Q_L$	Dimension - 6
$hZ^\mu\bar{\psi}_R\gamma_\mu\psi_R$	$i\left(H^\dagger\overleftrightarrow{D}_\mu H\right)\bar{u}_R\gamma^\mu u_R$	
$ih\widetilde{Z}_{\mu\nu}\left(\bar{\psi}_L\gamma^\mu\overleftrightarrow{\partial}^\nu\psi_L\right)$	$i\lvert H\rvert^2\widetilde{W}^{a\mu\nu}\left(\bar{Q}_L\gamma_\mu\sigma^a\overleftrightarrow{D}_\nu Q_L\right)$	Dimension - 8
$h\widetilde{Z}_{\mu\nu}\partial^\mu\left(\bar{\psi}_L\gamma^\nu\psi_L\right)$	$\lvert H\rvert^2\widetilde{W}^{a\mu\nu}D_\mu(\bar{Q}_L\gamma_\nu\sigma^a Q_L)$	
$ih\widetilde{Z}_{\mu\nu}\left(\bar{\psi}_R\gamma^\mu\overleftrightarrow{\partial}^\nu\psi_R\right)$	$i\lvert H\rvert^2\widetilde{B}^{\mu\nu}\left(\bar{u}_R\gamma_\mu\overleftrightarrow{D}_\nu u_R\right)$	
$h\widetilde{Z}_{\mu\nu}\partial^\mu\left(\bar{\psi}_R\gamma^\nu\psi_R\right)$	$\lvert H\rvert^2\widetilde{B}^{\mu\nu}D_\mu\left(\bar{u}_R\gamma_\nu u_R\right)$	

Chang, Chen, Liu and Luty, JHEP07(2023)030