

Flow in Light Ion Collisions from Kinetic Theory and Inferences on Hydrodynamisation

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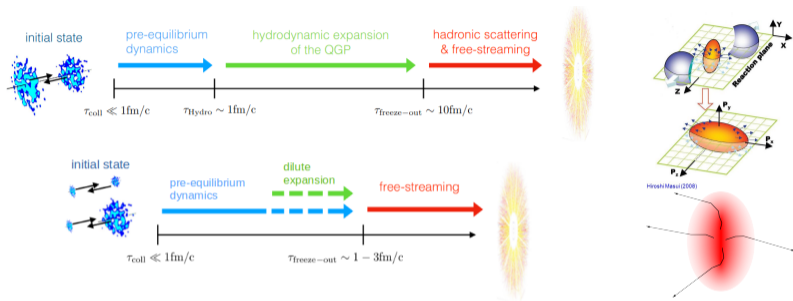
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Dynamical modelling in small vs. large systems

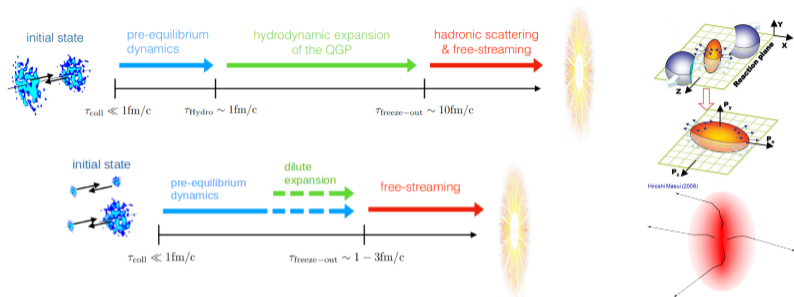
large systems: dominated by hydrodynamic QGP, strong collectivity
↓ inbetween: light ions
small systems: might not fully equilibrate, few scatterings



comparing **flow**: inaccuracies in **hydro** masked by **initial state** uncertainties ($v_2 = \kappa \epsilon_2$)

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Approximations in hydrodynamics

- gradient expansion: Knudsen number $\text{Kn} \sim \ell_{\text{mfp}}/L$
- expansion around equilibrium: inverse Reynolds number $\text{Re}^{-1} \sim |\pi_{\mu\nu}|/P$

but derivation does not give specific values for the regime of validity!

⇒ needs to be assessed in comparison to more microscopic description: kinetic theory!

RTA kinetic theory

We perform simulations of hadronic collisions using kinetic theory in conformal relaxation time approximation

- some microscopic info “integrated out”
⇒ no diluteness requirement
agreement in dense limit
- dynamics depends only on opacity $\hat{\gamma}$

Alexi Kurkela, Urs A. Wiedemann, Bin Wu, EPJC 79 (2019) 11, 965

$$p^\mu \partial_\mu f = -\frac{f - f_{\text{eq}}}{\tau_R}, \quad \tau_R = 5T \frac{\eta}{s}$$

$$\hat{\gamma} = \left(5 \frac{\eta}{s}\right)^{-1} \left(\frac{1}{a\pi} R \frac{dE_\perp^{(0)}}{d\eta}\right)^{1/4}$$

total interactions $\hat{\gamma} \sim \text{Kn}^{-1}$:
depends on **shear viscosity**,
transverse size and **energy scale**

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Aleksi Kurkela, Urs A. Wiedemann, Bin Wu, EPJC 79 (2019) 11, 965

and compare results for transverse flow to hydrodynamics

- conformal equation of state $\mathcal{E} = 3P = aT^4$
- transport coefficients from RTA
- elliptic flow of energy ε_p : extracted directly from hydro, no need for hadronization

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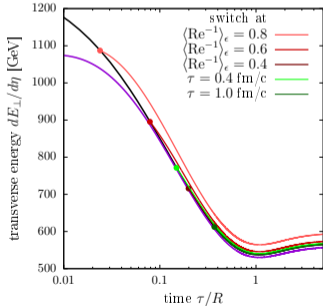
$$\varepsilon_p = \frac{\int_{\mathbf{x}_\perp} T^{xx} - T^{yy} + 2iT^{xy}}{\int_{\mathbf{x}_\perp} T^{xx} + T^{yy}}$$

$$c_{\varepsilon_p}\{2\} = \langle |\varepsilon_p|^2 \rangle \quad \text{etc.}$$

Comparing hydro to kinetic theory (for now: average profile)

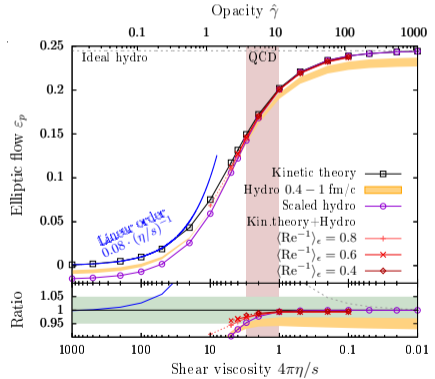
Hydro doesn't work at early times! We can

- scale initial condition of hydro
 - assumes equilibration before transverse expansion
- works for $\hat{\gamma}$ & 4



Victor E. Ambruş, Sören Schlichting, CW PRD 107 (2023) 094013

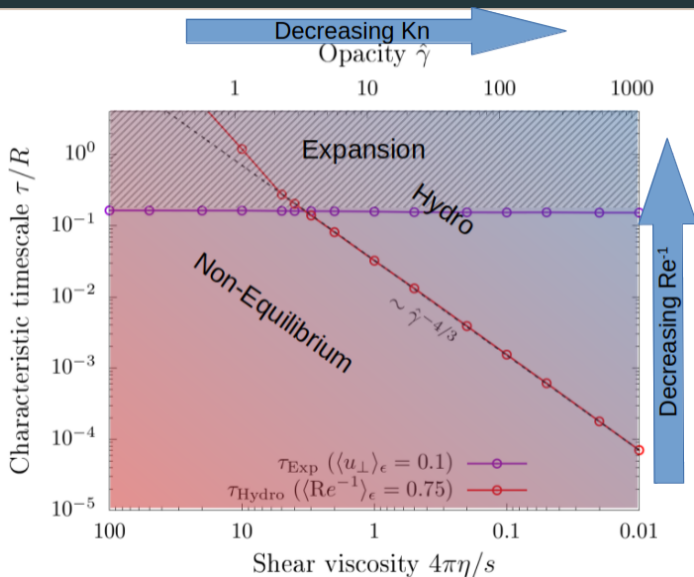
- set up hybrid simulations
 - later switch gives better agreement
 - switching at fixed Re^{-1} gives good handle on accuracy



Regime of applicability of hydrodynamics

timescale of hydrodynamization:
for $\hat{\gamma} \gtrsim 3$ (central OO?), need
non-equilibrium description of
transverse expansion

Victor E. Ambruş, Sören Schlichting, CW PRL 130
(2023) 152301

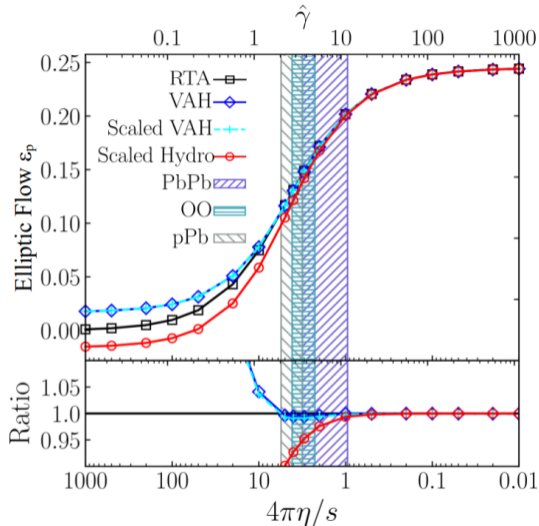


Applicability of viscous anisotropic hydrodynamics

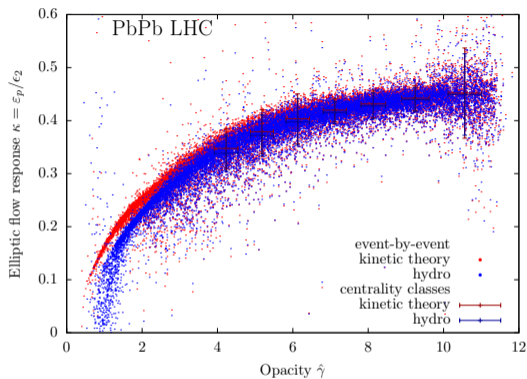
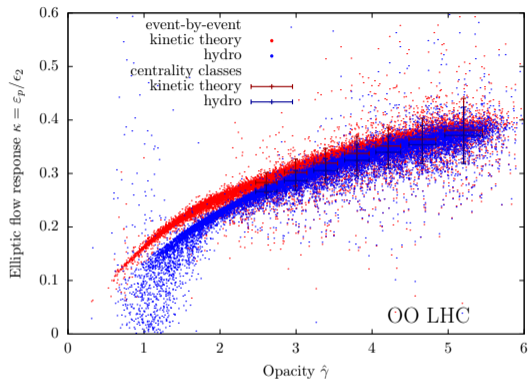
Same comparison of final state flow in the “VAH” code:

M. McNelis, D. Bazow, U. Heinz, *Comput. Phys. Commun.* 267, 108077

- Anisotropic reference state captures non-equilibrium pressure anisotropy
⇒ can initialize far from equilibrium
- accurate beyond traditional hydro:
 $\hat{\gamma} \ll 1$; covers high mult. pPb



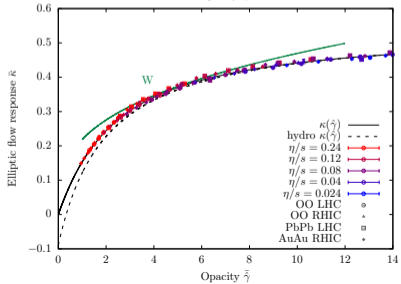
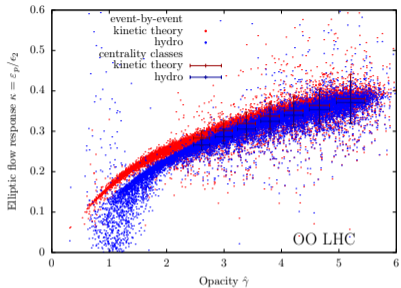
Event-by-event Flow Responses



- some event-by-event spread, but mainly following universal curves
- still hydro \rightarrow kinetic theory as $\hat{\gamma} \rightarrow \infty$

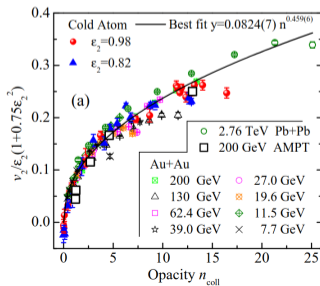
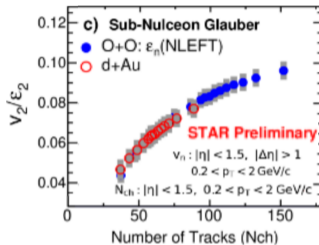
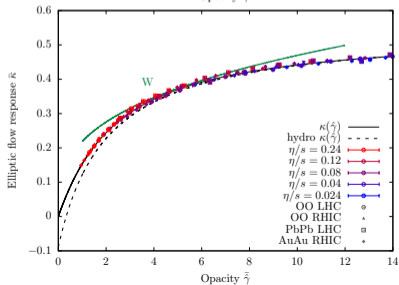
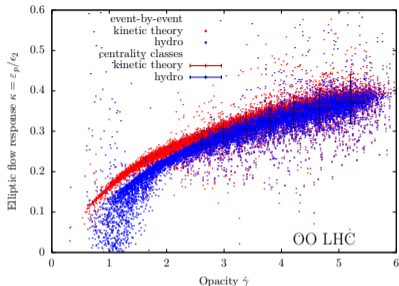
Universal flow response curve $v_2/\epsilon_2 = \kappa(\hat{\gamma})$

Hadronic collision simulations



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Experiment

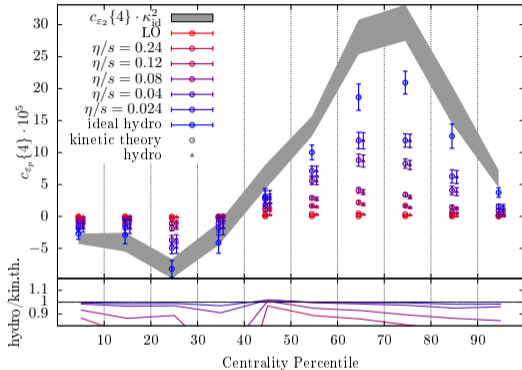
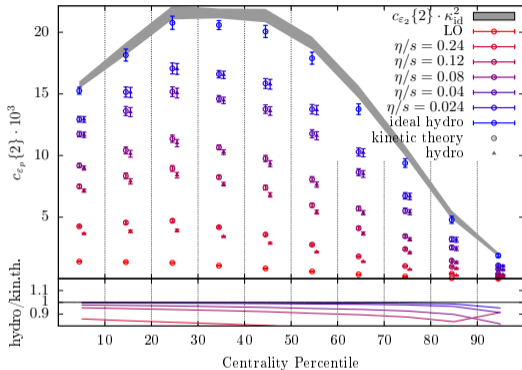
Hadronic collisions

Andrew Tamis @ IS25

Cold atoms

Kei Li, Hong-Fang Song,
Hao-Jie Xu, Yu-Liang Sun,
Fuqiang Wang,
arXiv:2405.02847

Flow Cumulants



- opacity \uparrow : flow response \uparrow , difference in flow response \downarrow
- centrality dependence of $\kappa(\hat{\gamma})$ introduces modulation
- flow fluct. dominated by avg. response to geometry fluct.

$$\langle (\epsilon_p)^n \rangle = \langle (\kappa \epsilon_2)^n \rangle = \bar{\kappa}^n \langle (\epsilon_2)^n \rangle + \dots$$

Hydrodynamization Observable: Definition

measuring hydrodynamization

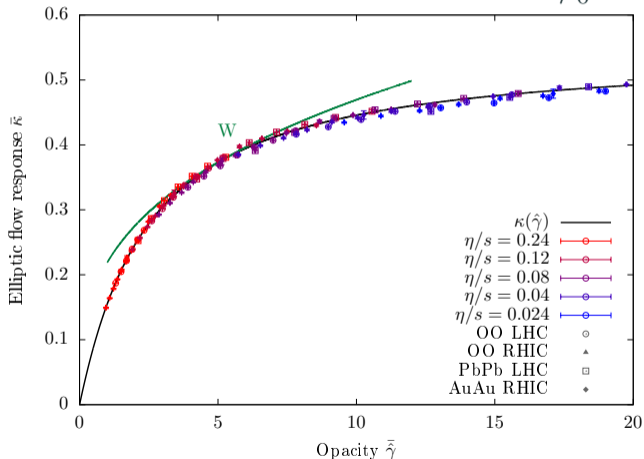
cancel geometry factors between similar systems:

$$\frac{\bar{\kappa}_{\text{RHIC}}^{2k}}{\bar{\kappa}_{\text{LHC}}^{2k}} \approx \frac{c_2^{\text{RHIC}}\{2k\}}{c_2^{\text{LHC}}\{2k\}}$$

$$\frac{\hat{\gamma}_{\text{RHIC}}}{\hat{\gamma}_{\text{LHC}}} \approx \left(\frac{\frac{dE_\gamma}{d\eta}_{\text{RHIC}}}{\frac{dE_\gamma}{d\eta}_{\text{LHC}}} \right)^{1/4}$$

to combine the two: log turns ratios into differences

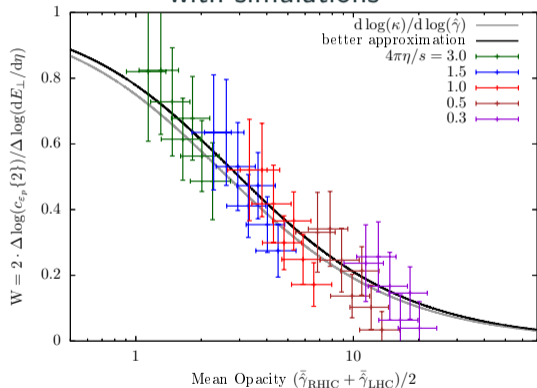
$$W = \frac{2}{k} \frac{\Delta \log(c_2\{2k\})}{\Delta \log(dE_\gamma/dy)} \approx \frac{d \log \kappa}{d \log \hat{\gamma}} \xrightarrow{\hat{\gamma}! \rightarrow 0} 1 \xrightarrow{\hat{\gamma}! \rightarrow 1} 0$$



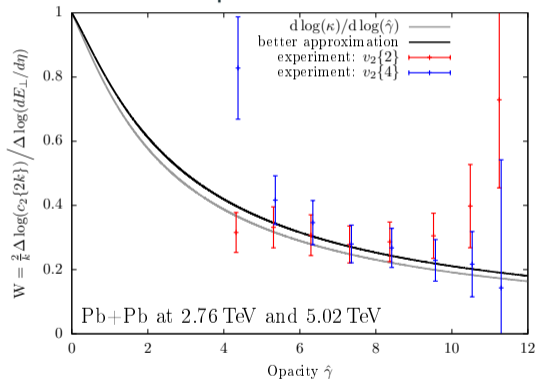
Hydrodynamization Observable: Proof of principle

crosscheck of W-observable:

with simulations

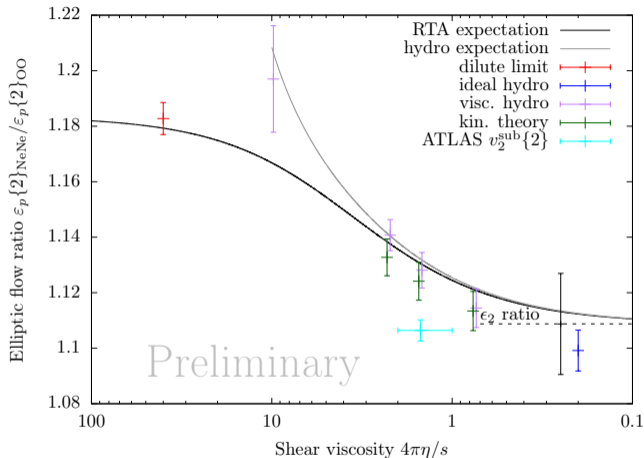


with experimental data



previous hydrodynamization criterion: $\hat{\gamma} \sim 3$ corresponds to $W \sim 0.5$

NeNe/00 ratio



- elliptic flow ratio $\sim \epsilon_2$ ratio \cdot ratio of responses
- difference in response varies with opacity scale and dynamical description

- simulated hadronic collisions in kinetic theory, assessed applicability of hydrodynamics
 - OO may lie on the border of applicability of (MIS) hydrodynamics
- flow results follow universal curve $\kappa(\hat{\gamma})$
 - can help understand e.g. flow ratio NeNe/OO
- hydrodynamization observable measures strength of dynamical flow response independent of initial state:

$$W = \frac{2}{k} \frac{\Delta \log(c_2\{2k\})}{\Delta \log(dE_{\perp}/dy)} \approx \frac{d \log \kappa}{d \log \hat{\gamma}}$$