Towards lattice simulations of strongly-correlated nuclear systems with high-fidelity chiral interactions

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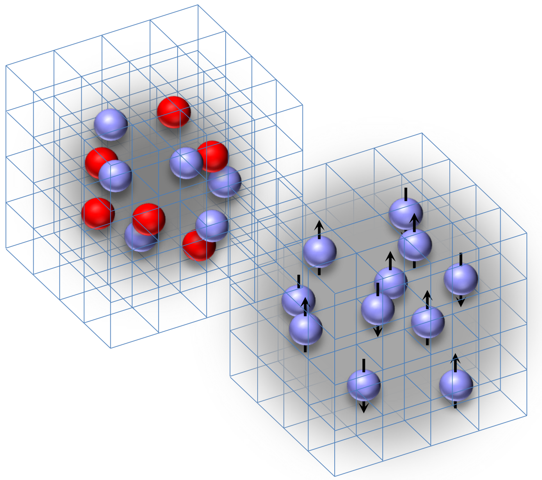
Nuclear Lattice EFT Collaboration



Light Ion Collisions at the LHC – 2025
CERN
December 1, 2025

FRIB
MICHIGAN STATE UNIVERSITY
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Outline

Nuclear lattice effective field theory (NLEFT)

Pinhole algorithm

^{16}O^{16}O collisions

^{16}O versus ^{20}Ne

Wavefunction matching

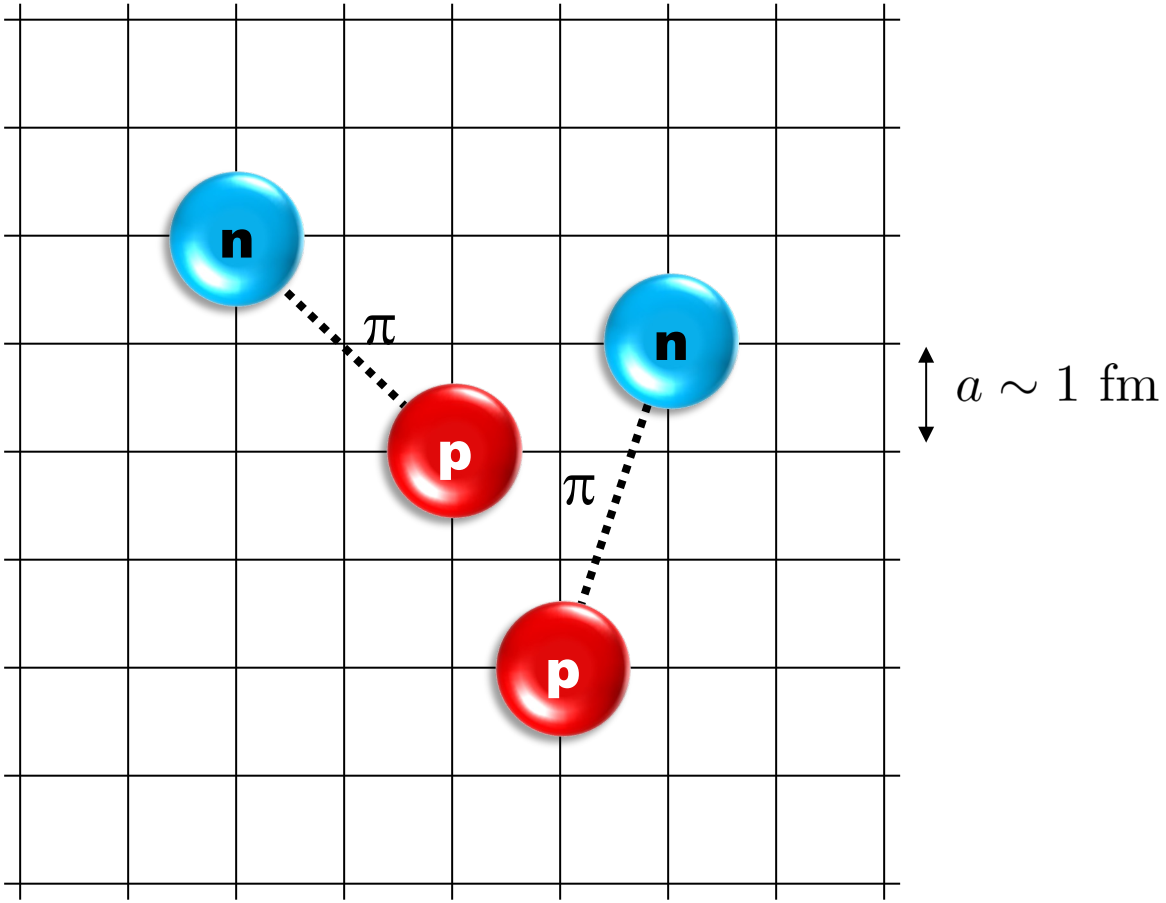
Structure of light and medium-mass nuclei

Pinhole sign problem

Markov tree products and mirror approximation

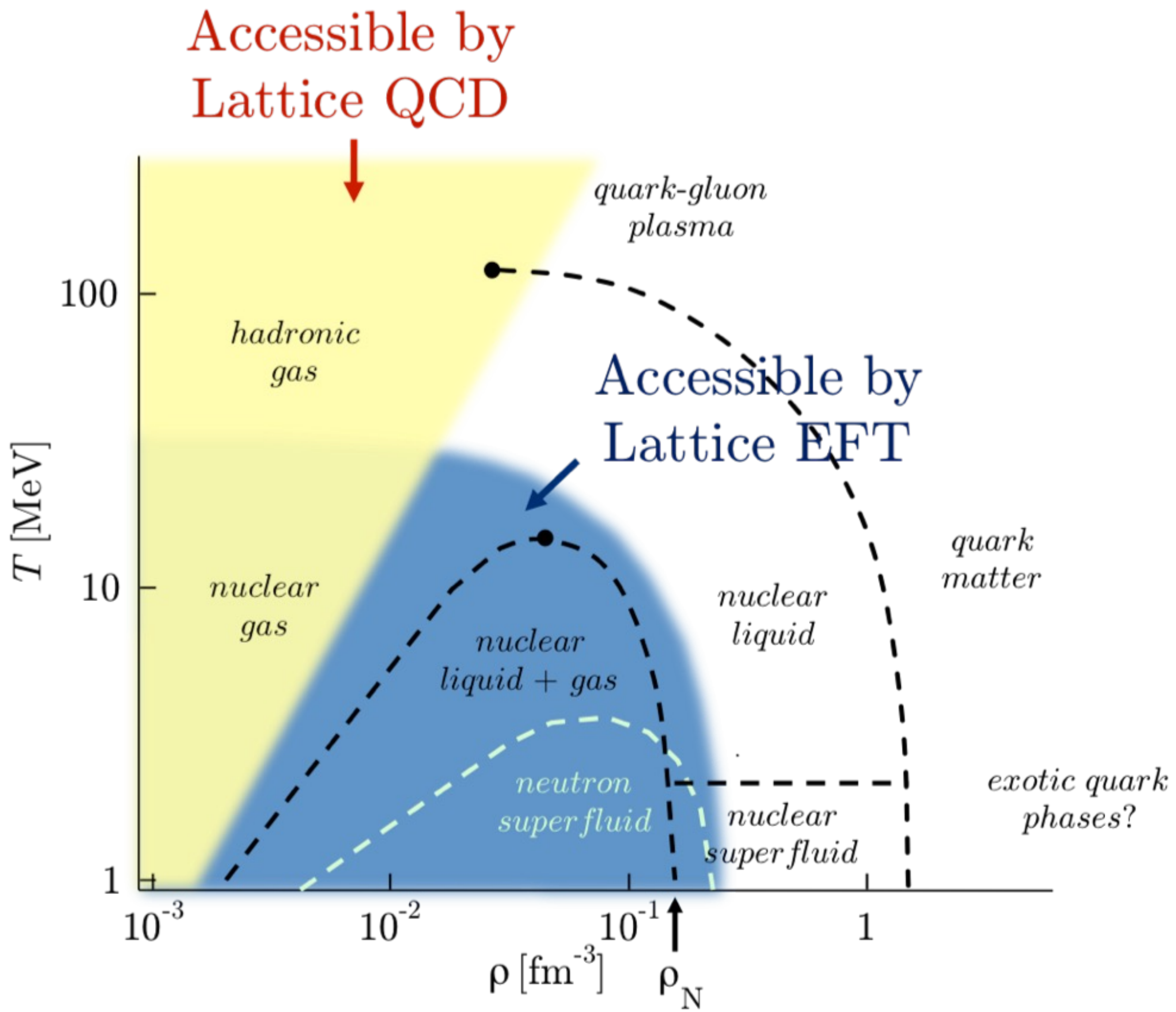
Summary and outlook

Nuclear lattice effective field theory (NLEFT)



D.L, Prog. Part. Nucl. Phys. 63 117-154 (2009); D.L., Annu. Rev. Nucl. Part. Sci. 75, 109 (2025)

Lähde, Meißner, Nuclear Lattice Effective Field Theory (2019), Springer


Accessible by
Lattice QCD
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Lattice EFT
quark-gluon
plasma
hadronic
gas
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quark
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neutron
superfluid
nuclear
superfluid
exotic quark
phases?
T [MeV]
100
10
1
10^{-3}
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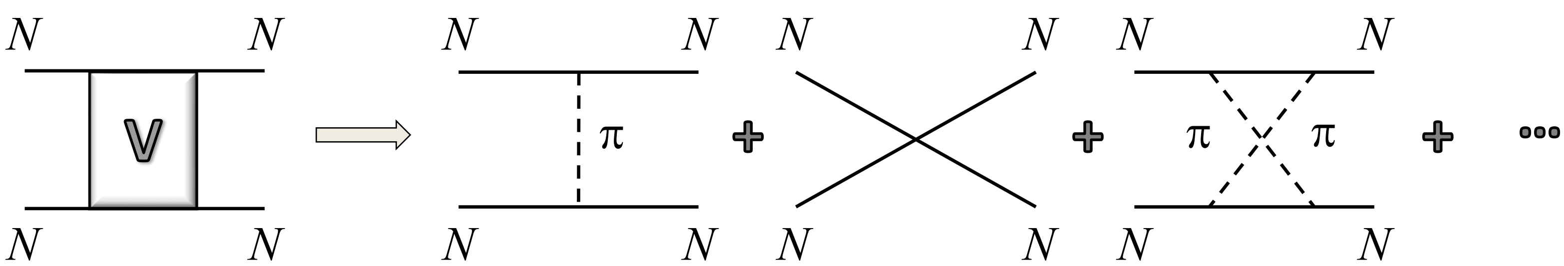
1
ρ [fm^{-3}]
ρ_N

Chiral effective field theory

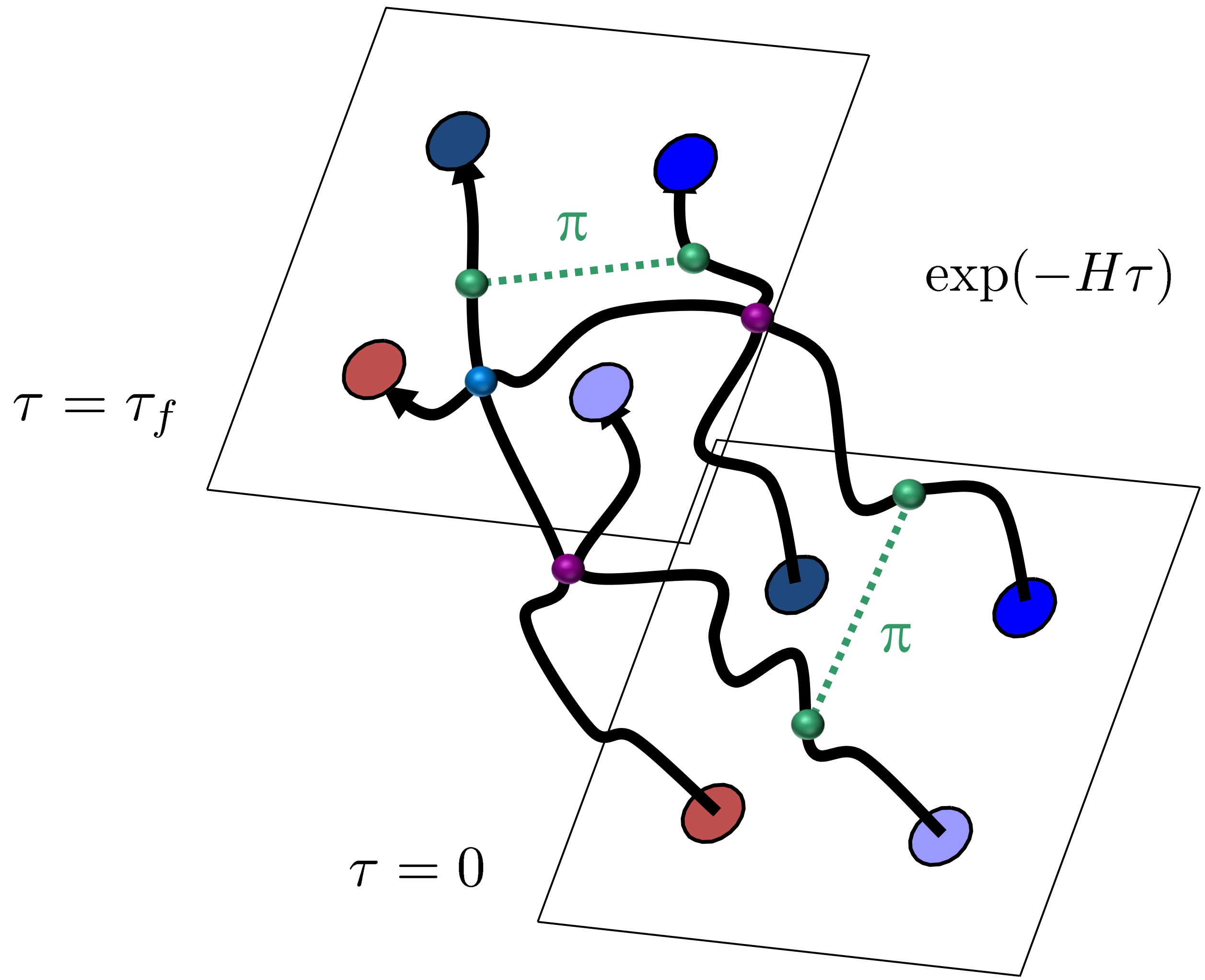
Construct the effective potential order by order



V^{OPEP} Contact interactions V^{TPEP}

Leading order (LO) Next-to-leading order (NLO)

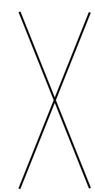
<u>Euclidean time projection</u>



Auxiliary field method

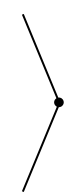
We can write exponentials of the interaction using a Gaussian integral identity

$$\exp\left[-\frac{C}{2}(N^\dagger N)^2\right]$$



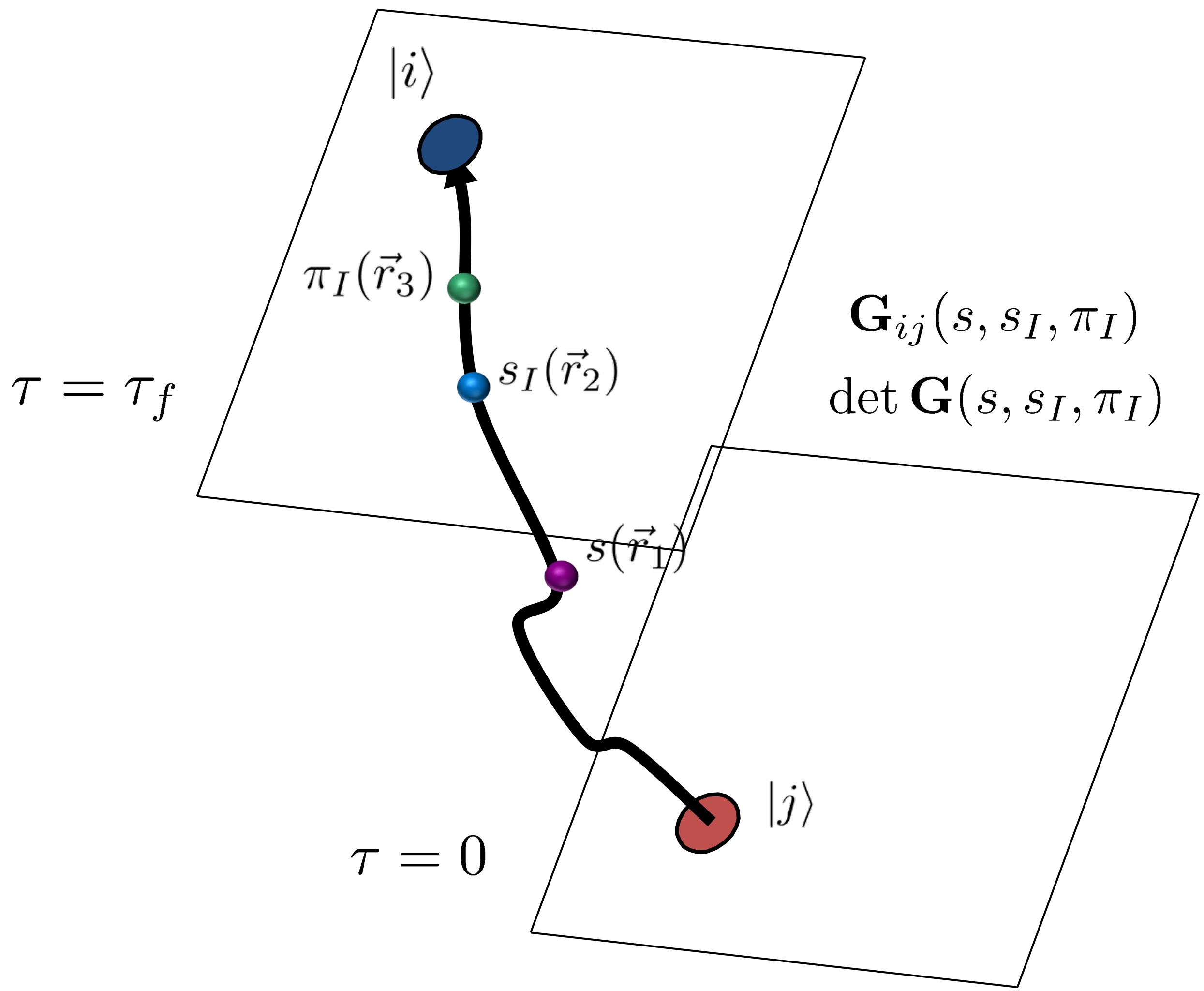
$$(N^\dagger N)^2$$

$$= \sqrt{\frac{1}{2\pi}}\int_{-\infty}^{\infty} ds \exp\left[-\frac{1}{2}s^2 + \sqrt{-C}\, s(N^\dagger N)\right]$$

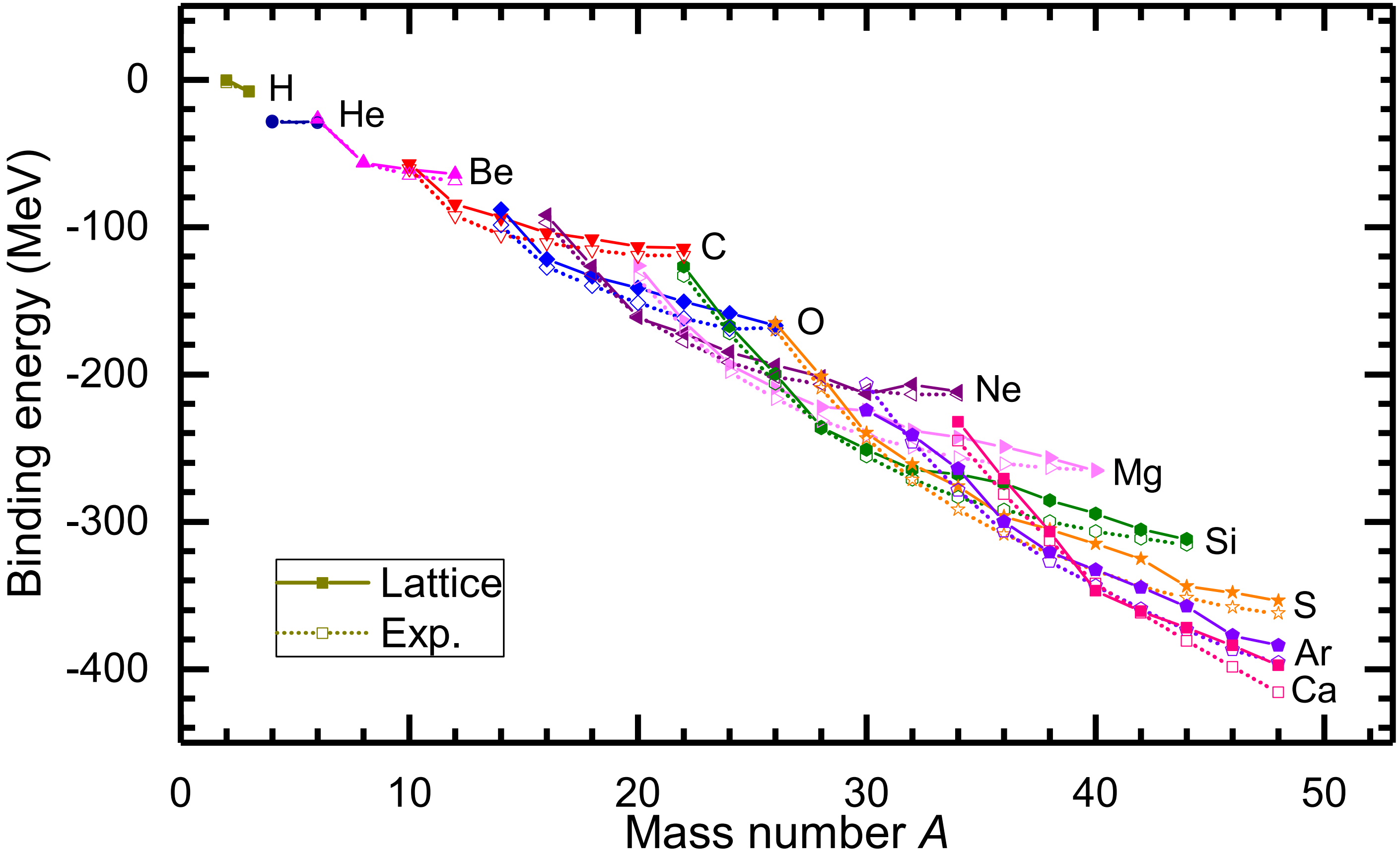


$$sN^\dagger N$$

We remove the interaction between nucleons and replace it with the interactions of each nucleon with a background field.


$|i\rangle$
$\pi_I(\vec{r}_3)$
$s_I(\vec{r}_2)$
$\tau = \tau_f$
$\mathbf{G}_{ij}(s, s_I, \pi_I)$
$\det \mathbf{G}(s, s_I, \pi_I)$
$s(\vec{r}_1)$
$|j\rangle$
$\tau = 0$

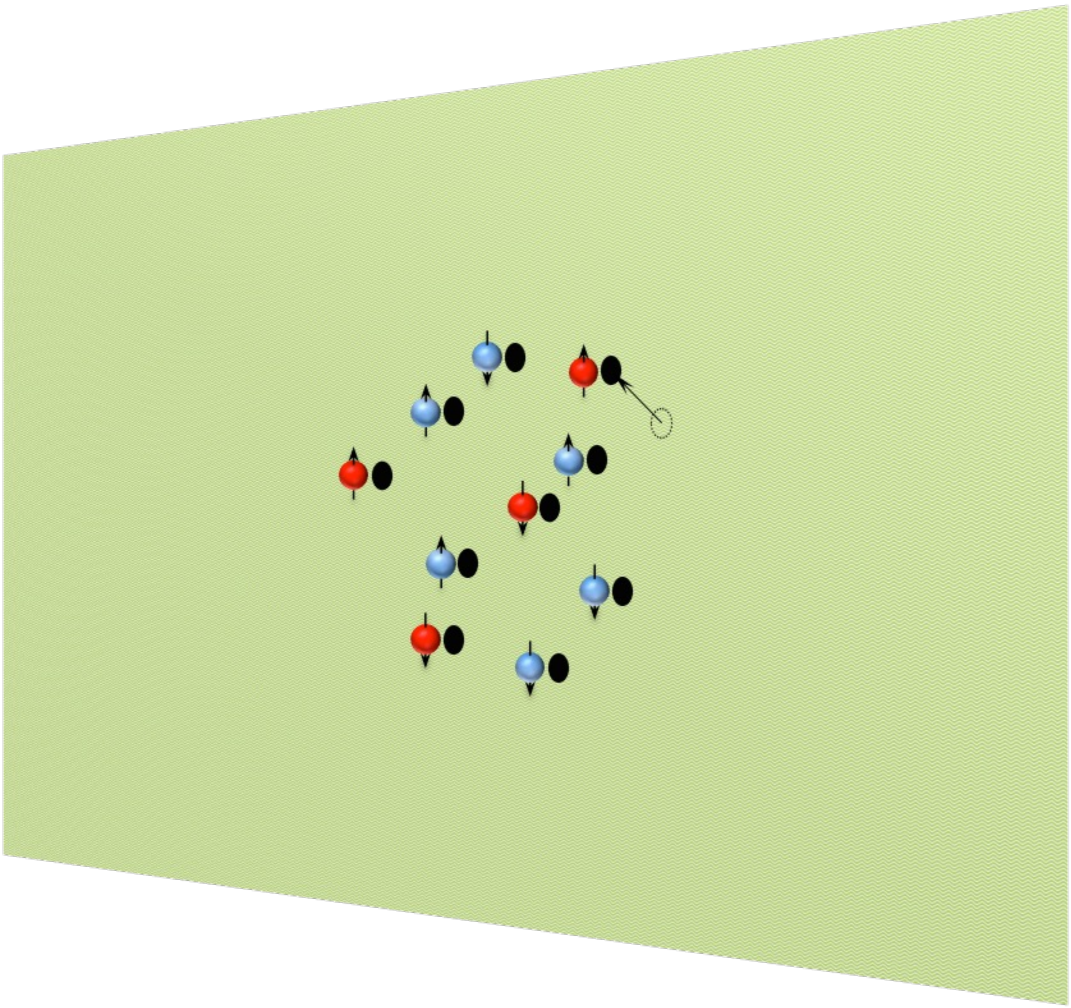
Essential elements for nuclear binding

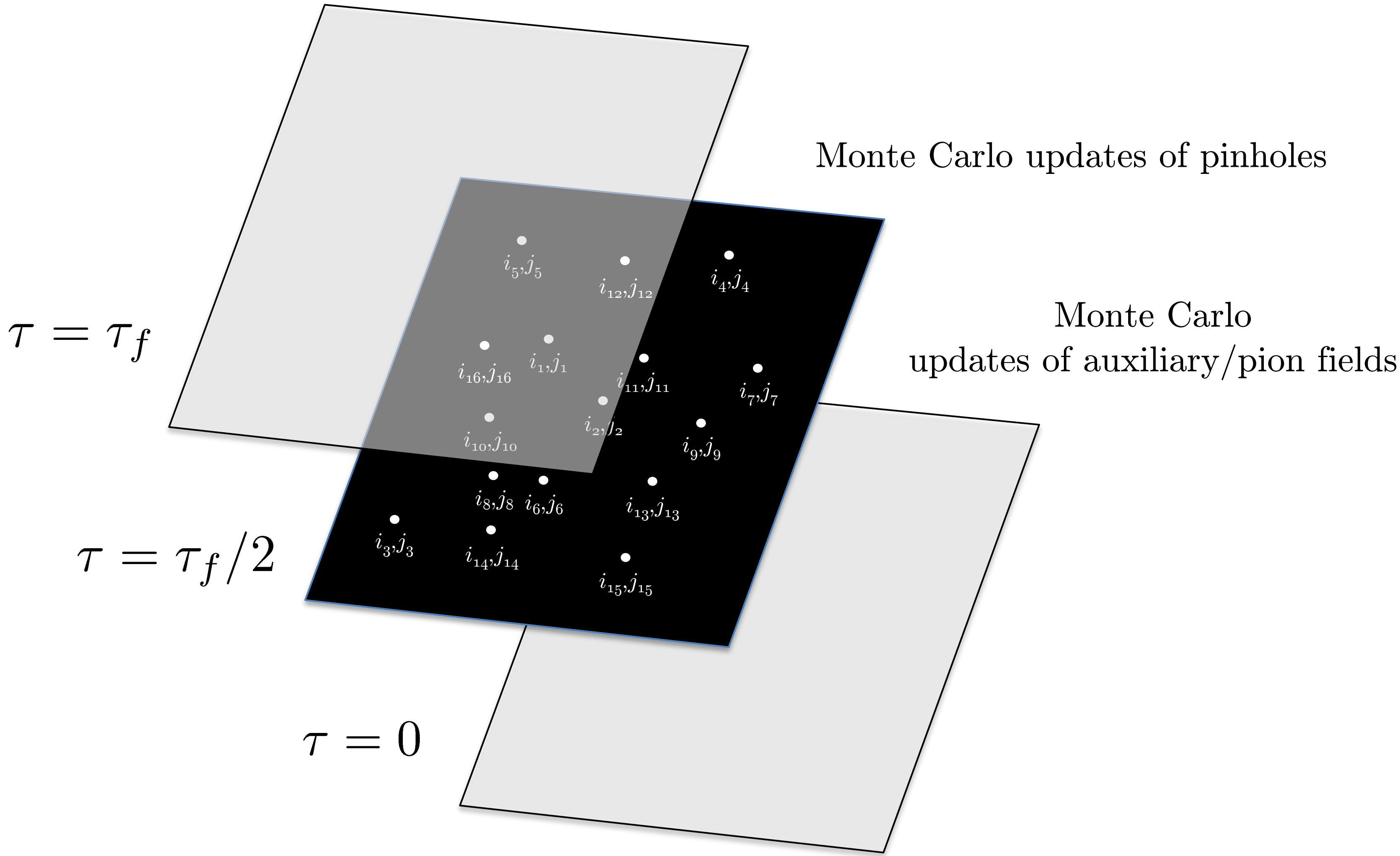


Lu, Li, Elhatisari, D.L., Epelbaum, Meißner, Phys. Lett. B 797, 134863 (2019)

	B	Exp.	R_{ch}	Exp.
^{3}H	8.48(2)(0)	8.48	1.90(1)(1)	1.76
^{3}He	7.75(2)(0)	7.72	1.99(1)(1)	1.97
^{4}He	28.89(1)(1)	28.3	1.72(1)(3)	1.68
^{16}O	121.9(1)(3)	127.6	2.74(1)(1)	2.70
^{20}Ne	161.6(1)(1)	160.6	2.95(1)(1)	3.01
^{24}Mg	193.5(02)(17)	198.3	3.13(1)(2)	3.06
^{28}Si	235.8(04)(17)	236.5	3.26(1)(1)	3.12
^{40}Ca	346.8(6)(5)	342.1	3.42(1)(3)	3.48

Pinhole algorithm





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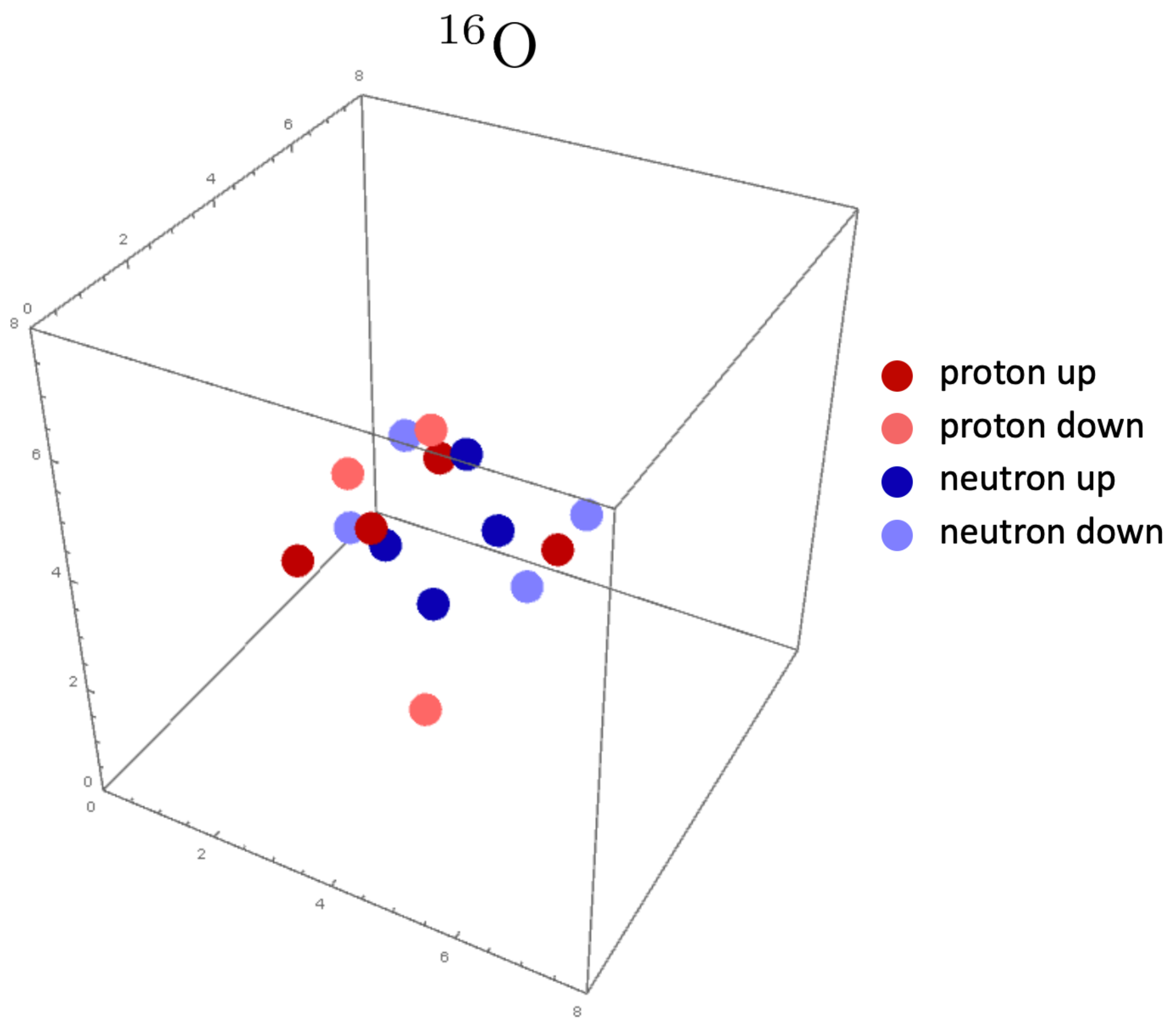
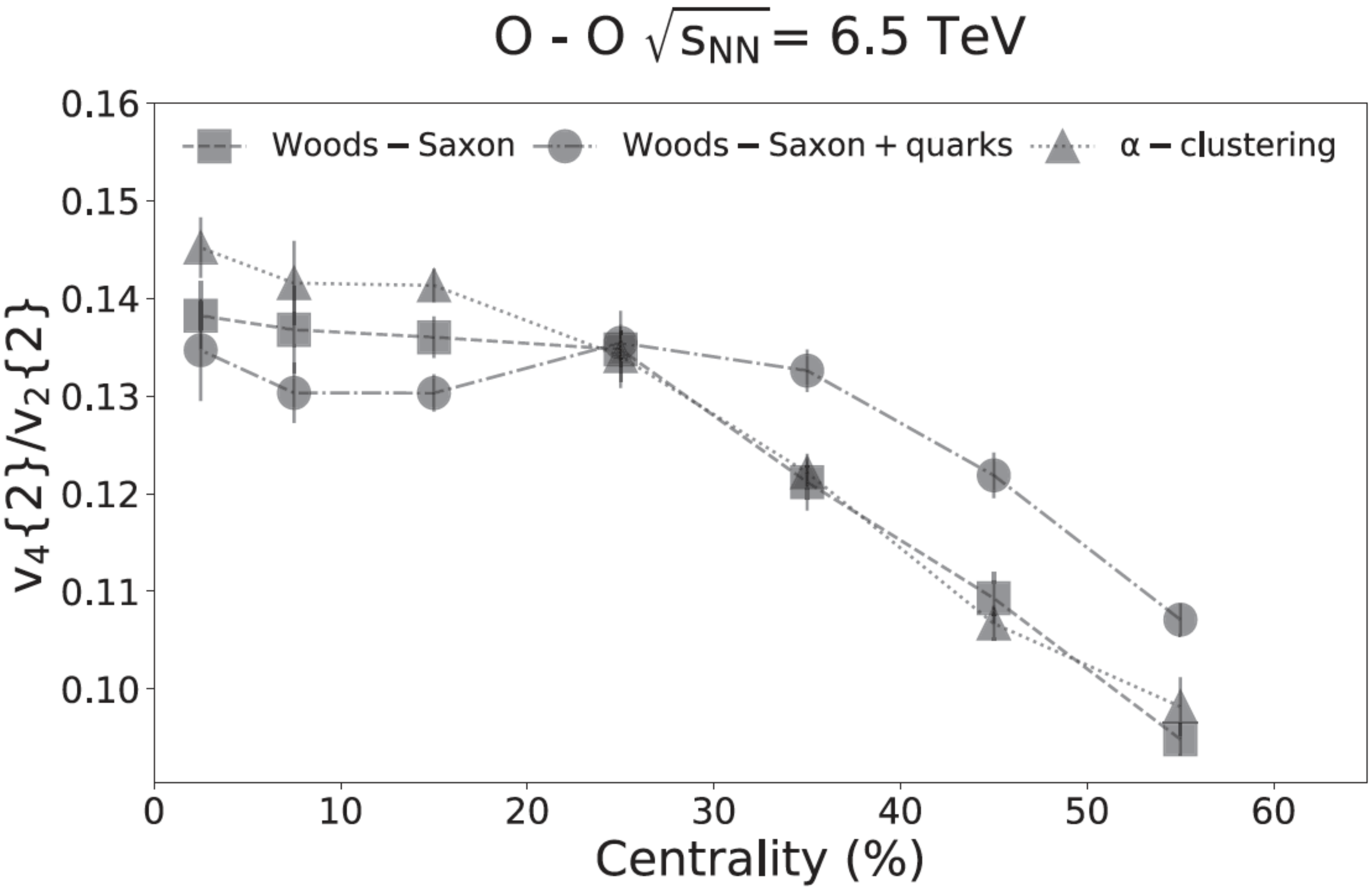
Pinhole configurations

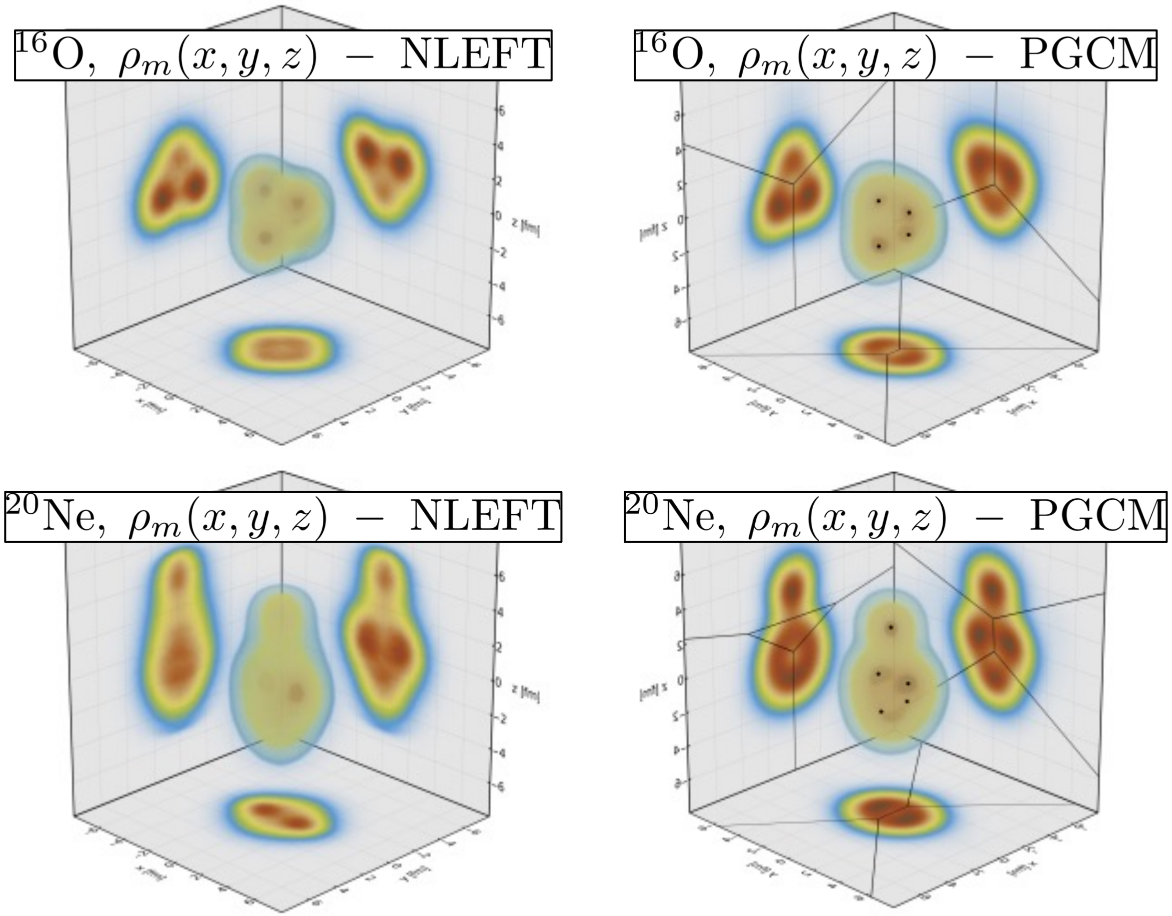


$^{16}O^{16}O$ collisions



Summerfield, Lu, Plumberg, D.L., Noronha-Hostler, Timmins, Phys. Rev. C 104, L041901 (2021)

Structure of ^{16}O and ^{20}Ne



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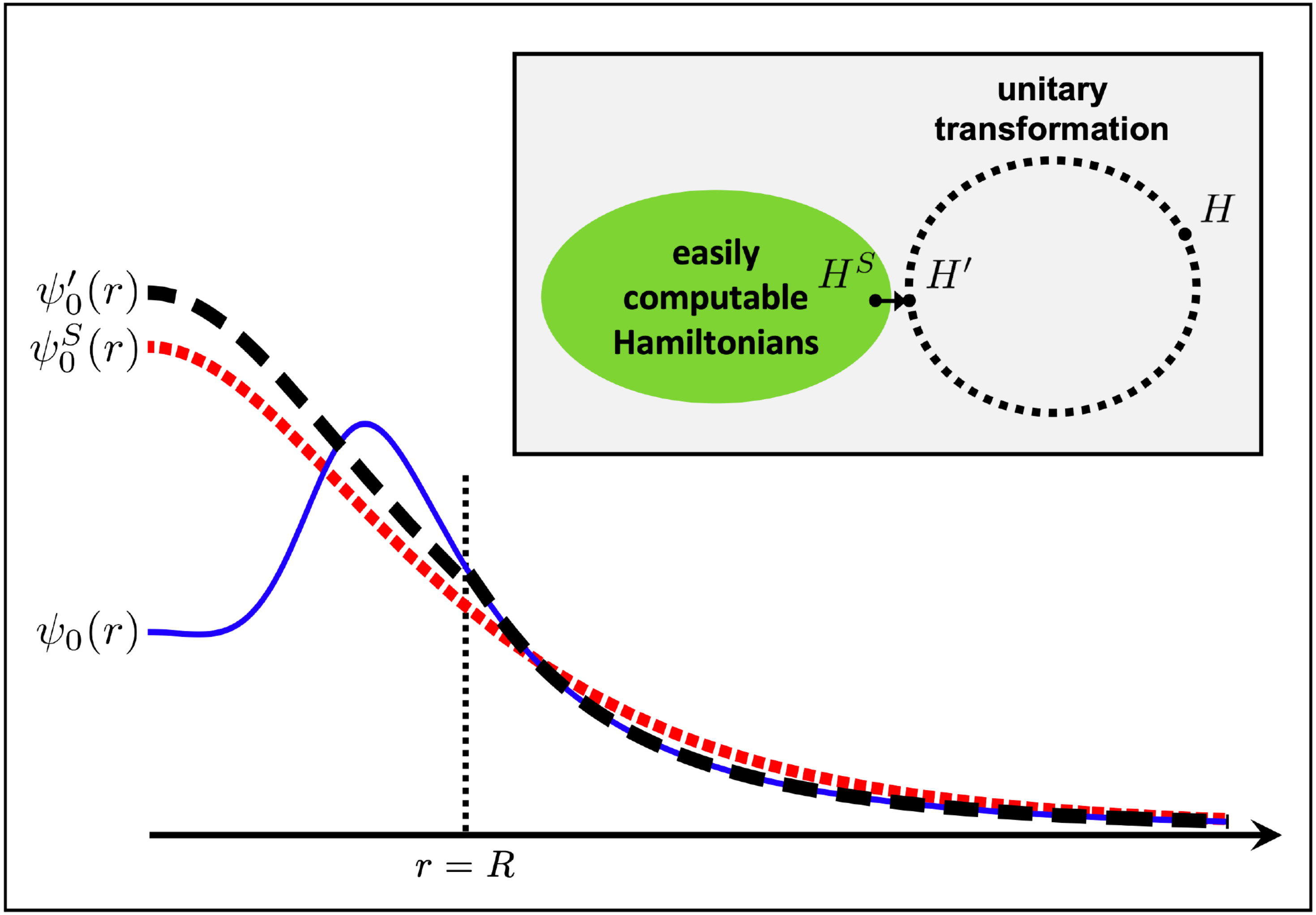
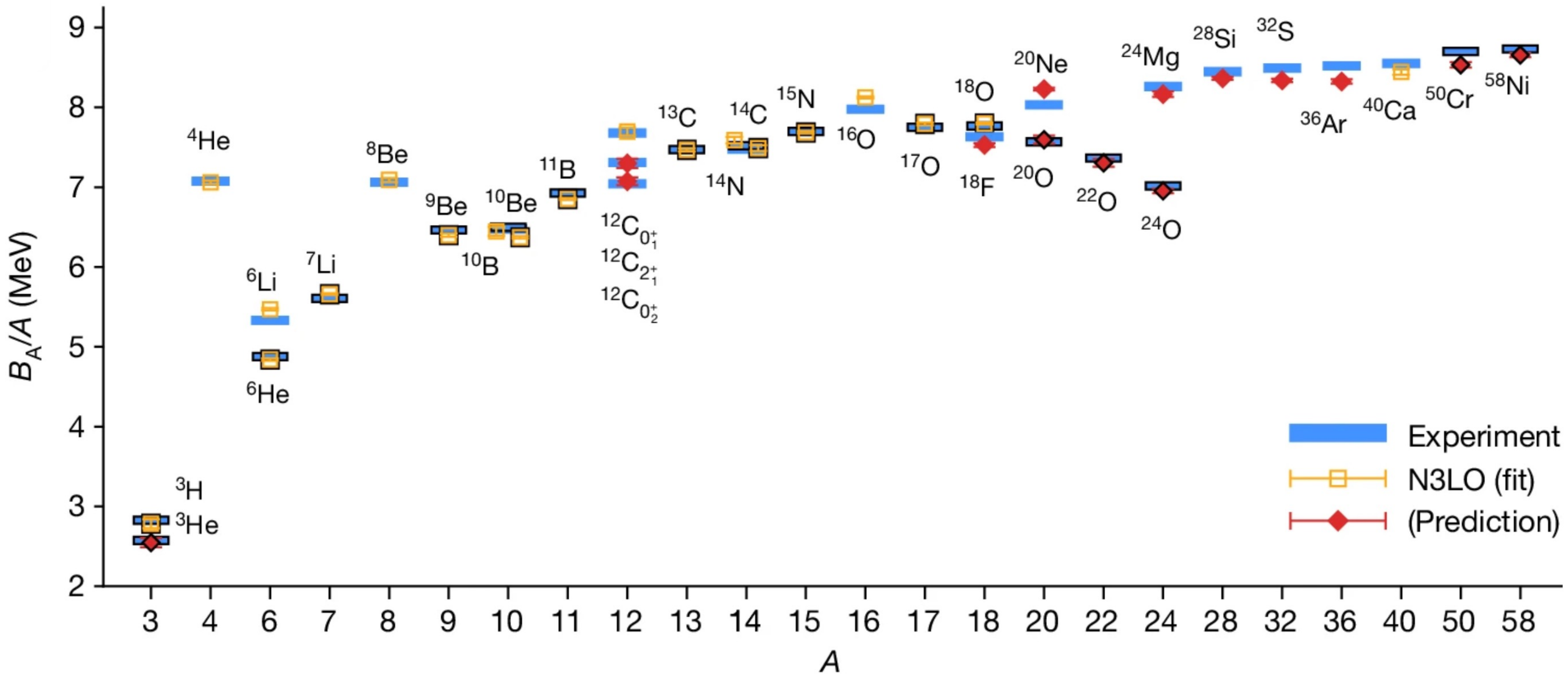
Wavefunction matching



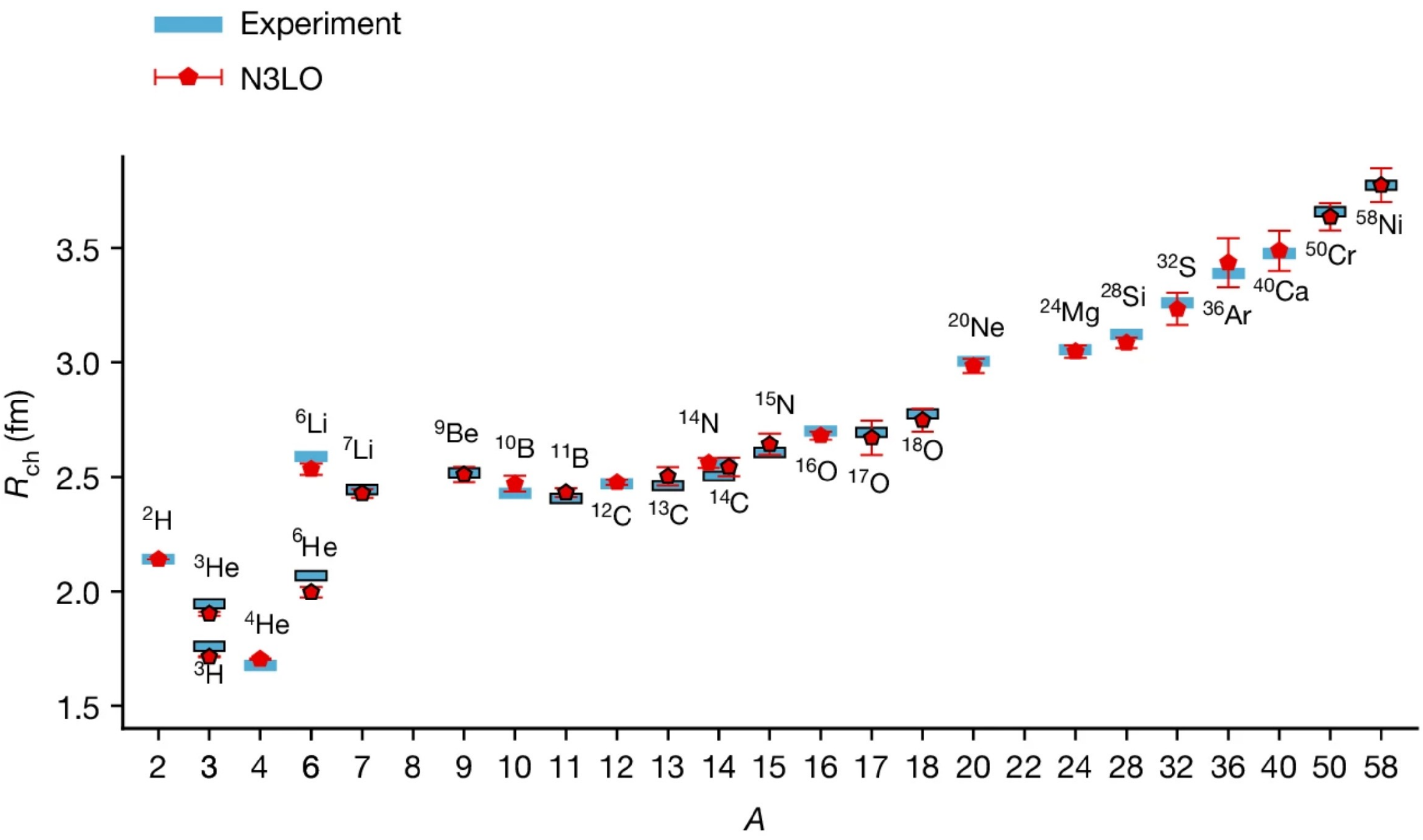
Elhatisari, Bovermann, Ma, Epelbaum, Frame, Hildenbrand, Krebs, Lähde, D.L., Li, Lu, M. Kim, Y. Kim, Meißner, Rupak, Shen, Song, Stellin, Nature 630, 59 (2024)

Binding energies



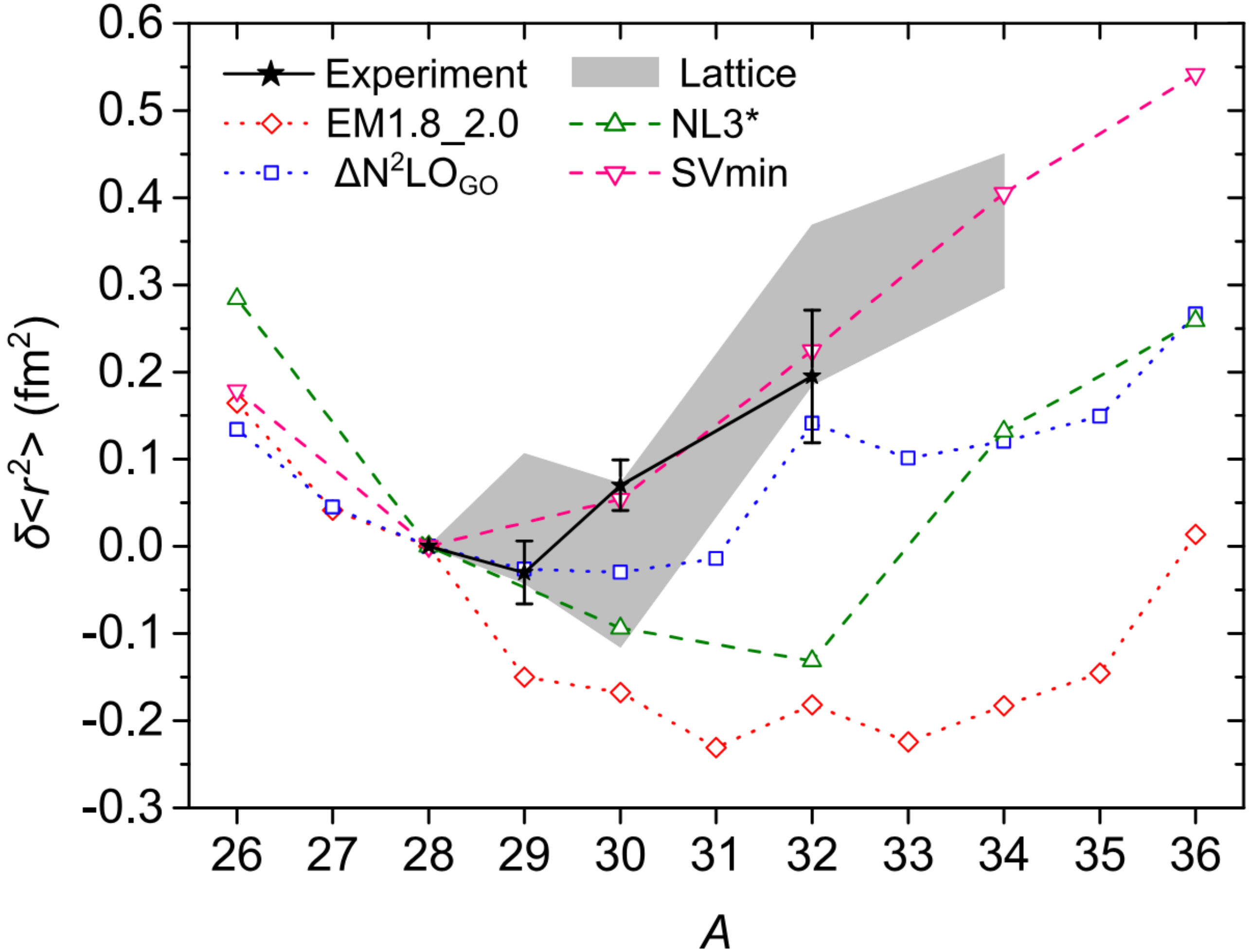
Elhatisari, Bovermann, Ma, Epelbaum, Frame, Hildenbrand, Krebs, Lähde, D.L., Li, Lu, M. Kim, Y. Kim, Meißner, Rupak, Shen, Song, Stellin, Nature 630, 59 (2024)

Charge radii



Elhatisari, Bovermann, Ma, Epelbaum, Frame, Hildenbrand, Krebs, Lähde, D.L., Li, Lu, M. Kim, Y. Kim, Meißner, Rupak, Shen, Song, Stellin, Nature 630, 59 (2024)

Charge radii of silicon isotopes



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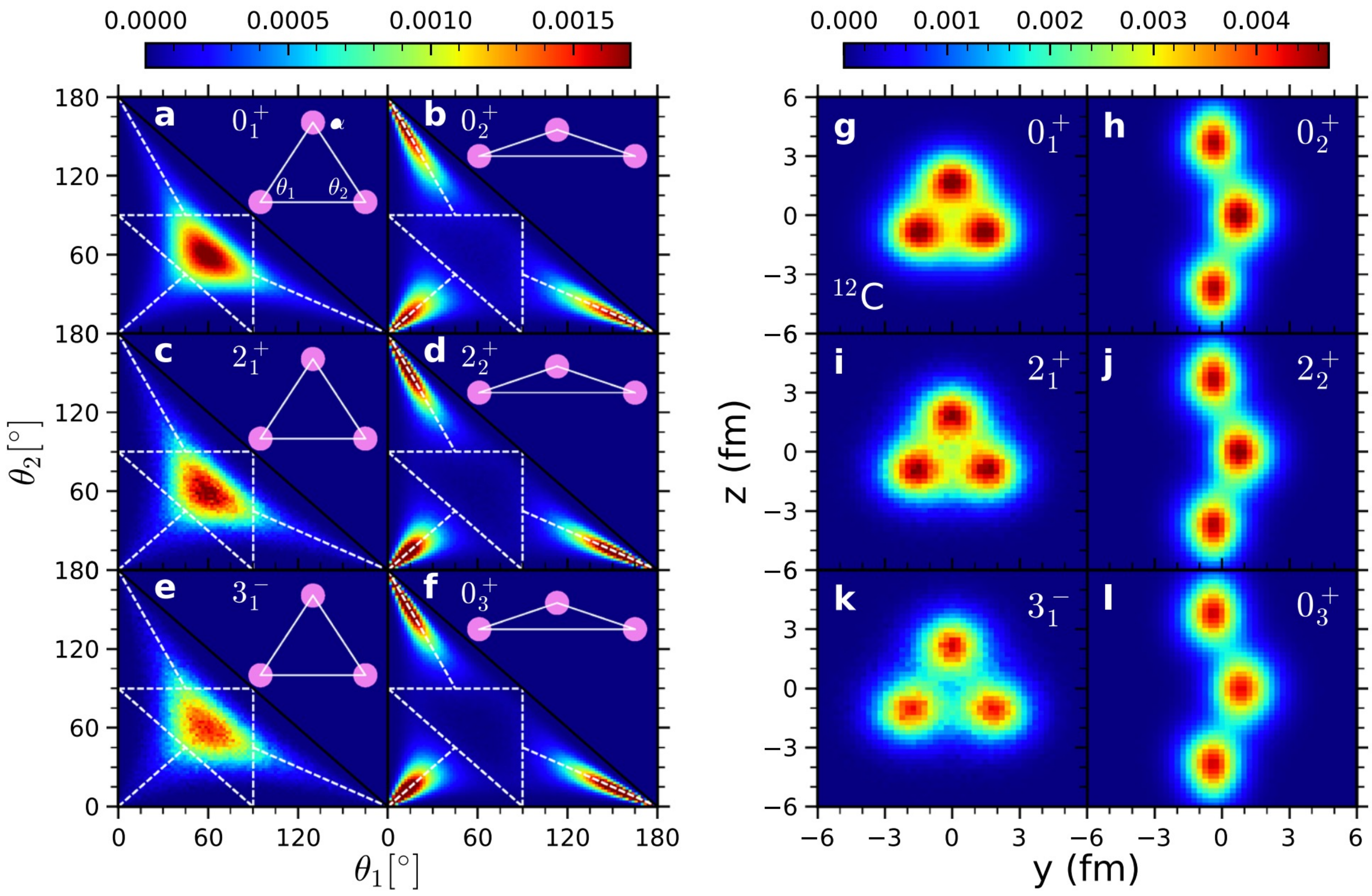
Emergent geometry and duality of ^{12}C



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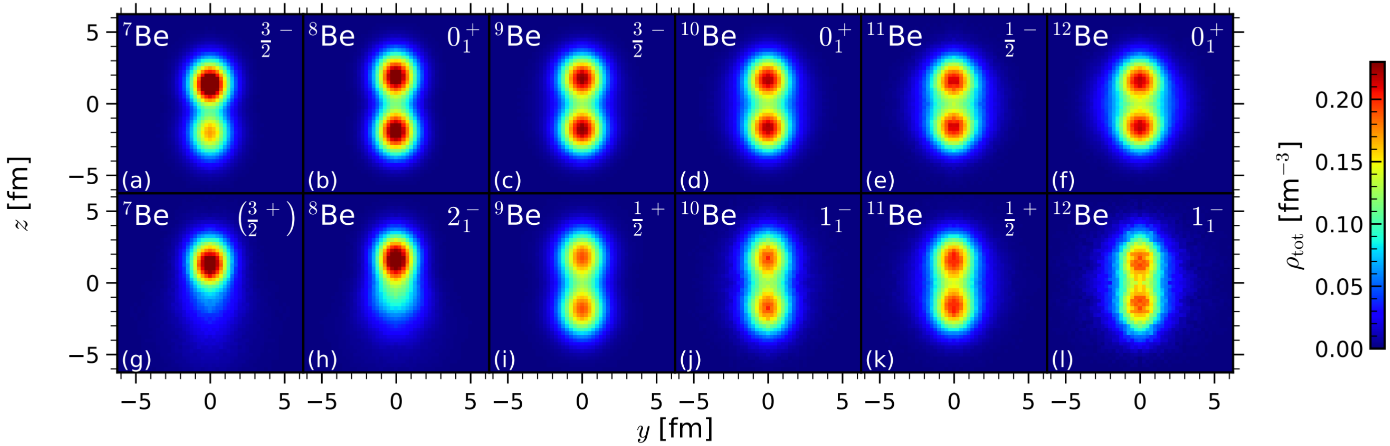
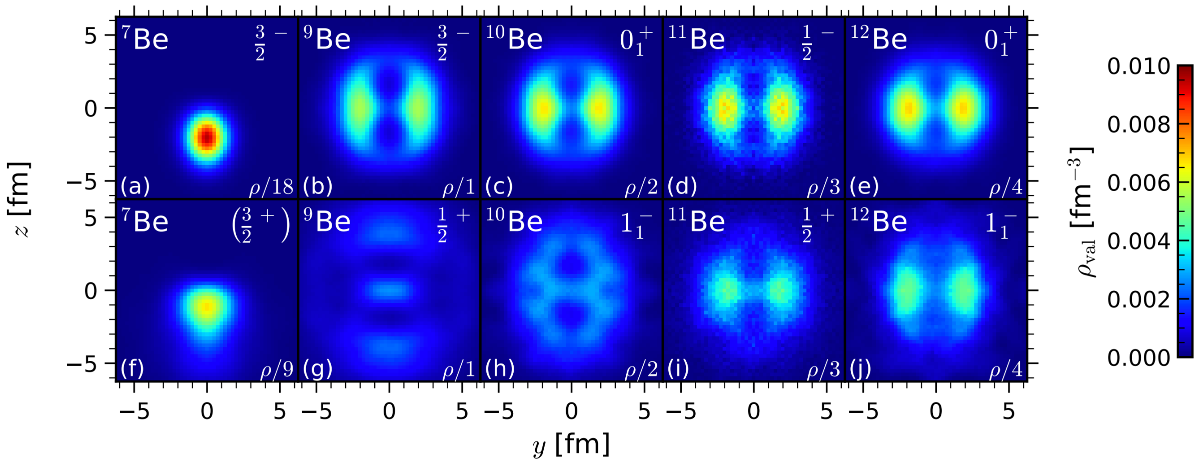
Beryllium isotopes





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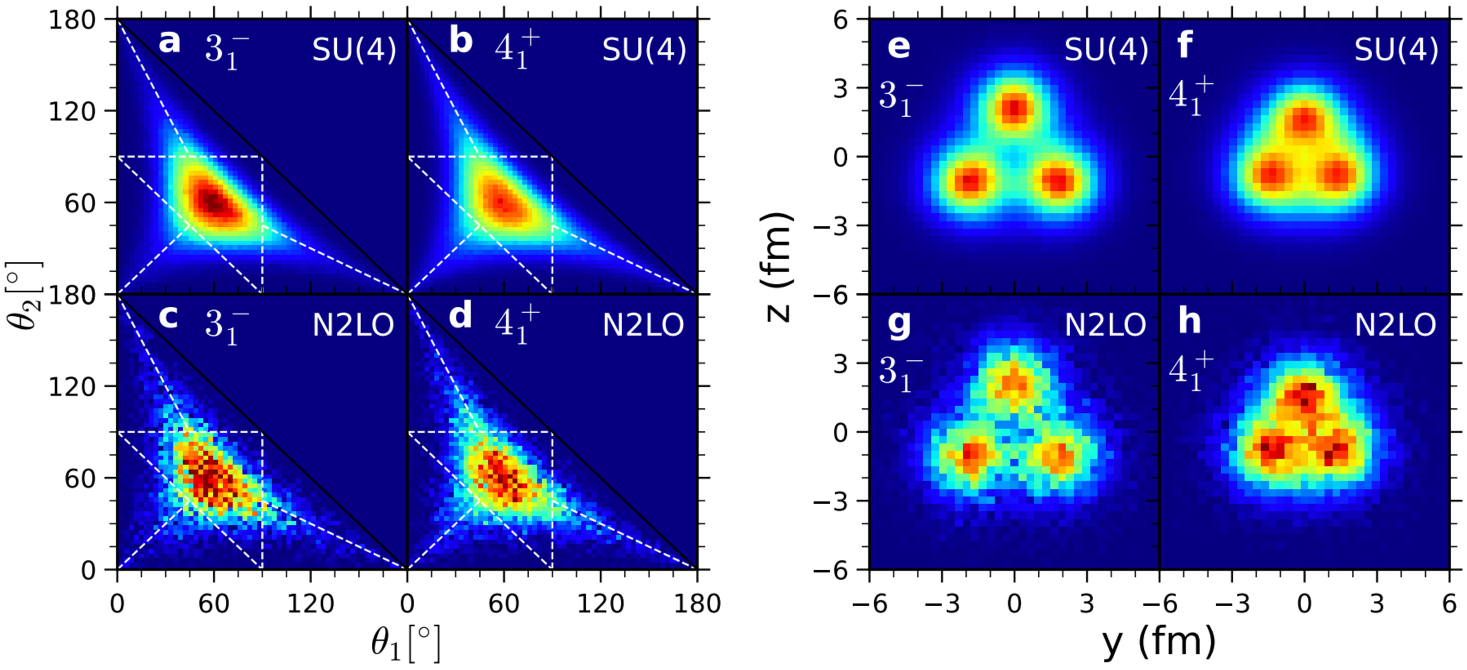
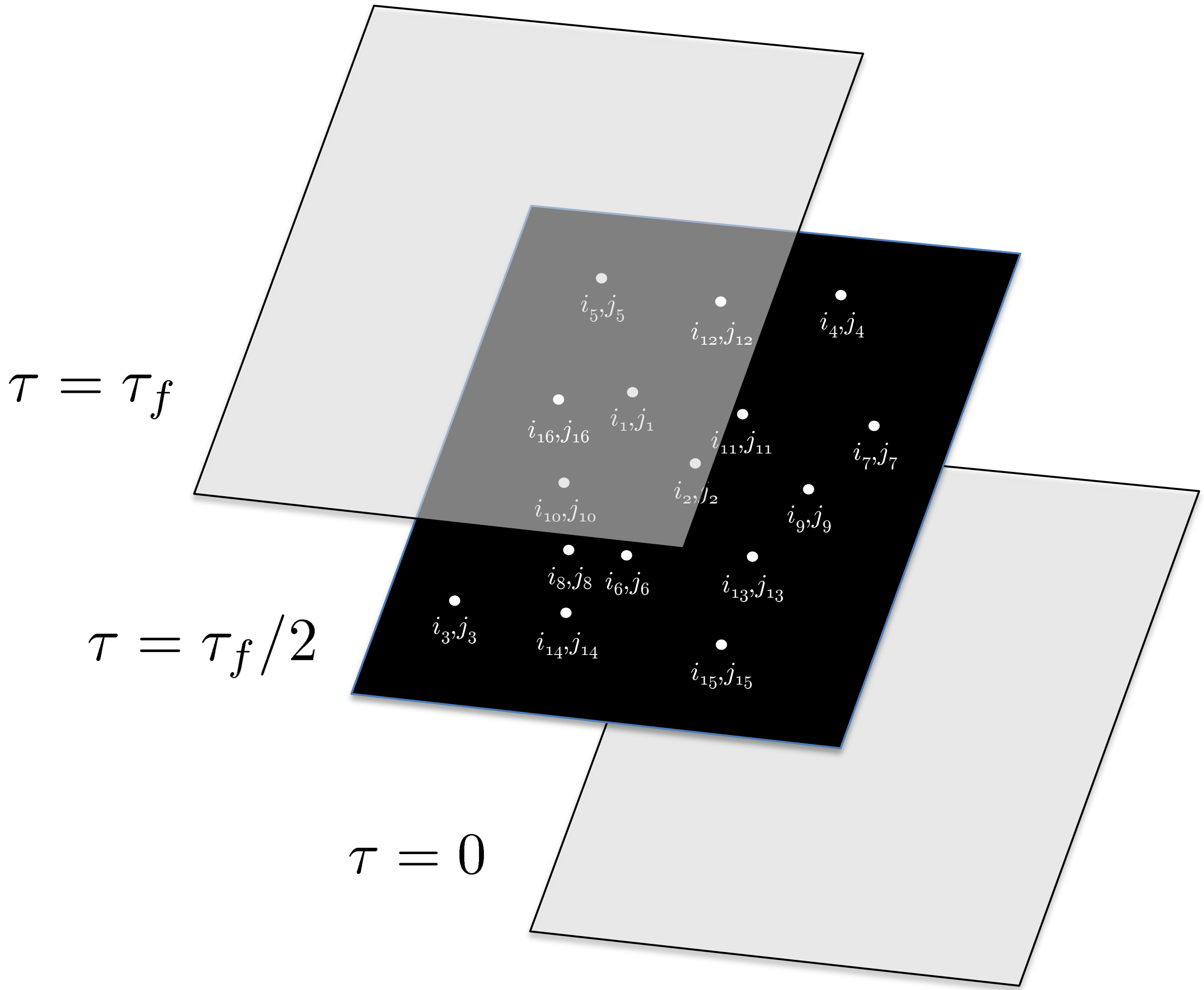
Pinhole sign problem



Shen, Elhatisari, Lähde, D.L., Lu, Meißner, Nature Commun. 14, 2777 (2023)

$$A_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\cdots\mathbf{n}_A,\tau_f) = \langle\Psi_I|e^{-H\tau_f/2}\rho_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\cdots\mathbf{n}_A)e^{-H\tau_f/2}|\Psi_I\rangle$$



Generalizations of Markov chains

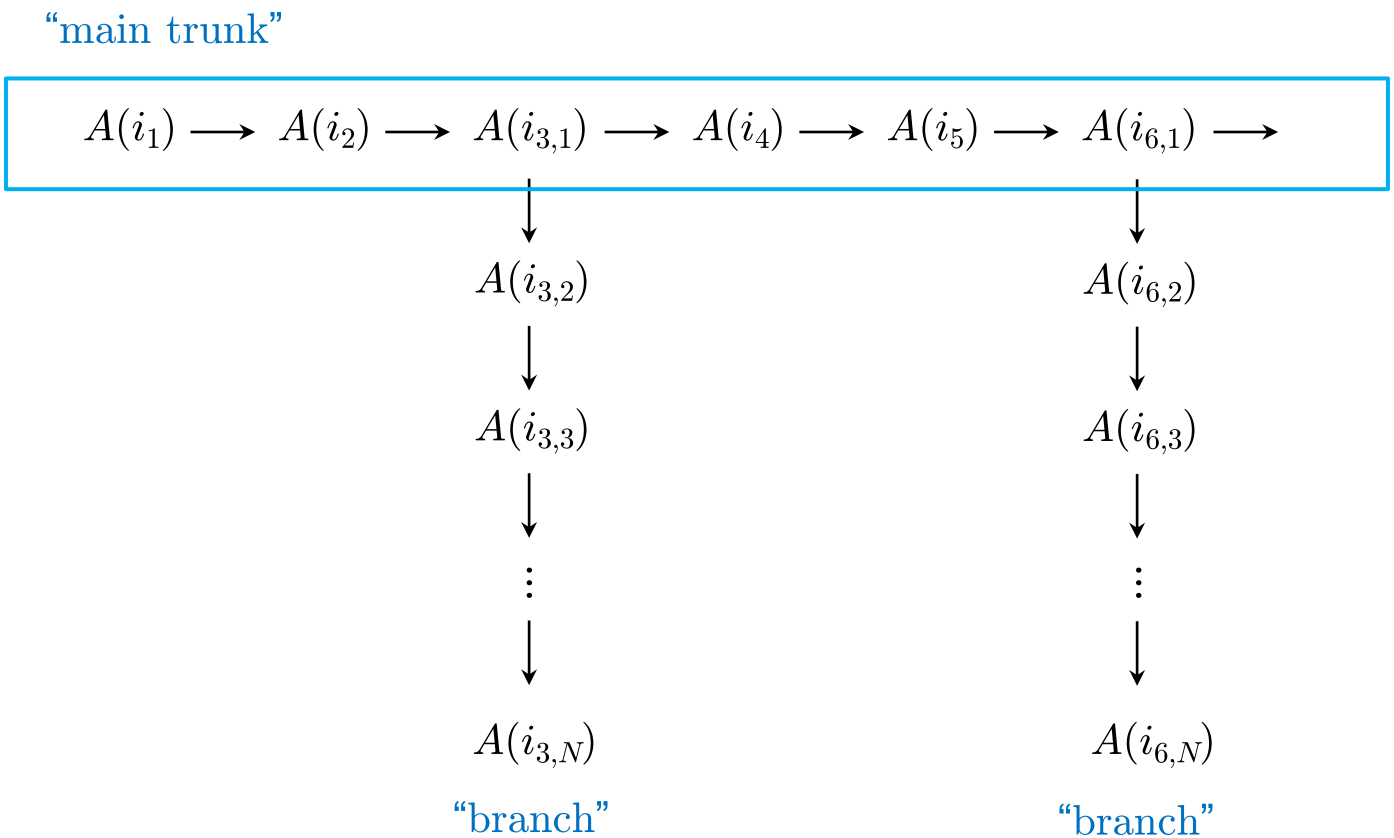
Markov chain:

A sequence of events where the probability of each future event depends only on the present state, not on the sequence of events that preceded it

$$A(i_1) \longrightarrow A(i_2) \longrightarrow A(i_3) \longrightarrow A(i_4) \longrightarrow A(i_5) \longrightarrow A(i_6) \longrightarrow$$

Shen, et al., work in progress

Markov tree with finite-length branches:



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Markov products:

$$A(i,j) = L(i)R(j)$$

$$\begin{array}{ccccccc}
A(i_1,j_1) & \longrightarrow & A(i_1,j_2) & \longrightarrow & A(i_1,j_3) & \longrightarrow & A(i_1,j_4) \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
A(i_2,j_1) & \longrightarrow & A(i_2,j_2) & \longrightarrow & A(i_2,j_3) & \longrightarrow & A(i_2,j_4) \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
A(i_3,j_1) & \longrightarrow & A(i_3,j_2) & \longrightarrow & A(i_3,j_3) & \longrightarrow & A(i_3,j_4) \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
A(i_4,j_1) & \longrightarrow & A(i_4,j_2) & \longrightarrow & A(i_4,j_3) & \longrightarrow & A(i_4,j_4)
\end{array}$$

$$\boxed{L(i_1) \longrightarrow L(i_2) \longrightarrow L(i_3) \longrightarrow L(i_4) \longrightarrow}$$

$$\times$$

$$\boxed{R(j_1) \longrightarrow R(j_2) \longrightarrow R(j_3) \longrightarrow R(j_4) \longrightarrow}$$

Symmetric Markov products:

$$A(i,j) = L(i)L(j)$$

$$\boxed{L(i_1) \longrightarrow L(i_2) \longrightarrow L(i_3) \longrightarrow L(i_4) \longrightarrow}$$

$$\times$$

$$\boxed{L(j_1) \longrightarrow L(j_2) \longrightarrow L(j_3) \longrightarrow L(j_4) \longrightarrow}$$

Mirror approximation of symmetric Markov products:

same

$$\boxed{L(i_1) \longrightarrow L(i_2) \longrightarrow L(i_3) \longrightarrow L(i_4) \longrightarrow}$$

$$\times$$

$$\boxed{L(i_1) \longrightarrow L(i_2) \longrightarrow L(i_3) \longrightarrow L(i_4) \longrightarrow}$$

Markov product trees:

$$A(i_1,j_1,k_1) \longrightarrow A(i_{2,1},j_2,k_{2,1}) \longrightarrow A(i_3,j_3,k_3) \longrightarrow$$

$$\swarrow \quad \searrow$$

$$A(i_{2,2},j_2,k_{2,1}) \quad A(i_{2,1},j_2,k_{2,2})$$

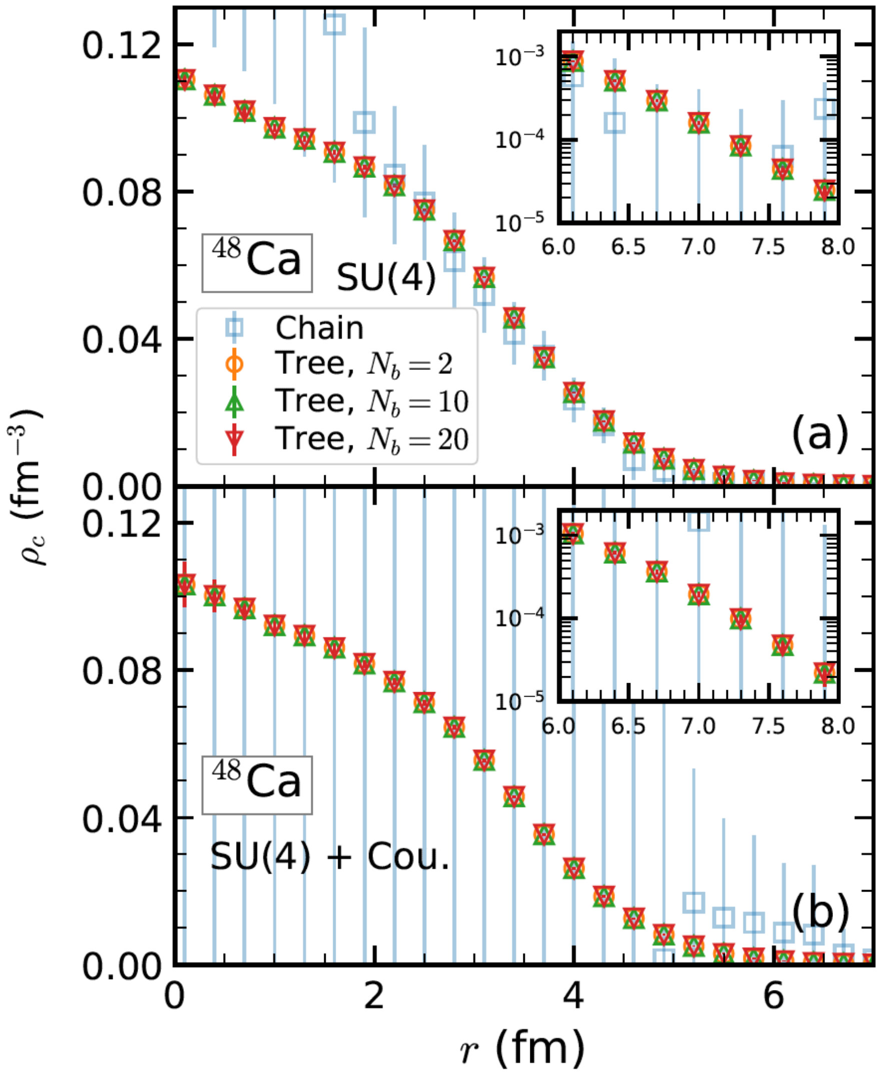
$$\swarrow \quad \searrow \qquad \swarrow \quad \searrow$$

$$A(i_{2,3},j_2,k_{2,1}) \qquad A(i_{2,2},j_2,k_{2,2}) \qquad A(i_{2,1},j_2,k_{2,3})$$

$$\swarrow \quad \searrow \qquad \swarrow \quad \searrow \qquad \swarrow \quad \searrow$$

We can greatly reduce the sign problem in pinhole simulations using Markov product trees and the mirror approximation.

Shen, et al., work in progress



Shen, et al., work in progress

Summary and Outlook

NLEFT is an *ab initio* approach to nuclear many-body systems that combines effective field theory with lattice methods. It has been benchmarked against precision low-energy experiments and is able to compute full A-body correlations and provide initial states for relativistic ion collisions.

Pinhole calculations with N3LO chiral interactions are in progress. After our precision benchmarks are met and the paper on Markov product trees and the mirror approximation is finished, we will make production runs of pinhole configurations with high-fidelity chiral interactions for light and medium-mass nuclei of interest to the relativistic ion community.