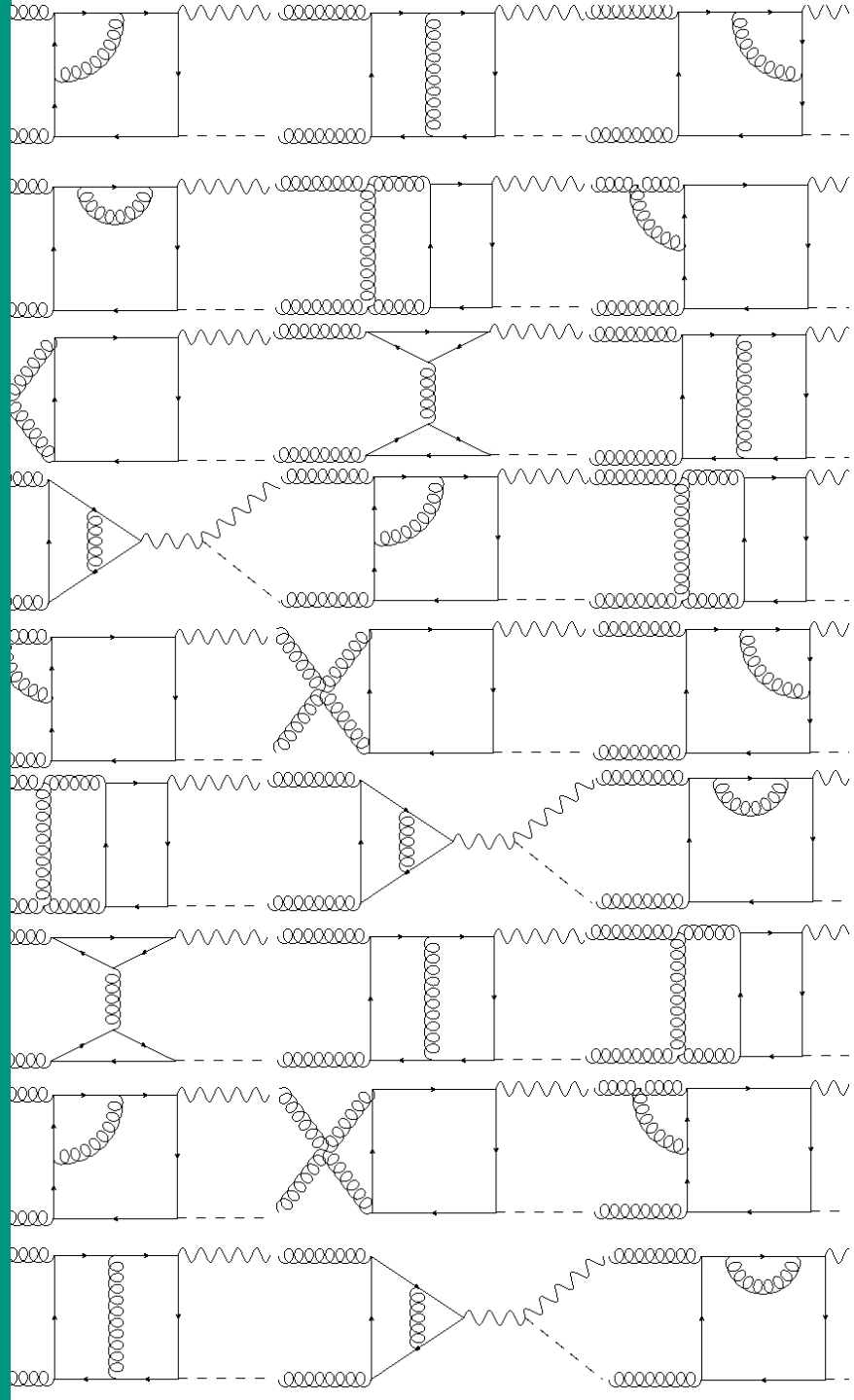
VIRTUAL TWO-LOOP QCD CORRECTIONS TO ZH and Z^*Z^* PRODUCTION VIA GLUON FUSION

[2509.07072]

Joshua Davies, Dominik Grau, Kay Schönwald,
Matthias Steinhauser, Daniel Stremmer, Marco Vitti

The 22nd Workshop of the LHC Higgs Working Group



Why ZH production?

- Third largest contribution to H production cross section
- Study of $H \to b\bar{b}$

Why gluon induced channel?

- Enhanced by large gluon luminosity at LHC:

$$\frac{\sigma_{gg\to ZH}}{\sigma_{pp\to ZH}} \approx 6\%$$

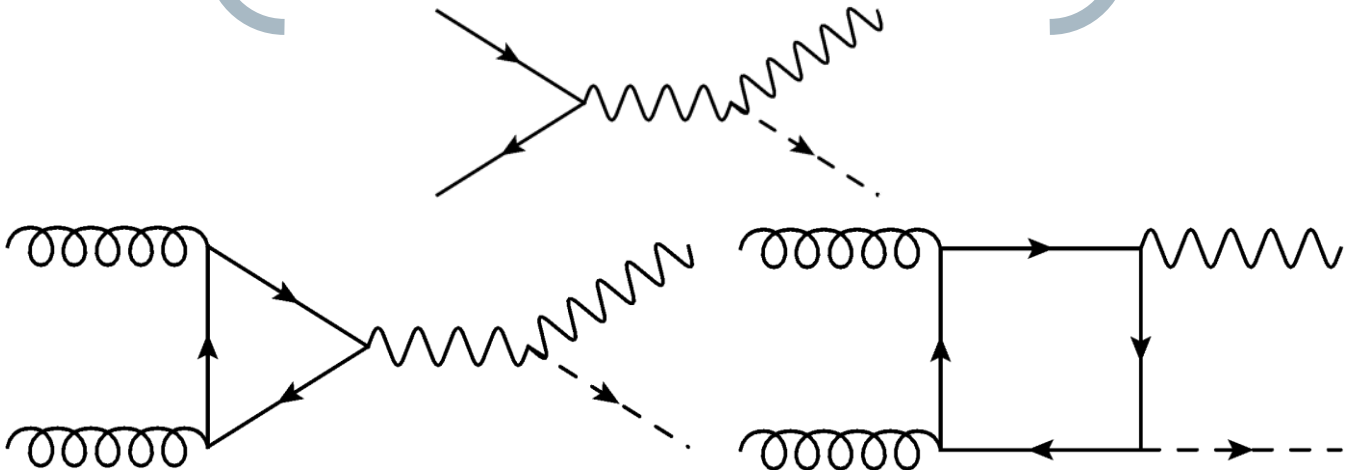
Why NLO corrections?

- Reduce scale uncertainties
- NLO is large: $\frac{\sigma_{gg-NLO}}{\sigma_{gg-LO}} \approx 2$

[Altenkamp, Dittmaier, Harlander, Rzehak, Zirke - 1211.5015]

Why Z^*Z^* production?

- Irreducible Background
- Interference of triangles and boxes constrains Higgs width



$\sqrt{S}$ [TeV]	m_H [GeV]	σ_{LO} [fb]	σ_{NLO} [fb]
13	125.0	$64.0^{+27+0\%}_{-20-23\%}$	$118.4^{+17+0\%}_{-14-13\%}$
13.6	125.0	$70.6^{+26+0\%}_{-20-24\%}$	$130.5^{+16+0\%}_{-14-13\%}$
14	125.0	$75.2^{+26+0\%}_{-20-24\%}$	$138.9^{+16+0\%}_{-14-13\%}$

R5

[Aveleira, Chen, Davies, Degrassi, Giardino, Gröber, Heinrich, Jones, Kerner, Schlenk, Steinhauser, Vitti - 2508.09905]

(ZH) Previous Works

Numerical Evaluation

- **pySecDec** — [Chen, Heinrich, Jones, Kerner, Klappert, Schlenk - 2011.12325]
- **Small-mass** — [Wang,Xu, Xu, Yang - 2107.08206]

Analytical Expansions

- **Large-Mass** $s/4\mathrm{m}_\mathrm{t}^2 \ll 1$ — [Hasselhuhn, Luthe, Steinhauser - 1611.05881] [Altenkamp, Dittmaier, Harlander, Rzehak, Zirke- 1211.5015]
- **Transverse Momentum** $t/4\mathrm{m}_\mathrm{t}^2 \ll 1$ — [Alasfar, Degrassi, Giardino, Groeber, Vitti - 2103.06225]
- **High-Energy** $s/4\mathrm{m}_\mathrm{t}^2 \gg 1 \quad \& \quad \mathrm{t}/4\mathrm{m}_\mathrm{t}^2 \gg 1$ — [Davies, Mishima, Steinhauser - 2011.12314]
- **t** $t/4\mathrm{m}_\mathrm{t}^2 \ll 1$ — Now: [2509.07072]

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t
$t/4m_t^2 \ll 1$
Now: [2509.07072]

Workflow

Generation of Amplitude

↓

Application of Projectors

↓

Expand Scalar Integrals

↓

Integration by Parts Reduction (IBP)

↓

Compute Master-Integrals via Differential–Eqs. (DEQ)

Forward Limit (t)

- Consider $m_h^2, m_Z^2, t < m_t^2, s$
- Expand simultaneously in m_h^2, m_Z^2, t

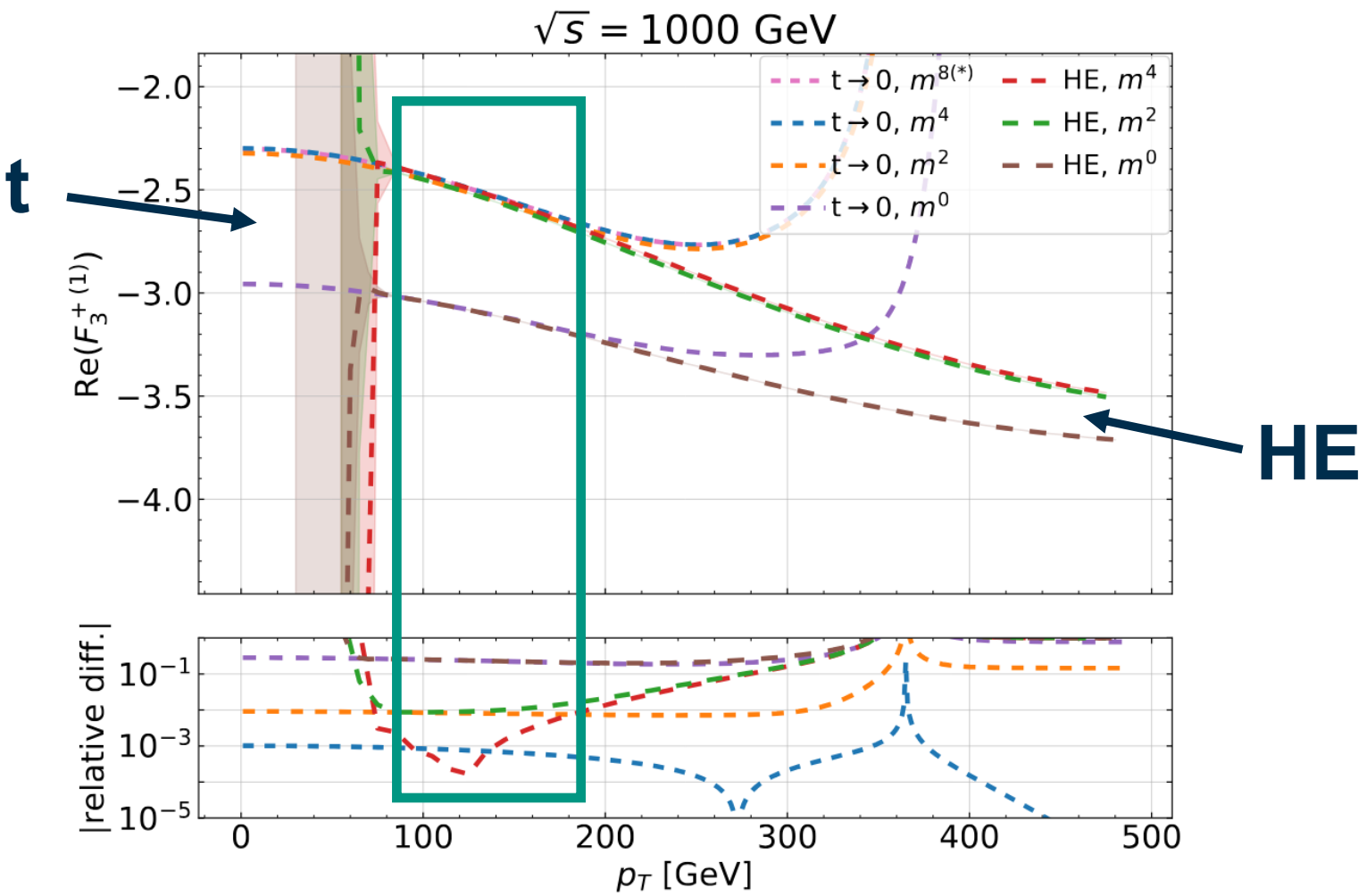
High Energy Expansion (HE)

- Consider $m_h^2, m_Z^2 < m_t^2 < s, t$
- Expand in external masses m_h^2, m_Z^2
- After IBP, solve DEQ in the limit $m_t \to 0$

(ZH) Results

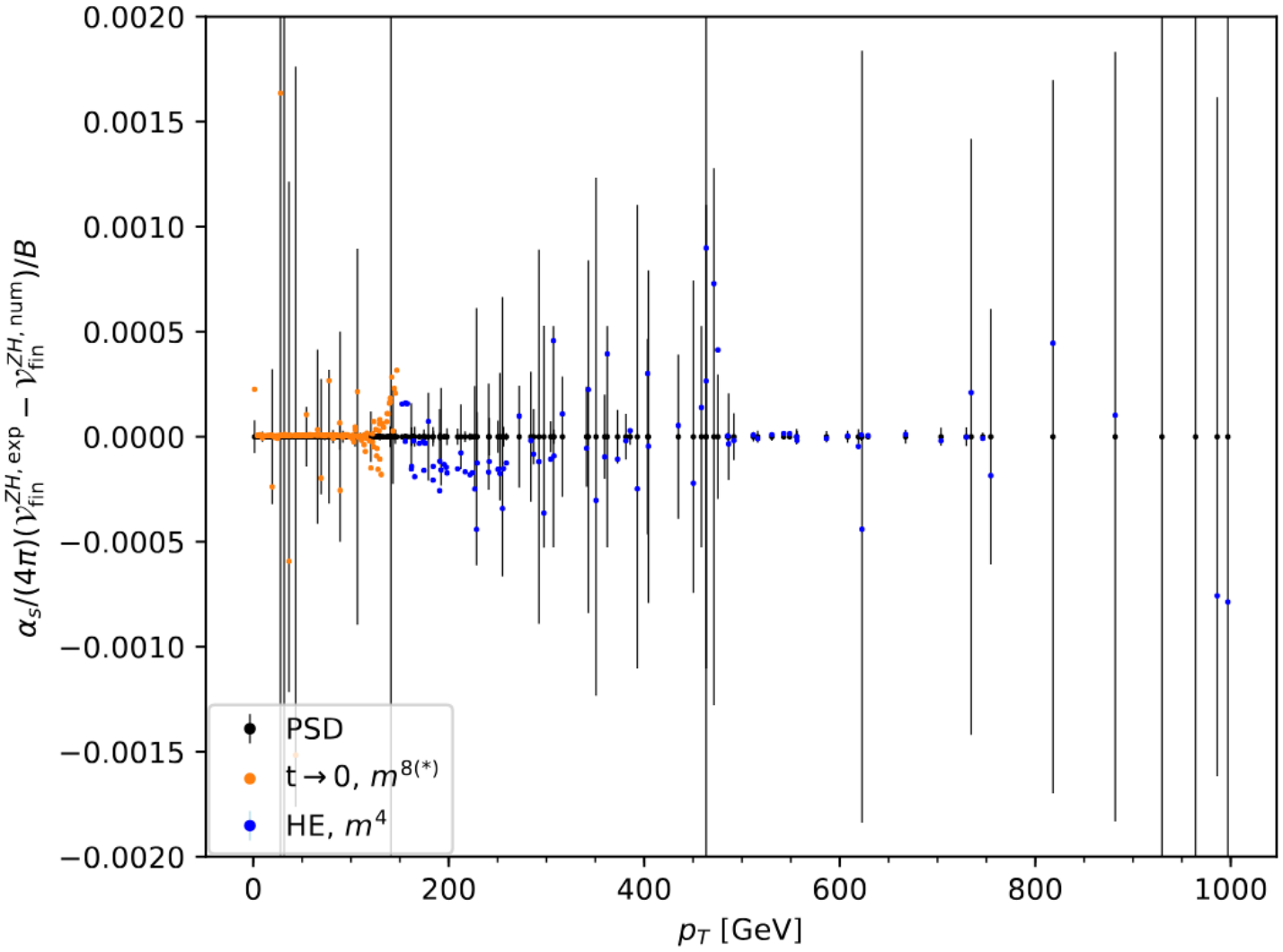
- Finite p_T intervall with matching HE- and t- expansion results

! Use:
t-exp if $p_T \leq 150$ GeV
HE-exp if $p_T \geq 150$ GeV



(ZH) Virtual finite cross section

$$|A|^2 = \left(\frac{\alpha_s}{4\pi}\right)^2 \mathcal{B} + \left(\frac{\alpha_s}{4\pi}\right)^3 \mathcal{V} + \mathcal{O}(\alpha_s^4)$$



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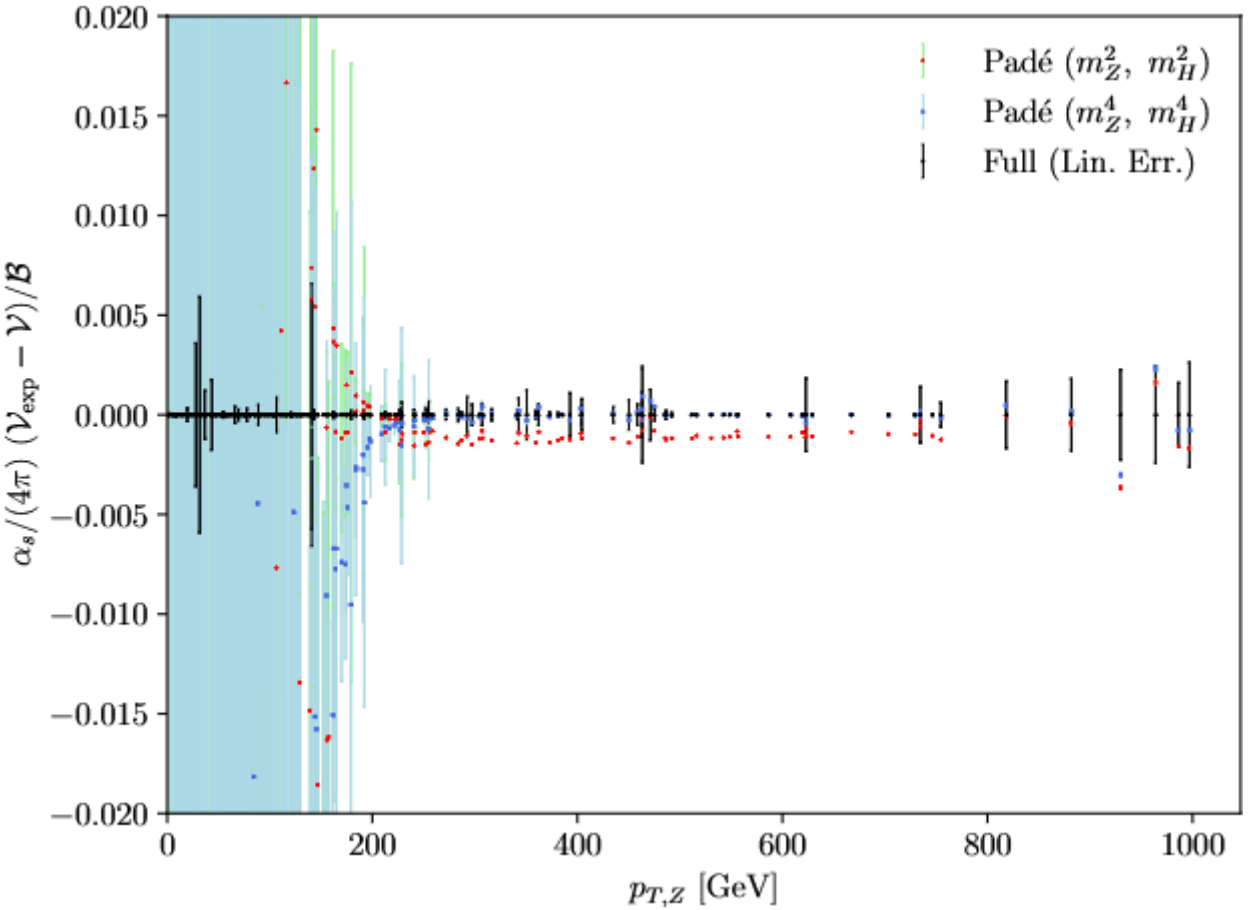
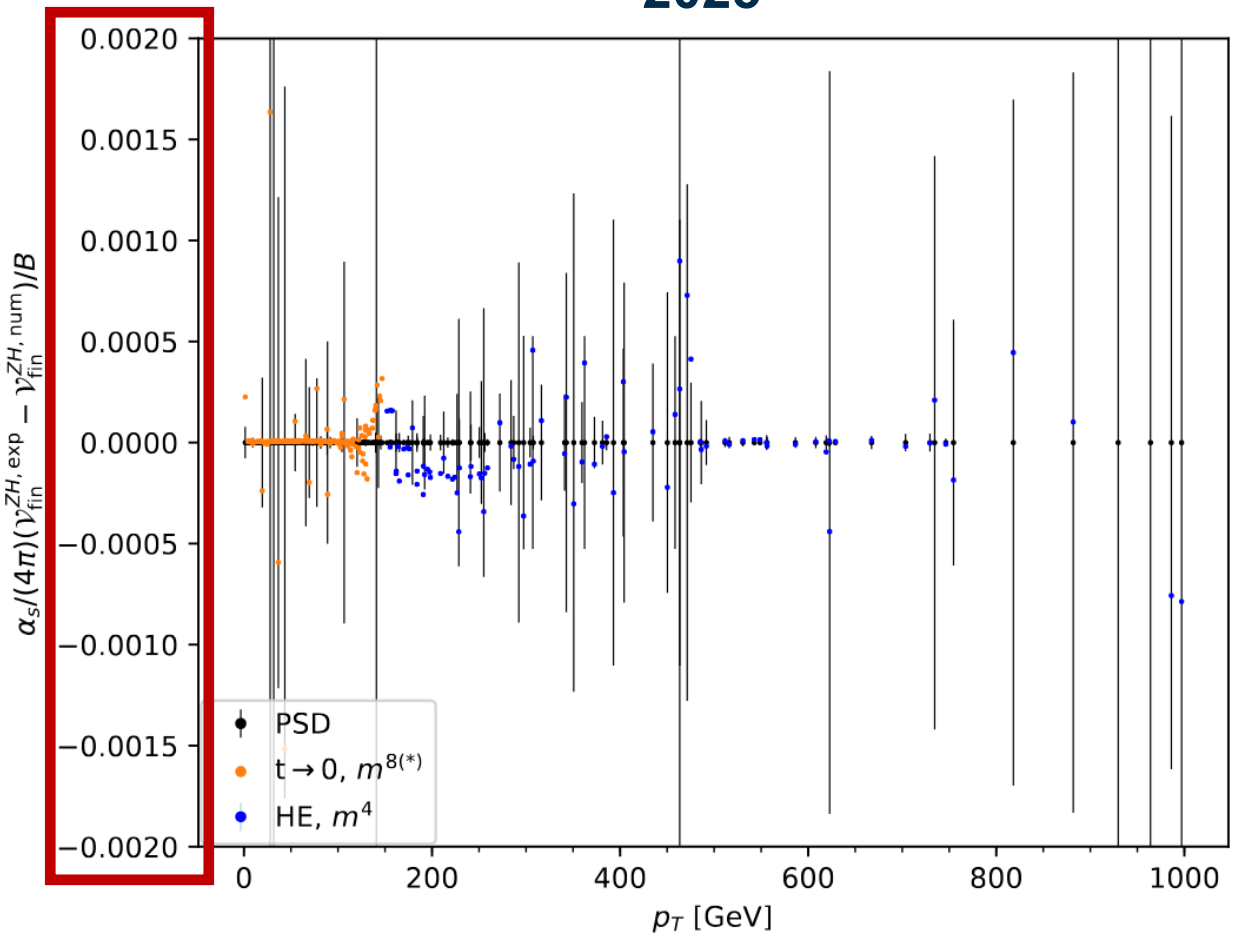
2022



[Chen, Davies, Heinrich, Jones, Kerner, Mishima, Schlenk, Steinhauser - 2204.05225]

2025



Summary

- Subpermille precision for whole phase space using analytical expansions
- Symbolic Results for $gg \to Z^*H^*, Z^*Z^*, Z^*\gamma^*, \gamma^*\gamma^*$
- Flexible w.r.t. renormalization scheme and parameter value

Whats next?

- Implementation into the C++ library $ggxy$ [Davies, Schönwald, Steinhauser, Stremmer - 2506.04323]
  - $\mathcal{O}(ms)$ runtime per phase space points
  - Includes real corrections using $RECOLA$ [Actis, Denner, Hofer, Scharf, Uccirati - 1211.6316]
- Application of analytical approximation also to EW corrections and to similiar processes (e.g. $gg \to H^*H^*, WW$)

R5

See talk of Kay Schönwald from yesterday

(ZH) Virtual finite cross section

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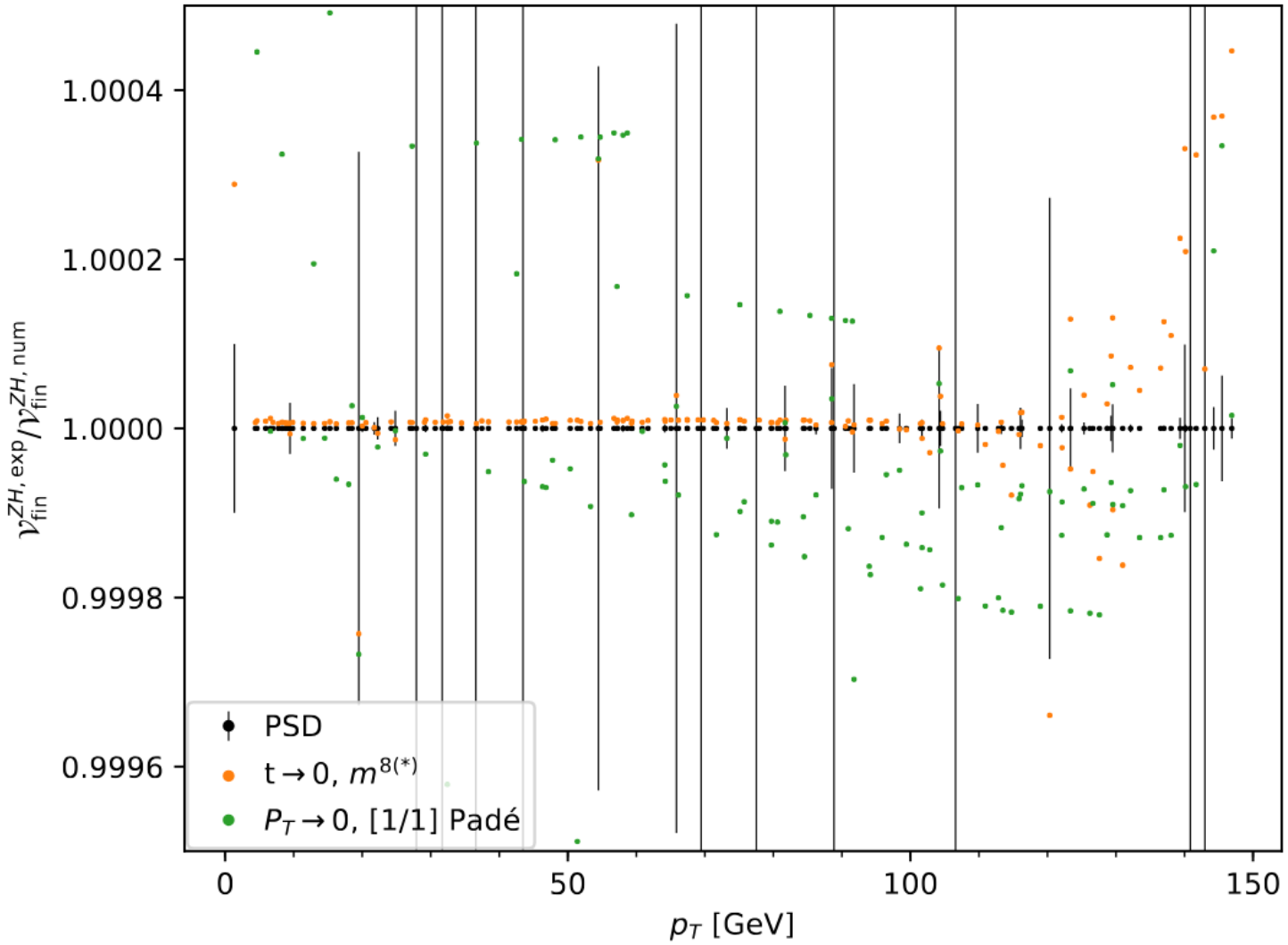

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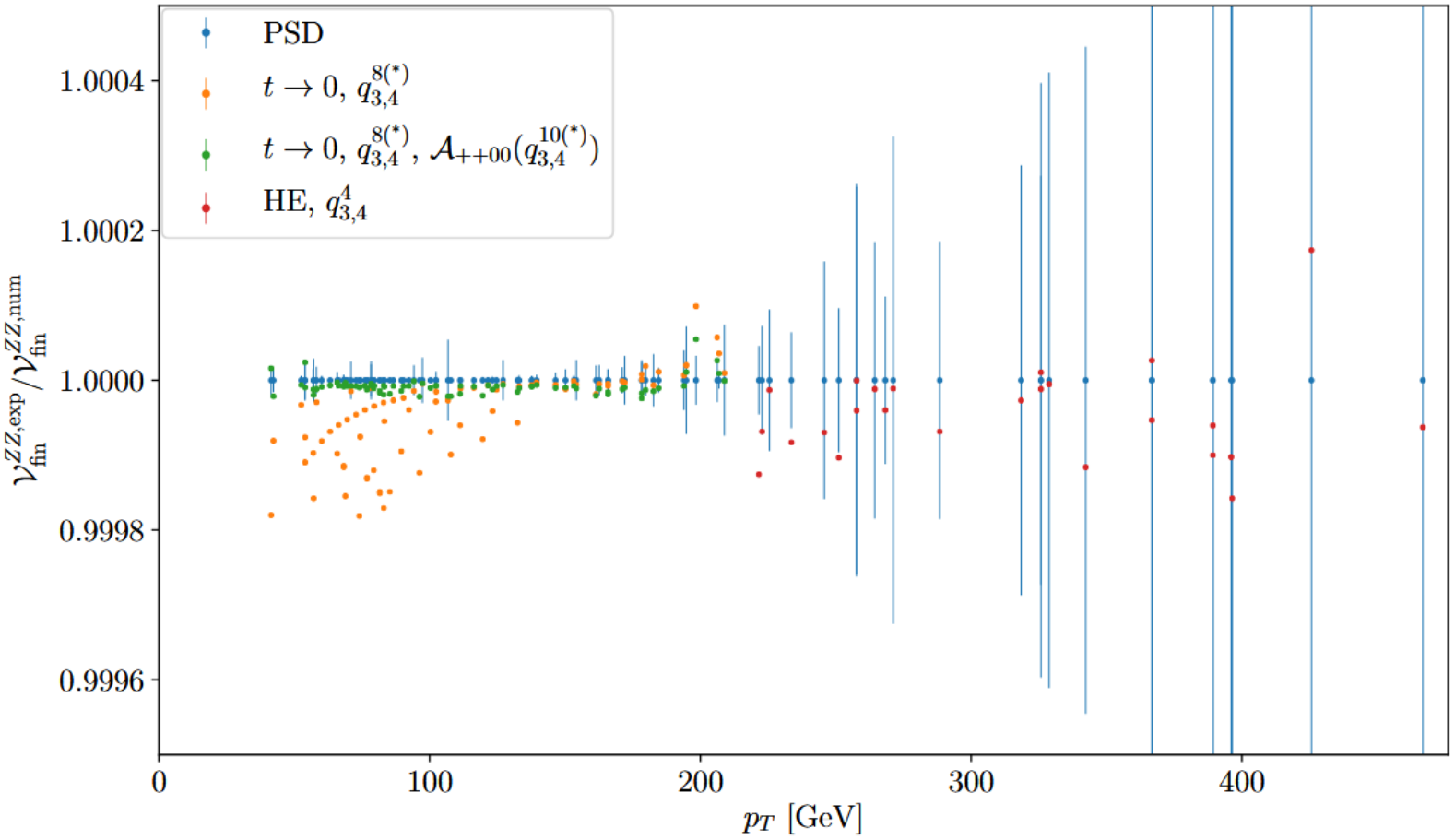
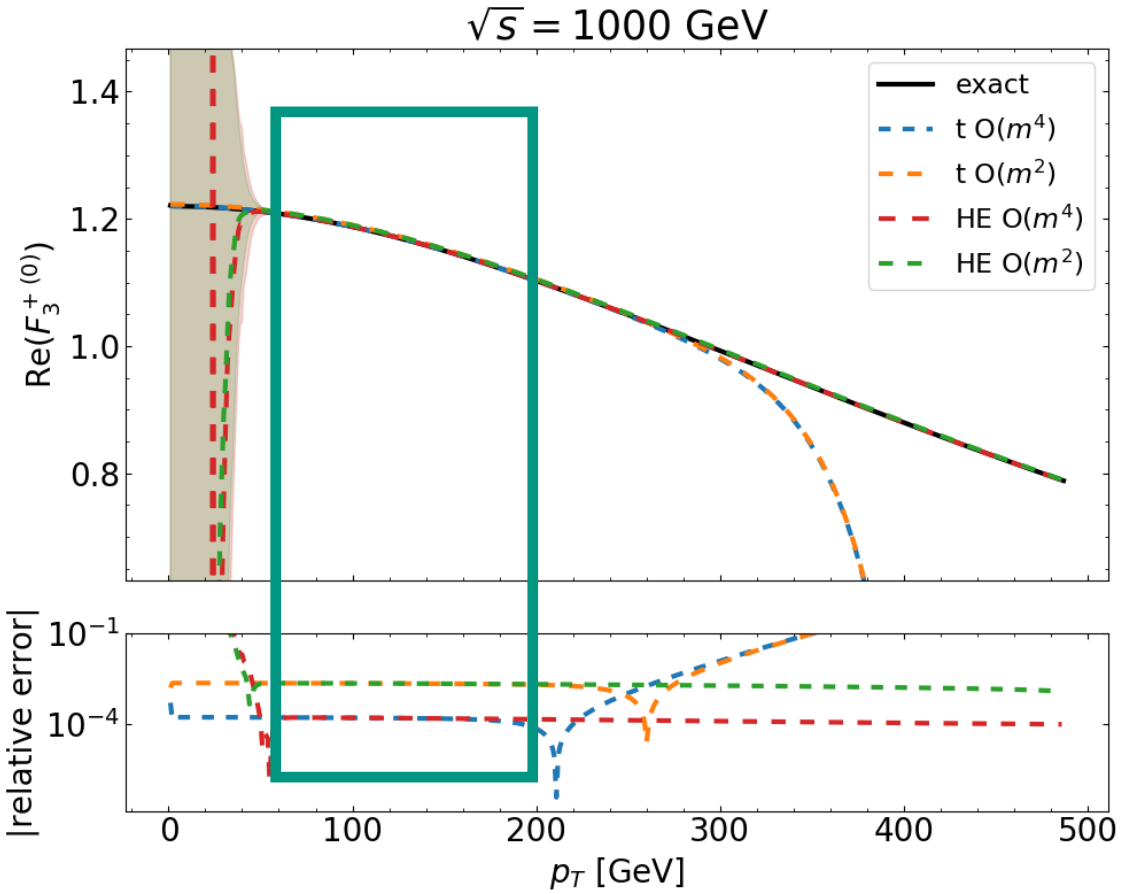


LO Results

- Quartic mass terms are sufficient for sub-percent precision

- Finite p_T intervall with matching HE- and t- expansion results

! Use:
t-exp if $p_T \leq 150$ GeV
HE-exp if $p_T \geq 150$ GeV



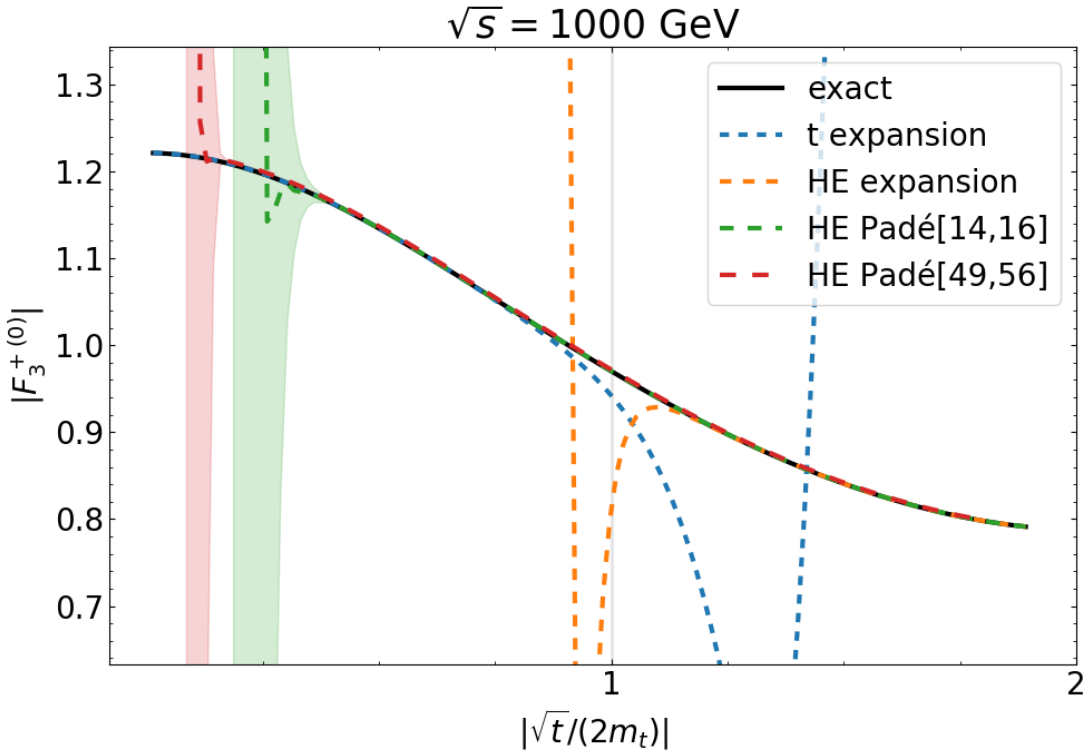
Pade Approximation

- Ansatz:

$$\left[\frac{n}{m}\right](x) = \frac{a_0 + a_1 x + \ldots + a_n x^n}{1 + b_1 x + \ldots + b_m x^m}$$

- Solve for:

$$N_{min} \leq n + m \leq N_{max} \quad \& \quad N_{min} \leq n + m - |n - m|$$



IBP – Integration by Parts

- **Example:** Massive Tadpole Integral:

$$F(a) = \int \frac{d^d k}{(m^2 - k^2)}$$

- The IBP relation is as follows:

$$\int d^d k \frac{\partial}{\partial k_\mu} \frac{k_\mu}{(m^2 - k^2)} = 0$$

- This leads to:

$$(d - 2a)F(a) - 2am^2 F(a+1) = 0$$

$$\Rightarrow F(a) = \frac{d - 2a + 2}{2(a-1)m^2} F(a-1)$$

Generalization:

- Derivatives w.r.t. all loop momenta
- Integrand multiplied with all loop + external momenta

→ System of linear Equation

→ Solve with Laporta Algorithm

→ From knowing the Master Integral F(1) all other integrals can be calculated recursively

Computer Programs

- Amplitude Generation & Projection
  - QGRAF [Nogueira‘91]
  - TAPIR [Gerlach, Herren, Lang – 2201.05618]
  - EXP [Harlander, Seidensticker, Steinhauser‘98]
  - in house FORM code

- IBP-Reduction:
  - KIRA [Maierhoefer, Usovitsch, Uwer - 1705.05610]
  - FIRE [Smirnov, Chukharev - 1901.07808]
  - LITERED [Lee – 1212.2685]
  - REDUZE [Manteuffel, Studerus – 1201.4330]

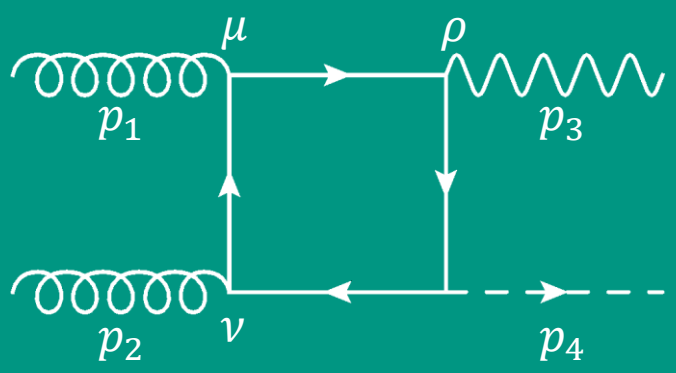
Old slides

Calculation

Step 1

Generate the Amplitude

$$A = \delta_{ab}\epsilon_\mu^a(p_1)\epsilon_\nu^b(p_2)\epsilon_\rho^*(p_3)A^{\mu\nu\rho}$$



Step 2

Apply Projectors

$$P_i^{\mu\nu\rho} = \sum_j c_{ij} S_j^{\mu\nu\rho}$$

to extract scalar form factors

$$F_i = P_i^{\mu\nu\rho} A_{\mu\nu\rho}$$

in order to avoid calculating tensor integrals

Step 3

Calculate Loop Integrals

$$I(s, t, m_H, m_Z, m_t)$$

- Easy at LO
- Not so easy at NLO

Calculation of the Loop Integrals - Analytic Expansion

Step 3.1

Expand in the external masses

$$\{I_i(s,t,m_H,m_Z,m_t)\}_{7k}$$

$$\rightarrow \{\widetilde{I}_i(s,t,m_t)\}_{700k}$$

- Simplifies integrals
- Proliferates intermediate expressions
- Generates spurious poles

Step 3.2

Apply IBP Reduction

$$\{\widetilde{I}_i(s,t,m_t)\}_{700k}$$

$$\rightarrow \{M_i(s,t,m_t)\}_{161}$$

Reduces integrals to a minimal set of „Master Integrals“

FIRE [Smirnov, Chukharev - 1901.07808]
KIRA [Maierhoefer, Usovitsch, Uwer - 1705.05610]
REDUZE [Manteuffel, Studerus – 1201.4330]

Step 3.3

Compute Master Integrals using DEQ

$$\partial_x \vec{M} = A\vec{M}, \qquad x = t, m_t$$

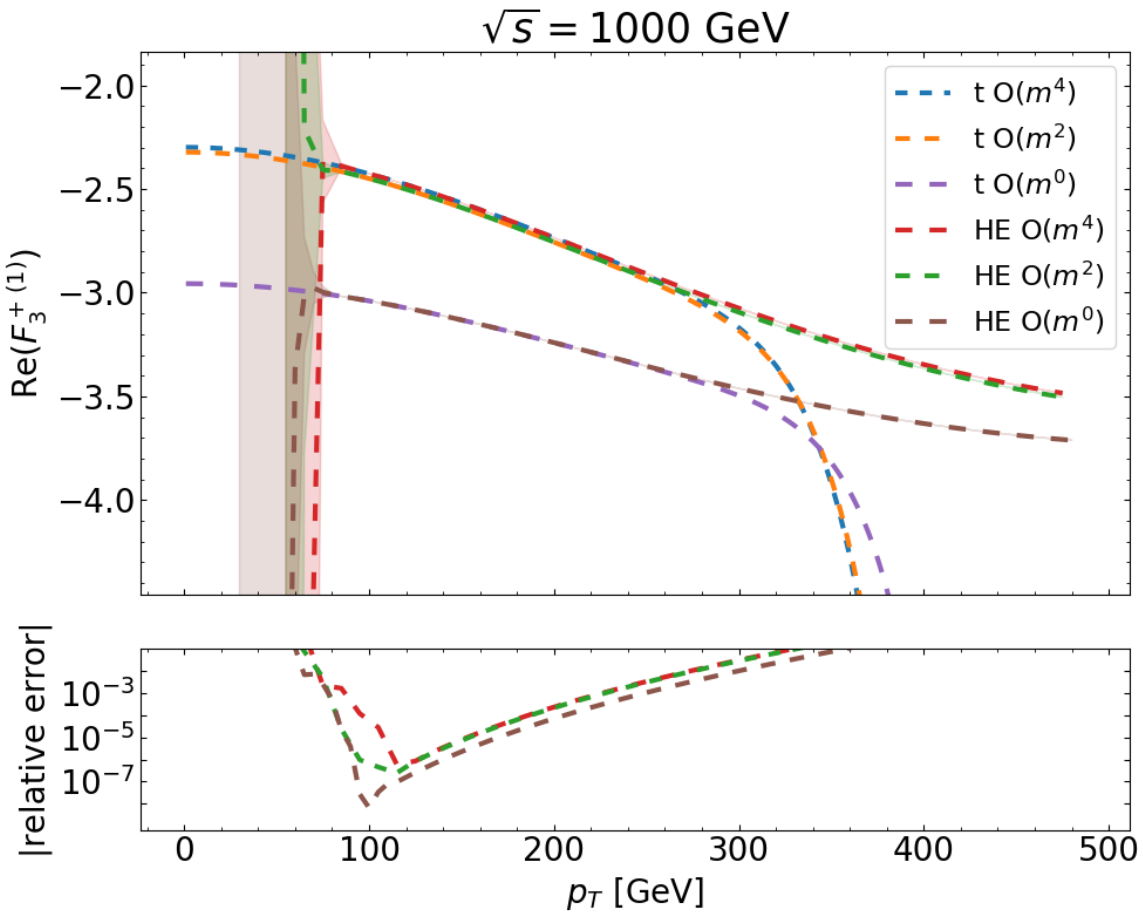
t-Ansatz:

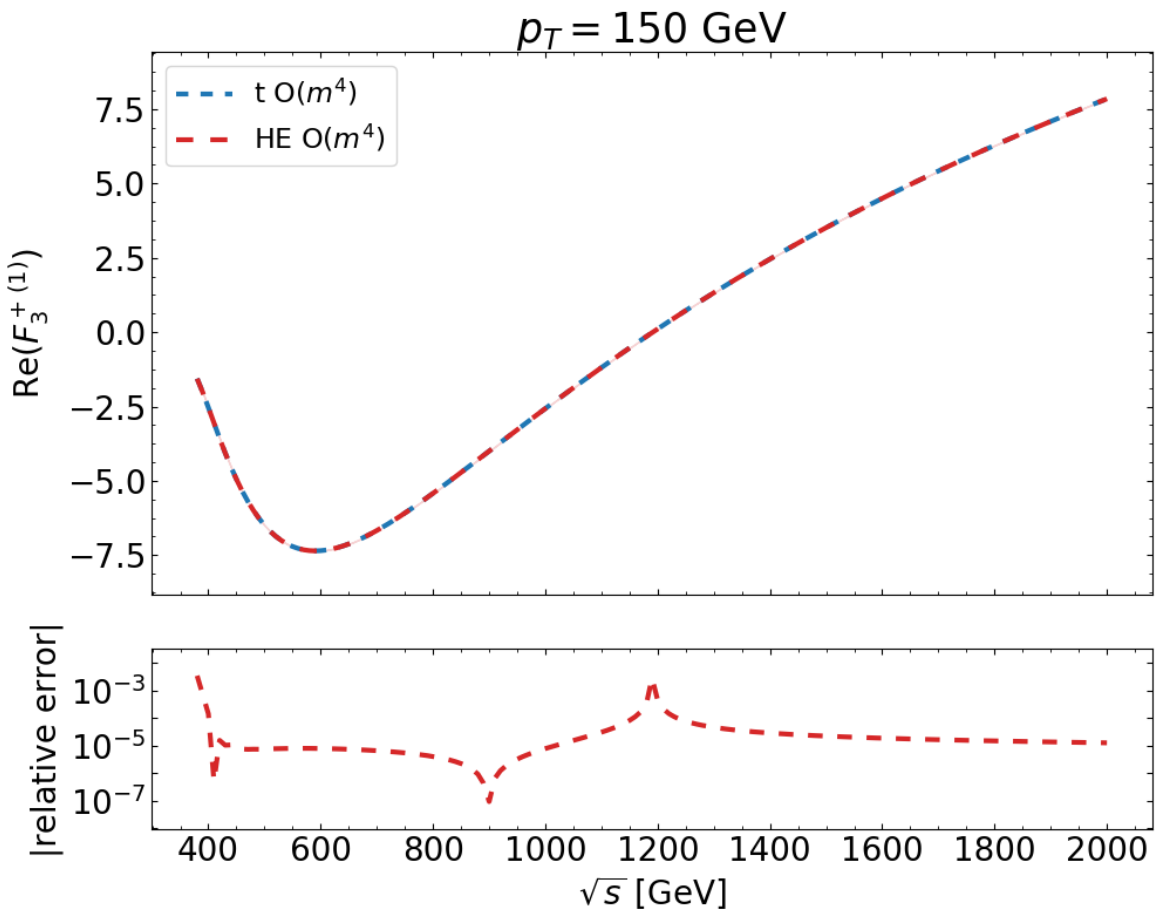
$$M_i = \sum_{a,b} C_{ab}^{(i)}(s,m_t)\epsilon^a \left(\frac{t}{m_t^2}\right)^b$$

HE-Ansatz:

$$M_i = \sum_{a,b,c} C_{abc}^{(i)}(s,t)\epsilon^a \left(\frac{m_t}{\sqrt{s}}\right)^b \left(\ln\frac{m_t^2}{s}\right)^c$$

NLO Results





! Agreement of HE- and t-expansion also holds at NLO

Calculation of the Loop Integrals

Analytical Expansions

- **Large-Mass**
  $s/4m_t^2 \ll 1$
- **Transverse Momentum**
  $t/4m_t^2 \ll 1$
- **High-Energy**
  $s/4m_t^2 \gg 1 \quad \& \quad t/4m_t^2 \gg 1$
- **t**
  $t/4m_t^2 \ll 1$

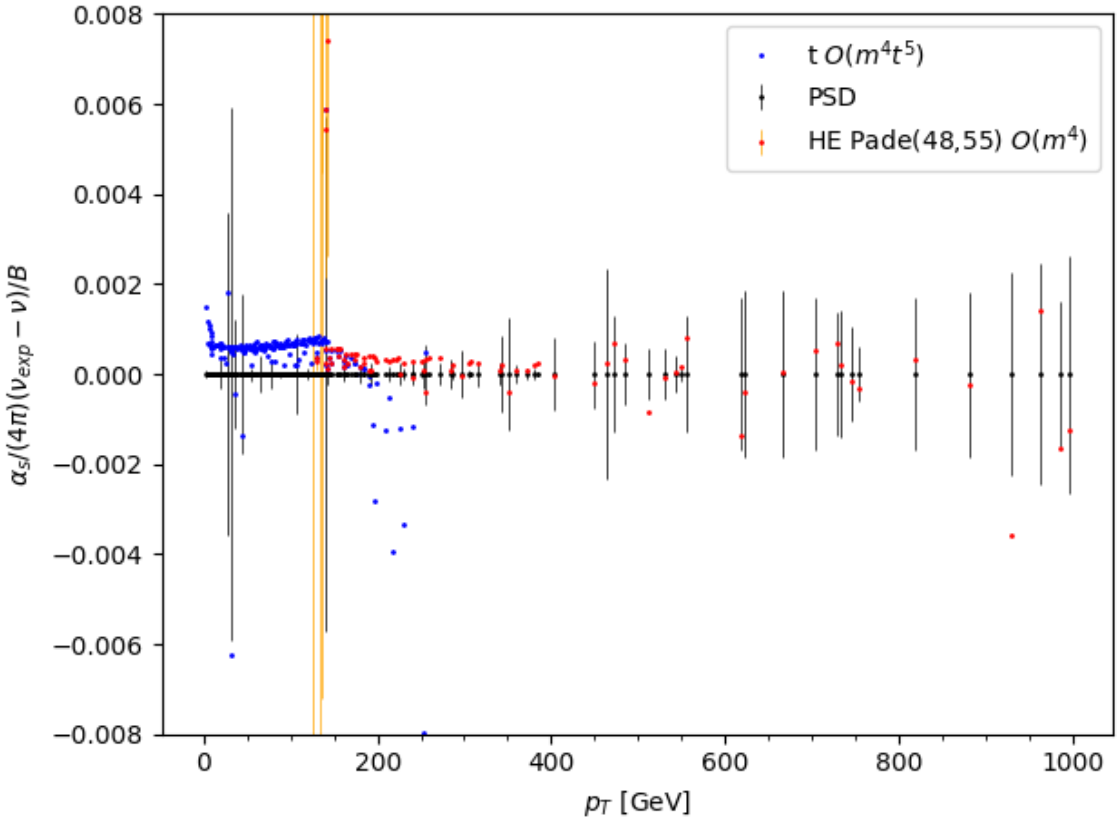
Radius of convergence is limited by two-top thresholds

$$s = 4m_t^2 \qquad \& \qquad t = 4m_t^2$$

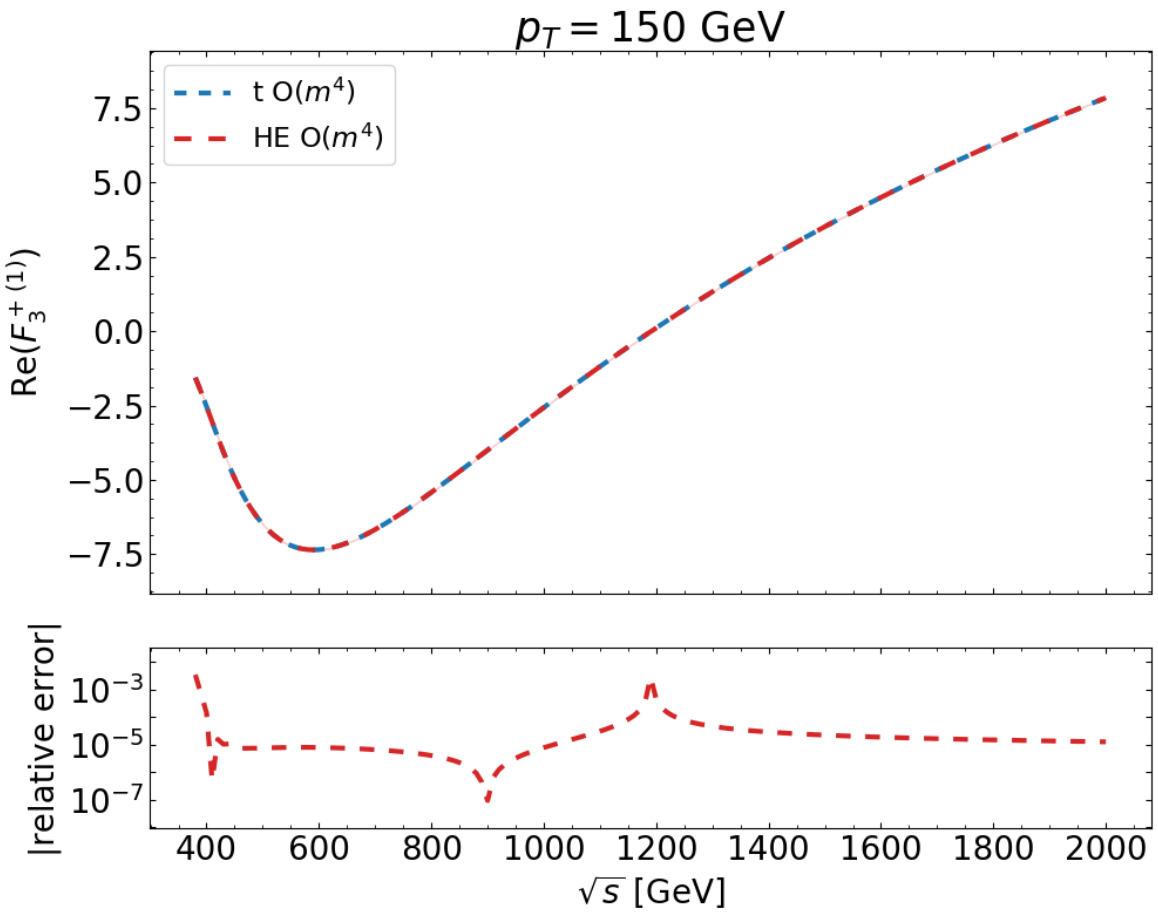
→Combine results of different expansions to cover the full phase space

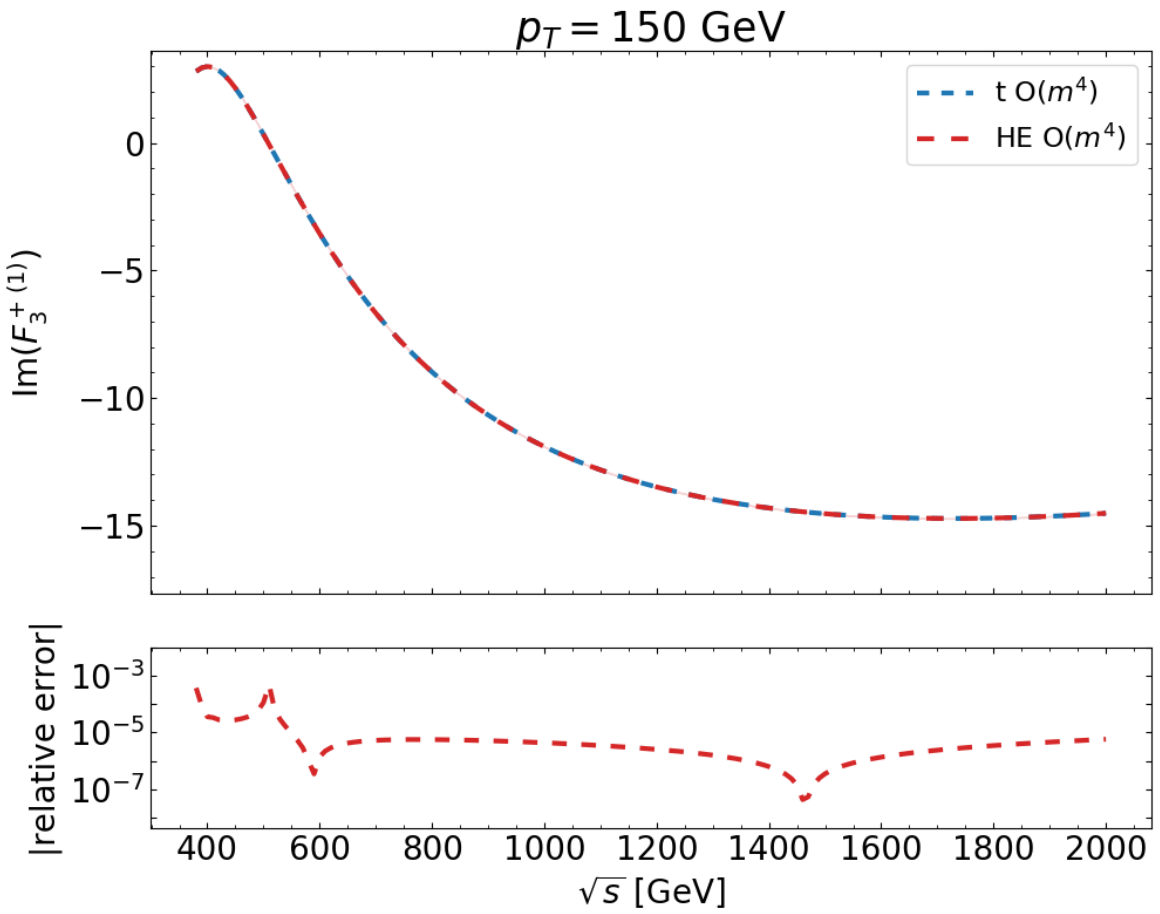
Virtual finite cross section

$$v_{\text{fin}} = \frac{G_F^2 m_Z^2}{16}\left(\frac{\alpha_s}{\pi}\right)^2 \left\{ \sum_\lambda \underbrace{2\text{Re}\left(\left|A^{(0)}A^{(1)*}\right|\right)}_{\text{Interference of the LO and NLO amplitudes}} + \overbrace{\frac{C_A}{2}\left(\pi^2 - \log^2\frac{\mu^2}{s}\right)\left|A^{(0)}\right|^2}^{\text{finite contribution of the IR counterterms}} \right\}$$



NLO Results





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