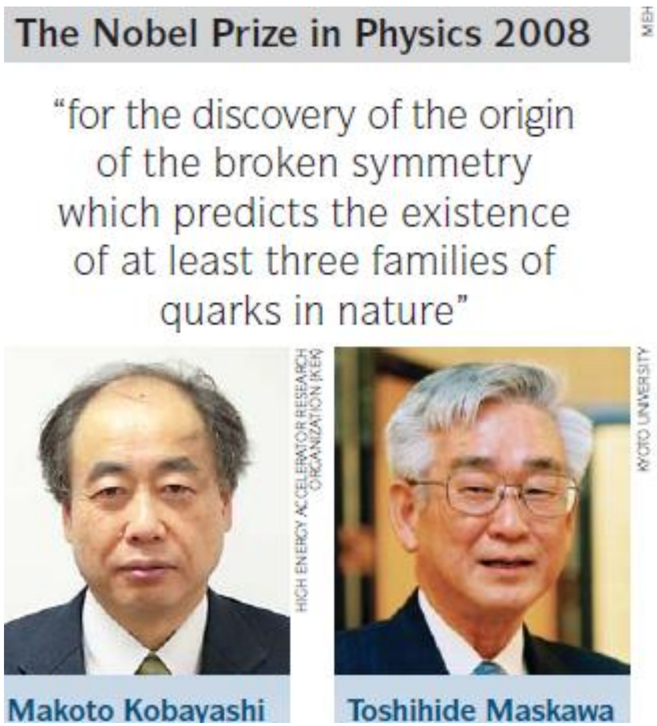
Could a 4^{th} Generation still evade the LHC limits ?

Mathieu Buchkremer, J.-M. Gérard, F. Maltoni

19th January 2012

Workshop on Very Heavy Quarks at the LHC

Introduction

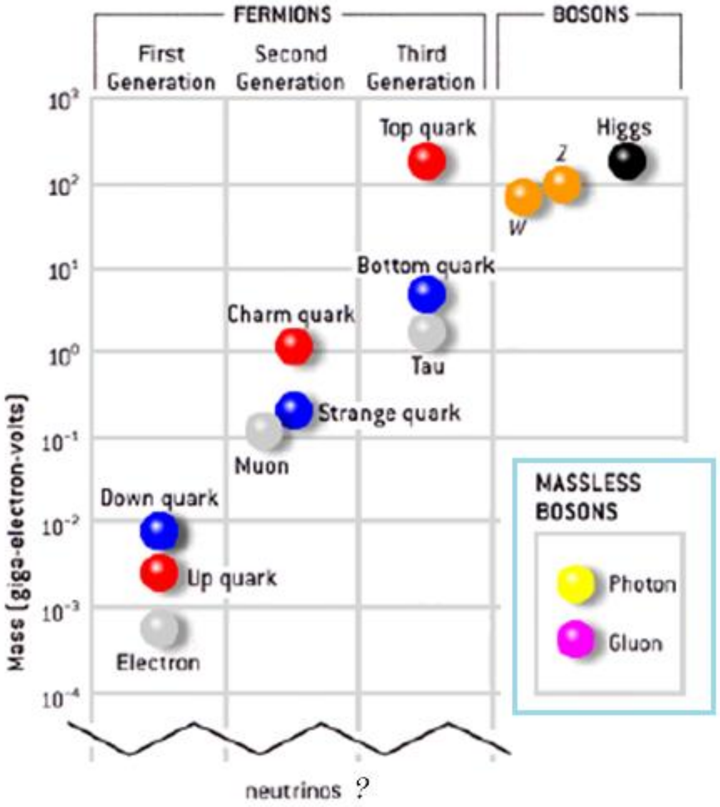


The Nobel Prize in Physics 2008

“for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature”

Makoto Kobayashi

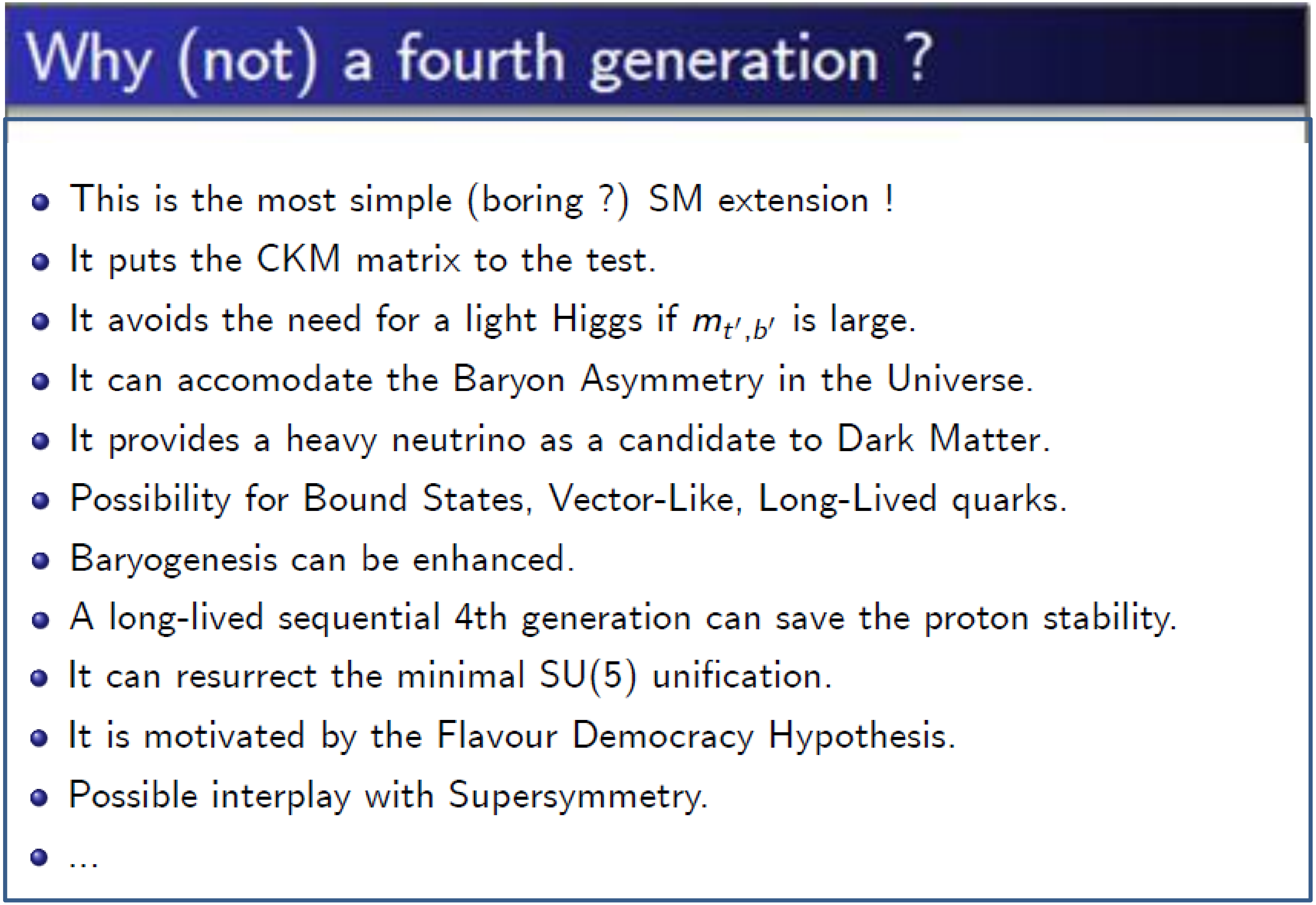
Toshihide Maskawa



But this theoretical framework is still incomplete :

- The origin of the mass spectrum is unexplained.
- The pattern of fermion mixings is still a mystery.
- Why is the top quark so heavy ⇔ Why is the electron so light ?
- Why should there be 3 families of matter particles ?

Introduction

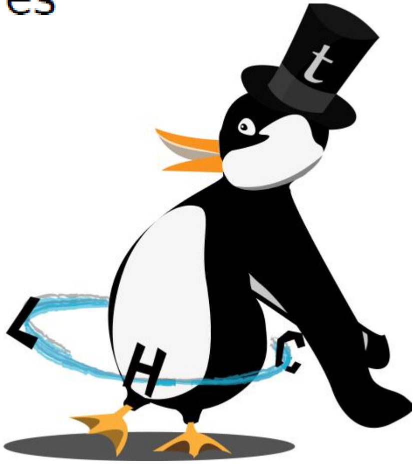


Why (not) a fourth generation ?

- This is the most simple (boring ?) SM extension !
- It puts the CKM matrix to the test.
- It avoids the need for a light Higgs if $m_{t',b'}$ is large.
- It can accomodate the Baryon Asymmetry in the Universe.
- It provides a heavy neutrino as a candidate to Dark Matter.
- Possibility for Bound States, Vector-Like, Long-Lived quarks.
- Baryogenesis can be enhanced.
- A long-lived sequential 4th generation can save the proton stability.
- It can resurrect the minimal SU(5) unification.
- It is motivated by the Flavour Democracy Hypothesis.
- Possible interplay with Supersymmetry.
- ...

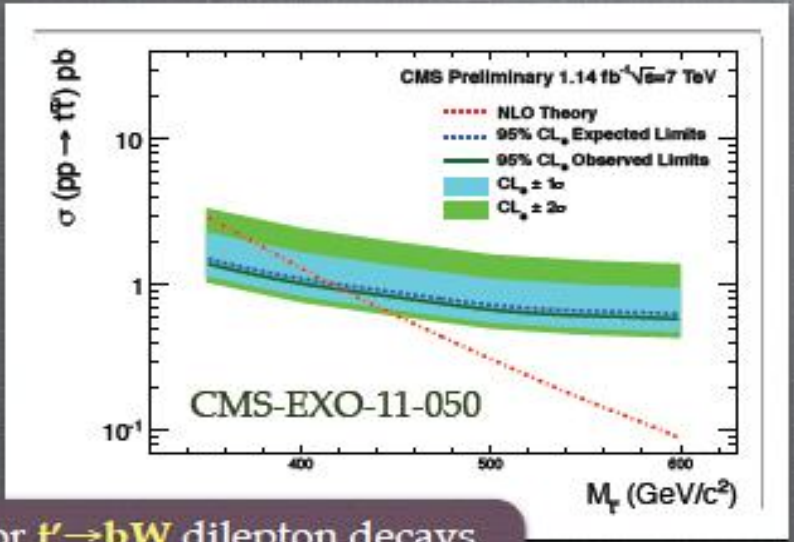
Outline

1. Experimental Constraints :
    1. Direct Searches at the LHC
    2. Interplay with Higgs Physics
2. Electroweak Precision Observables :
    1. Formalism
    2. Global Analysis
    3. New Channels for 4th Generation Searches
3. Conclusions
4. Epilogue : Consequences for V_{tb}

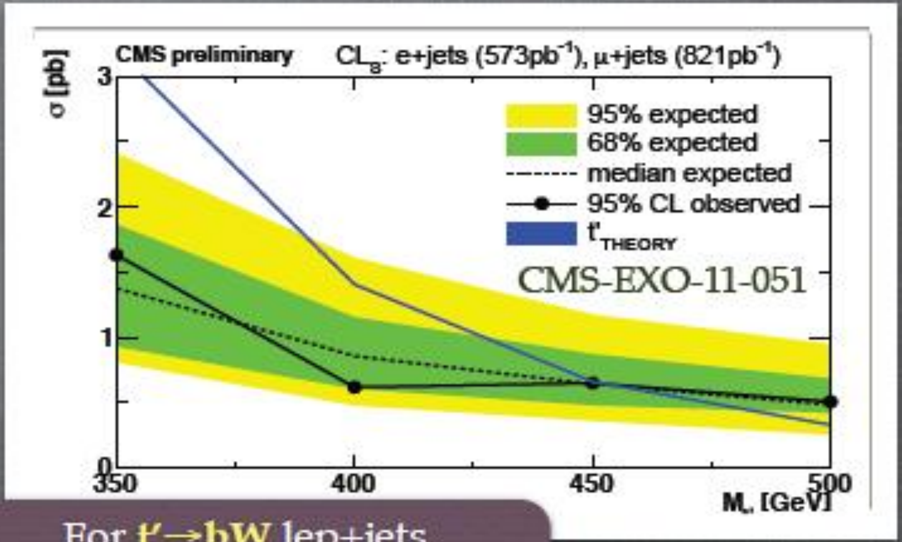


1. Direct Searches

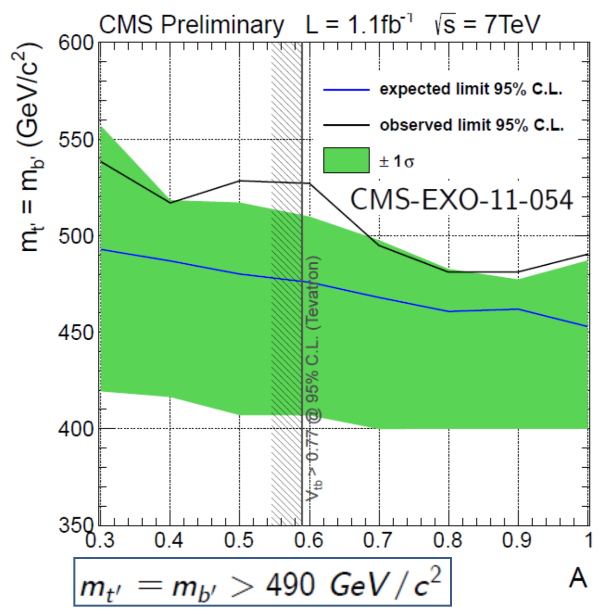
$$m_Q < \sqrt{\frac{4\pi}{3}}v \simeq 500\ GeV$$



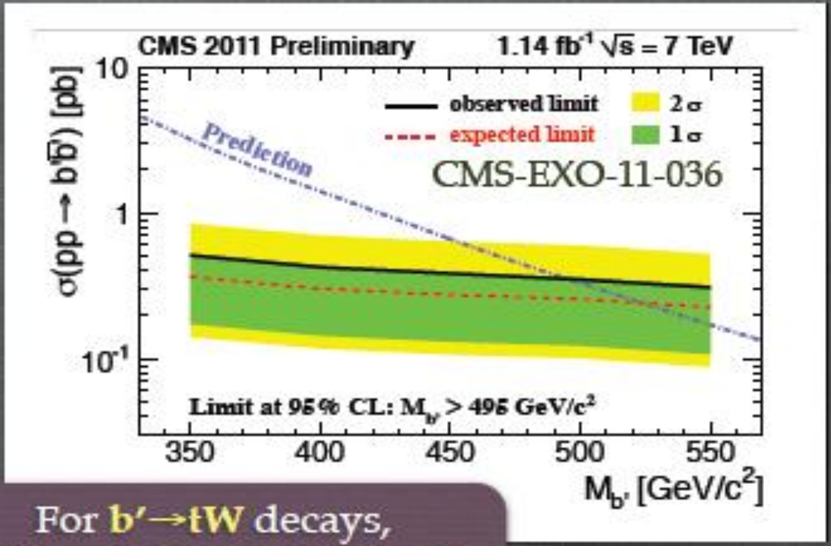
For **t′→bW** dilepton decays,
M(t′) > 422 GeV at 95% C.L.



For **t′→bW** lep+jets,
M(t′) > 450 GeV at 95% C.L.



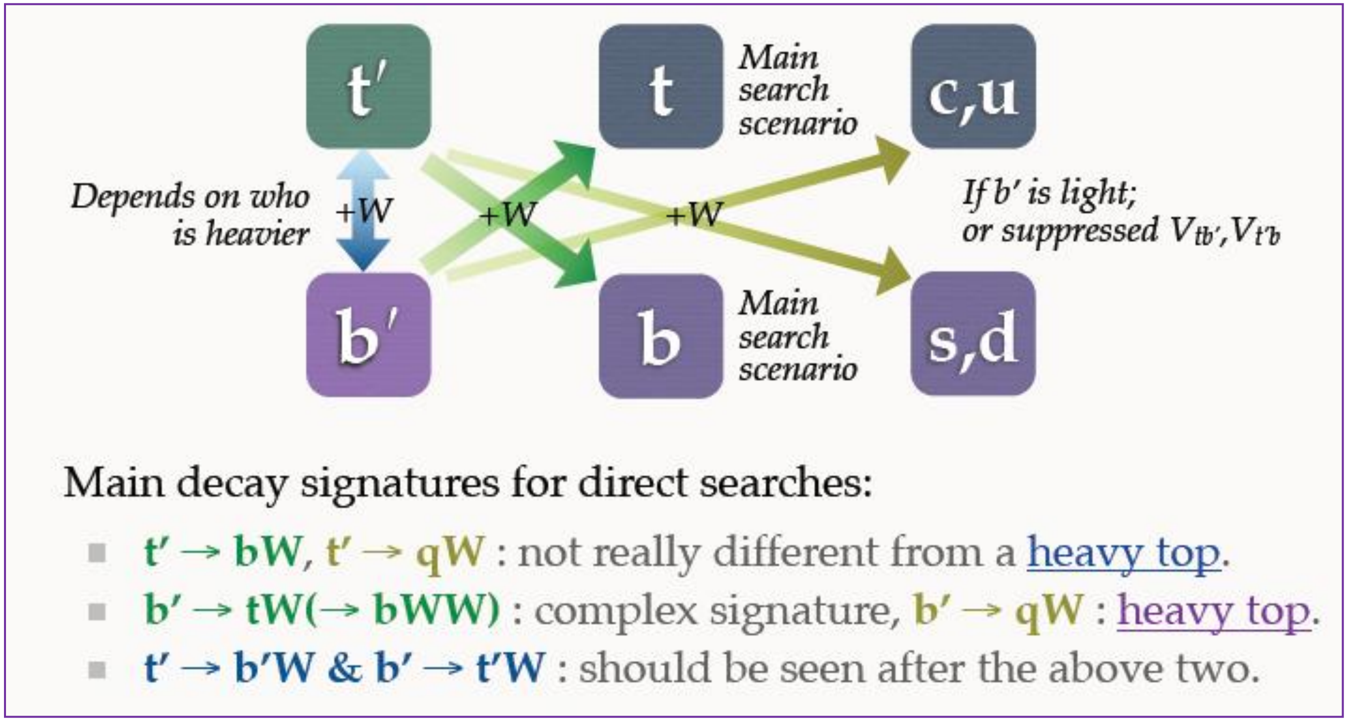
$m_{t'} = m_{b'} > 490\ GeV/c^2$



For **b′→tW** decays,
M(b′) > 495 GeV at 95% C.L.

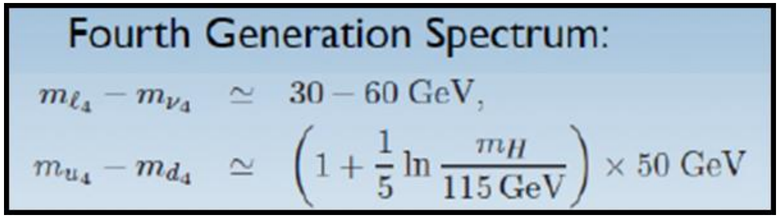
Kai-Feng Chen
National Taiwan University

1. Direct Searches



Main decay signatures for direct searches:

- **t′ → bW, t′ → qW** : not really different from a heavy top.
- **b′ → tW(→ bWW)** : complex signature, **b′ → qW** : heavy top.
- **t′ → b′W & b′ → t′W** : should be seen after the above two.



Fourth Generation Spectrum:

$$m_{\ell_4} - m_{\nu_4} \simeq 30 - 60\ \text{GeV},$$

$$m_{u_4} - m_{d_4} \simeq \left(1 + \frac{1}{5}\ln\frac{m_H}{115\ \text{GeV}}\right) \times 50\ \text{GeV}$$

Limitations

- Excludes the inverted hierarchies $m_{b'} > m_{t'}$ and $m_{\nu'} > m_{\tau'}$
- Does not include CKM mixings with the 3 known generations
- Dismisses small and large quark mass splittings

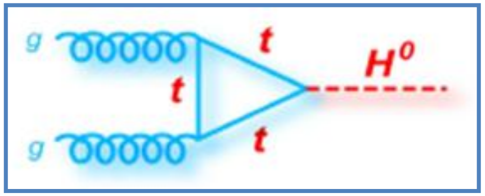
$\boxed{(\Delta m_{q'} < 50\ GeV)}$ $\boxed{(\Delta m_{q'} > 70\ GeV)}$

4th generation parameter space from :

G. D. Kribs et al., “Four Generations and Higgs Physics”, Phys. Rev. D.76. 075016

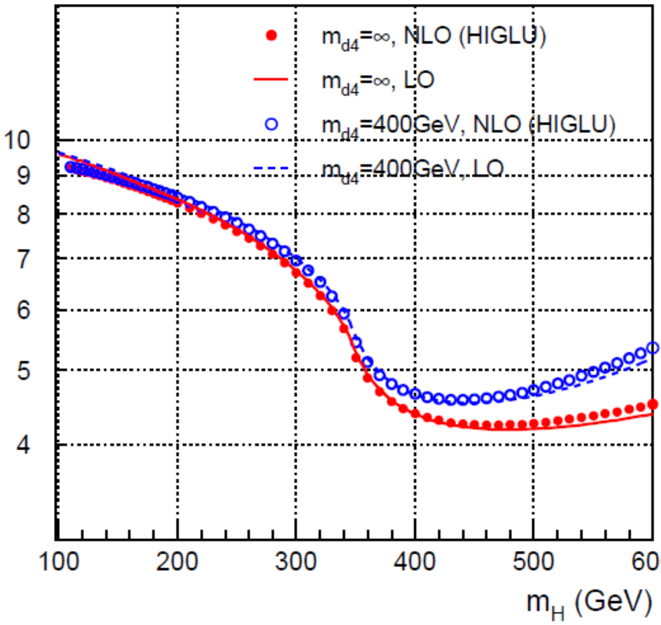
1. Higgs Physics

SM4 : enhancement factor due to t' and b' loops (X. Ruan, Z. Zhang, arXiv:hep-ph/11051634)

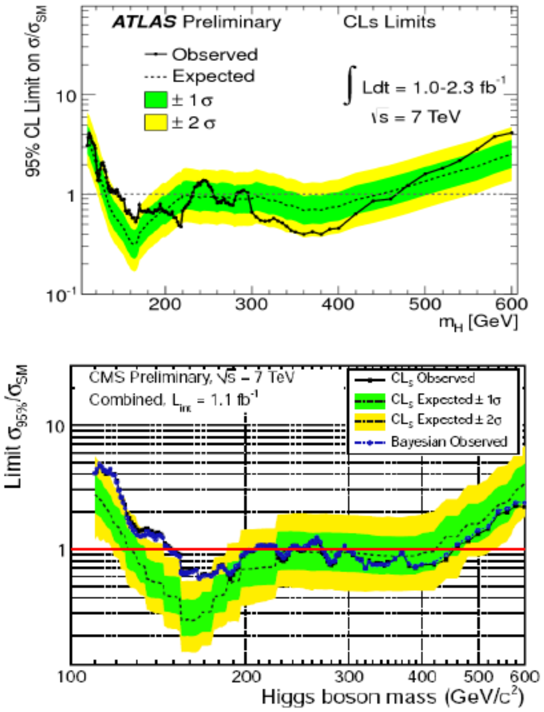


- $\sigma_{SM4}/\sigma_{SM3} \simeq 4$ for $m_H \in [350, 600]\ GeV$
- $\sigma_{SM4}/\sigma_{SM3} \simeq 9$ for $m_H \in [120, 150]\ GeV$

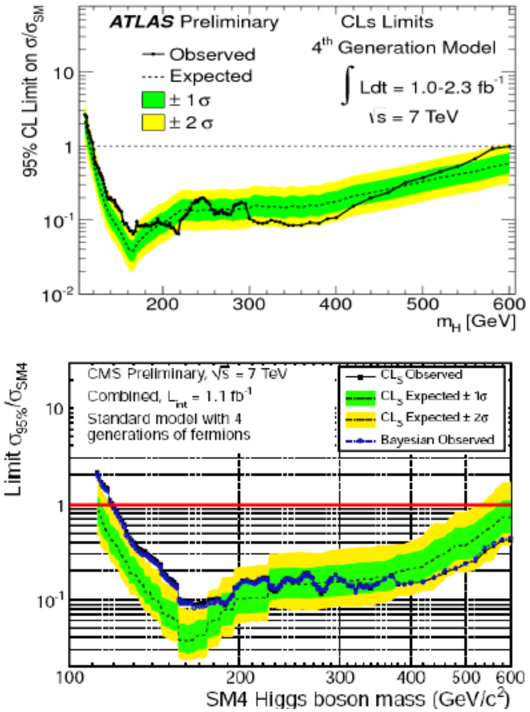
$\sigma(gg\to H)_{SM4}/\sigma(gg\to H)_{SM}$



- **Combined limit 3SM**



Combined limit: 4SM (heavy masses)



$120\ GeV < m_H < 600\ GeV \Longrightarrow$ 4th generation is excluded ?

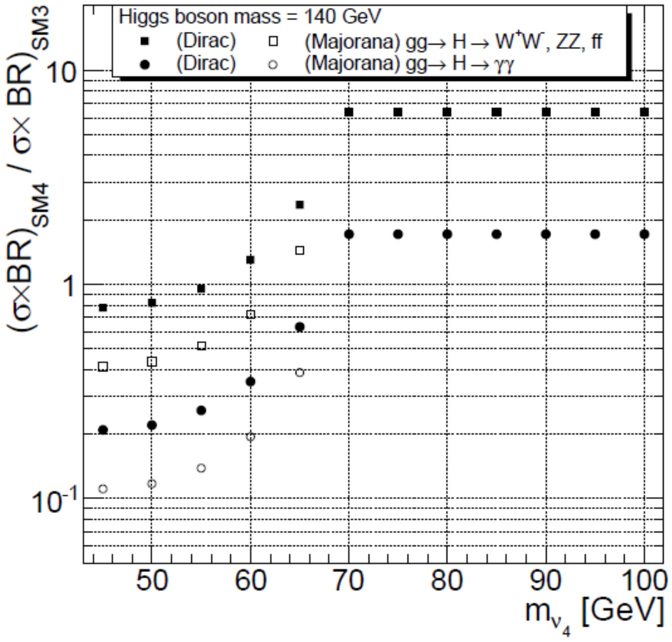
1. Higgs Physics

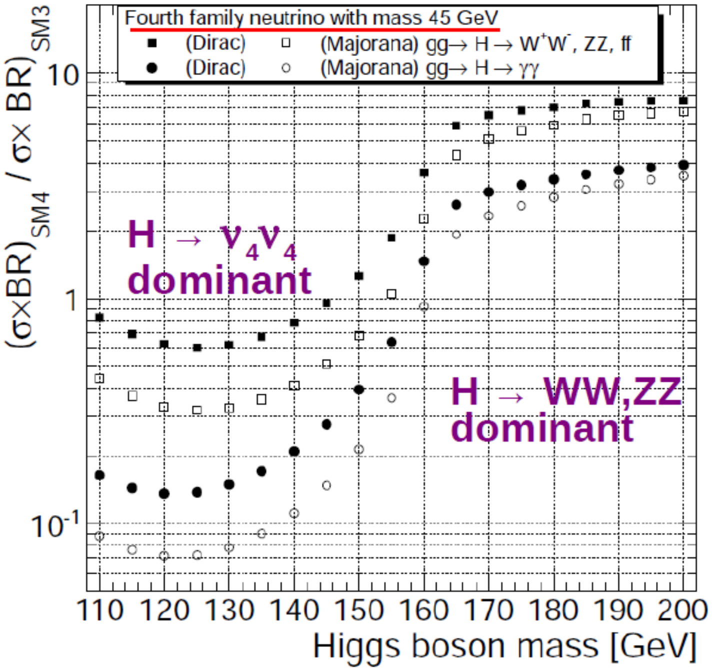
- This assumes that the Higgs does not decay to new fermions
- BR($h \to WW/ZZ$) can be smaller in SM4 if $h \to \nu'\bar{\nu}'$ is kinematically allowed

(S. A. Cetin et al., arXiv:hep-ph/11084071)

$m_{\nu'} > 45\ (39.5)\ GeV$ if stable Dirac (Majorana) neutrino

$m_{\nu'} > 90.3\ (62.5)\ GeV$ if unstable Dirac (Majorana) neutrino



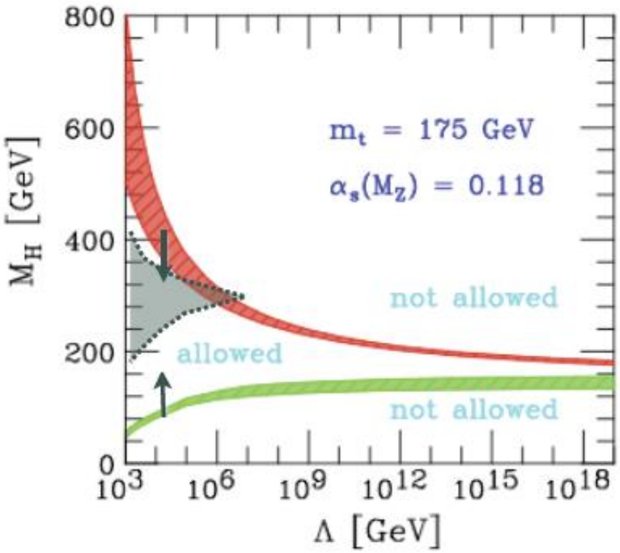


$H \to \nu_4\nu_4$ dominant

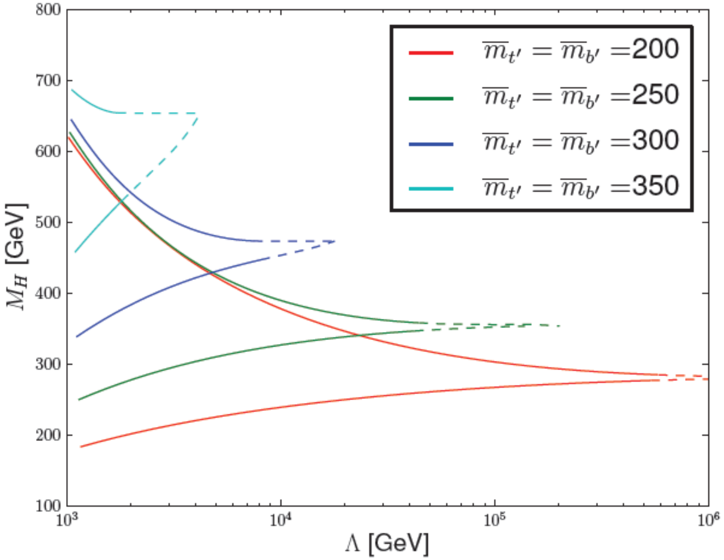
$H \to WW,ZZ$ dominant

+ Large EW NLO corrections for SM4 at the LHC (G. Passarino et al., arXiv:hep-ph/11082025)

1. Higgs Physics



SM4: $\overline{m}_{\tau'}$ =200 $\overline{m}_{\nu'}$ =80 $\theta_{34}^{CKM} = 0°$



Lower bound for m_H: vacuum stability

$$\lambda(\mu^2) > 0$$

Upper bound for m_H: triviality bound

$$\lambda(\mu^2) = \frac{\lambda(m_H^2)}{1 - \frac{3}{4\pi^2}\lambda(m_H^2)\log\frac{\mu^2}{m_H^2}}$$ singular when $\mu^2 \approx \Lambda_L^2 \equiv m_H^2 \exp\left[\frac{4\pi^2}{3\lambda(m_H^2)}\right]$

Running of λ

$$\frac{d\lambda(\mu)}{d\log\mu^2} = \frac{1}{16\pi^2}\left[12\lambda^2 + \frac{3}{8}g^4 + \frac{3}{16}\left(g^2+g'^2\right)^2 - 3h_t^4 - 3\lambda g^2 - \frac{3}{2}\lambda\left(g^2+g'^2\right) + 6\lambda h_t^2\right]$$

The running of λ is directly sensitive to new heavy fermions.

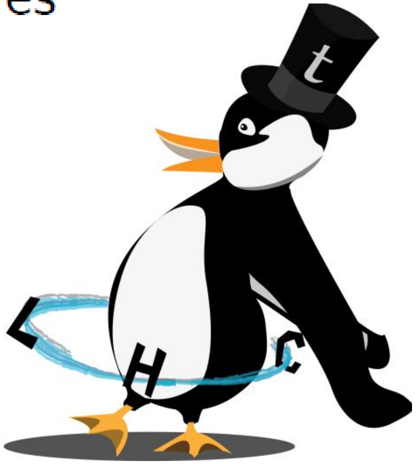
Conclusions :

- $m_H^{SM4} > 700\ GeV$: perturbativity is lost below 1 TeV
- $m_{t',b',\tau',\nu'} > 400\ GeV$: perturbativity is lost below 1 TeV
- ...

4^{th} generation with perturbative Yukawa couplings is almost ruled out.

Outline

1. Experimental Constraints :
    1. Direct Searches at the LHC
    2. Interplay with Higgs Physics
2. Electroweak Precision Observables :
    1. Formalism
    2. Global Analysis
    3. New Channels for 4th Generation Searches
3. Conclusions
4. Epilogue : Consequences for V_{tb}



2. Electroweak Precision Observables

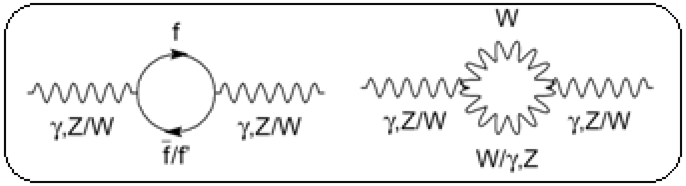
History of the PDG reviews

- 1994: "one heavy generation of ordinary fermions is allowed at 95% CL"
- 1998: "an extra generation of ordinary fermions is now excluded at the 99.2% CL"
- 2002: "an extra generation of ordinary fermions is excluded at the 99.8% CL on the basis of the S parameter alone. [. . .] This result assumes [. . .] that any new families are degenerate. This restriction can be relaxed [. . .] to 95%."
- 2010: "an extra generation of ordinary fermions is excluded at the 6 σ level on the S parameter alone. This result assumes [. . .] that any new families are degenerate. [. . .] a fourth family is disfavored but not excluded by current data."

[Erler/Langacker]

(Borrowed from Alexander Lenz)

2. Electroweak Precision Observables



$$\begin{cases} S = \dfrac{\alpha}{4s^2}\left[c^2\left(\dfrac{\Pi_{ZZ}(m_Z^2)}{m_Z^2} - \dfrac{\Pi_{ZZ}(0)}{m_Z^2} - \dfrac{\Pi_{\gamma\gamma}(m_Z^2)}{m_Z^2}\right) - \dfrac{c}{s}(c^2 - s^2)\left(\dfrac{\Pi_{\gamma Z}(m_Z^2)}{m_Z^2} - \dfrac{\Pi_{\gamma Z}(0)}{m_Z^2}\right)\right] \\ T = \dfrac{1}{\alpha}\left[\dfrac{\Pi_{WW}(0)}{m_W^2} - \dfrac{\Pi_{ZZ}(0)}{m_Z^2} - 2\dfrac{s}{c}\dfrac{\Pi_{\gamma Z}(m_Z^2)}{m_Z^2}\right] \\ U = \dfrac{\alpha}{4s^2}\left[\dfrac{\Pi_{WW}(m_W^2)}{m_W^2} - \dfrac{\Pi_{WW}(0)}{m_W^2} - c^2\left(\dfrac{\Pi_{ZZ}(m_Z^2)}{m_Z^2} - \dfrac{\Pi_{ZZ}(0)}{m_Z^2}\right) - s^2\dfrac{\Pi_{\gamma\gamma}(m_Z^2)}{m_Z^2} - 2sc\left(\dfrac{\Pi_{\gamma Z}(m_Z^2)}{m_Z^2} - \dfrac{\Pi_{\gamma Z}(0)}{m_Z^2}\right)\right] \end{cases}$$

***S* parameter** : sensitive to the number of LH and RH fermions carrying a weak isospin

➔*constraints the number of new chiral matter content (extra fermion doublets).*

T parameter : sensitive to the difference between the loop corrections to the Z and W self-energies

➔*Constraints the mass splitting between the two partners within a given isospin doublet.*

U parameter : usually negligible for most NP models

(for a review, see Gfitter's arXiv:hep-ph/11070975)

$$\rho = \frac{m_W^2}{m_Z^2\ Cos\theta_W^2} + \Delta\rho$$

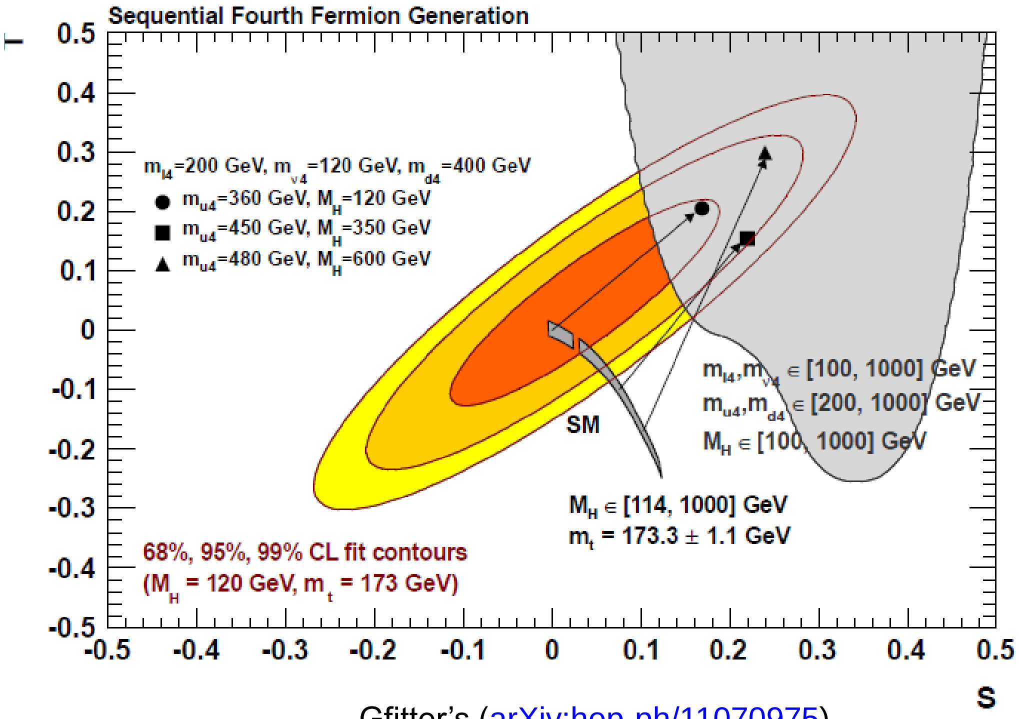
2. Electroweak Precision Observables

- Shift in S and T from introducing a sequential 4th generation :

$$\boxed{\begin{aligned}\Delta S_4 &\simeq \frac{N_C}{6\pi}\left(1-2Y\left[\ln\frac{m_{t'}^2}{m_{b'}^2}-\ln\frac{m_{\nu_{\tau'}}^2}{m_{\tau'}^2}\right]\right)\\ \Delta T_4 &\simeq \frac{N_C}{12\pi\, s_W^2\, c_W^2}\left[\left(\frac{m_{t'}^2-m_{b'}^2}{m_Z^2}\right)^2+\left(\frac{m_{\nu_{\tau'}}^2-m_{\tau'}^2}{m_Z^2}\right)^2\right]\end{aligned}}$$

- Including Higgs corrections :

$$\boxed{\begin{aligned}S_H &= \frac{1}{12\pi}\ln\frac{M_H^2}{(117\,\mathrm{GeV})^2},\\ T_H &= -\frac{3}{16\pi \bar{x}_W}\ln\frac{M_H^2}{(117\,\mathrm{GeV})^2}\end{aligned}}$$



Gfitter's (arXiv:hep-ph/11070975)

- Mass splitting corrections :

$$\boxed{m_{t'}-m_{b'} \simeq \left(1+\frac{1}{5}\ln\frac{m_H}{115\ GeV}\right)\times 50\ GeV}$$

Caution should be exercised !

- But does not include the 4 × 4 CKM mixings !

$$\boxed{V_{CKM4} \overset{?}{=} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{ub'} \\ V_{cd} & V_{cs} & V_{cb} & V_{cb'} \\ V_{td} & V_{ts} & V_{tb} & V_{tb'} \\ V_{t'd} & V_{t's} & V_{t'b} & V_{t'b'} \end{pmatrix}}$$

2. Electroweak Precision Observables

- Number of parameters : $\boxed{(N-1)^2 = \underbrace{\frac{N(N-1)}{2}}_{\text{Euler angles}} + \underbrace{\frac{(N-1)(N-2)}{2}}_{\text{Complex phases}}}$

generations	3	4	5
angles	3	6	10
phases	1	3	6

- Iterative definition for $n_g = 4$

$$\begin{aligned} V_{CKM4} &= U_4 \times U_3 \\ &= R_{34}(\theta_u) \times R_{24}(\theta_v) \times R_{14}(\theta_w) \times R_{23} \times R_{13} \times R_{12}. \end{aligned}$$

$$\left\{ \begin{array}{lcl} V_{ui} & = & \cos\theta_w V_{ui}^{(0)} \\ V_{ci} & = & \cos\theta_v V_{ci}^{(0)} - \sin\theta_v \sin\theta_w V_{ui}^{(0)} \\ V_{ti} & = & \cos\theta_u V_{ti}^{(0)} - \sin\theta_u \sin\theta_v V_{ci}^{(0)} - \sin\theta_u \cos\theta_v \sin\theta_w V_{ui}^{(0)} \\ V_{t'i} & = & \sin\theta_u V_{ti}^{(0)} + \cos\theta_u \sin\theta_v V_{ci}^{(0)} + \cos\theta_u \cos\theta_v \sin\theta_w V_{ui}^{(0)} \end{array} \right.$$

F.J. Botella and L.L. Chau,
Phys. Lett. B168, 97 (1986)

H. Fritzsch and J. Plankl,
Phys. Rev. D 35, 1732 (1987)

L. Lavoura,
Phys. Rev. D 40, 2440 (1989)

…

- Base line scenario :

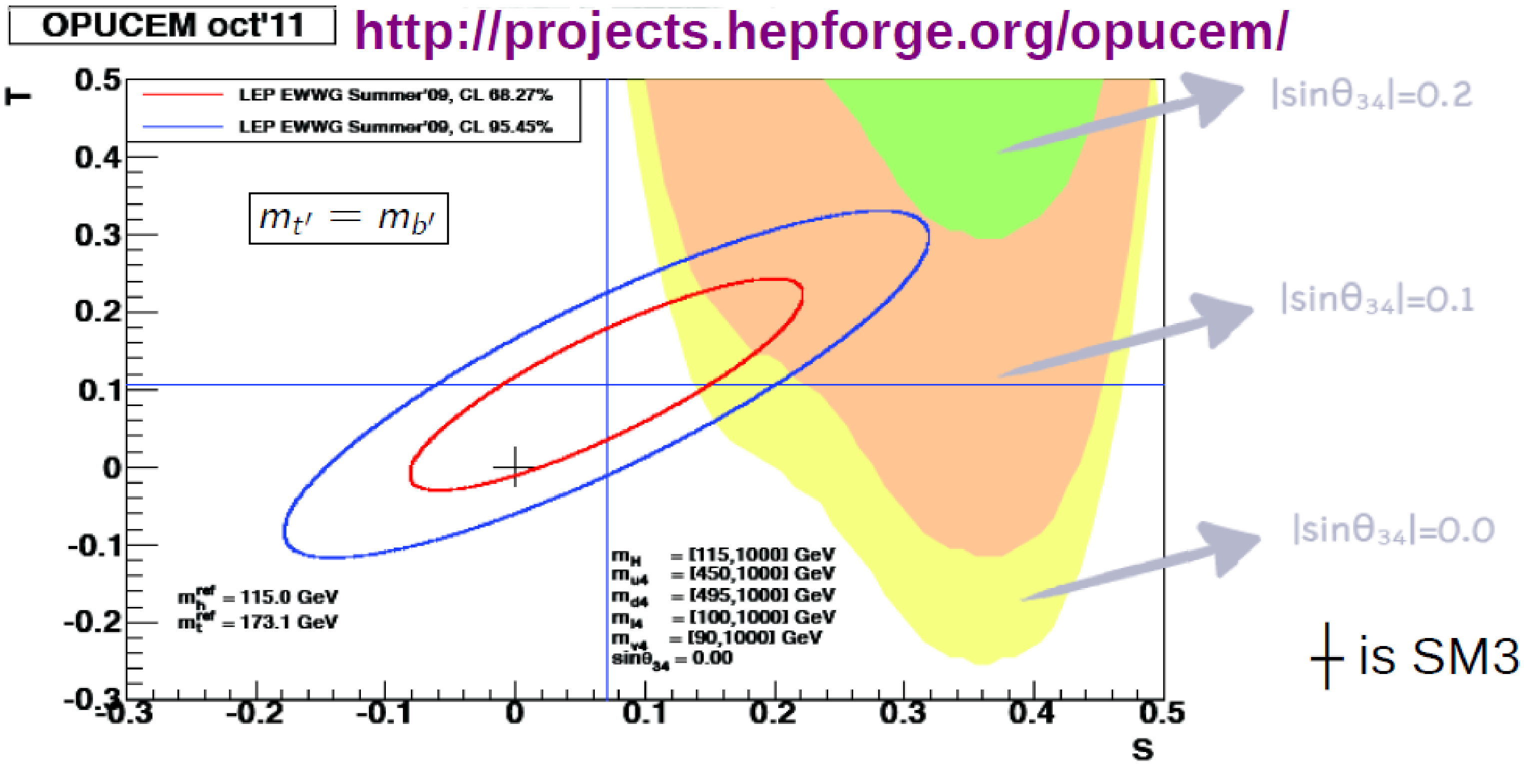
$$V_{CKM}^{4\times 4} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_{34} & s_{34} \\ 0 & 0 & -s_{34} & c_{34} \end{pmatrix} \left(\begin{array}{ccc|c} & & & 0 \\ & V_{CKM}^{3\times 3} & & 0 \\ & & & 0 \\ \hline 0 & 0 & 0 & 1 \end{array} \right) \sim \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & V_{tb} & V_{tb'} \\ 0 & 0 & V_{t'b} & V_{t'b'} \end{pmatrix}$$

$$\boxed{s_{34} = |V_{t'b}| \simeq |V_{tb'}|}$$

2. Electroweak Precision Observables

arXiv:hep-ph/11120507

http://projects.hepforge.org/opucem/

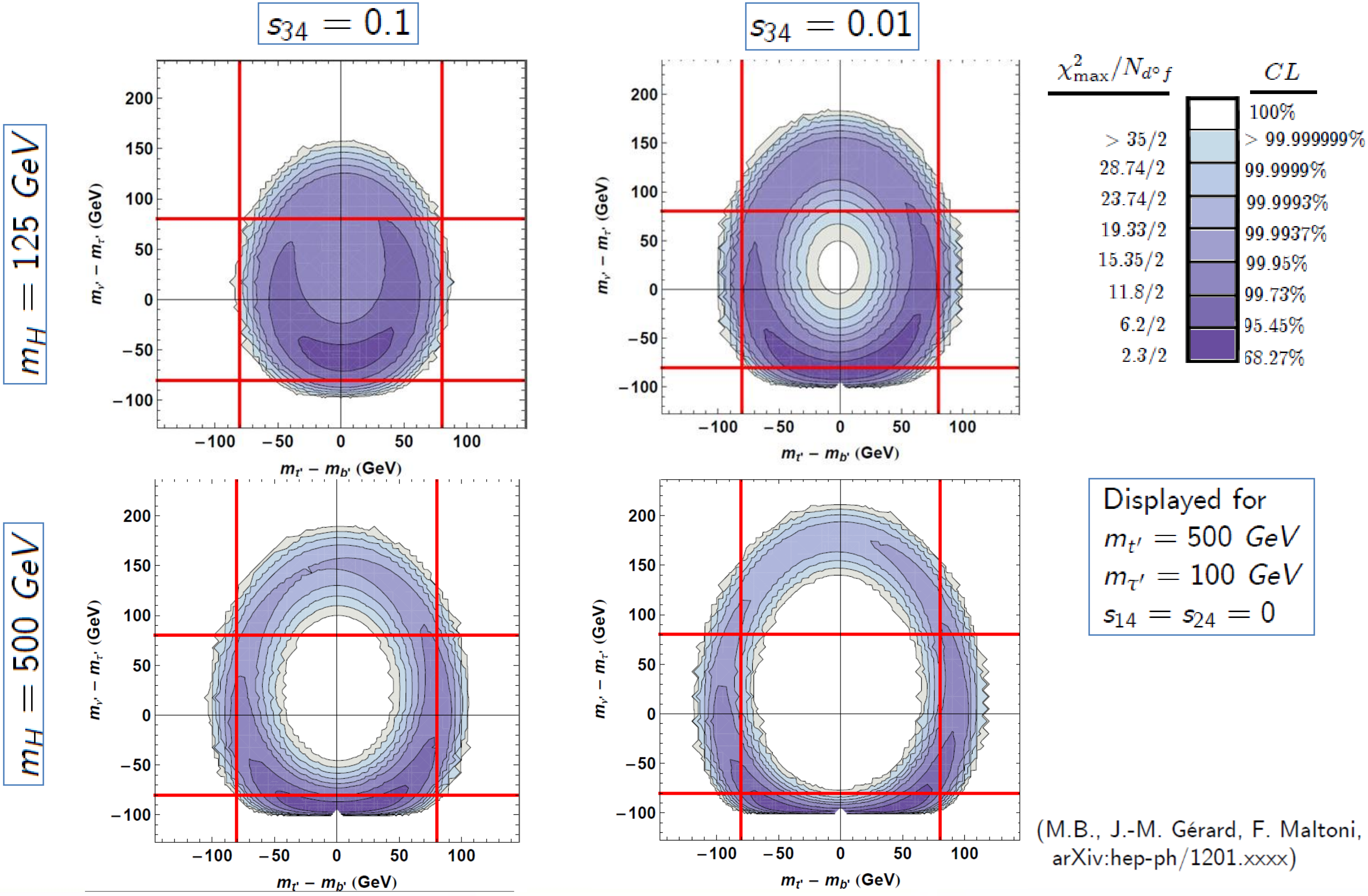


+ is SM3

- Flavour fits still allow for large mixing in the SM4 sector ...
  [Bobrowski, Lenz, Riedl, Rohrwild, arXiv:hep-ph/09024883]
- ... but scenarios with large s_{34} are ruled out by EPO fits !
  [M. Chanowitz, arXiv:hep-ph/09043570]
  [Eberhardt, Lenz, Rohrwild, arXiv:hep-ph/10053505]

$\Longrightarrow$ There is a non-trivial interplay between constraints from flavour and EPO !

2. Electroweak Precision Observables



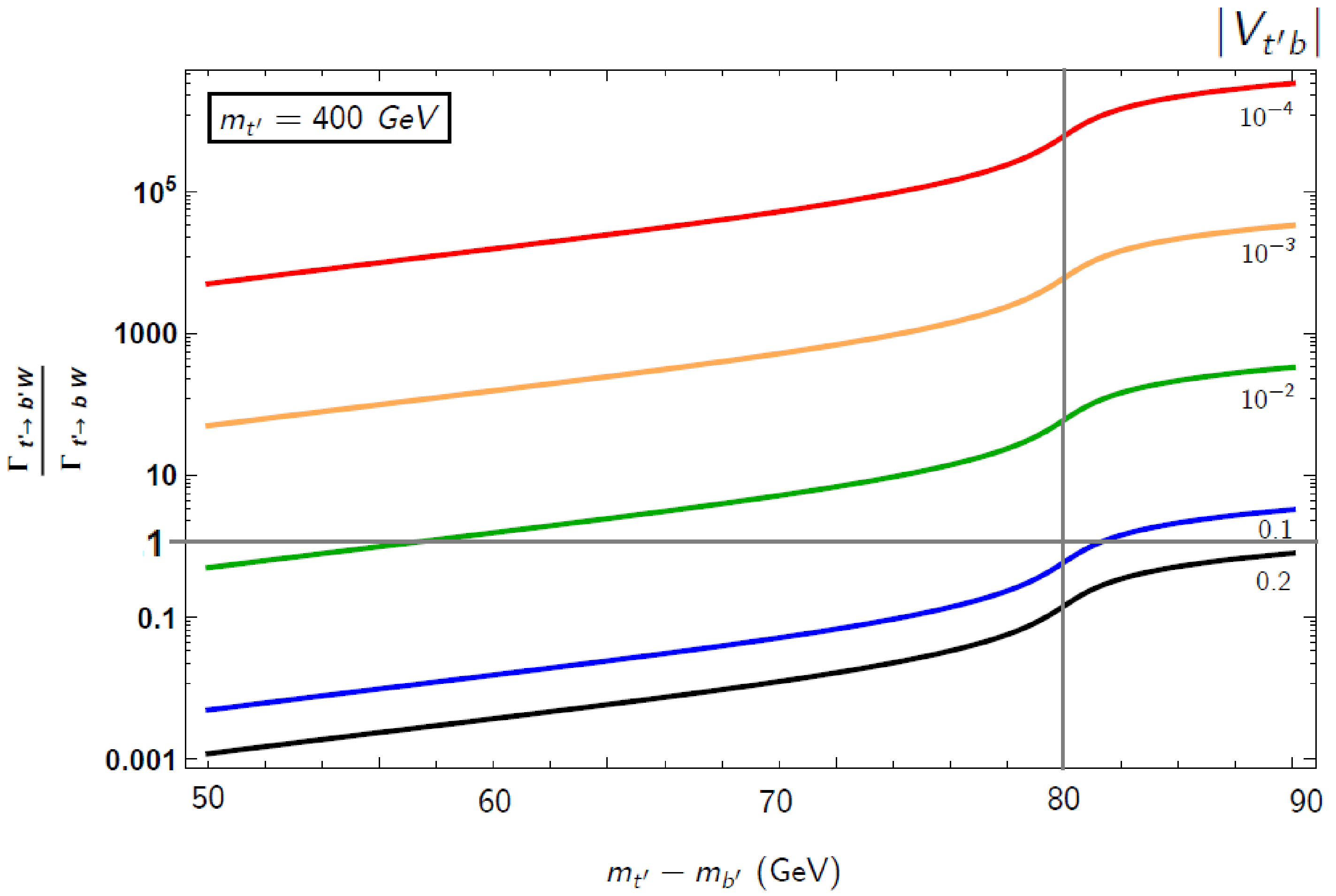
Displayed for
$m_{t'} = 500\ GeV$
$m_{\tau'} = 100\ GeV$
$s_{14} = s_{24} = 0$

(M.B., J.-M. Gérard, F. Maltoni, arXiv:hep-ph/1201.xxxx)

2. Electroweak Precision Observables

$m_{t'} > m_{b'}$ with $|m_{t'} - m_{b'}| \simeq m_W$

(M.B., J.-M. Gérard, F. Maltoni,
arXiv:hep-ph/1201.xxxx)



$t' \longrightarrow b'W^{(*)}$
Phase-space suppressed
CKM favored

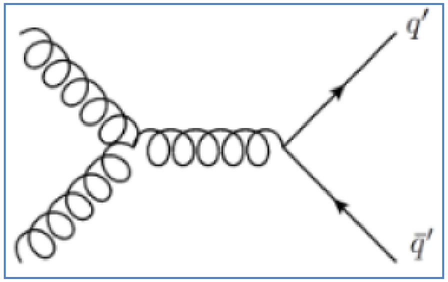
$t' \longrightarrow bW$
Phase-space favored
CKM suppressed

2. Electroweak Precision Observables

Assuming $|V_{t'b'}| \gg |V_{t'b}| \sim |V_{tb'}|$

- Pair production

Classical hierarchy	$m_{t'} - m_{b'} \simeq m_W$	$t'\bar{t}' \to bb\ 3W^+3W^-$
		$b'\bar{b}' \to b\bar{b}\ 2W^+2W^-$
Inverted hierarchy	$m_{b'} - m_{t'} \simeq m_W$	$t'\bar{t}' \to bb\ W^+W^-$
		$b'\bar{b}' \to b\bar{b}\ 2W^+2W^-$

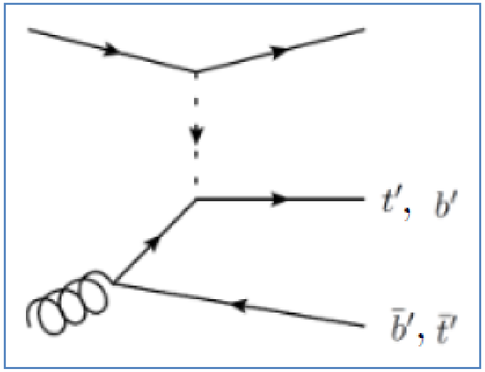


$$\sigma_{q'\bar{q}'}(m_{t'} = 500\ GeV, \Delta m_{q'} = 75\ GeV, \sqrt{s} = 7\ (8)\ TeV) \simeq 300\ (500)\ fb$$

$N = \mathcal{L} \cdot \sigma \simeq 9000\ (15000)$ events for $\mathcal{L} = 30\ fb^{-1}$

- Electroweak production

Classical hierarchy	$m_{t'} - m_{b'} \simeq m_W$	$t'b' \to bb\ 3W^+2W^-$
		$\bar{t}'b' \to b\bar{b}\ 3W^-2W^+$
Inverted hierarchy	$m_{b'} - m_{t'} \simeq m_W$	$t'b' \to bb\ W^+2W^-$
		$\bar{t}'b' \to b\bar{b}\ 2W^+W^-$



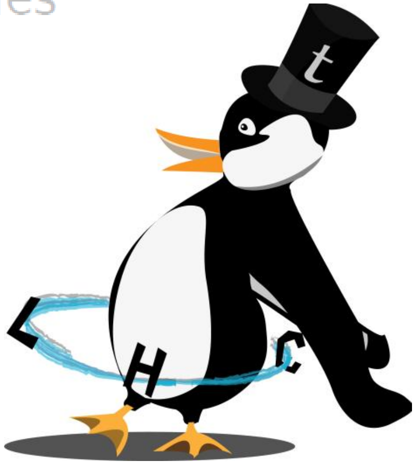
$$\sigma_{t'\bar{b}'+\bar{t}'b'}(m_{t'} = 500GeV, \Delta m_{q'} = 75GeV, \sqrt{s} = 7\ (8)\ TeV) \simeq 50\ (100)\ fb$$

$N = \mathcal{L} \cdot \sigma \simeq 1500\ (3000)$ events for $\mathcal{L} = 30\ fb^{-1}$

(M.B., J.-M. Gérard, F. Maltoni,
arXiv:hep-ph/1201.xxxx)

Outline

1. Experimental Constraints :
    1. Direct Searches at the LHC
    2. Interplay with Higgs Physics
2. Electroweak Precision Observables :
    1. Formalism
    2. Global Analysis
    3. New Channels for 4th Generation Searches
3. Conclusions
4. Epilogue : Consequences for V_{tb}



3. Conclusions : Evading the LHC limits …

… is possible :

- if 4^{th} generation is long-lived ($s_{34} \ll 1$) : t' (b') decays outside the detector.
- if the $4-2$ ($4-1$) mixing occurs to be large : $t' \to cW, uW$ ($b' \to sW, dW$) competes.
- if additional New Physics enters the game (2HDM, VL quarks, ...)

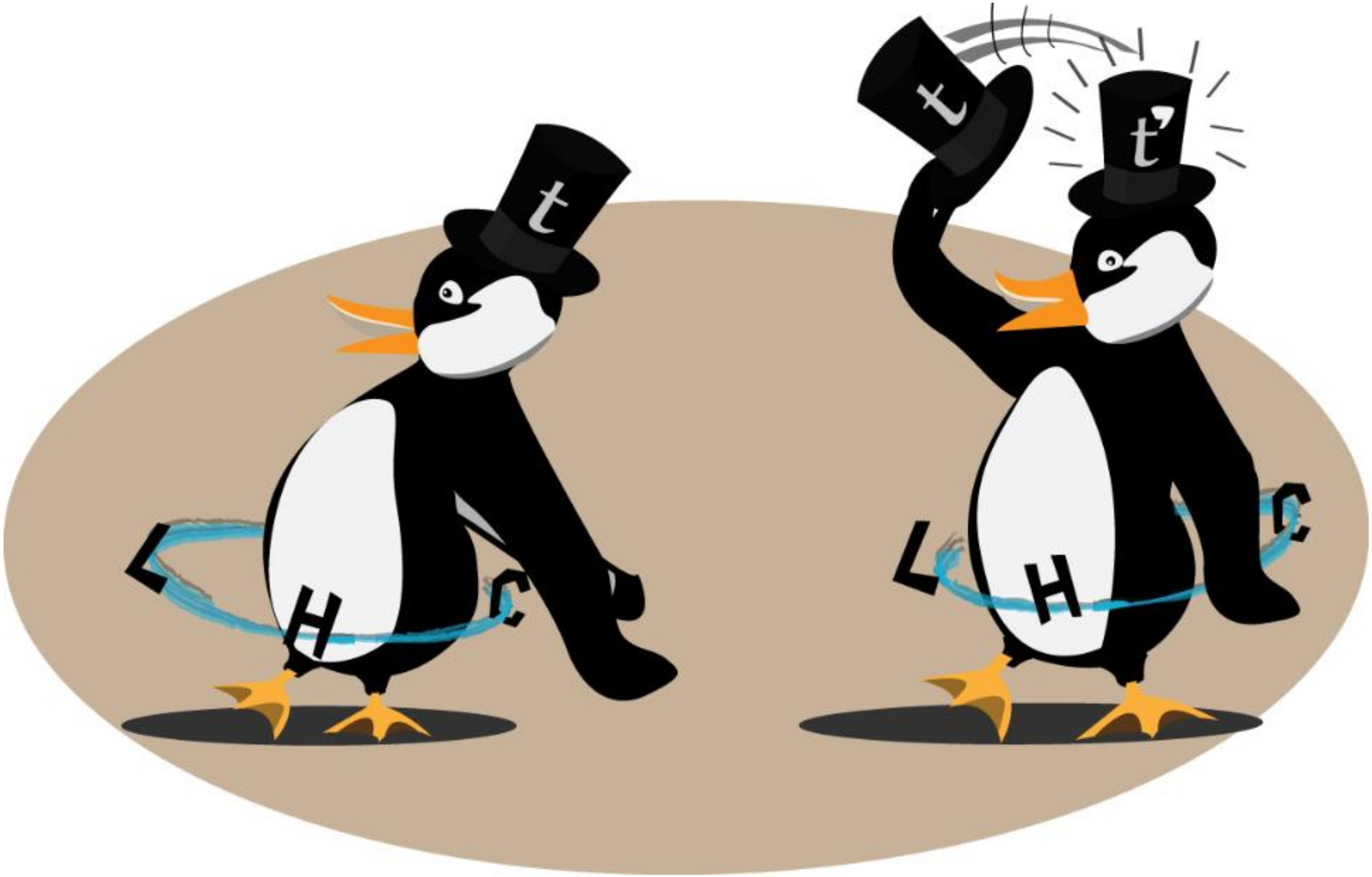
Most especially :

- if the quark mass splitting is large ($|m_{t'} - m_{b'}| \gtrsim m_W$ still allowed by EPO !)
- if the $4-3$ mixing angle is small ($s_{34} \lesssim 0.1$ still allowed by flavour !)

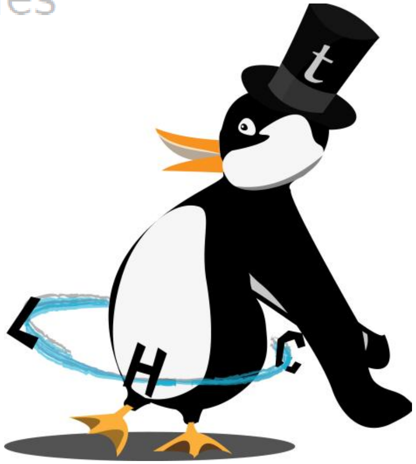
Then, the new channels $t' \to b'W^{(*)}$ / $b' \to t'W^{(*)}$ could be dominant and searched for !

And even if it is not there, 4^{th} generation already told us a lot about V_{tb} !

Thanks for your attention !



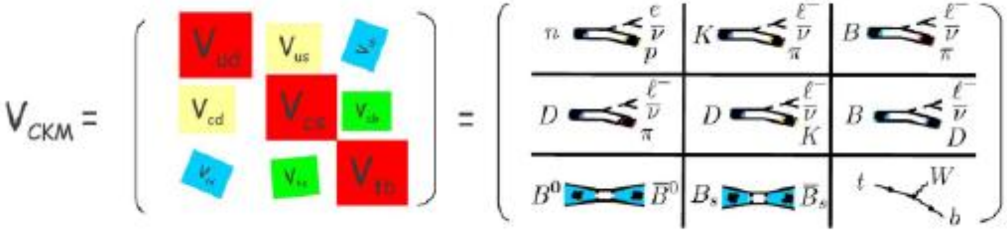
Outline



4. Consequences for Vtb

Unitarity constraints

(ICHEP '10)



$$V_{CKM4} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{ub'} \\ V_{cd} & V_{cs} & V_{cb} & V_{cb'} \\ V_{td} & V_{ts} & V_{tb} & V_{tb'} \\ V_{t'd} & V_{t's} & V_{t'b} & V_{t'b'} \end{pmatrix} \quad ?$$

	absolute value	relative error	direct measurement from
V_{ud}	0.97418 ± 0.00027	0.028%	nuclear beta decay
V_{us}	0.2255 ± 0.0019	0.84%	semi-leptonic K-decay
V_{ub}	0.00393 ± 0.00036	9.2%	semi-leptonic B-decay
V_{cd}	0.230 ± 0.011	4.8%	semi-leptonic D-decay
V_{cs}	1.04 ± 0.06	5.8%	(semi-)leptonic D-decay
V_{cb}	0.0412 ± 0.0011	2.7%	semi-leptonic B-decay
V_{tb}	> 0.74		(single) top-production

K.Nakamura et al.(2010), "Review of Particle Physics: The CKM Quark–Mixing Matrix".

- 1^{st} row unitarity :
  $1 - (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2) = 0.0001 \pm 0.0011$
- 2^{nd} row unitarity :
  $1 - (|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2) = -0.1362 \pm 0.1249$
- 1^{st} column unitarity :
  $1 - |V_{ud}|^2 - |V_{cd}|^2 = -0.0019 \pm 0.0051$
- 2^{nd} column unitarity :
  $1 - |V_{us}|^2 - |V_{cs}|^2 = -0.1324 \pm 0.1248$

$$\boxed{|V_{ub'}| \lesssim 0.04 \quad ; \quad |V_{cb'}| \lesssim 0.11 \quad ; \quad |V_{tb'}| \lesssim m_W \,/\, m_{t'}}$$

- $|V_{td}|$ and $|V_{ts}|$ are not accessible
- $|V_{ub'}|/|V_{cb'}|$ can still be larger than $|V_{ub}|/|V_{cb}|$
- $|V_{t'b}| \simeq |V_{tb'}| \lesssim \lambda$, but is favored to be small if $m_{t',b'}$ is large

4. Consequences for Vtb

Extracting $|V_{tb}|$ from t-channel single top production

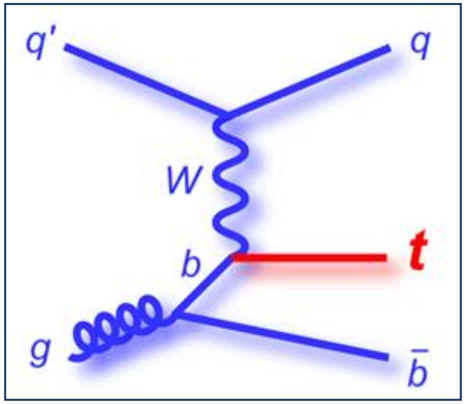
$$\boxed{R \equiv \frac{\Gamma(t \to Wb)}{\Gamma(t \to Wq)} = \frac{|V_{tb}|^2}{|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2}}$$

- If three generations are assumed, $V^{\dagger}_{CKM} V_{CKM} = 1$

$$|V_{td}|, |V_{ts}| \ll |V_{tb}| \implies R \simeq |V_{tb}|^2 \simeq 1$$

- Experimental measurement (FERMILAB-PUB-09-059-E) :

$$\begin{aligned} R_{CDF} &= 1.12^{+0.27}_{-0.23} \;;\; R_{D\varnothing} = 1.03^{+0.19}_{-0.17} \\ &\implies R > 0.61 \text{ à } 95\% \text{ CL} \\ &\implies |V_{tb}| = \sqrt{R} > 0.78 \text{ à } 95\% \text{ CL} \end{aligned}$$



4. Consequences for Vtb

Extracting $|V_{tb}|$ from t-channel single top production

$$\boxed{R \equiv \frac{\Gamma(t \to Wb)}{\Gamma(t \to Wq)} = \frac{|V_{tb}|^2}{|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2}}$$

- If three generations are assumed, $V^{\dagger}_{CKM} V_{CKM} = 1$

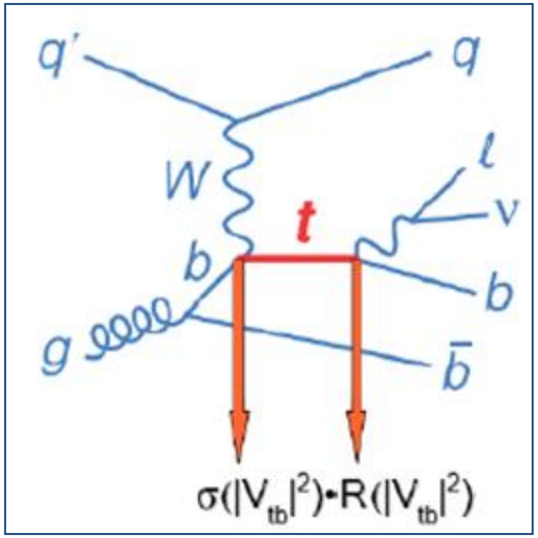
$$|V_{td}|, |V_{ts}| \ll |V_{tb}| \implies R \simeq |V_{tb}|^2 \simeq 1$$

- Experimental measurement (FERMILAB-PUB-09-059-E) :

$$\begin{aligned} R_{CDF} &= 1.12^{+0.27}_{-0.23} \;\; ; \;\; R_{D\emptyset} = 1.03^{+0.19}_{-0.17} \\ &\implies R > 0.61 \text{ à } 95\% \text{ CL} \\ &\implies |V_{tb}| = \sqrt{R} > 0.78 \text{ à } 95\% \text{ CL} \end{aligned}$$

- Without appealing to unitarity,
  $(|V_{td}| = 0.0086, |V_{ts}| = 0.0403, R > 0.61)$:

$$\sqrt{R} \neq |V_{tb}| = \sqrt{\frac{R(|V_{td}|^2 + |V_{ts}|^2)}{1 - R}} > 0.052 \text{ !}$$

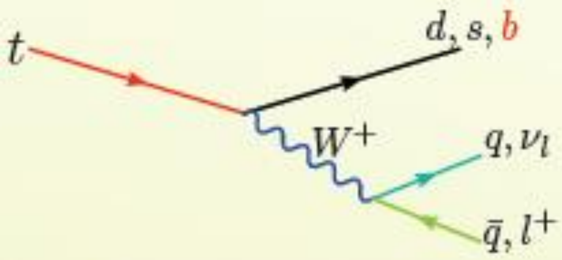


J. Alwall, R. Frederix, J.–M. Gerard, A.Giammanco,
M. Herquet, S. Kalinin, E. Kou, V. Lemaitre, F. Maltoni
"Is $V_{tb} \simeq 1$?", Eur.Phys.J. C49 (2007) 791-801.

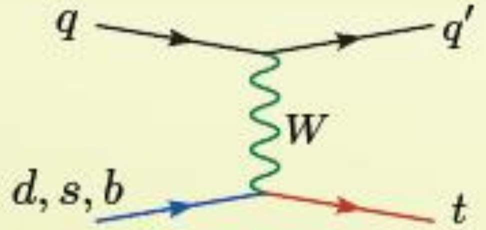
4. Consequences for Vtb

- From single top observables to CKM matrix elements :

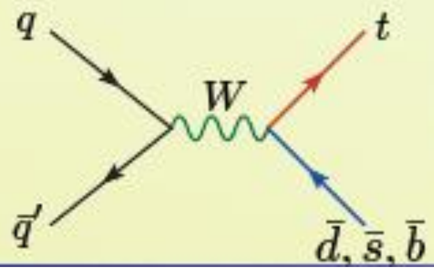
$$\{ \ R, \sigma(s), \sigma(t) \ \} \to \{ \ |V_{td}|, |V_{ts}|, |V_{tb}| \ \}$$



$$\sim R = \frac{\Gamma(t \to Wb)}{\Gamma(t \to Wq(= d, s, b))} = \frac{|V_{tb}|^2}{|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2}$$



$$\sim |V_{td}|^2 \sigma_d^{\text{t-ch}} + |V_{ts}|^2 \sigma_s^{\text{t-ch}} + |V_{tb}|^2 \sigma_b^{\text{t-ch}}$$



$$\sim (|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2) \sigma^{\text{s-ch}}$$

4. Consequences for Vtb

- From single top observables to CKM matrix elements :

$$\{\ R, \sigma(s), \sigma(t)\ \} \rightarrow \{\ |V_{td}|, |V_{ts}|, |V_{tb}|\ \}$$

(June '11 update)

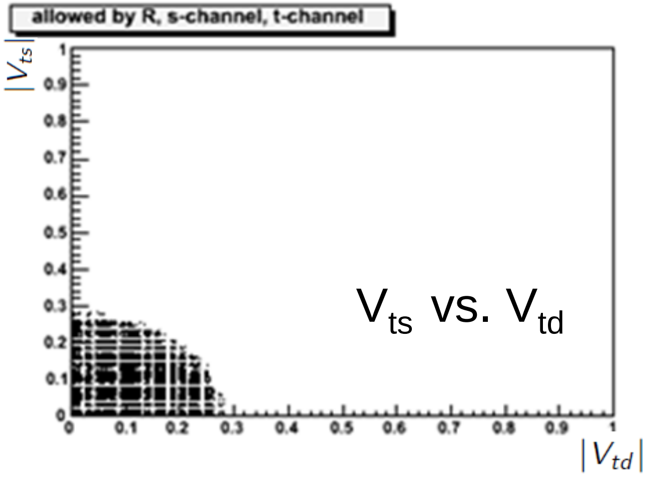
$\boxed{|V_{tb}| > 0.7 \text{ at } 95\% \text{ C.L.}}$

$R > 0.92$ with 5.4 fb^{-1} (FERMILAB PUB-11-300-E)

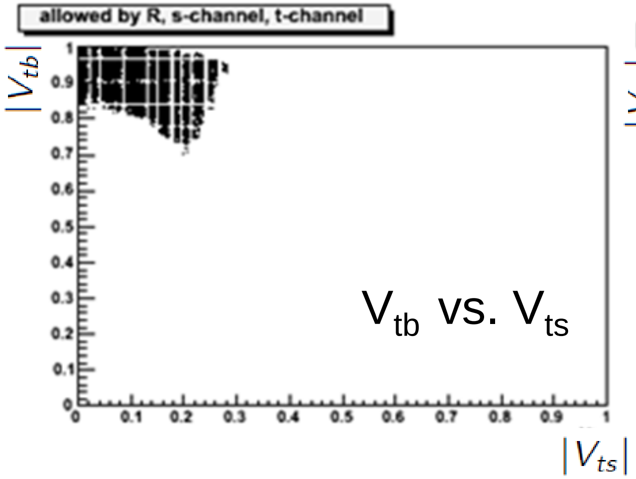
$\sigma(tb + tqb) = 3.84^{+0.89}_{-0.83}\ pb$ with 4.8 fb^{-1} (PLB 690, 5 (2010))

$\sigma(p\bar{p} \rightarrow tb) = 0.98 \pm 0.63\ pb$ with 5.4 fb^{-1} (FERMILAB PUB-11-216-E)

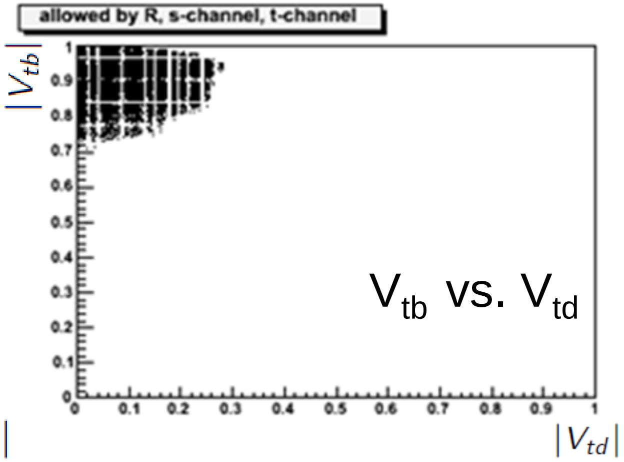
$\sigma(p\bar{p} \rightarrow tb) = 2.9 \pm 0.59\ pb$ with 5.4 fb^{-1} (FERMILAB PUB-11-216-E)



V_{ts} vs. V_{td}



V_{tb} vs. V_{ts}



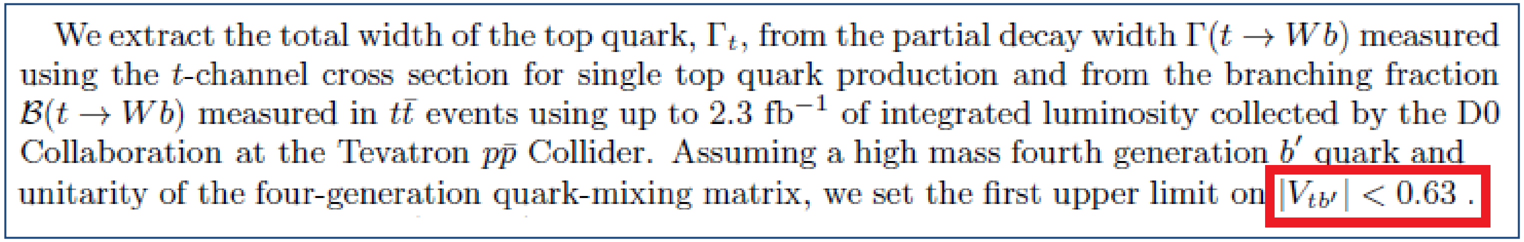
V_{tb} vs. V_{td}

4. Consequences for Vtb

FERMILAB-PUB-10/394-E

Determination of the width of the top quark

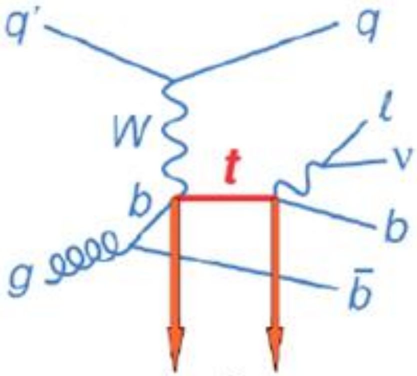
(The D0 Collaboration*)



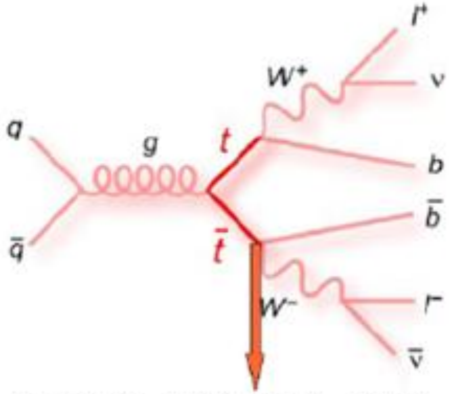
We extract the total width of the top quark, Γ_t, from the partial decay width $\Gamma(t \to Wb)$ measured using the t-channel cross section for single top quark production and from the branching fraction $\mathcal{B}(t \to Wb)$ measured in $t\bar{t}$ events using up to 2.3 fb^{-1} of integrated luminosity collected by the D0 Collaboration at the Tevatron $p\bar{p}$ Collider. Assuming a high mass fourth generation b' quark and unitarity of the four-generation quark-mixing matrix, we set the first upper limit on $\boxed{|V_{tb'}| < 0.63}$.

$\boxed{|V_{tb}| > 0.77 \text{ at } 95\% \text{ C.L.}}$

Improvement !



$\sigma(|V_{tb}|^2)\cdot R$



R = BR(t→Wb) / BR(t→Wq)

- Single top production

$$\begin{aligned}\sigma_t^{\text{exp}} &= \sigma_t^{th}|V_{tb}|^2 BR[t \to Wb] \\ &= \sigma_t^{th}|V_{tb}|^2 R.BR[t \to Wq]\end{aligned}$$

- Pair production

$$\begin{aligned}\sigma_{t\bar{t}}^{\text{exp}} &= \sigma_{t\bar{t}}^{th}.BR[t \to Wb].BR[t \to Wb] \\ &= \sigma_{t\bar{t}}^{th}.BR^2[t \to Wb] \\ &= \sigma_{t\bar{t}}^{th}.R^2.BR^2[t \to Wq]\end{aligned}$$

$$\boxed{|V_{tb}|^2 = \frac{(\sigma_t^{\text{exp}})^2.(\sigma_{t\bar{t}}^{th})}{(\sigma_{t\bar{t}}^{\text{exp}}).(\sigma_t^{th})^2}}$$

Backup

Long-Lived quarks (I)

- Full decay rate for a heavy top to lighter quarks ($m_q \neq 0$) :

$$\Gamma(t' \rightarrow Wq) = \frac{G_F\, m_{t'}^3}{8\pi\sqrt{2}} |V_{t'q}|^2\, \kappa\, \left[\left(1 - \frac{m_q^2}{m_{t'}^2}\right)^2 + \frac{m_W^2}{m_{t'}^2}\left(1 + \frac{m_q^2}{m_{t'}^2}\right) - 2\frac{m_W^4}{m_{t'}^4}\right]$$

$$\kappa = \frac{\sqrt{[m_{t'}^2 - (m_W + m_q)^2][m_{t'}^2 - (m_W - m_q)^2]}}{m_{t'}^2}$$

- Asymptotic expression for $m_{t'} \gg m_q$:

$$\boxed{\Gamma(t' \rightarrow Wq) \simeq 180\ MeV\ \ |V_{t'q}|^2 \frac{m_{t'}^3}{m_W^3}}$$

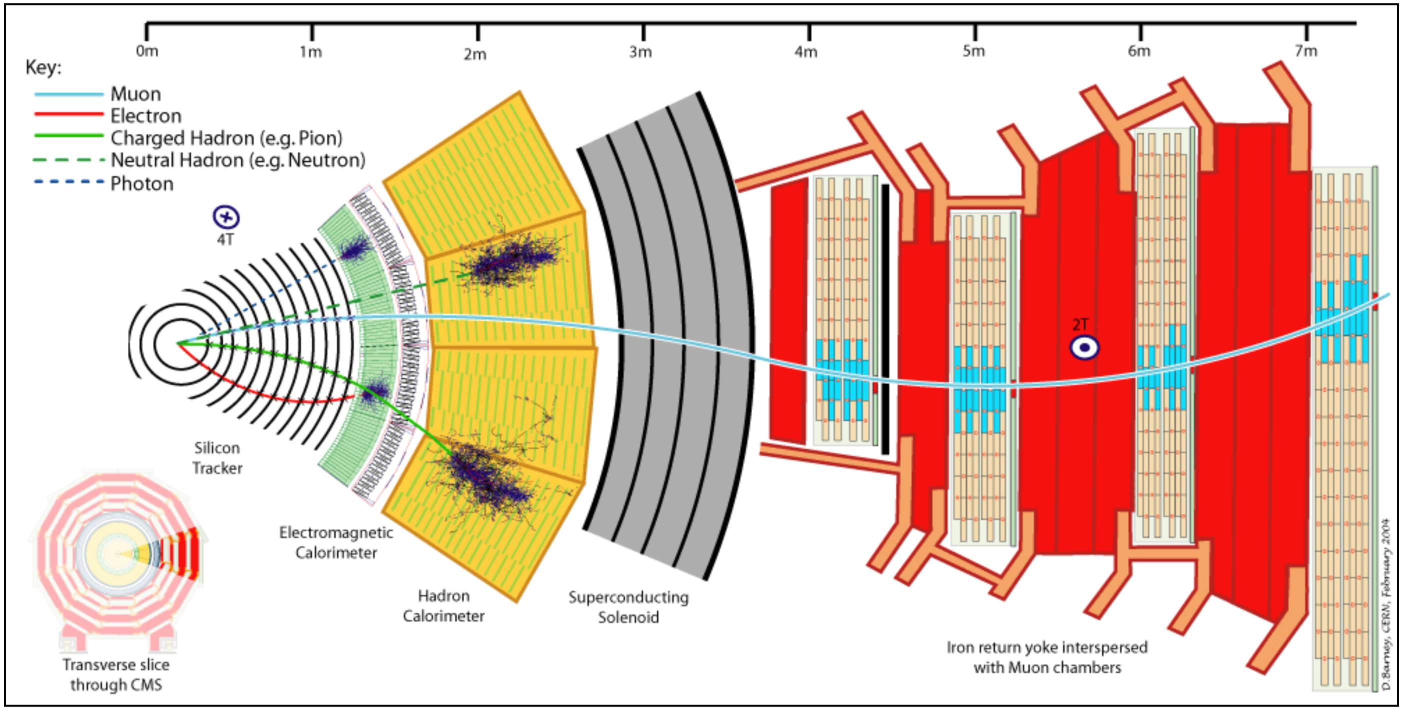
- Condition for forming $\langle t'\bar{t}'\rangle$ ($\langle t'\bar{q}\rangle$) bound states ($t \geq \tau_{QCD} \simeq 10^{-23}\ s$) :

$$\boxed{m_{t'} \lesssim 125\ GeV\ (100\ GeV)\ |V_{t'q}|^{-\frac{2}{3}}}$$

$$\boxed{\tau = \frac{6.67.10^{-25}\ GeV}{\Gamma(GeV)}\ \ s}$$

Long-Lived quarks (II)

CMS



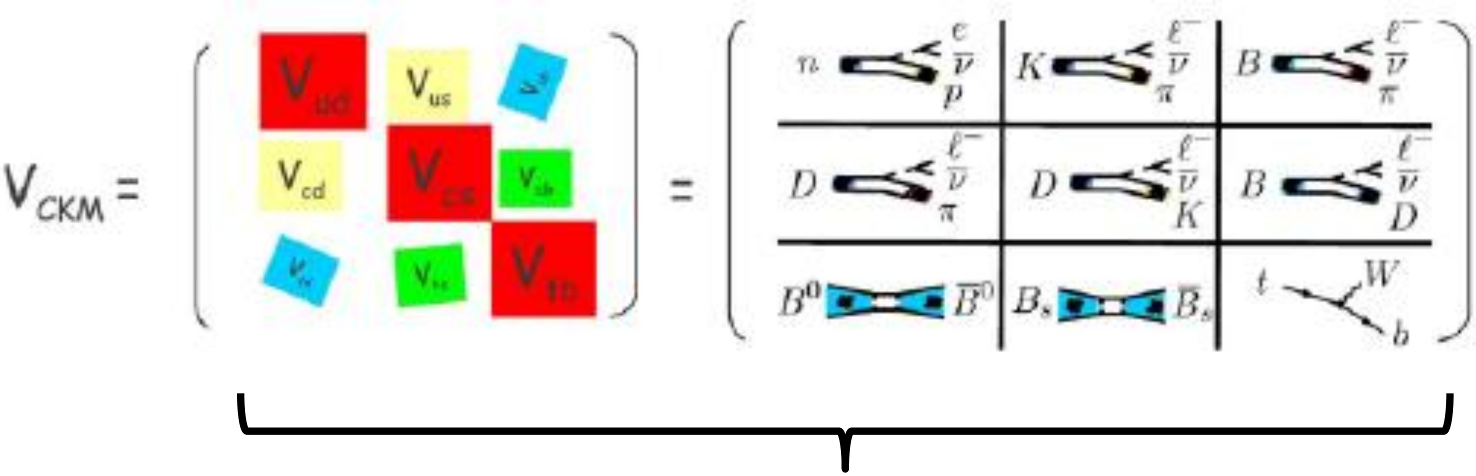
- Metastability bounds : $1\ s > \tau_{t'} > 1\ ns$
- Translating this as bounds on the mixing angle : $10^{-13} < |s_{34}| < 10^{-9}$

  (Larger values unacessible for lifetimes below the nanosecond level for $m_{t'} \in [300, 800]$)
- No scheduled seaches for fractionally-charged particles at CMS

CKM Matrix in SM4 (I)

- Number of parameters : $\boxed{(N-1)^2 = \underset{\text{Euler angles}}{\frac{N(N-1)}{2}} + \underset{\text{Complex phases}}{\frac{(N-1)(N-2)}{2}}}$

generations	3	4	5
angles	3	6	10
phases	1	3	6



$$V_{\text{CKM}} = \begin{pmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.0011}_{-0.0007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.000045} \end{pmatrix}$$

K.Nakamura et al.(2010), "*Review of Particle Physics: The CKM Quark–Mixing Matrix*".

CKM Matrix in SM4 (III)

Four generations : most common choice

$$V_{CKM4} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{ub'} \\ V_{cd} & V_{cs} & V_{cb} & V_{cb'} \\ V_{td} & V_{ts} & V_{tb} & V_{tb'} \\ V_{t'd} & V_{t's} & V_{t'b} & V_{t'b'} \end{pmatrix}$$

1) It is defined iteratively :

$$V = R_{n-1,n}\tilde{R}_{n-2,n}\cdots\tilde{R}_{2,n}\tilde{R}_{1,n}\cdots R_{k-1,k}\tilde{R}_{k-2,k}\cdots\tilde{R}_{2,k}\tilde{R}_{1,k}\cdots R_{23}\tilde{R}_{13}R_{12}\,.$$

$n=3$: $(V_{ij}) = R_{23}\tilde{R}_{13}R_{12}$

$n=4$: $(V_{ij}) = R_{34}\tilde{R}_{24}\tilde{R}_{14}R_{23}\tilde{R}_{13}R_{12}$

$n=5$: $V = R_{45}\tilde{R}_{35}\tilde{R}_{25}\tilde{R}_{15}R_{34}\tilde{R}_{24}\tilde{R}_{14}R_{23}\tilde{R}_{13}R_{12}$

2) The Standard Model is easily « factored out »

$$\begin{aligned} V_{CKM4} &= U_4 \times U_3 \\ &= R_{34}(\theta_u) \times R_{24}(\theta_v) \times R_{14}(\theta_w) \times R_{23} \times R_{13} \times R_{12}. \end{aligned}$$

3) The 1st row and the 4th column are very simply written

$$\begin{aligned} V_{ui} &= \cos\theta_w V_{ui}^{(0)} \\ V_{ci} &= \cos\theta_v V_{ci}^{(0)} - \sin\theta_v \sin\theta_w V_{ui}^{(0)} \\ V_{ti} &= \cos\theta_u V_{ti}^{(0)} - \sin\theta_u \sin\theta_v V_{ci}^{(0)} - \sin\theta_u \cos\theta_v \sin\theta_w V_{ui}^{(0)} \\ V_{t'i} &= \sin\theta_u V_{ti}^{(0)} + \cos\theta_u \sin\theta_v V_{ci}^{(0)} + \cos\theta_u \cos\theta_v \sin\theta_w V_{ui}^{(0)} \end{aligned}$$

CKM Matrix in SM4 (IV)

Four generations : most common choice

$$V_{CKM4} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{ub'} \\ V_{cd} & V_{cs} & V_{cb} & V_{cb'} \\ V_{td} & V_{ts} & V_{tb} & V_{tb'} \\ V_{t'd} & V_{t's} & V_{t'b} & V_{t'b'} \end{pmatrix}$$

$$\begin{pmatrix}
c_{12}c_{13}c_{14} & c_{13}c_{14}s_{12} & c_{14}s_{13}e^{-i\delta_{13}} & s_{14}e^{-i\delta_{14}} \\[2ex]
\begin{array}{c} -c_{23}c_{24}s_{12} - c_{12}c_{24}s_{13}s_{23}e^{i\delta_{13}} \\ -c_{12}c_{13}s_{14}s_{24}e^{i(\delta_{14}-\delta_{24})} \end{array} &
\begin{array}{c} c_{12}c_{23}c_{24} - c_{24}s_{12}s_{13}s_{23}e^{i\delta_{13}} \\ -c_{13}s_{12}s_{14}s_{24}e^{i(\delta_{14}-\delta_{24})} \end{array} &
\begin{array}{c} c_{13}c_{24}s_{23} \\ -s_{13}s_{14}s_{24}e^{-i(\delta_{13}+\delta_{24}-\delta_{14})} \end{array} &
c_{14}s_{24}e^{-i\delta_{24}} \\[2ex]
\begin{array}{c} -c_{12}c_{23}c_{34}s_{13}e^{i\delta_{13}} + c_{34}s_{12}s_{23} \\ -c_{12}c_{13}c_{24}s_{14}s_{34}e^{i\delta_{14}} \\ +c_{23}s_{12}s_{24}s_{34}e^{i\delta_{24}} \\ +c_{12}s_{13}s_{23}s_{24}s_{34}e^{i(\delta_{13}+\delta_{24})} \end{array} &
\begin{array}{c} -c_{12}c_{34}s_{23} - c_{23}c_{34}s_{12}s_{13}e^{i\delta_{13}} \\ -c_{12}c_{23}s_{24}s_{34}e^{i\delta_{24}} \\ -c_{13}c_{24}s_{12}s_{14}s_{34}e^{i\delta_{14}} \\ +s_{12}s_{13}s_{23}s_{24}s_{34}e^{i(\delta_{13}+\delta_{24})} \end{array} &
\begin{array}{c} c_{13}c_{23}c_{34} \\ -c_{13}s_{23}s_{24}s_{34}e^{i\delta_{24}} \\ -c_{24}s_{13}s_{14}s_{34}e^{i(\delta_{14}-\delta_{13})} \end{array} &
c_{14}c_{24}s_{34} \\[2ex]
\begin{array}{c} -c_{12}c_{13}c_{24}c_{34}s_{14}e^{i\delta_{14}} \\ +c_{12}c_{23}s_{13}s_{34}e^{i\delta_{13}} \\ +c_{23}c_{34}s_{12}s_{24}e^{i\delta_{24}} - s_{12}s_{23}s_{34} \\ +c_{12}c_{34}s_{13}s_{23}s_{24}e^{i(\delta_{13}+\delta_{24})} \end{array} &
\begin{array}{c} -c_{12}c_{23}c_{34}s_{24}e^{i\delta_{24}} + c_{12}s_{23}s_{34} \\ -c_{13}c_{24}c_{34}s_{12}s_{14}e^{i\delta_{14}} \\ +c_{23}s_{12}s_{13}s_{34}e^{i\delta_{13}} \\ +c_{34}s_{12}s_{13}s_{23}s_{24}e^{i(\delta_{13}+\delta_{24})} \end{array} &
\begin{array}{c} -c_{13}c_{23}s_{34} \\ -c_{13}c_{34}s_{23}s_{24}e^{i\delta_{24}} \\ -c_{24}c_{34}s_{13}s_{14}e^{i(\delta_{14}-\delta_{13})} \end{array} &
c_{14}c_{24}c_{34}
\end{pmatrix}$$

F.J. Botella and L.L. Chau, Phys. Lett. B168, 97 (1986)

H. Fritzsch, J. Plankl, Phys. Rev. D 35, 1732 (1987)

L. Lavoura, Phys. Rev. D 40, 2440 (1989)

…

CKM Matrix in SM4 (V)

Remarks

$$V_{CKM4} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{ub'} \\ V_{cd} & V_{cs} & V_{cb} & V_{cb'} \\ V_{td} & V_{ts} & V_{tb} & V_{tb'} \\ V_{t'd} & V_{t's} & V_{t'b} & V_{t'b'} \end{pmatrix}$$

1) 4th column is defined via the 4x4 unitarity :

$$|V_{ub'}|^2 = 1 - |V_{ud}|^2 - |V_{us}|^2 - |V_{ub}|^2$$
$$|V_{cb'}|^2 = 1 - |V_{cd}|^2 - |V_{cs}|^2 - |V_{cb}|^2$$
$$|V_{tb'}|^2 = 1 - |V_{td}|^2 - |V_{ts}|^2 - |V_{tb}|^2$$

$$\left|V^{4\times4}_{CKM}\right| = \begin{pmatrix} 0.97418 & 0.2255 & 0.0039 & <0.035 \\ 0.22 & >0.83 & 0.041 & <0.52 \\ <0.052 & <0.28 & >0.78 & <0.62 \\ <0.11 & <0.52 & <0.63 & >0.73 \end{pmatrix}$$

Limits @ ~2σ

(Directly measured)

2) H. Lacker's & A. Menzel's works :

$$|V^{4x4}_{CKM}| = \begin{pmatrix} 0.97414^{+0.00032}_{-0.00023} & 0.2245^{+0.0012}_{-0.0012} & (4.200^{+0.090}_{-0.910}) \cdot 10^{-3} & 0.025^{+0.011}_{-0.025} \\ 0.2256^{+0.0011}_{-0.0059} & 0.9717^{+0.0024}_{-0.0105} & (41.09^{+0.45}_{-1.45}) \cdot 10^{-3} & 0.057^{+0.097}_{-0.057} \\ 0.001^{+0.035}_{-0.001} & 0.062^{+0.044}_{-0.062} & 0.910^{+0.079}_{-0.080} & 0.41^{+0.15}_{-0.27} \\ 0.013^{+0.039}_{-0.013} & 0.04^{+0.12}_{-0.04} & 0.41^{+0.14}_{-0.27} & 0.910^{+0.078}_{-0.083} \end{pmatrix}$$

(CKM Fitter)

3) Current constraints might suffer from the PMNS sector, that can modify the already determined V_{CKM} elements

$$\left|V^{4\times4}_{CKM}\right| = \begin{pmatrix} \begin{array}{c} <0.977 \\ >0.80 \end{array} & \begin{array}{c} <0.226 \\ >0.183 \end{array} & \begin{array}{c} <0.0046 \\ >0.0022 \end{array} & <0.57 \\ \begin{array}{c} <0.249 \\ >0.174 \end{array} & \begin{array}{c} <0.977 \\ >0.796 \end{array} & \begin{array}{c} <0.0419 \\ >0.0325 \end{array} & <0.57 \\ <0.47 & <0.48 & >0.54 & <0.65 \\ <0.47 & <0.49 & <0.82 & >0.31 \end{pmatrix}$$

Heiko Lacker & Andreas Menzel
Second Workshop on
Beyond 3 Generation Standard Model

Combined CKM, PMNS & e.w. precision fit needed (M_W & $\Gamma(Z \to ll)$ depend on G_F @LO)

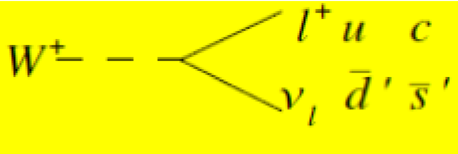
CKM Matrix in SM4 (VI)

H. Lacker, A. Menzel, "Simultaneous Extraction of the Fermi constant and PMNS matrix elements in the presence of a fourth generation", arXiv: hep-ph/1003.4532v2

- **The lepton mass splitting interplays with the quarks within S and T !**

- **Going from 3 to 4 generations influence the extraction methods of the known V_{CKM3} elements.**

  - |V$_{tb}$ | : cfr. single top at Tevatron ; one extracts $|V_{tb}|\frac{|V_{tb}|}{\sqrt{|V_{td}|^2+|V_{ts}|^2+|V_{tb}|^2}}$ rather than $|V_{tb}|$!
  - |V$_{cs}$ | : extraction from W leptonic branching fractions relies on assuming lepton universality and 3 generations of leptons :



$$\frac{\Gamma(W\rightarrow l\,\nu)}{\Gamma(W\rightarrow All)}\approx\frac{1}{3+3\sum_{i=u,c}\sum_{j=d,s,b}|V_{ij}|^2(1+\alpha_s(M_W)/\pi)}$$

  - |V$_{ud}$ | : extraction from the measurement of the Fermi constant, which influences its determination from superallowed beta decays in 0+➔0+ transitions or muon decays ; one extracts $|V_{ud}|\sqrt{(1-|U_{e4}|^2)}G_F^{4SM}/G_F^{3SM}$ rather than $|V_{ud}|$!

Combined CKM, PMNS & e.w. precision fit needed

SM4 expressions for the Oblique parameters

$$S = \frac{N_c}{6\pi}\sum_{f=1}^{4}\left[1-\frac{1}{3}\ln\frac{m_{u_f}^2}{m_{d_f}^2}\right]+\frac{1}{6\pi}\sum_{f=1}^{4}\left[1+\ln\frac{m_{\nu_f}^2}{m_{l_f}^2}\right],$$

$$T = \frac{N_c}{16\pi x_W \bar{x}_W M_Z^2}\left[\sum_{q=u,d,s,\dots,t',b'} m_q^2-\sum_{f=1}^{4}\sum_{f'=1}^{4}|V_{u_f d_{f'}}|^2 F_T(m_{u_f}^2, m_{d_{f'}}^2)\right]$$

$$+\frac{1}{16\pi x_W \bar{x}_W M_Z^2}\left[\sum_{l=\nu_e,e^-,\dots,\nu_4,l_4^-} m_l^2-\sum_{f=1}^{4}\sum_{f'=1}^{4}|V_{\nu_f l_{f'}}|^2 F_T(m_{\nu_f}^2, m_{l_{f'}}^2)\right],$$

$$U = \frac{N_c}{3\pi}\left[\sum_{f=1}^{4}\sum_{f'=1}^{4}|V_{u_f d_{f'}}|^2 F_U(m_{u_f}^2, m_{d_{f'}}^2)-\frac{5}{6}\sum_{f=1}^{4}1\right]$$

$$+\frac{1}{3\pi}\left[\sum_{f=1}^{4}\sum_{f'=1}^{4}|V_{\nu_f l_{f'}}|^2 F_U(m_{\nu_f}^2, m_{l_{f'}}^2)-\frac{5}{6}\sum_{f=1}^{4}1\right].$$

O. Eberhardt, A. Lenz, J. Rohrwild,
Less space for a new family of fermions,
Phys. Rev. D82 : 095006,2010

with

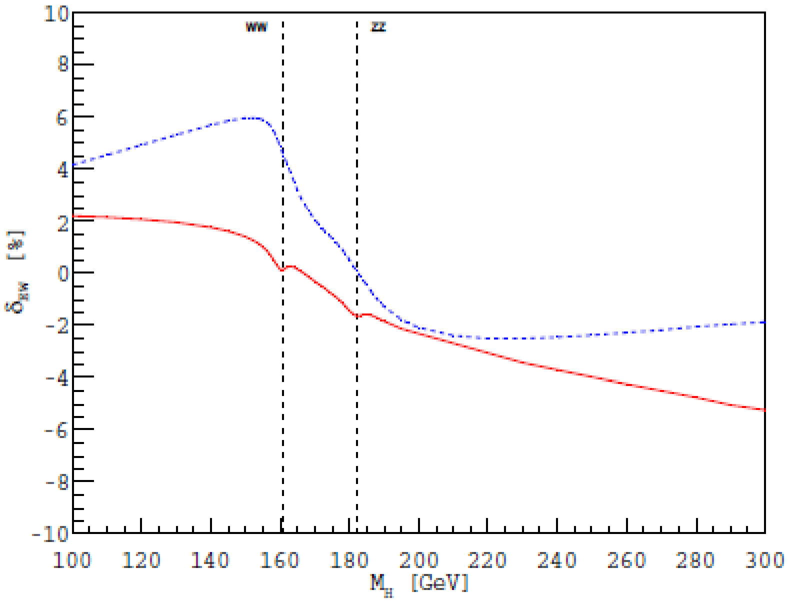
$$F_T(m_1^2, m_2^2) := 2\frac{m_1^2 m_2^2}{m_1^2-m_2^2}\ln\frac{m_1^2}{m_2^2},$$

$$F_U(m_1^2, m_2^2) := 2\frac{m_1^2 m_2^2}{(m_1^2-m_2^2)^2}+\left(\frac{m_1^2+m_2^2}{2(m_1^2-m_2^2)}-\frac{m_1^2 m_2^2(m_1^2+m_2^2)}{(m_1^2-m_2^2)^3}\right)\ln\frac{m_1^2}{m_2^2}.$$

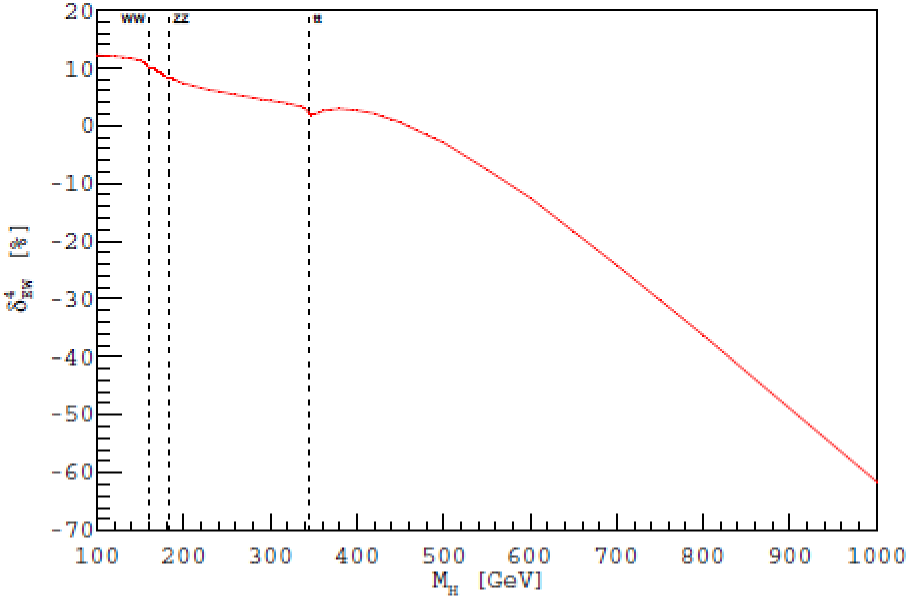
Comments about N(N)LO EW corrections (not available yet !)

A precise knowledge of the NLO electroweak corrections is important !

$$\sigma_{\mathrm{SM4}}(gg \to H) = \sigma^{\mathrm{LO}}_{\mathrm{SM4}}(gg \to H)\left(1 + \delta^4_{EW}\right)$$



$$m_{t'} = m_{b'} = 400\ \mathrm{GeV}$$
$$(t'\!-\!b'\ \text{quarks only})$$



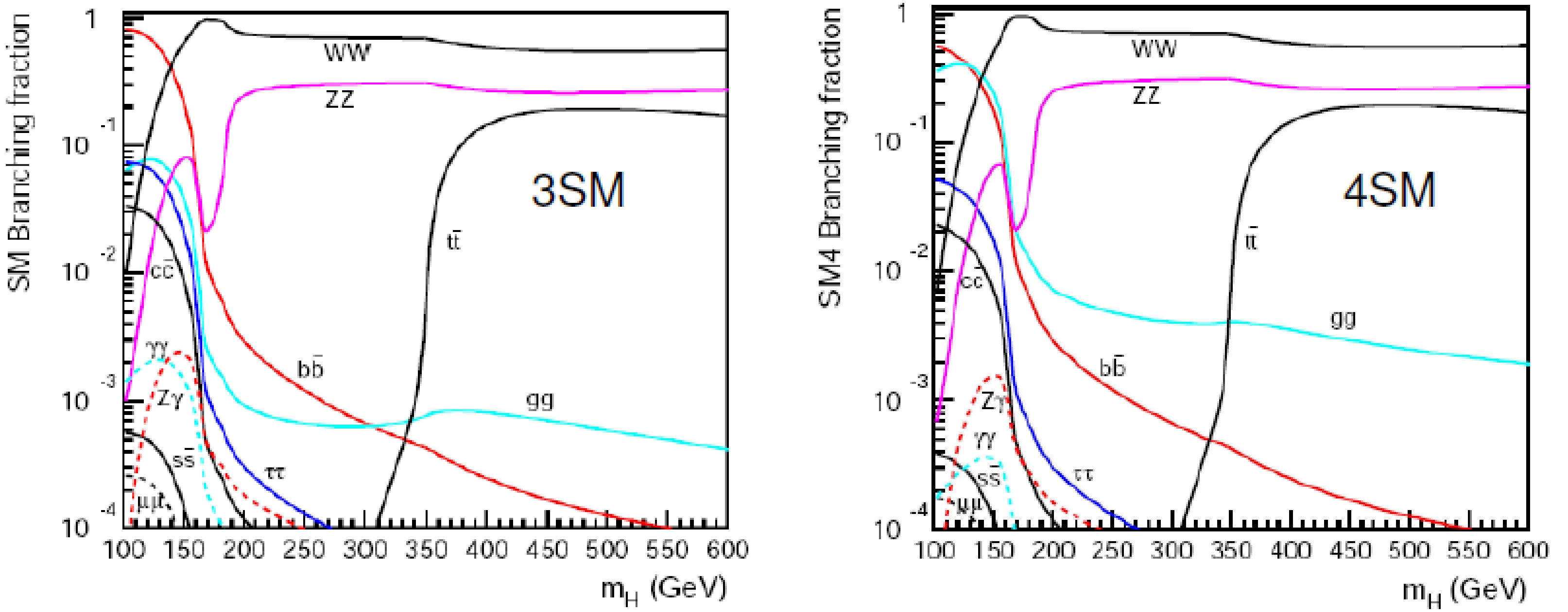
$$m_{t'} = m_{b'} = m_{l'} = m_{\nu_l'} = 600\ \mathrm{GeV}.$$

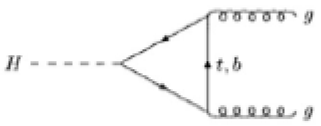
“Complete Electroweak Corrections to Higgs production in a Standard Model with four generations at the LHC”, Passarino et al. (ArXiv:1108.2025)

δ^4_{EW}
= +4% for mt’ = 172 GeV
= -12% for mt’ = 600 GeV
= -60% for mt’ = 1 TeV

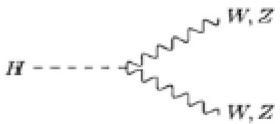
SM3 Higgs vs SM4 Higgs at LHC7

Ruan & Zhang (2011): $m_{t'}=m_{b'}=m_{l4}$ =400 GeV, $m_{\nu4}$ =? (likely large)

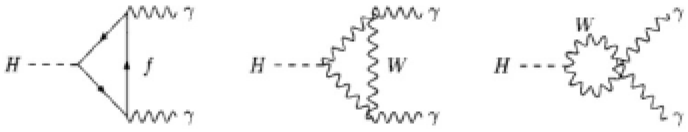




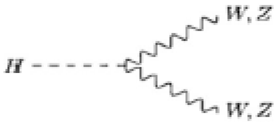
$\Gamma(H\to gg)$ enhanced



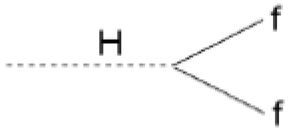
$\Gamma(H\to WW/ZZ)$ unchanged



$\Gamma(H\to\gamma\gamma)$ reduced due to destructive interference



$\Gamma(H\to WW/ZZ)$ unchanged



New partial rate added if $m_H > 2\,m_f$