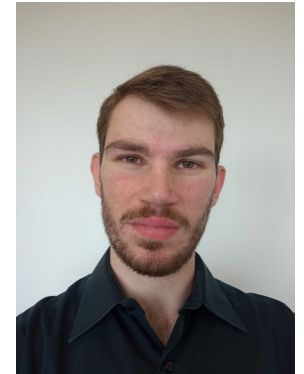


Visualizing nuclear correlations: *ab initio* “shapes” of nuclei and application to ^{22}Ne

Benjamin Bally

Light ion collisions at the LHC 2025 - CERN

01 December 2025



Stavros Bofos



TECHNISCHE
UNIVERSITÄT
DARMSTADT



DeformedNuclei
(A. Tichai)



Motivations

- Interface between low- and high-energy nuclear physics

- New and mutually beneficial possibilities

- Determine the initial geometry of collisions

- Select nuclear species to collide

- Gain information about the structure of nuclei

- A lot of activity over the past few years

- Giacalone, PRL 127, 242301 (2021)

- Bally, PRL 128, 082301 (2022)

- Jia, PRL 131, 022301 (2023)

- Ryssens, PRL 130, 212302 (2023)

- and others...

- EMMI RRTF 2022 (Heidelberg)

- INT Program 23-1a (Seattle)

- Several workshops (Saclay, CERN, Beijing)

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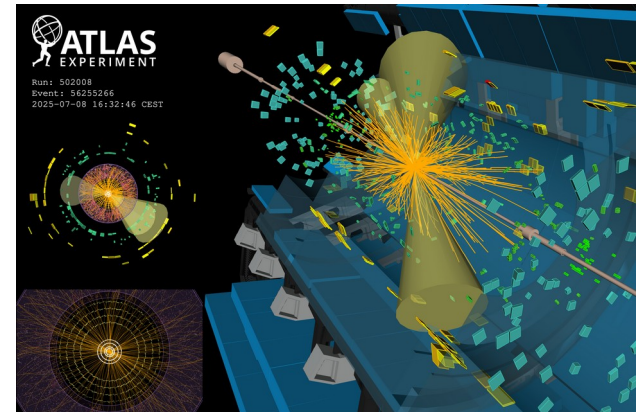
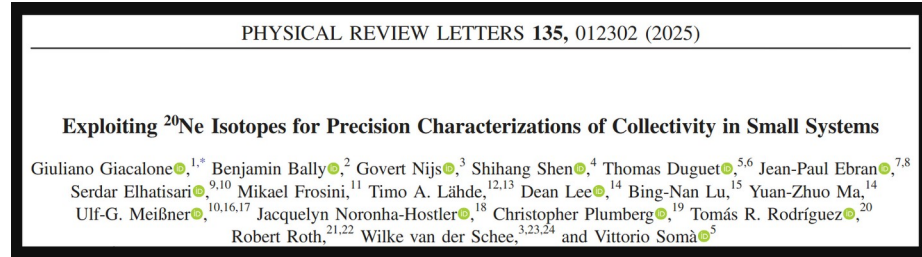
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- **Culminated on the 8th of July with the $^{20}\text{Ne}+^{20}\text{Ne}$ collisions at LHC**

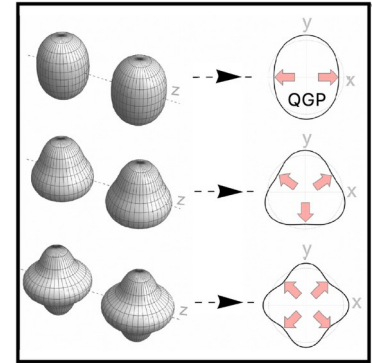
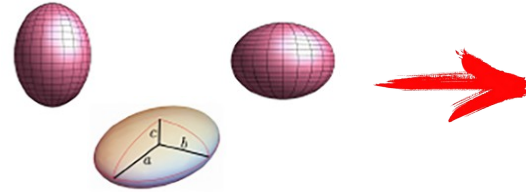


Nuclear arrangements → spatial anisotropies

- Protons and neutrons inside nuclei can adopt various spatial configurations

Nuclear arrangements → spatial anisotropies

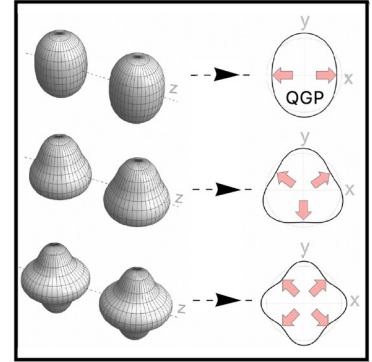
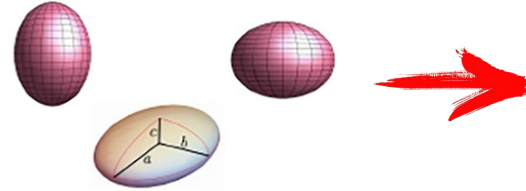
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- Nuclear deformation (quadrupole, octupole, ...)



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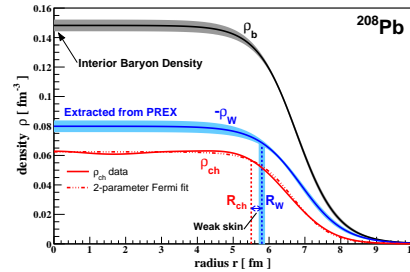
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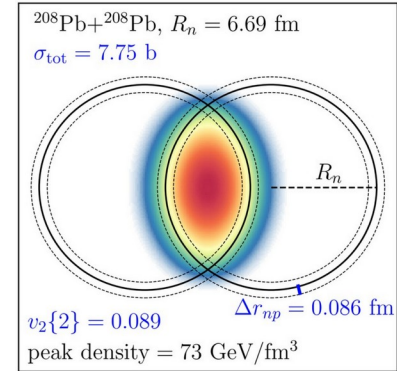


- Neutron-skin

→ directly connected to the EoS



Adhikari, PRL 126, 172502 (2021)

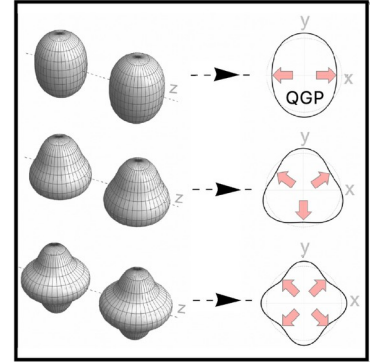
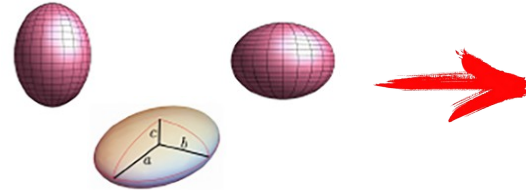


Gialcone, PRL 131, 202302 (2023)

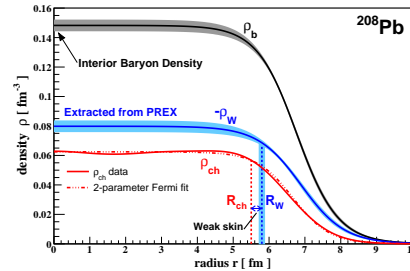
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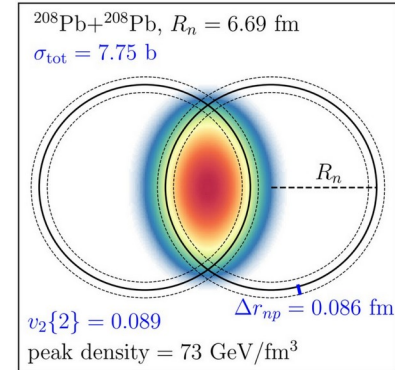
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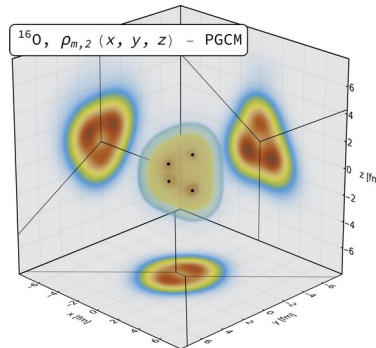


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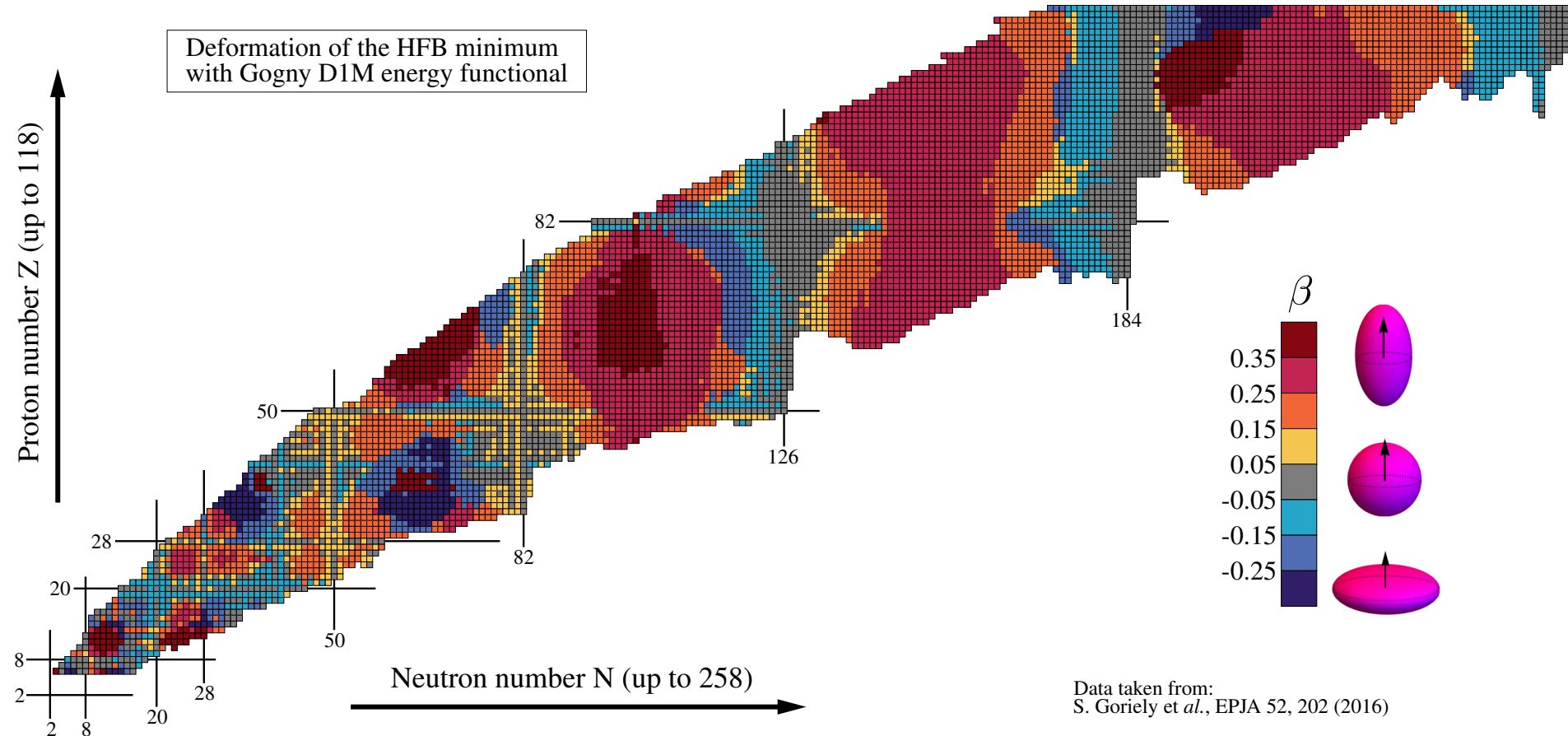


Giacalone, PRL 131, 202302 (2023)

- Clustering (e.g. α -clusters)



Deformation correlations are (almost) everywhere

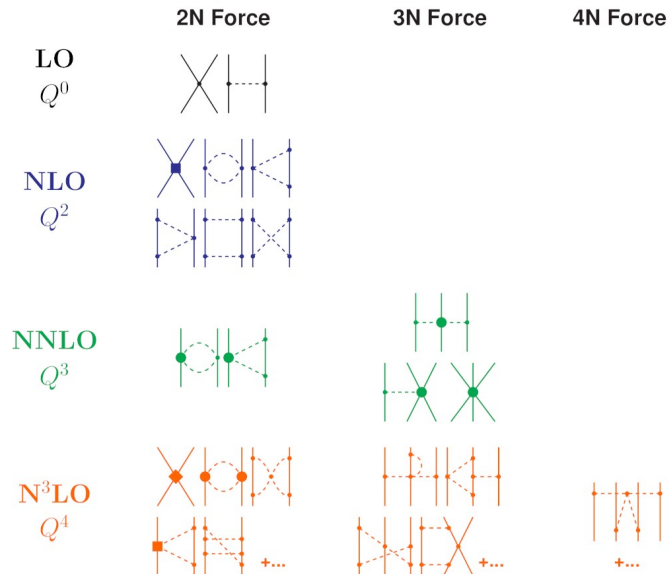


Ab initio nuclear theory in a nutshell

1) Nuclei made of A interacting structureless nucleons (Z protons, N neutrons)

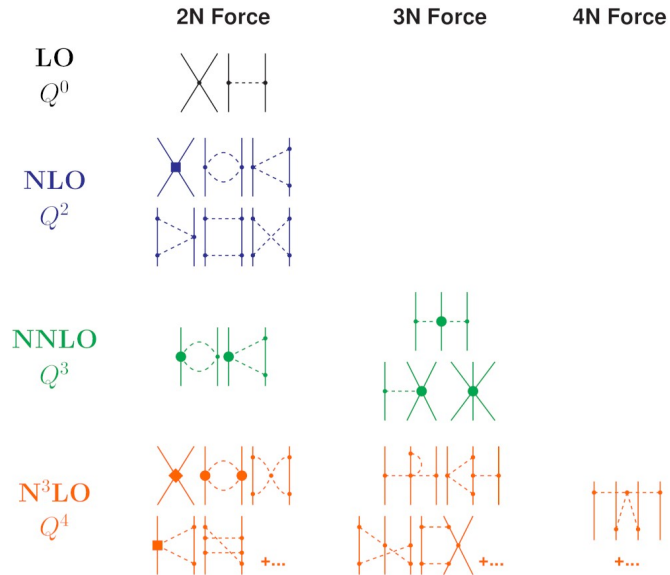
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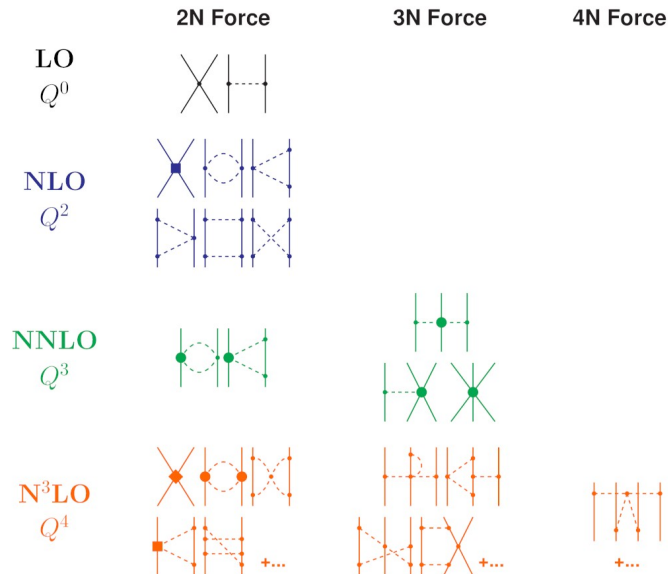
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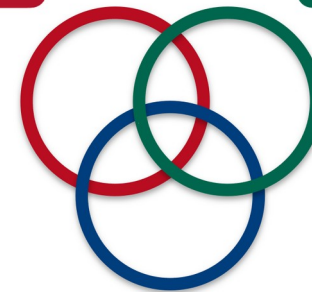
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Nuclear interaction

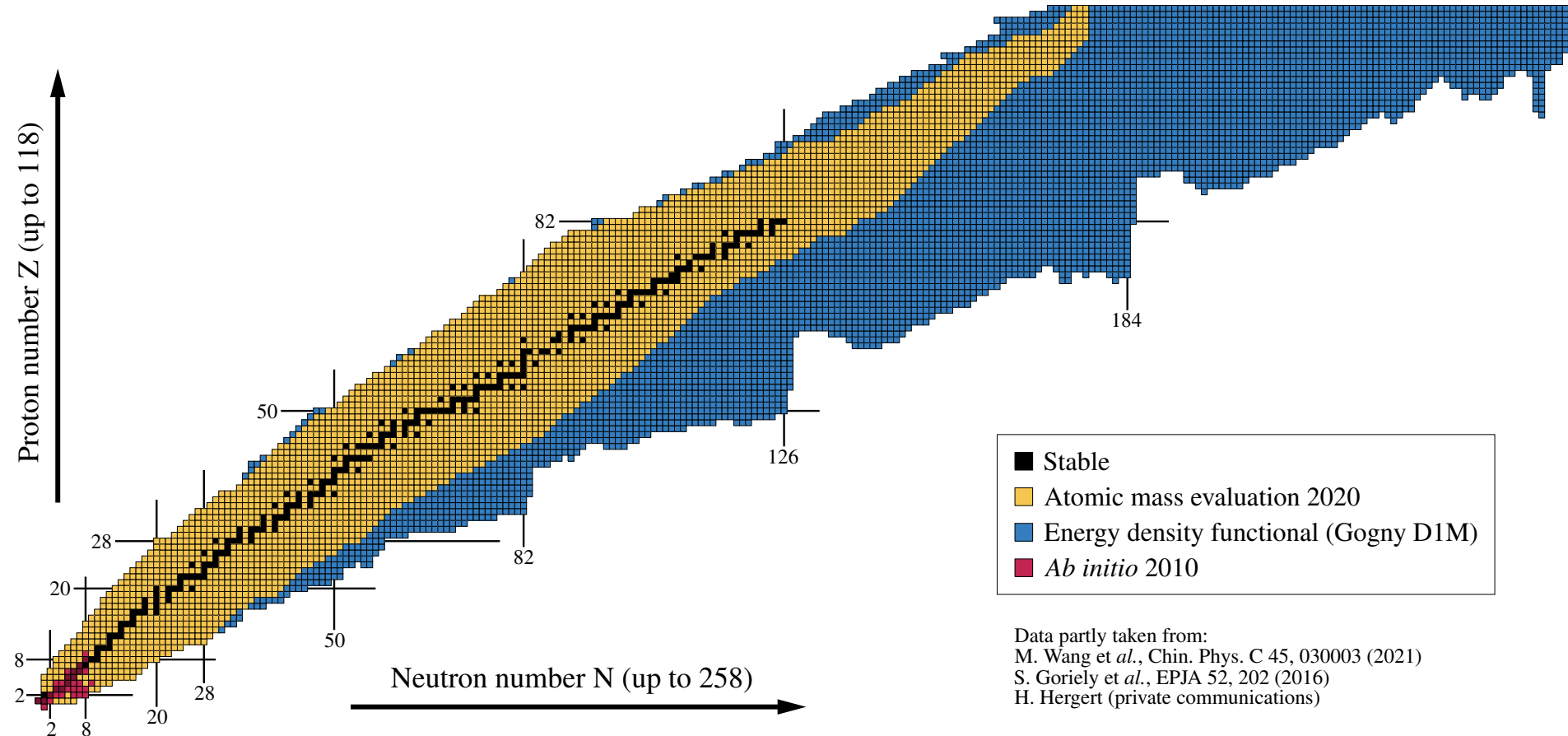
Many-body method



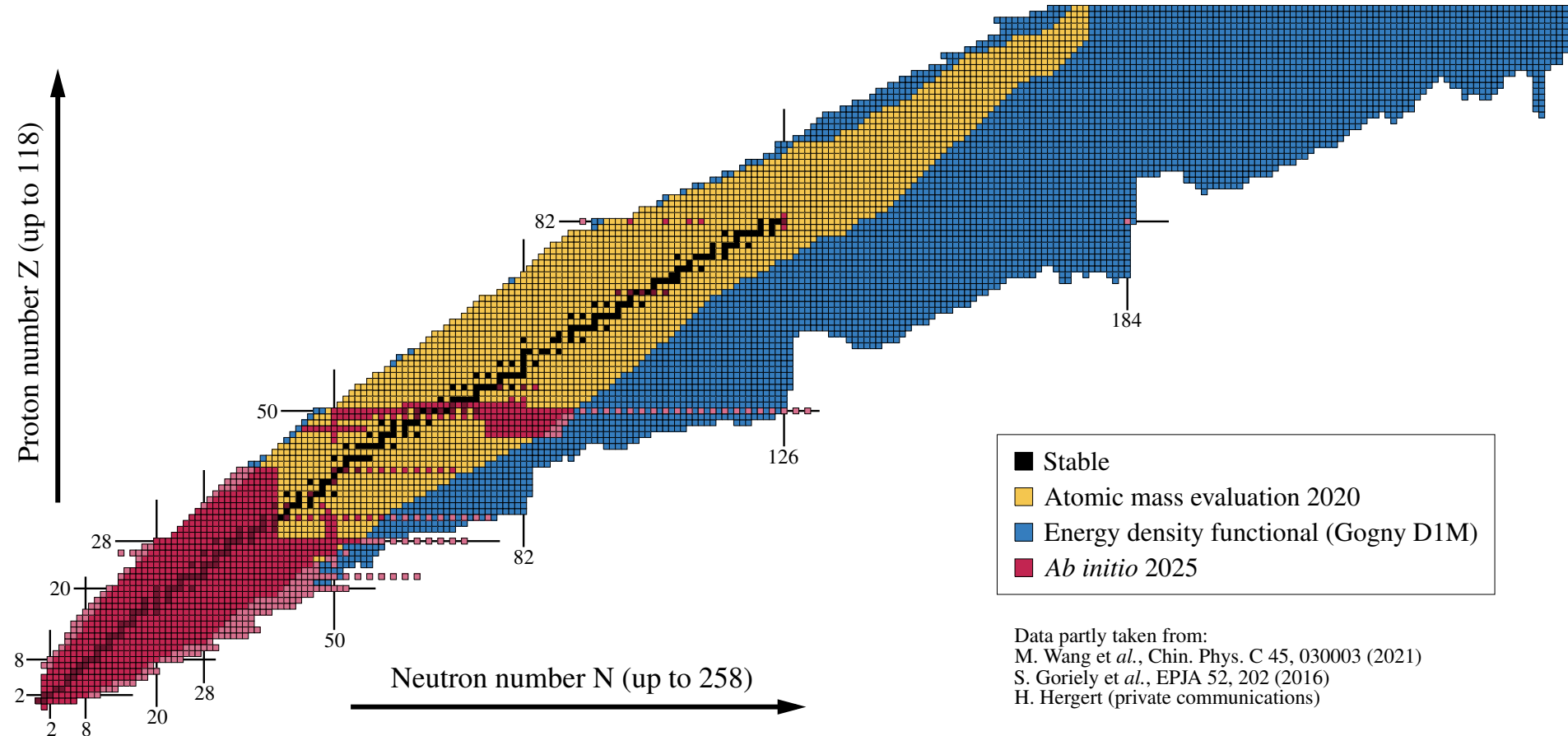
Numerical method

Courtesy of P. Arthuis

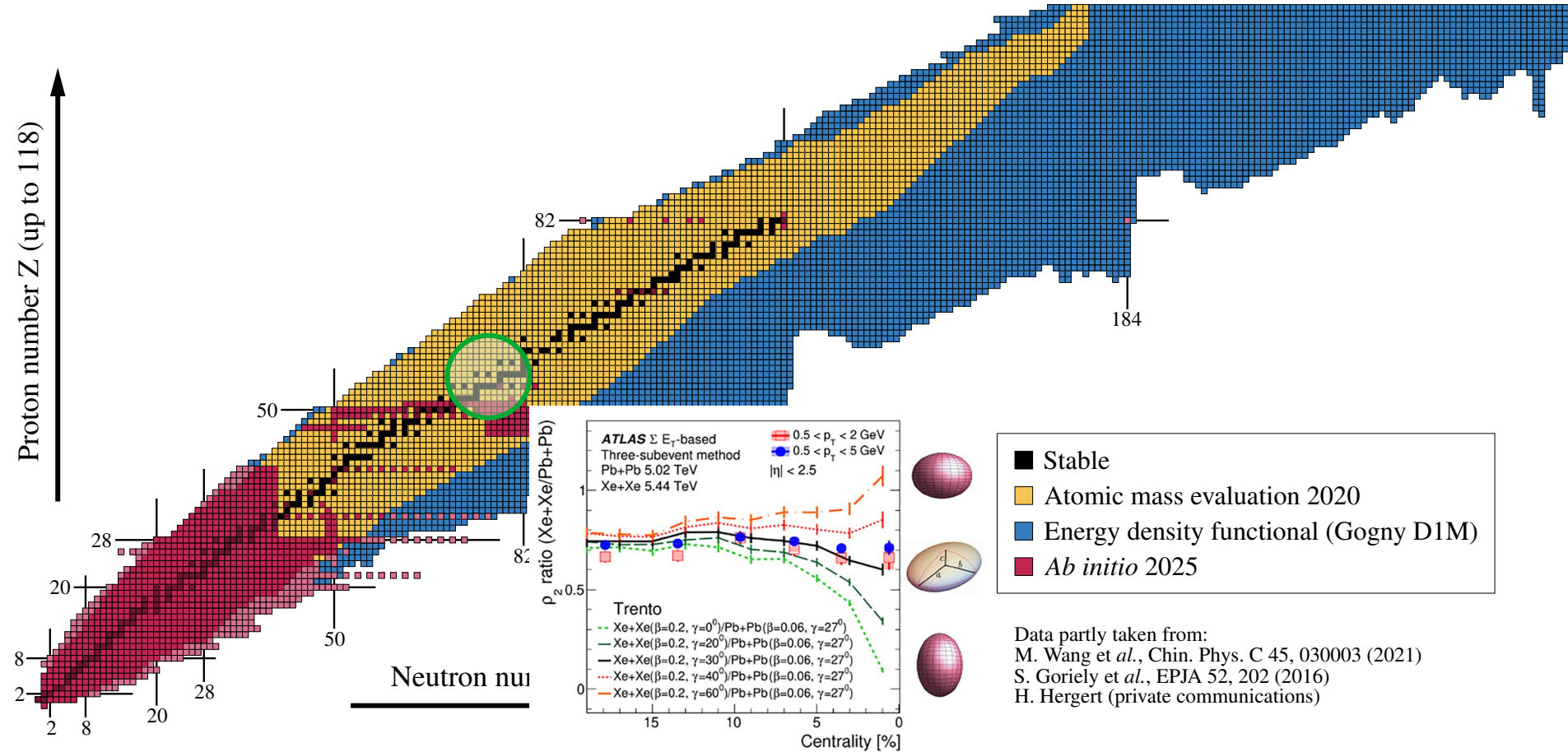
Reach of *ab initio* nuclear theory (2010)



Reach of *ab initio* nuclear theory (2025)

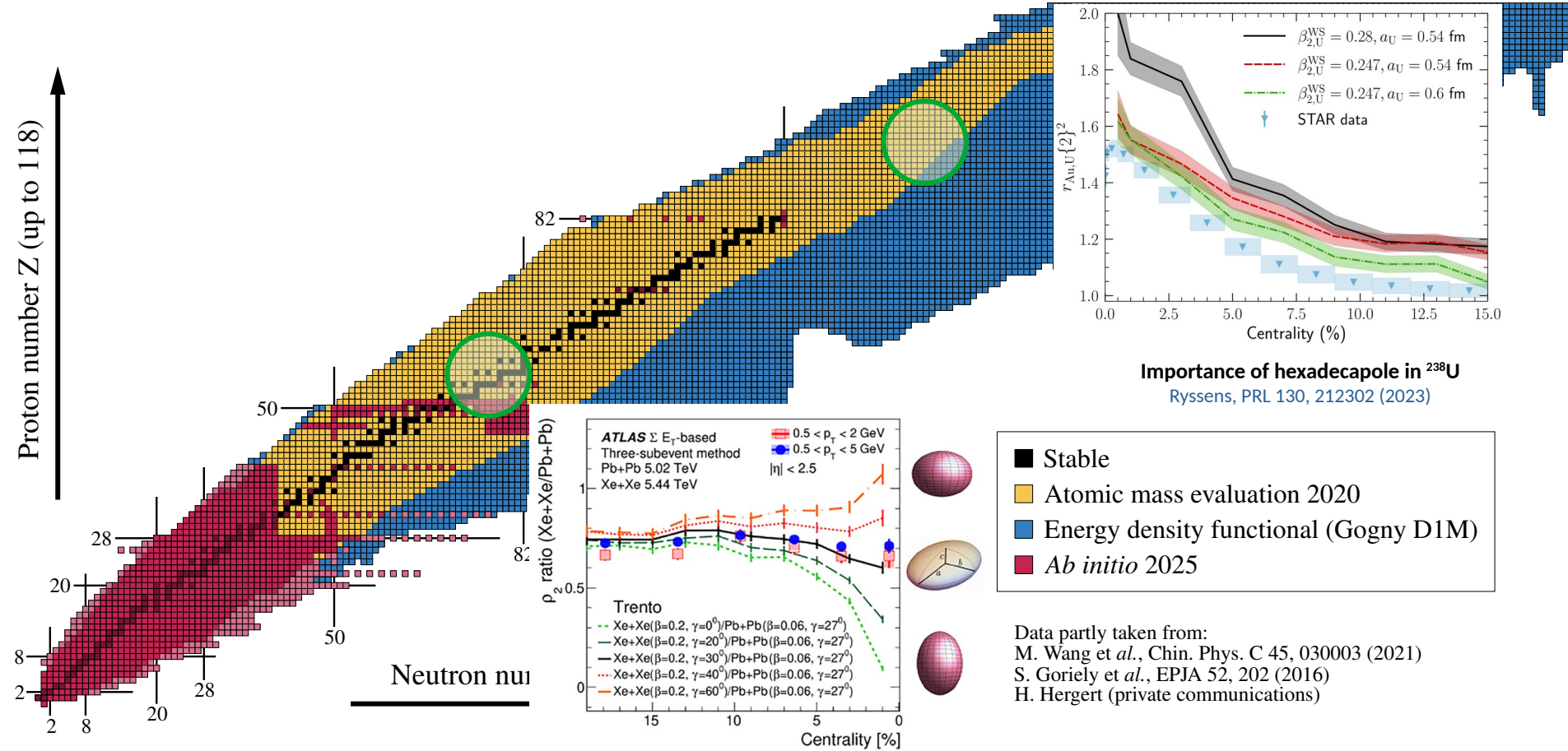


Also contributions from density functionals



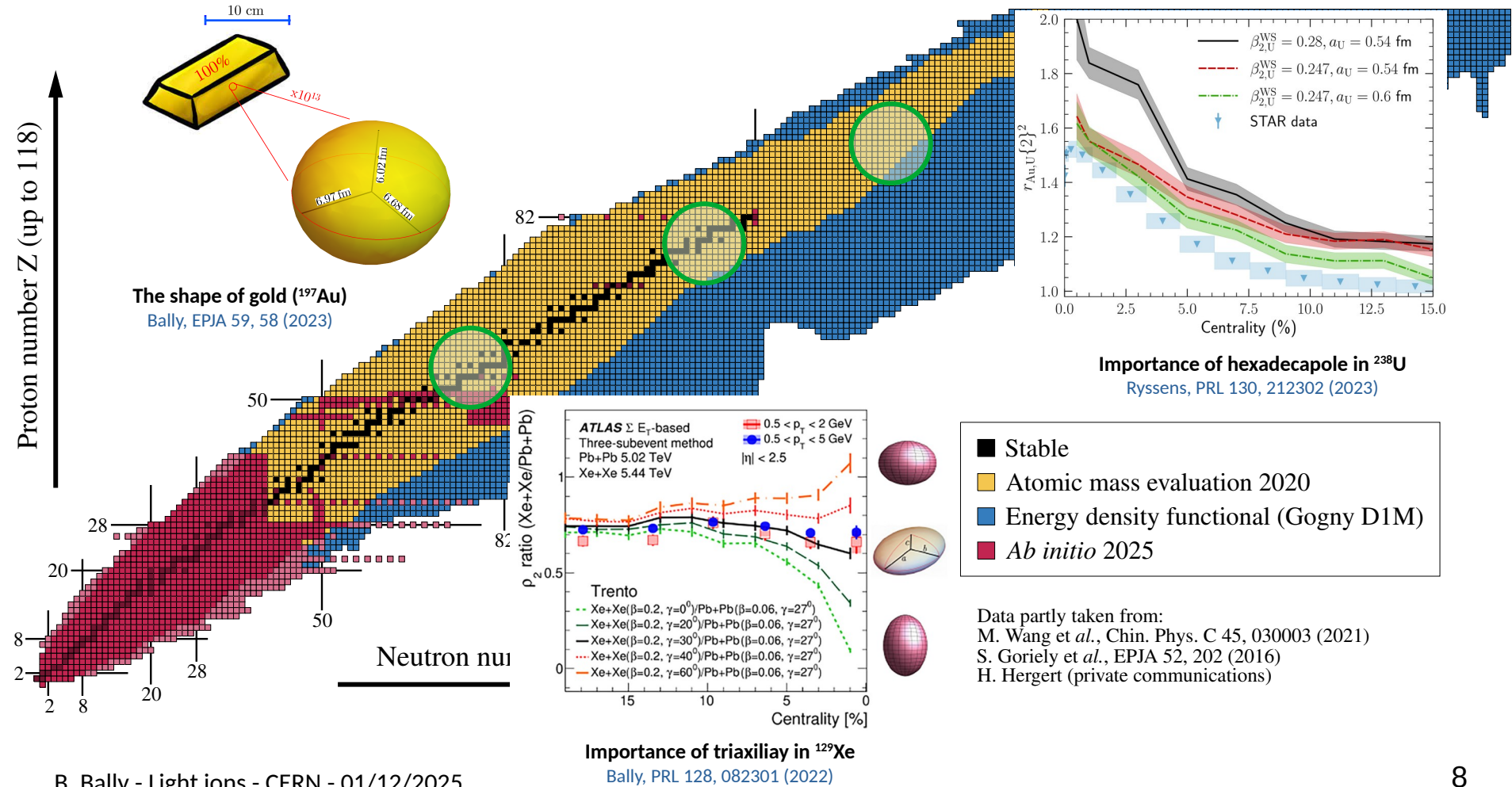
Importance of triaxiliay in ^{129}Xe
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Projected Generator Coordinate Method (PGCM)

- Ansatz for the nuclear many-body wave function

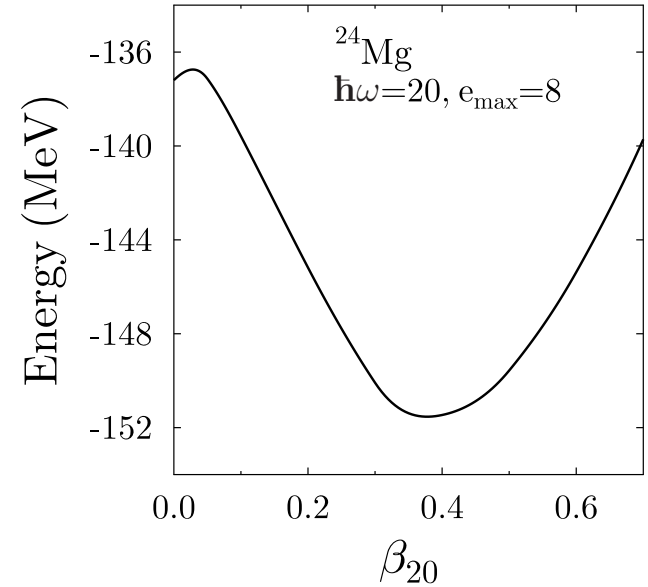
$$|\Psi_\epsilon^{\sigma M}\rangle = \sum_{qK} f_{\epsilon, qK}^{\sigma M} P_{MK}^\sigma |\Phi(q)\rangle \quad \text{where } \sigma \equiv Z, N, J, \pi$$

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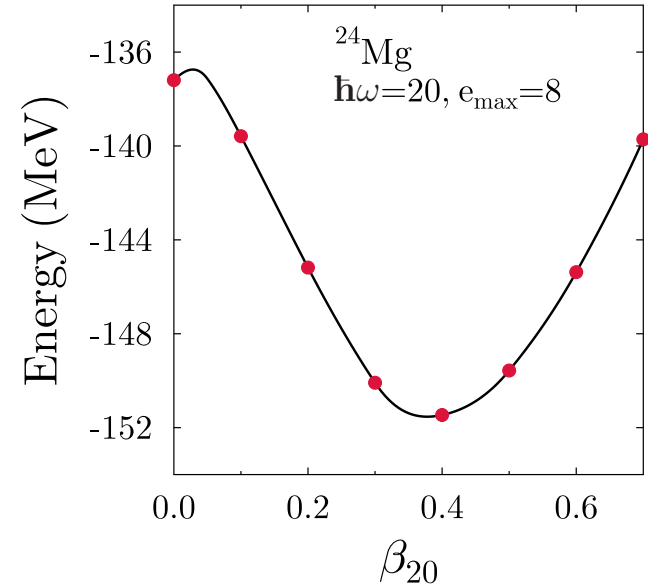


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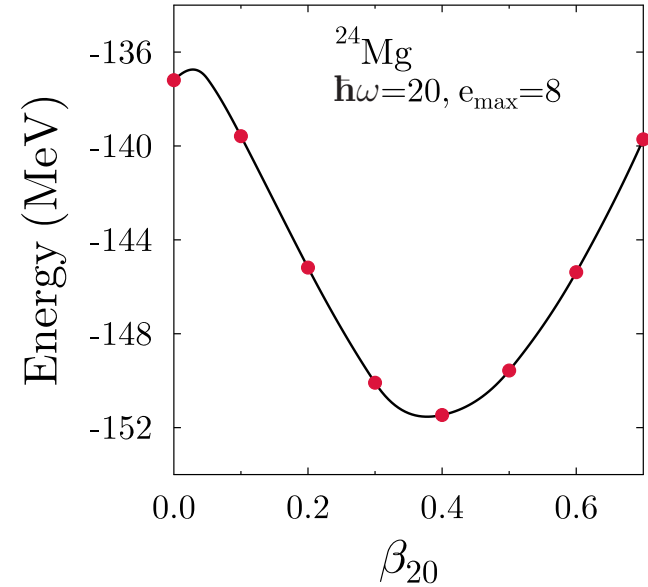
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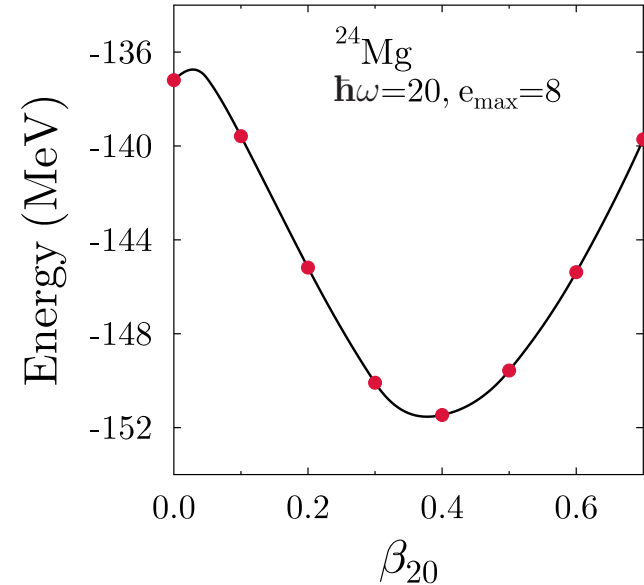
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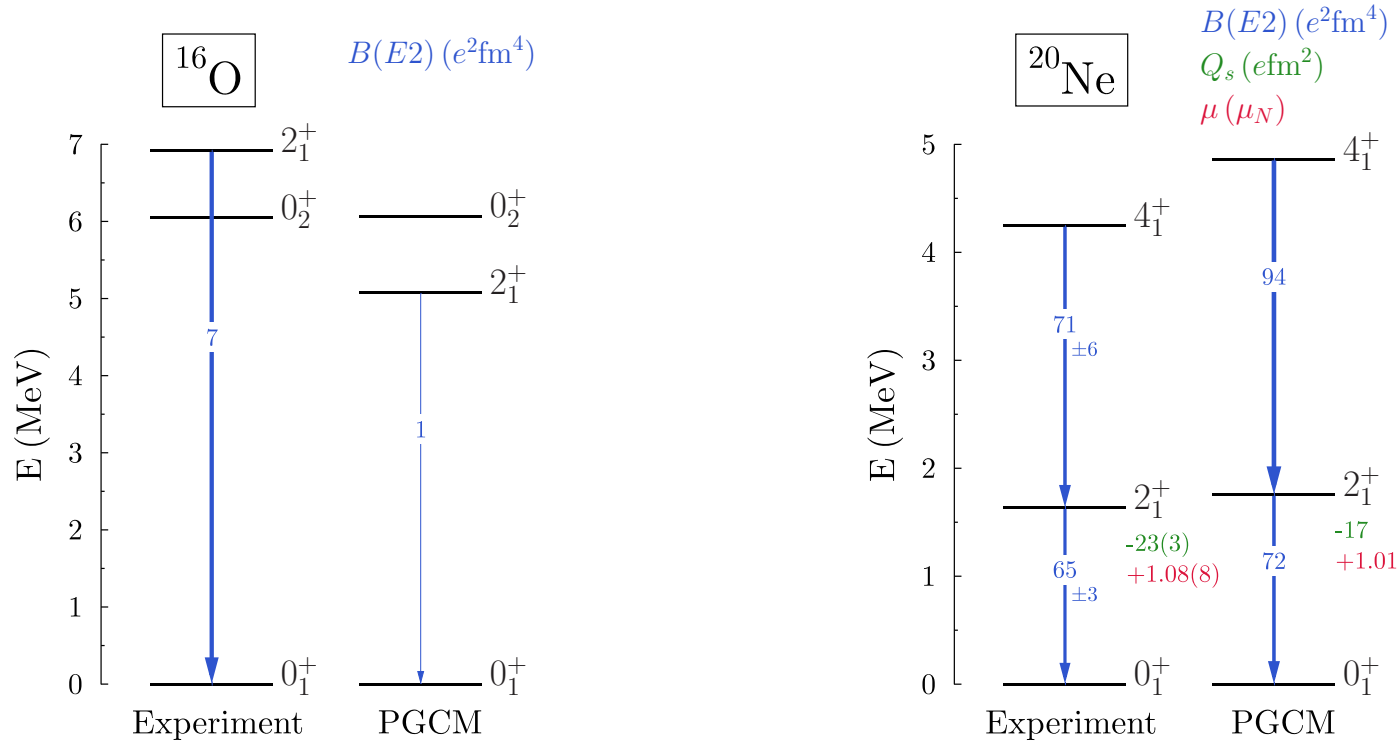
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- Calculations for ^{16}O and ^{20}Ne are the largest ever performed (to my knowledge)
 - Simultaneous exploration of $\beta_{20}, \beta_{22}, \beta_{30}, \beta_{32}$ + symmetry restoration (total: 10 dimensions)
 - Chiral EFT interaction at N3LO
- [Giacalone, PRL 135, 012302 \(2025\)](#)



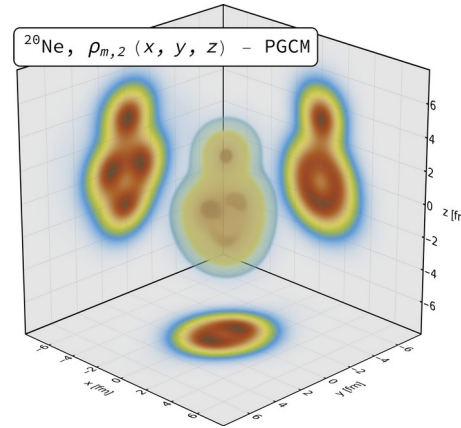
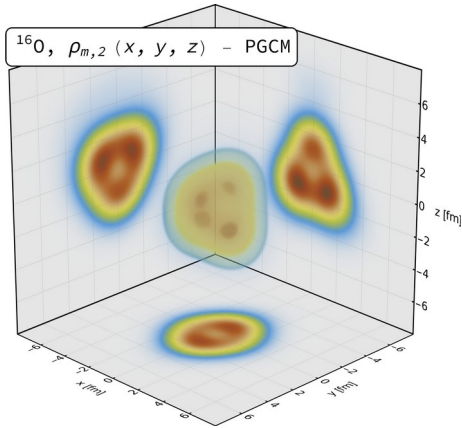
Low-energy spectroscopy



Average one-body density for ^{16}O and ^{20}Ne

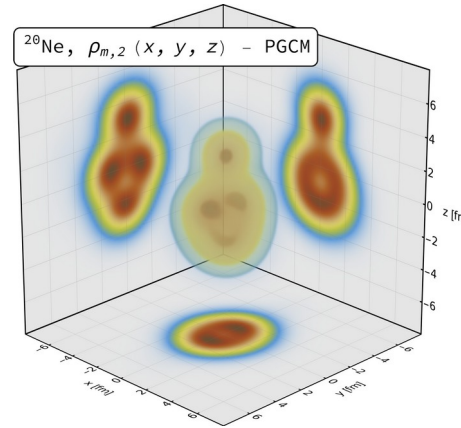
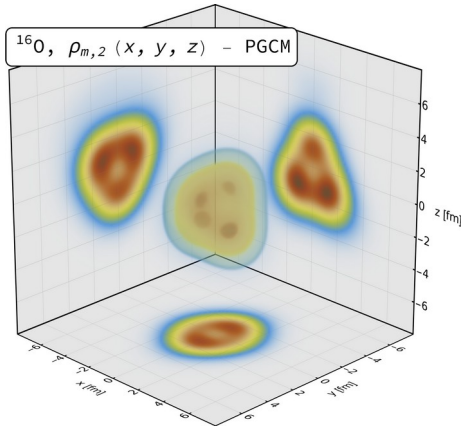
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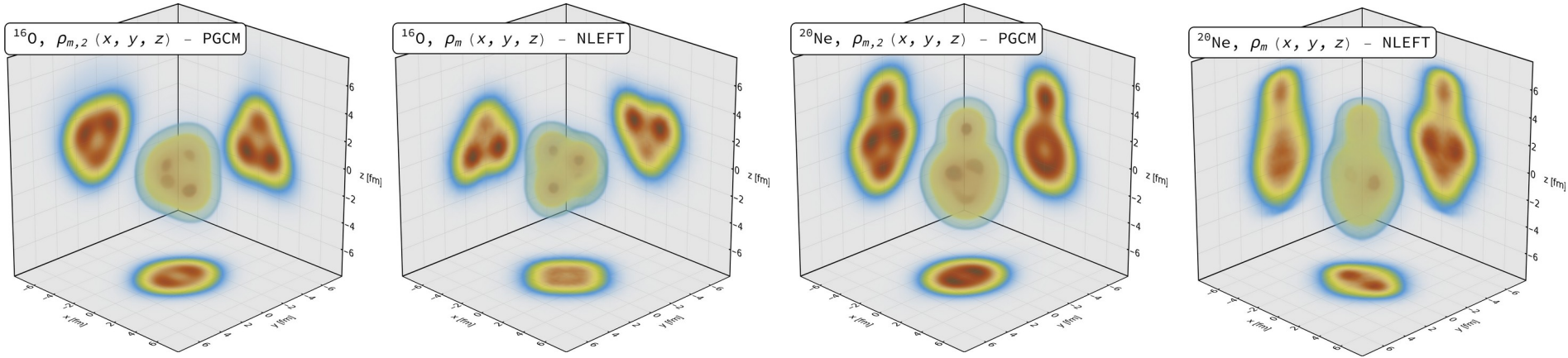
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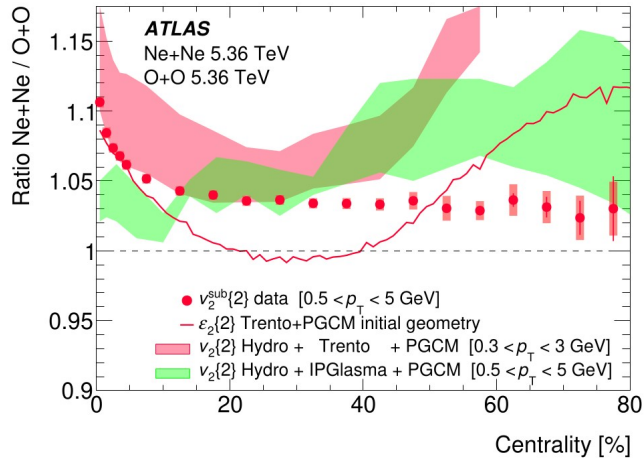
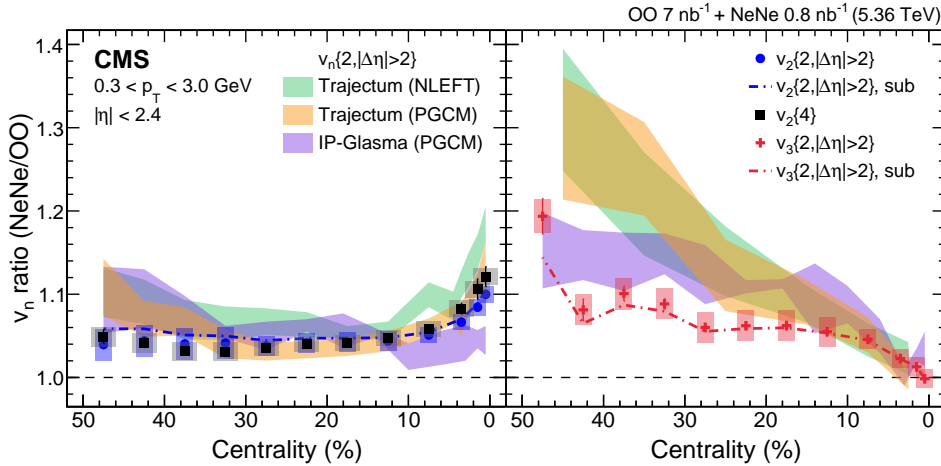
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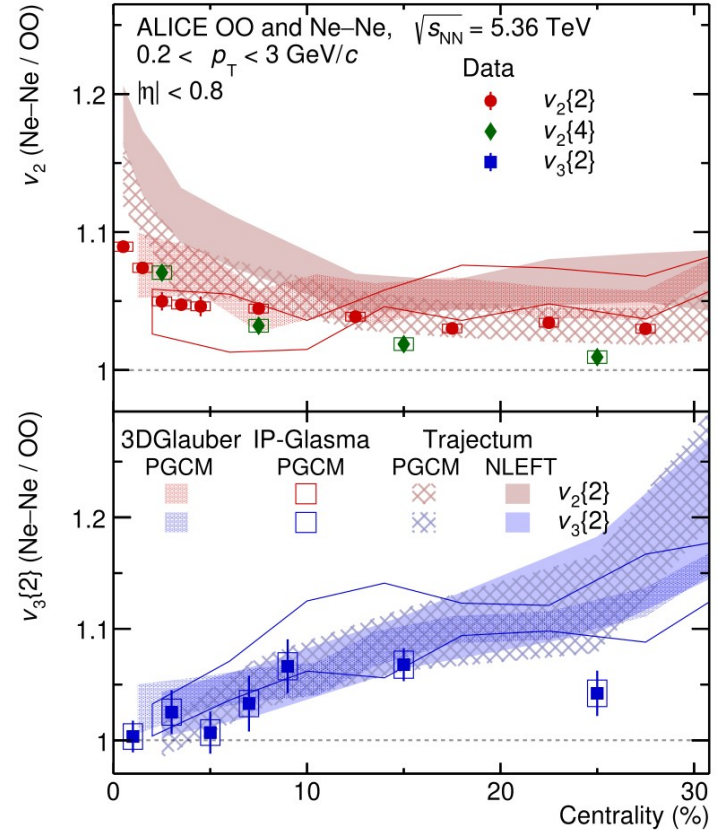


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- Help visualize the matter distribution of the nucleus
- Agreement with NLEFT calculations (reduced to a one-body)

Impressive results!

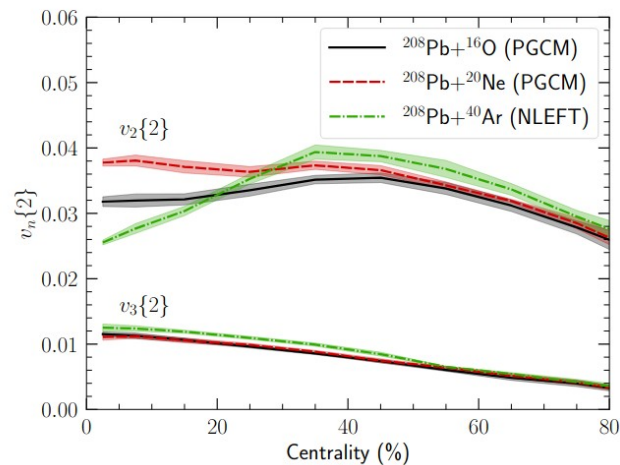
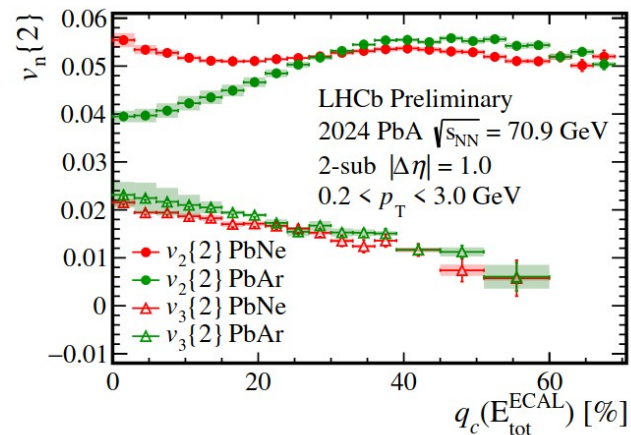


CMS: arXiv:2510.02580
 ATLAS: arXiv:2509.05171
 ALICE: arXiv:2509.06428



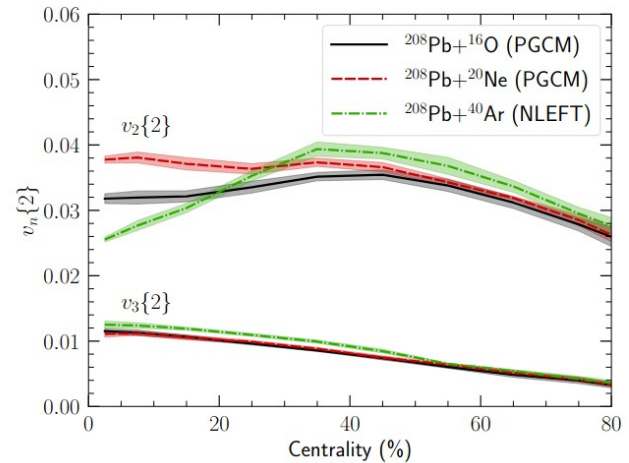
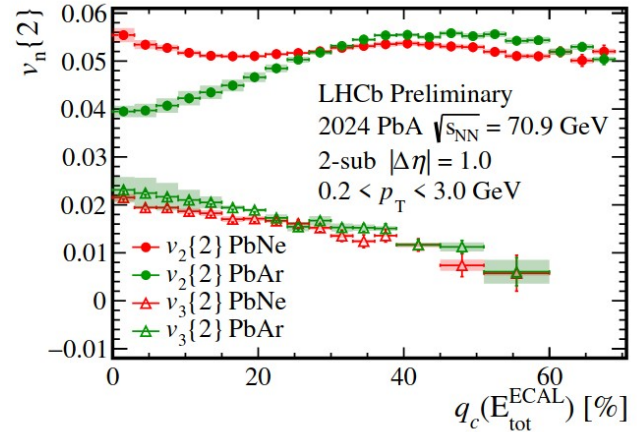
New application to ^{22}Ne

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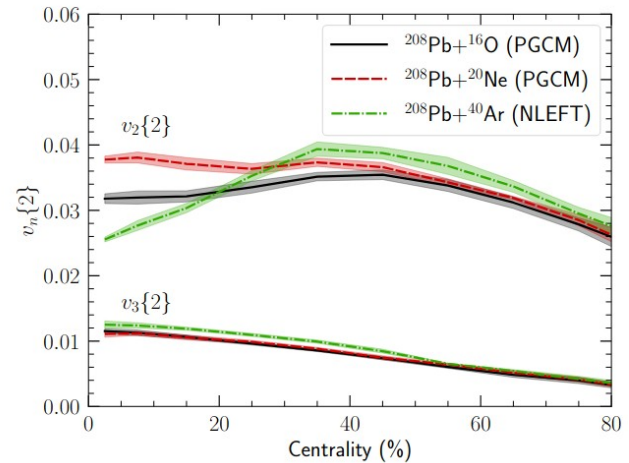
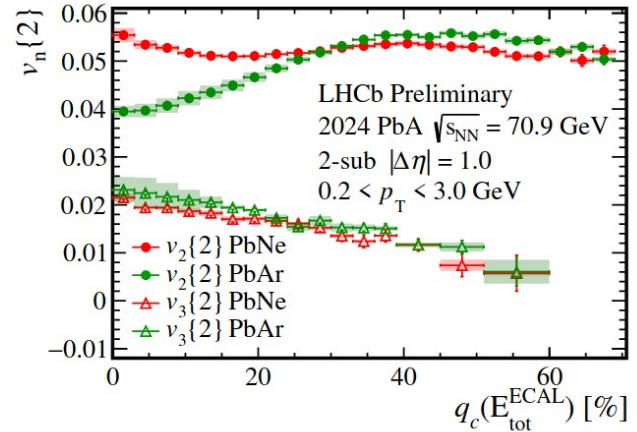
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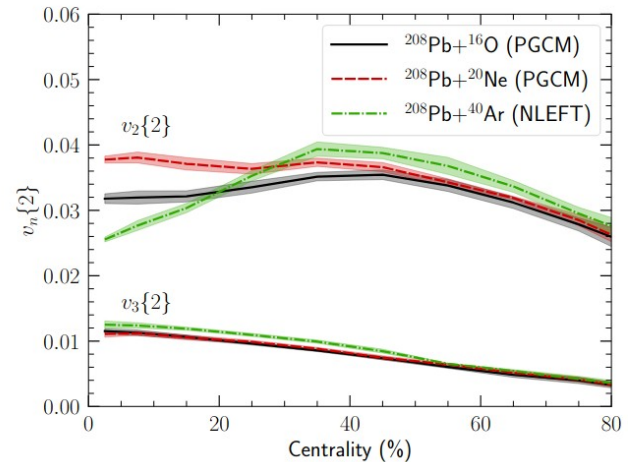
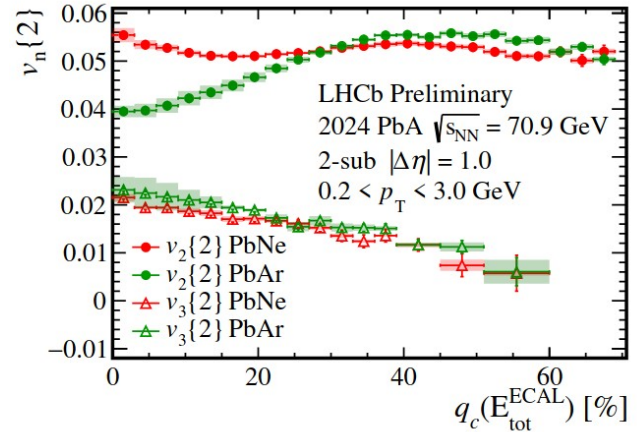
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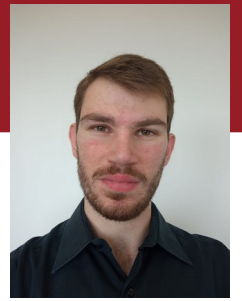


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[Giacalone, PRL 134, 082301](https://arxiv.org/abs/2508.2301) (2025)
- Deformation often changes along an isotopic chain
→ Need calculation of ^{22}Ne



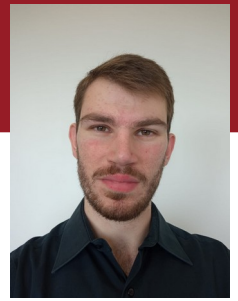
New formalism: PGCM-SVD



Stavros Bofos

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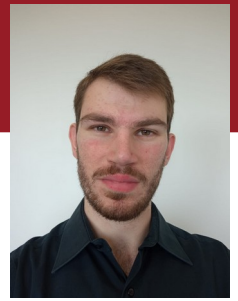


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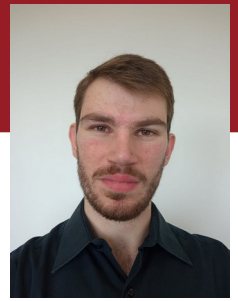
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- New dimension reduction method: PGCM with Singular Value Decomposition (PGCM-SVD)
Bofos, in preparation (2026)

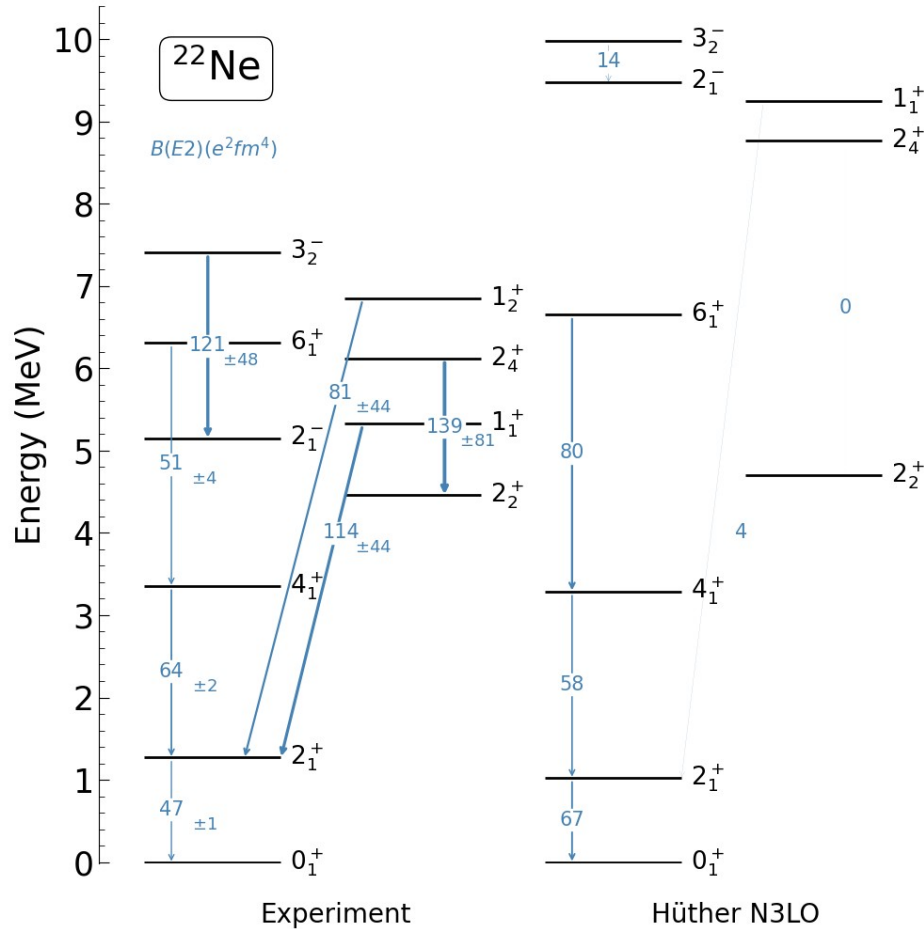
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Bofos, in preparation (2026)
- Reduces the computational burden
→ heavier systems and more systematic calculations

Low-energy spectroscopy

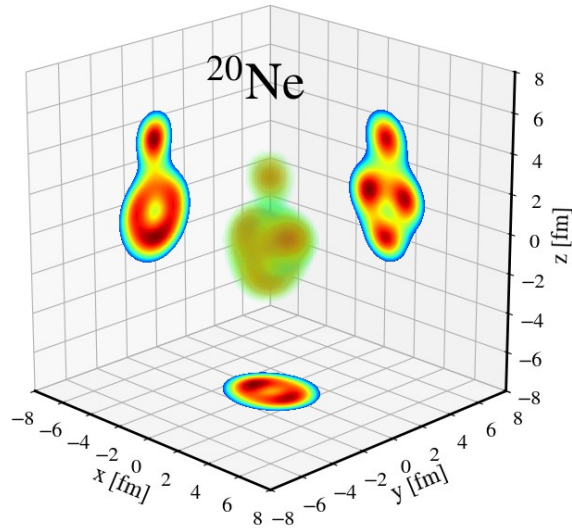


Preliminary

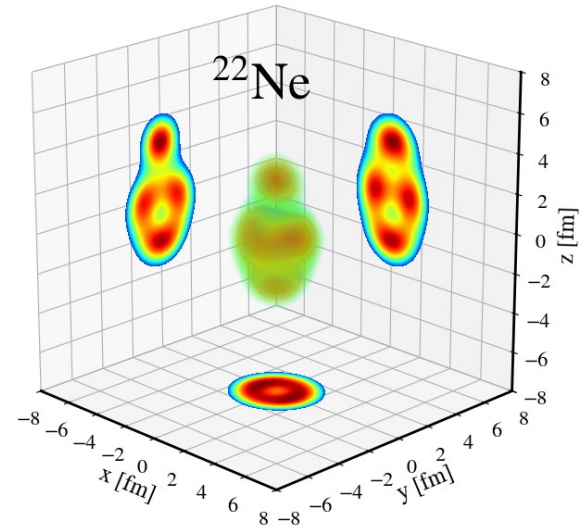
	Exp.	PGCM
$r_{\text{rms}}(\text{fm})$	2.952	3.065
$Q_s(2_1^+)(e\text{fm}^2)$	-21.15(1)	-16.66
$\mu(2_1^+)(\mu_N)$	+0.65(2)	+0.65

One-body density at average deformation

Hüther N3LO, $e_{\max} = 6$, $\hbar\omega = 12$



Hüther N3LO, $e_{\max} = 6$, $\hbar\omega = 12$



- Similar shape but certain components (e.g., octupole) are smaller in ^{22}Ne

Eccentricity operator $\varepsilon_2(\mathbf{r}_1, \mathbf{r}_2)$

- New operator recently proposed that access the two-body density of the ground state

Duguet, PRL 135, 182301 (2025)

$$\hat{\mathcal{E}}_n(\mathbf{r}_1, \mathbf{r}_2) = c_n^{-1} r_1^n Y_n^n(\Omega_1) c_{-n}^{-1} r_2^n Y_n^{-n}(\Omega_2)$$

- Resemblance with “Kumar invariants” known in nuclear structure

Kumar, PRL 28, 249 (1972)

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Duguet, PRL 135, 182301 (2025)

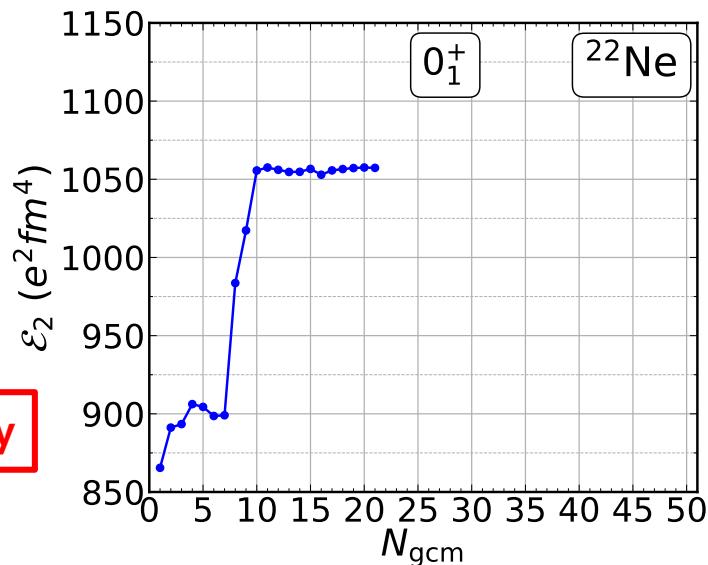
$$\hat{\mathcal{E}}_n(\mathbf{r}_1, \mathbf{r}_2) = c_n^{-1} r_1^n Y_n^n(\Omega_1) c_{-n}^{-1} r_2^n Y_n^{-n}(\Omega_2)$$

- Resemblance with “Kumar invariants” known in nuclear structure

Kumar, PRL 28, 249 (1972)

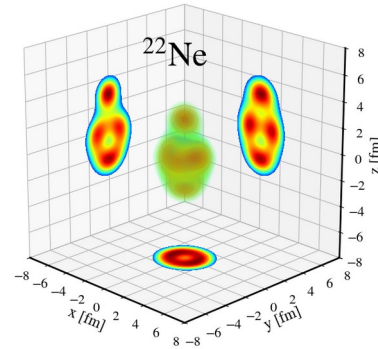
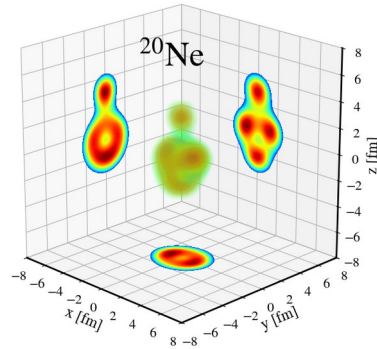
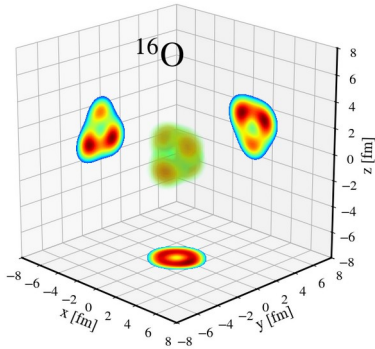
- Implementation on going with application to ^{22}Ne

Bofos, in preparation (2026)



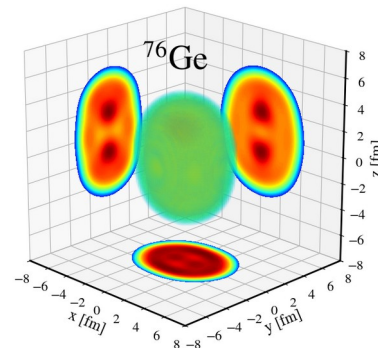
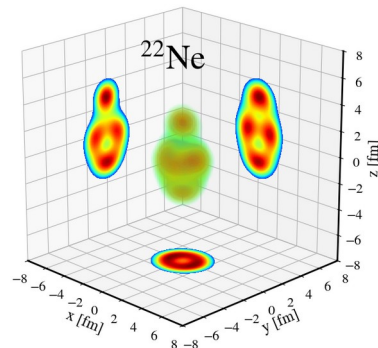
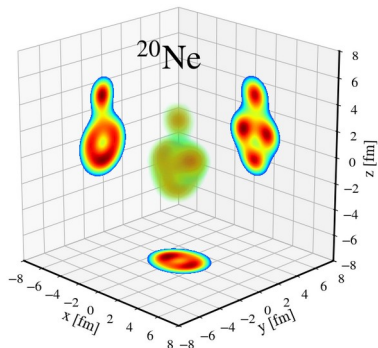
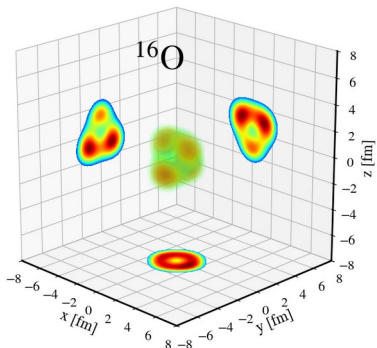
Preliminary

Conclusion



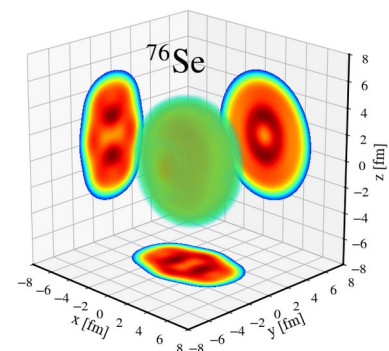
- Very useful approximation
 - Starting point to determine the initial geometry
 - Capture part of the correlations at moderate cost

Conclusion

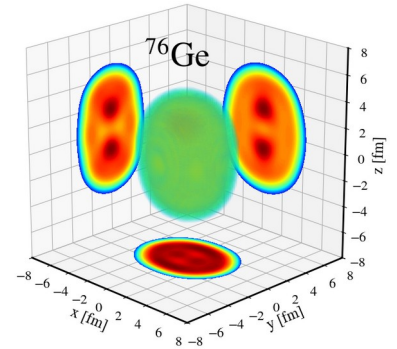
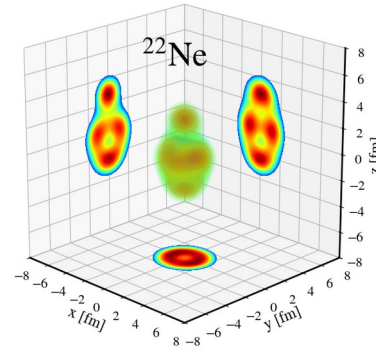
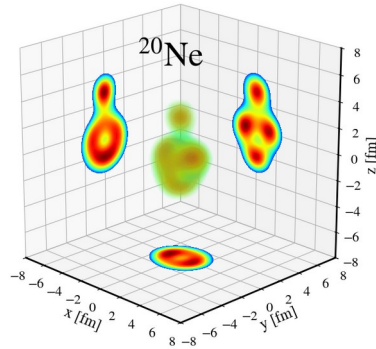
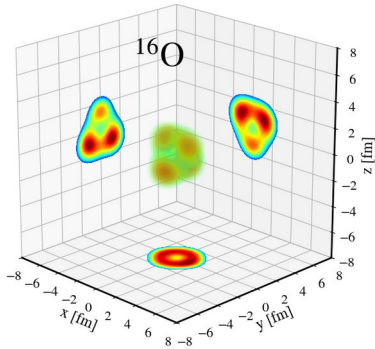


→ Giuliano's talk

- Very useful approximation
 - Starting point to determine the initial geometry
 - Capture part of the correlations at moderate cost

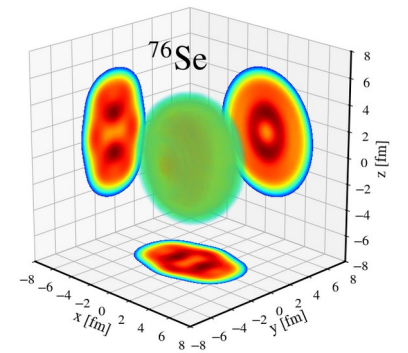


Conclusion



→ Giuliano's talk

- Very useful approximation
 - Starting point to determine the initial geometry
 - Capture part of the correlations at moderate cost
- **Nucleus is a strongly correlated quantum system with symmetries**
 - Can't fully be absorbed in a one-body density



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