Strong First-Order Phase Transitions and Exotic Phases in the CP-Conserving 2HDM

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Collaborative Research Center TRR 257
Particle Physics Phenomenology after the Higgs Discovery

An ongoing project together with:
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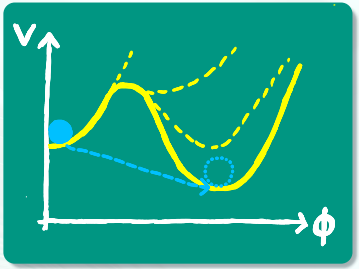
CERN – 04/12/2025

Outline

1. Motivation
   - Evolution of the early Universe
   - The CP-conserving 2HDM
2. One-loop effective potential & temperature corrections
   - Finite ‘on-shell-like’ renormalisation
3. Parameter scan and results

How did the early Universe evolve around the electroweak epoch?

- Appearance of **strong** first-order phase transitions? (SFOPT)
  - 1-step: **S**ymmetric (high T) → **EW**-broken ($T = 0$)
  - multi-step: **S** → broken → ...→ **EW**



[Athron et al., 2305.02357]

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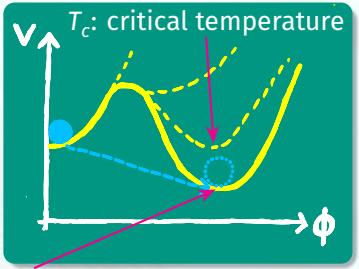

$T_n / T_p / T_{\text{compl}} < T_c$:
nucleation/percolation/
completion temperature

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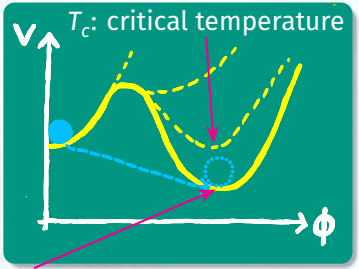

- Appearance of exotic intermediate phases: CP-violating and charge-breaking (CB) vacua?
  [Aoki, Biermann, CB, Ivanov, Mühlleitner, 2308.04141]
- Gravitational-wave signals from FOPTs?
- Compatibility with recent collider constraints?

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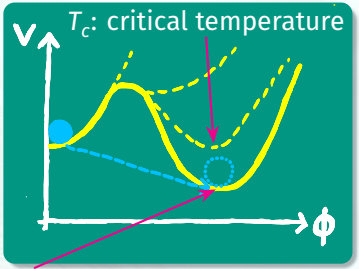

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Excellent testbed for BSM physics with extended scalar sectors

For the CP-conserving 2HDM e.g.: ... *[Basler et al., '16][Dorsch et al., '17][Andersen et al., '17][Bernon et al., '17][Su et al., '20] [Aoki et al., '21][Gonçalves et al., '21][Biekötter et al., '22-'23][Bittar et al., '25][Anisha et al., '25][Lee et al., '25][Bhatnagar et al., '25]* ...
Theory uncertainties e.g.: ... *[Ramsey-Musolf et al., '20-'24][Croon et al., '20][Athron et al., '22][Keus et al., '25][Biekötter et al., '25]*...

The CP-conserving 2HDM

Tree-level scalar potential of the real 2HDM (with softly broken $\mathbb{Z}_2$ symmetry):

$$V_{\text{tree}} = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 \left(\Phi_1^\dagger \Phi_2 + h.c.\right) + \frac{\lambda_1}{2}\left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger \Phi_2\right)^2$$
$$+ \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) + \frac{\lambda_5}{2}\left[\left(\Phi_1^\dagger \Phi_2\right)^2 + \text{h.c.}\right]$$

with

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \rho_1 + i\eta_1 \\ \zeta_1 + \omega_1 + i\psi_1 \end{pmatrix}, \qquad \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \rho_2 + \omega_{\text{CB}} + i\eta_2 \\ \zeta_2 + \omega_2 + i\left(\psi_2 + \omega_{\text{CP}}\right) \end{pmatrix}$$

and real fields ρ_i, η_i, ζ_i, ψ_i $(i = 1, 2)$, and VEVs ω_j $(j = 1, 2, \text{CP}, \text{CB})$

Present-day EW-breaking vacuum at zero temperature $T = 0$ (with $v_j \equiv \omega_j|_{T=0}$):

$$v_{\text{CB}} = v_{\text{CP}} = 0 \quad \text{and} \quad v^2 \equiv v_1^2 + v_2^2 = (246.22\ \text{GeV})^2 \quad \text{and} \quad \tan\beta \equiv v_2/v_1$$

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and real fields ρ_i, η_i, ζ_i, ψ_i $(i = 1, 2)$, and VEVs ω_j $(j = 1, 2, \text{CP}, \text{CB})$

Physical input parameters:

$$M_h, M_H, M_A, M_{H^\pm}, m_{12}^2, \tan\beta, \alpha, v$$

* "Normal hierarchy": $M_h = M_h^{\text{SM}} < M_H$
* "Inverted hierarchy": $M_h < M_H = M_h^{\text{SM}}$

with the mass eigenstates h, H: CP-even, A: CP-odd, $H^\pm$: charged, and the CP-even mixing angle α

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Effective potential at finite temperatures T

Full 1-loop effective potential including thermal corrections:

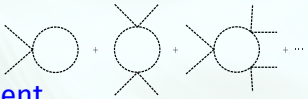
$$V_{\text{eff}}(T) = V_{\text{tree}} + V_{\text{CW}} + V_{\text{CT}} + V_T(T)$$

with

- V_{CW}: 1-loop Coleman-Weinberg potential ($\overline{\text{MS}}$)
- V_{CT}: (finite) counterterm potential

} T-independent

- $V_T(T)$: 1-loop thermal corrections at finite T



[Quiros, hep-ph/9901312]

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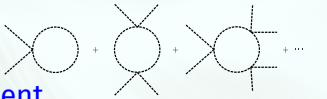
- $V_T(T)$: 1-loop thermal corrections at finite T + daisy resummation



[Quiros, hep-ph/9901312]

Perturbative expansion becomes unreliable at high T

- Resum 'daisy' diagrams to recover perturbativity [Arnold, Espinosa, '92][Parwani, '92]

⇒ Mass eigenvalues obtain T-dependent Debye corrections

(Other methods e.g.: **Partial Dressing** [Bittar et al., '25], **dimensional reduction/3D EFT** [Farakos et al., '94-'95] [Braaten et al., '95][Ekstedt et al., '22])

Finite 'on-shell-like' renormalisation

Counterterm potential:

$$V_{CT} = \delta m_{11}^2 \Phi_1^\dagger \Phi_1 + \delta m_{22}^2 \Phi_2^\dagger \Phi_2 - \delta m_{12}^2 \left(\Phi_1^\dagger \Phi_2 + h.c.\right) + \frac{\delta\lambda_1}{2}\left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{\delta\lambda_2}{2}\left(\Phi_2^\dagger \Phi_2\right)^2$$
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$$+ \delta T_1(\zeta_1 + \omega_1) + \delta T_2(\zeta_2 + \omega_2) + \delta T_{CP}(\psi_2 + \omega_{CP}) + \delta T_{CB}(\rho_2 + \omega_{CB})$$

Finite counterterms are fixed by the 'on-shell-like' renormalisation conditions:

$$\left.\frac{\partial\left(V_{CW} + V_{CT}\right)}{\partial\phi_i}\right|_{\Phi=\langle\Phi\rangle, T=0} \stackrel{!}{=} 0\,, \qquad \left.\frac{\partial^2\left(V_{CW} + V_{CT}\right)}{\partial\phi_i \partial\phi_j}\right|_{\Phi=\langle\Phi\rangle, T=0} \stackrel{!}{=} 0\,,$$

with $\phi_i, \phi_j \in \{\rho_1, \eta_1, \zeta_1, \psi_1, \rho_2, \eta_2, \zeta_2, \psi_2\}$ and $\langle\Phi_1\rangle_{T=0} = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_1\end{pmatrix}$, $\langle\Phi_2\rangle_{T=0} = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_2\end{pmatrix}$

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Counterterm potential:

$$
\begin{aligned}
V_{\text{CT}} = {} & \delta m_{11}^2 \Phi_1^\dagger \Phi_1 + \delta m_{22}^2 \Phi_2^\dagger \Phi_2 - \delta m_{12}^2 \left(\Phi_1^\dagger \Phi_2 + h.c.\right) + \frac{\delta\lambda_1}{2}\left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{\delta\lambda_2}{2}\left(\Phi_2^\dagger \Phi_2\right)^2 \\
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$$

Conditions keep the physical masses and mixing angles as well as the VEVs at $T = 0$ at their tree-level values $\Rightarrow$ convenient for large parameter scans!

Scanning procedure

ScannerS *[Mühlleitner, Sampaio, Santos, Wittbrodt ('13-)'20]*

- Generate parameter points in R2HDM T1 (random/targeted/ML-improved scans)
- Constraints from:
  - *theory*: bounded-from-below *[Branco et al., '11]*, perturbative unitarity, absolute stability *[Barroso et al., '13]*
  - *experiment*: *B*-physics *[Haller et al., '18]*, `HiggsTools` *[Bechtle et al., '08-'21]*, *STU*-parameters *[Peskin et al., '92]*

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⇓

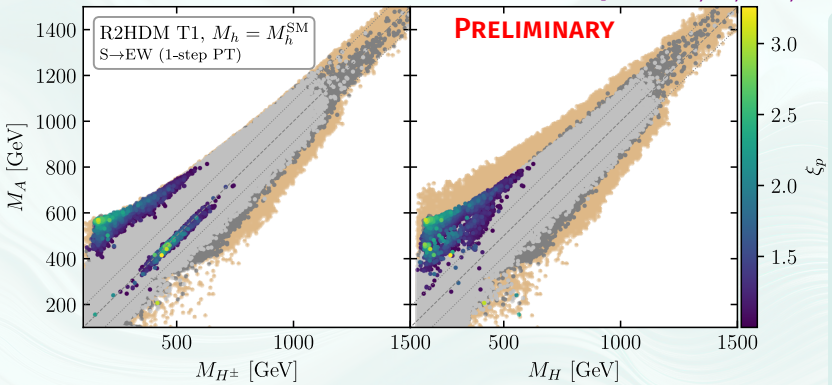
BSMPTv3 *[Basler, Biermann, Mühlleitner, Müller, Santos, Viana ('18-)'24]*

- Phase tracking from $T_{\text{high}} = 1$ TeV down to $T_{\text{low}} = 0$ GeV
- Test for electroweak symmetry restoration (EWSR) at high $T \to \infty$
- Calculation of bounce solution, characteristic (critical, nucleation, percolation, completion) temperatures and VEVs, GW signals → `CalcGW`

⇒ Sort points according to transition history, **S→EW**, **S→N**(eutral broken)**→EW**, ...

One-step phase transitions, $M_h = M_h^{\rm SM} < M_H$ case

[Biermann, CB, Boto, Mühlleitner, Santos, Viana; in preparation]



- light-brown points: full ScannerS sample
- dark-grey points: S→EW points without EWSR
- light-grey points: S→EW points with EWSR, but $\xi_p < 1$
- coloured points: S→EW points with EWSR and $\xi_p \geq 1$

Normal hierarchy: $M_h = M_h^{\rm SM}$

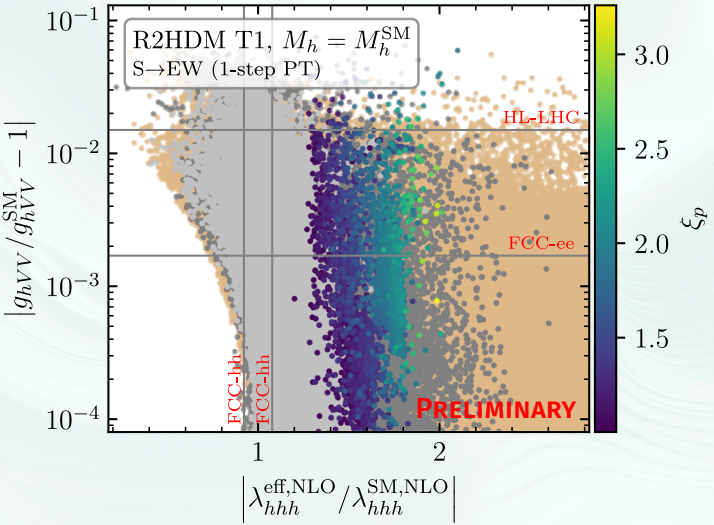
For SFOPT points:

- $M_{H^\pm} \lesssim 700$ GeV
- Degeneracy between $M_{H^\pm}$ and M_A (M_H)
- Typically, $M_A > M_H + M_Z$, so $A \to HZ$ possible
- FOPT strength $\xi_p \lesssim 3$

$$\xi_p = \frac{\sqrt{\sum_i \left[\omega_{i,\rm true}(T_p) - \omega_{i,\rm false}(T_p)\right]^2}}{T_p} \qquad (i = 1, 2, \mathrm{CB}, \mathrm{CP})$$

SM-like single- and di-Higgs couplings, $M_h = M_h^{SM} < M_H$ case

[Biermann, CB, Boto, Mühlleitner, Santos, Viana; in preparation]

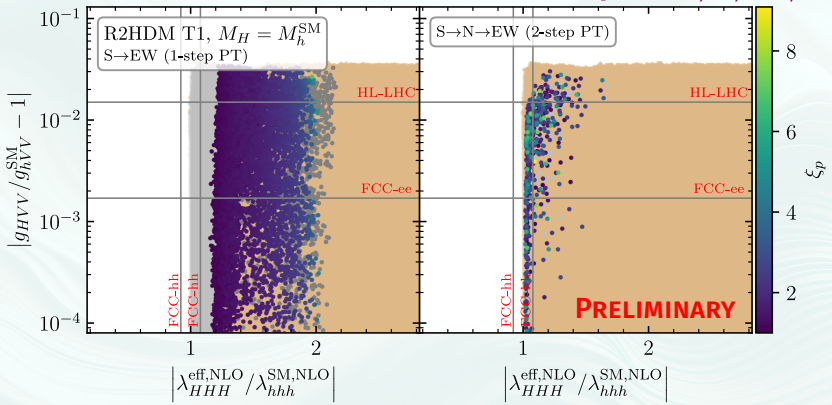


Normal hierarchy: $M_h = M_h^{SM}$

- $\kappa_\lambda = \left|\lambda_{hhh}^{eff,NLO}/\lambda_{hhh}^{SM,NLO}\right|$
- $g_{hVV}/g_{hVV}^{SM} = \sin(\beta - \alpha)$
- FOPT strength ξ_p increases with increasing κ_λ
- Generally little dependence on single-Higgs coupling
- Future projections: possibility to constrain parameter space with 1-step SFOPTs

SM-like single- and di-Higgs couplings, $M_h < M_H = M_h^{\rm SM}$ case

[Biermann, CB, Boto, Mühlleitner, Santos, Viana; in preparation]



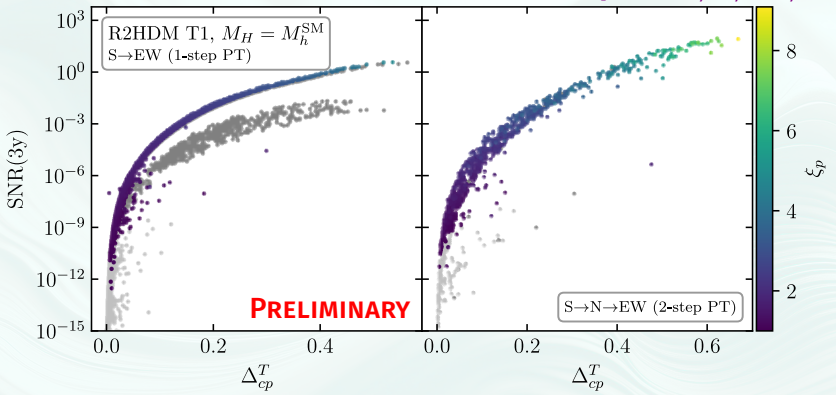
Inverted hierarchy:
$M_H = M_h^{\rm SM}$

- $g_{HVV}/g_{hVV}^{\rm SM} = \cos(\beta - \alpha)$
- Strong 2-step FOPTs with $\xi_p \lesssim 8$ and smaller κ_λ than for 1-step SFOPTs

For very SM-like single-Higgs coupling, 2-step SFOPTs pushed towards $\kappa_\lambda = 1$

Gravitational-wave signals, $M_h < M_H = M_h^{\rm SM}$ case

[Biermann, CB, Boto, Mühlleitner, Santos, Viana; in preparation]



$$\text{SNR} = \sqrt{\mathcal{T} \int_{f_{\rm min}}^{f_{\rm max}} df \left[\frac{h^2 \Omega_{\rm GW}(f)}{h^2 \Omega_{\rm Sens}(f)} \right]^2}$$

Inverted hierarchy:
$M_H = M_h^{\rm SM}$

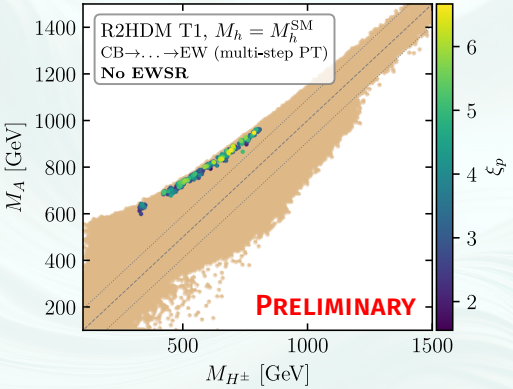
- SNR(3y): $\mathcal{T}$ = 3-year signal-to-noise ratio of the LISA experiment
- 'Duration' of PT:

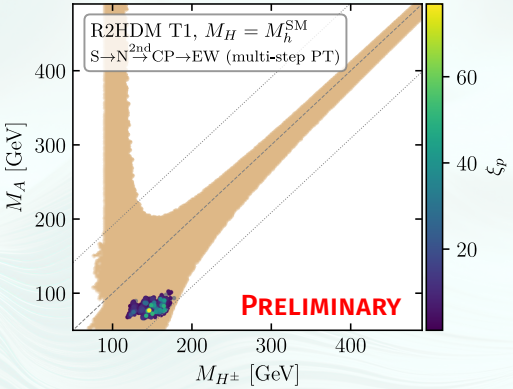
$$\Delta_{cp}^{T} \equiv \frac{T_c - T_p}{T_c}$$

- Both 1- and 2-step FOPTs reach SNRs > 1 for large Δ_{cp}^{T} values ⇒ supercooled scenarios

Exotic intermediate phases

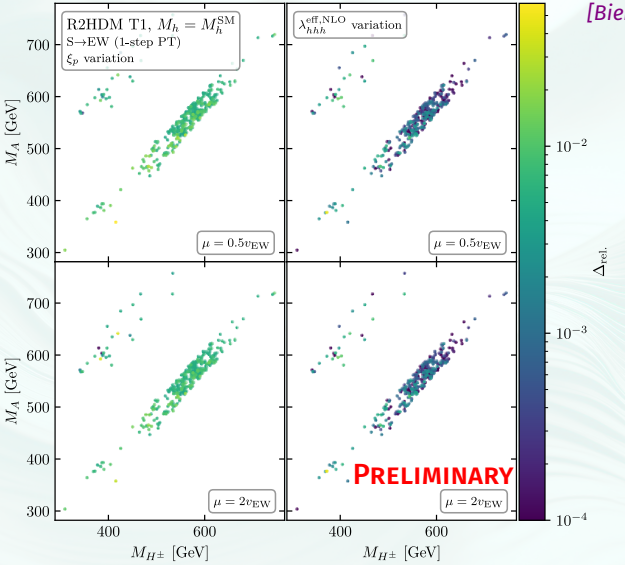
[Biermann, CB, Boto, Mühlleitner, Santos, Viana; in preparation]





- CB phases with intermediate symmetry restoration, e.g. CB→S→EW, but no EWSR
- Intermediate CP phases with very strong CP→EW transitions $\xi_p > 10$ ⇒ large SNRs?

Renormalisation-scale dependence



[Biermann, CB, Boto, Mühlleitner, Santos, Viana; in preparation]

Variation of scale parameter in $V_{\rm CW} + V_{\rm CT}$

Relative difference wrt. central scale $\mu_0 = v_{\rm EW} = 246.22$ GeV and $\mu = \{0.5\mu_0, 2\mu_0\}$:

$$\Delta_{\rm rel.} \equiv \left| 1 - \frac{X(\mu)}{X(\mu_0)} \right|$$

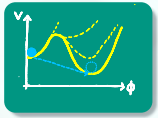
with $X \in \{\xi_p, \lambda_{hhh}^{\rm eff,NLO}\}$

- Subset of S→EW points for normal hierarchy with no change in PT history
- Differences typically around $\mathcal{O}(1\%)$

⇒ Scale dependence for selected points mild in OS-like scheme

Summary

First-order phase transitions in the CP-conserving 2HDM



- Comprehensive parameter scan in the R2HDM T1 with the **BSMPTv3** code
- Study of ‘normal’ and ‘inverted’ mass hierarchy scenarios
- Appearance of 1-step and multi-step SFOPTs, testable by future colliders?
- Exotic phases involving charge-breaking and CP-violating intermediate vacua
- Typically, multi-step FOPTs reach noticeably higher strengths than 1-step ones

⇒ **Potentially detectable GW signals for multi-step (exotic?) FOPTs**

✍ Analysis of variation of renormalisation scale/scheme: **in progress**

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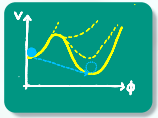

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Thank You for Your Attention! ☺

Backup

$\mathbb{Z}_2$ symmetry

To avoid dangerously large flavour-changing neutral currents at tree level:

- Impose discrete $\mathbb{Z}_2$ symmetry:

$$\Phi_1 \longrightarrow \Phi_1, \quad \Phi_2 \longrightarrow -\Phi_2$$

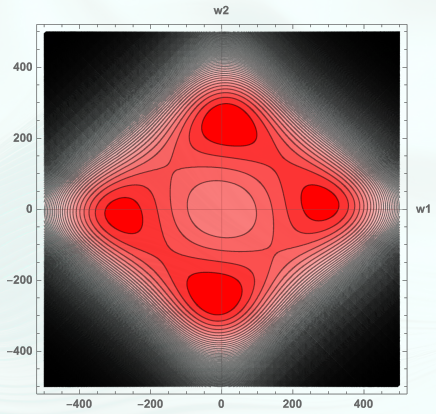
- Depending on $\mathbb{Z}_2$ charges of fermion fields, four possible types:

	u-type	d-type	leptons
Type I	Φ_2	Φ_2	Φ_2
Type II	Φ_2	Φ_1	Φ_1
Lepton-Specific	Φ_2	Φ_2	Φ_1
Flipped	Φ_2	Φ_1	Φ_2

- m_{12}^2 term in V breaks $\mathbb{Z}_2$ symmetry softly

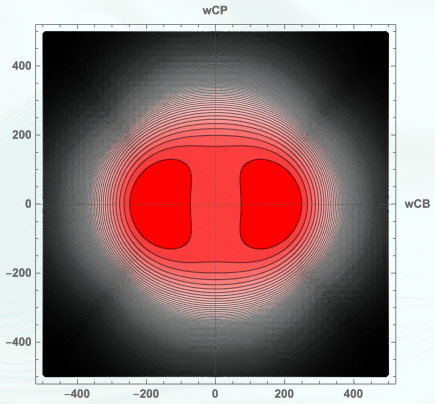
Types of vacua in the 2HDM

Type of vacuum	$\sqrt{2}\langle\Phi_1\rangle$	$\sqrt{2}\langle\Phi_2\rangle$
Symmetric	$\begin{pmatrix}0\\0\end{pmatrix}$	$\begin{pmatrix}0\\0\end{pmatrix}$
Neutral EW-breaking	$\begin{pmatrix}0\\\omega_1\end{pmatrix}$	$\begin{pmatrix}0\\\omega_2\end{pmatrix}$
CP-breaking	$\begin{pmatrix}0\\\omega_1\end{pmatrix}$	$\begin{pmatrix}0\\\omega_2 + i\omega_{CP}\end{pmatrix}$
Charge-breaking (CB)	$\begin{pmatrix}0\\\omega_1\end{pmatrix}$	$\begin{pmatrix}\omega_{CB}\\\omega_2\end{pmatrix}$



Types of vacua in the 2HDM

Type of vacuum	$\sqrt{2}\langle\Phi_1\rangle$	$\sqrt{2}\langle\Phi_2\rangle$
Symmetric	$\begin{pmatrix}0\\0\end{pmatrix}$	$\begin{pmatrix}0\\0\end{pmatrix}$
Neutral EW-breaking	$\begin{pmatrix}0\\\omega_1\end{pmatrix}$	$\begin{pmatrix}0\\\omega_2\end{pmatrix}$
CP-breaking	$\begin{pmatrix}0\\\omega_1\end{pmatrix}$	$\begin{pmatrix}0\\\omega_2 + i\omega_{CP}\end{pmatrix}$
Charge-breaking (CB)	$\begin{pmatrix}0\\\omega_1\end{pmatrix}$	$\begin{pmatrix}\omega_{CB}\\\omega_2\end{pmatrix}$



One-loop thermal corrections for $T \to \infty$

$$V_T \overset{T\to\infty}{\approx} -\sum_k n_k \begin{cases} \frac{\pi^2}{90}T^4 - \frac{1}{24}m_k^2T^2 + \frac{1}{12\pi}\overline{m}_k^3T & k = H^\pm, h, H, A, W_L, Z_L, \gamma_L \\ \frac{\pi^2}{90}T^4 - \frac{1}{24}m_k^2T^2 + \frac{1}{12\pi}m_k^3T & k = W_T, Z_T \\ \frac{7\pi^2}{720}T^4 - \frac{1}{48}m_k^2T^2 & k = t, b, \tau \end{cases}$$

with

- n_k: number of d.o.f.s of field k
- $\overline{m}_k$ (m_k): mass eigenvalue for field k including (excluding) thermal Debye corrections from Daisy resummation

Thermal evolution

Full one-loop effective potential including thermal corrections:

$$V_{\text{eff}}(T) = V_{\text{tree}} + V_{\text{CW}} + V_{\text{CT}} + V_T(T)$$

CW: Coleman-Weinberg potential
CT: counterterm potential
T: thermal corrections

In high-T limit: T dependence in V_{eff} from

$$m_{ii}^2(T) = m_{ii}^2 + c_i T^2$$

for m_{11}^2 and m_{22}^2 and with

$$c_i = \frac{1}{12}\left(3\lambda_i + 2\lambda_3 + \lambda_4\right) + \frac{1}{16}\left(3g^2 + g'^2\right) + \delta_{i2}\frac{1}{12}\left(y_\tau^2 + 3y_b^2 + 3y_t^2\right)$$

including gauge and Yukawa couplings

Electroweak symmetry (non-)restoration in the 2HDM

Is the EW symmetry always restored at $T \to \infty$? $\Rightarrow$ Check for minimum at origin

Extract leading term $\propto T^2$ for $T \to \infty$:

$$\text{Hessian } H_{ij} \equiv \left.\frac{\partial^2 V_T}{\partial\bar{\omega}_i \partial\bar{\omega}_j}\right|_{\bar{\omega}_{i,j}=0} \quad \Rightarrow \quad \mathcal{H} \equiv \lim_{T\to\infty} \frac{H}{T^2} = \lim_{T\to\infty} \frac{1}{T^2} \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} = \begin{pmatrix} \mathcal{H}_{11} & 0 \\ 0 & \mathcal{H}_{22} \end{pmatrix}$$

with

$$\mathcal{H}_{11} = c_1 - \frac{1}{16\pi}\left[\sqrt{2}\left(3g^3 + g'^3\right) + 4\left(3\sqrt{c_1}\lambda_1 + \sqrt{c_2}\left(2\lambda_3 + \lambda_4\right)\right)\right]$$

$$\mathcal{H}_{22} = c_2 - \frac{1}{16\pi}\left[\sqrt{2}\left(3g^3 + g'^3\right) + 4\left(3\sqrt{c_2}\lambda_2 + \sqrt{c_1}\left(2\lambda_3 + \lambda_4\right)\right)\right]$$

$\Rightarrow$ **Condition for a minimum at the origin:** $\mathcal{H}_{11} > 0$ and $\mathcal{H}_{22} > 0$

Finite renormalisation constants for the 2HDM

From *[Basler et al., 1803.02846]*

$$\delta m_{11}^2 = \frac{1}{2} H^{\text{CW}}_{\zeta_1,\zeta_1} + H^{\text{CW}}_{\psi_1,\psi_1} - \frac{5}{2} H^{\text{CW}}_{\rho_1,\rho_1} + \frac{1}{2}\frac{v_2}{v_1}\left(H^{\text{CW}}_{\zeta_1,\zeta_2} - H^{\text{CW}}_{\eta_1,\eta_2}\right) + t v_2^2$$

$$\delta m_{22}^2 = \frac{1}{2}\left(H^{\text{CW}}_{\zeta_2,\zeta_2} - 3H^{\text{CW}}_{\eta_2,\eta_2}\right) + \frac{1}{2}\frac{v_1}{v_2}\left(H^{\text{CW}}_{\zeta_1,\zeta_2} - H^{\text{CW}}_{\eta_1,\eta_2}\right) + \frac{v_1^2}{v_2^2}\left(H^{\text{CW}}_{\psi_1,\psi_1} - H^{\text{CW}}_{\rho_1,\rho_1}\right) + v_1^2 t$$

$$\delta m_{12}^2 = H^{\text{CW}}_{\eta_1,\eta_2} + \frac{v_1}{v_2}\left(H^{\text{CW}}_{\psi_1,\psi_1} - H^{\text{CW}}_{\rho_1,\rho_1}\right) + v_1 v_2 t$$

$$\delta\lambda_1 = \frac{1}{v_1^2}\left[2H^{\text{CW}}_{\rho_1,\rho_1} - H^{\text{CW}}_{\zeta_1,\zeta_1} - H^{\text{CW}}_{\psi_1,\psi_1}\right] - \frac{v_2^2}{v_1^2} t$$

$$\delta\lambda_2 = \frac{1}{v_2^2}\left[H^{\text{CW}}_{\eta_2,\eta_2} - H^{\text{CW}}_{\zeta_2,\zeta_2}\right] + \frac{v_1^2}{v_2^4}\left(H^{\text{CW}}_{\rho_1,\rho_1} - H^{\text{CW}}_{\psi_1,\psi_1}\right) - \frac{v_1^2}{v_2^2} t$$

$$\delta\lambda_3 = \frac{1}{v_1 v_2}\left(H^{\text{CW}}_{\eta_1,\eta_2} - H^{\text{CW}}_{\zeta_1,\zeta_2}\right) + \frac{1}{v_2^2}\left(H^{\text{CW}}_{\rho_1,\rho_1} - H^{\text{CW}}_{\psi_1,\psi_1}\right) - t$$

$$\delta\lambda_4 = t$$

$$\delta\lambda_5 = \frac{2}{v_2^2}\left(H^{\text{CW}}_{\psi_1,\psi_1} - H^{\text{CW}}_{\rho_1,\rho_1}\right) + t$$

$$N^{\text{CW}}_{\phi_i} \equiv \frac{\partial V_{\text{CW}}}{\partial \phi_i} \qquad H^{\text{CW}}_{\phi_i\phi_j} \equiv \frac{\partial^2 V_{\text{CW}}}{\partial \phi_i \partial \phi_j}$$

$$\delta T_1 = H^{\text{CW}}_{\eta_1,\eta_2} v_2 + H^{\text{CW}}_{\rho_1,\rho_1} v_1 - N^{\text{CW}}_{\zeta_1}, \quad \delta T_2 = H^{\text{CW}}_{\eta_1,\eta_2} v_1 + H^{\text{CW}}_{\eta_2,\eta_2} v_2 - N^{\text{CW}}_{\zeta_2}, \quad \delta T_{\text{CP}} = \frac{v_1^2}{v_2} H^{\text{CW}}_{\zeta_1,\psi_1} + H^{\text{CW}}_{\zeta_1,\psi_2} v_1 - N^{\text{CW}}_{\psi_2}, \quad \delta T_{\text{CB}} = -N^{\text{CW}}_{\rho_2}$$

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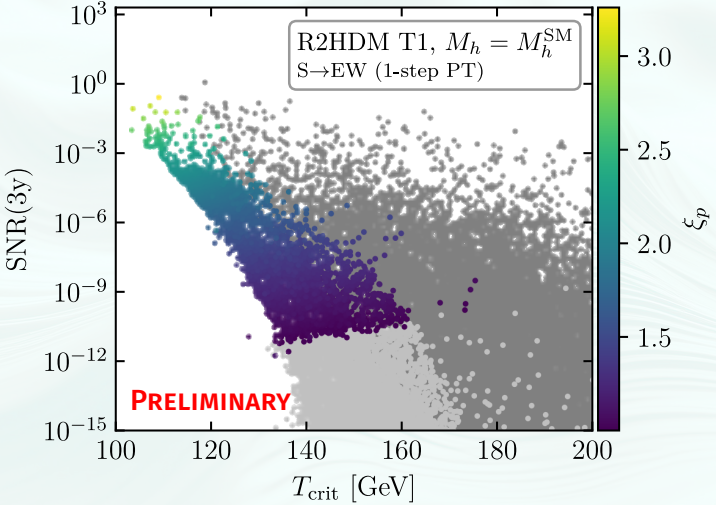
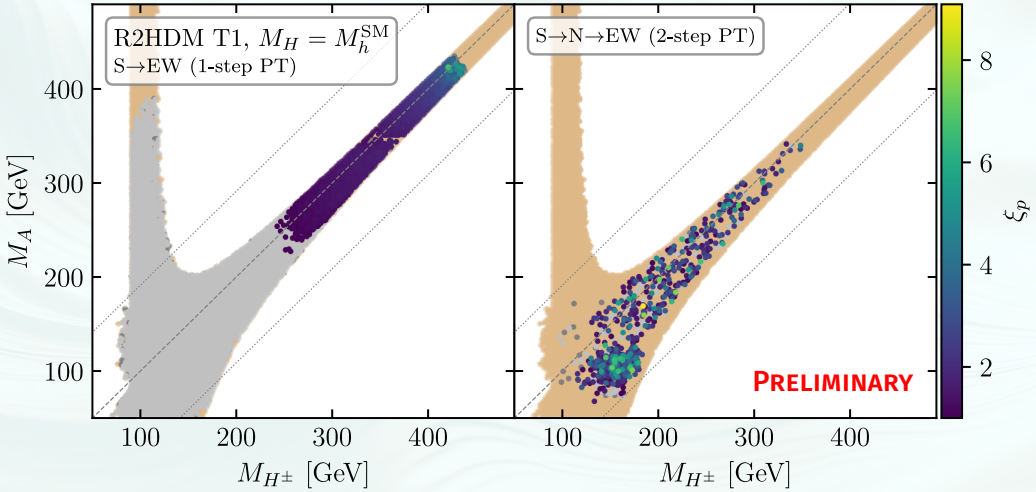
Gravitational-wave signals, $M_h = M_h^{SM} < M_H$

[Biermann, CB, Boto, Mühlleitner, Santos, Viana; in preparation]



One- and two-step phase transitions, $M_h < M_H = M_h^{\mathrm{SM}}$

[Biermann, CB, Boto, Mühlleitner, Santos, Viana; in preparation]