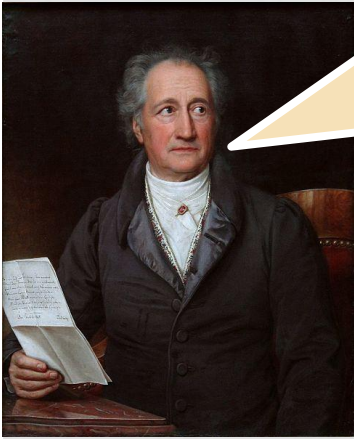


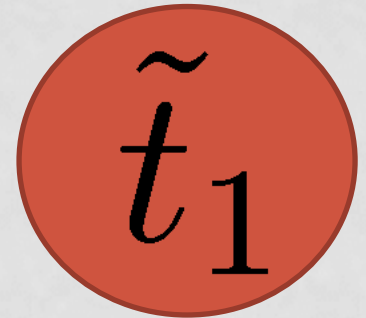
# More Light Stop

vs. Lightest Higgs at 125 GeV



Goethe

Yukihiro Mimura (NTU)



Sequence of PreWorkShop on very heavy quarks at the LHC

Focus Workshop on Heavy Quarks at LHC (2012.1.19)

# Menu

1. Why are stop masses important in Radiative EW symmetry breaking?
2. Naturalness issue
3. 125 GeV Higgs in MSSM

# Stop masses are important!

$$\frac{M_Z^2}{2} = -\mu^2 + \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} \simeq -\mu^2 - m_{H_u}^2 \equiv -m_2^2$$

(tree level)

$$V_{\text{tree}} = (m_{H_d}^2 + |\mu|^2)|H_d^0|^2 + (m_{H_u}^2 + |\mu|^2)|H_u^0|^2 - (B\mu H_u^0 H_d^0 + h.c.) \\ + \frac{g^2 + g'^2}{8} (|H_u^0|^2 - |H_d^0|^2)^2$$

Correction from  $\frac{\partial \Delta V}{\partial v_u}$ ,  $\frac{\partial \Delta V}{\partial v_d}$  is small.



It should be evaluated at  $Q = m_{\tilde{t}} \equiv \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$

$$M_Z^2 = -2m_2^2(Q = m_{\tilde{t}})$$

$$M_Z^2 = 2 \frac{dm_2^2}{\ln Q} \ln \frac{Q_0}{m_{\tilde{t}}} \quad [m_2^2(Q_0) = 0]$$

$$\simeq \frac{3y_t^2}{4\pi^2} (m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 + A_t^2) \ln \frac{Q_0}{m_{\tilde{t}}}$$

Sensitive to the stop mass parameters

## RGEs

$$\frac{dm_{H_u}^2}{d \ln Q} = \frac{3y_t^2}{8\pi^2} (m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 + A_t^2) - \frac{1}{8\pi^2} (3g_2^2 M_2 + g'^2 M_1) + \frac{1}{16\pi^2} g'^2 S$$

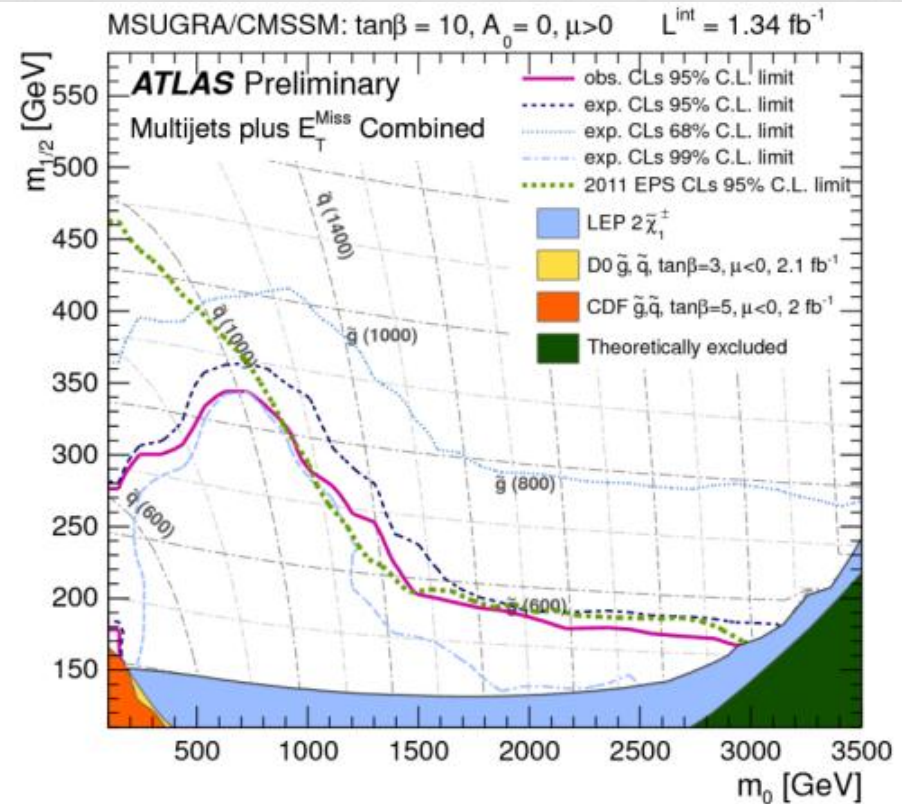
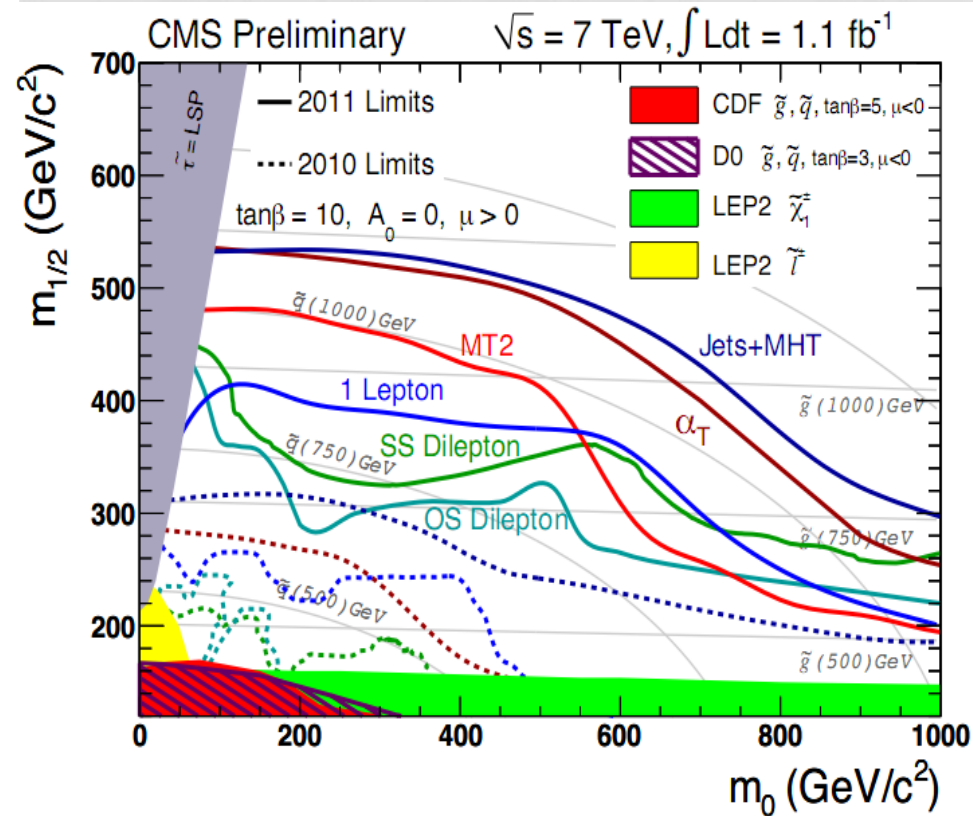
$$\frac{d\mu^2}{d \ln Q} = \frac{1}{8\pi^2} (3y_t^2 + 3y_b^2 + y_\tau^2 - g'^2 - 3g_2^2) \mu^2$$

$$S = \sum Y_i m_i^2$$

Hypercharge weighted  
Trace of scalar masses



# minimal Supergravity (mSUGRA)/Constrained MSSM (CMSSM)



## mSUGRA/CMSSM parameters

1. Unified Gaugino masses at GUT scale  $m_{1/2}$
2. Universal scalar mass (at Planck/GUT scale)  $m_0$
3. Scalar trilinear coupling (A-term)  $A_0$
4. Ratio of up- and down-type Higgs vev  $\tan \beta$
5. Higgsino mass  $\text{sgn}(\mu)$

( $|\mu|$  is fixed by the minimization of Higgs potential.)

$$\frac{M_Z^2}{2} = -\mu^2 + \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} \simeq -\mu^2 - m_{H_u}^2 \equiv -m_2^2$$

(tree level)

RGE solution:

$$m_{H_u}^2 = -0.05m_0^2 - 1.75M_{1/2}^2 - 0.34M_{1/2}A_0 - 0.10A_0^2$$

Cancellation sensitivity  $\Delta \equiv \left| \frac{\partial \ln M_Z}{\partial \ln \mu} \right|^{-1}$

$$\Delta = \frac{M_Z^2}{2\mu^2} = \frac{91^2}{2 \times 1.75(500)^2} = 0.009464$$

for  $M_{1/2} = 500$  GeV

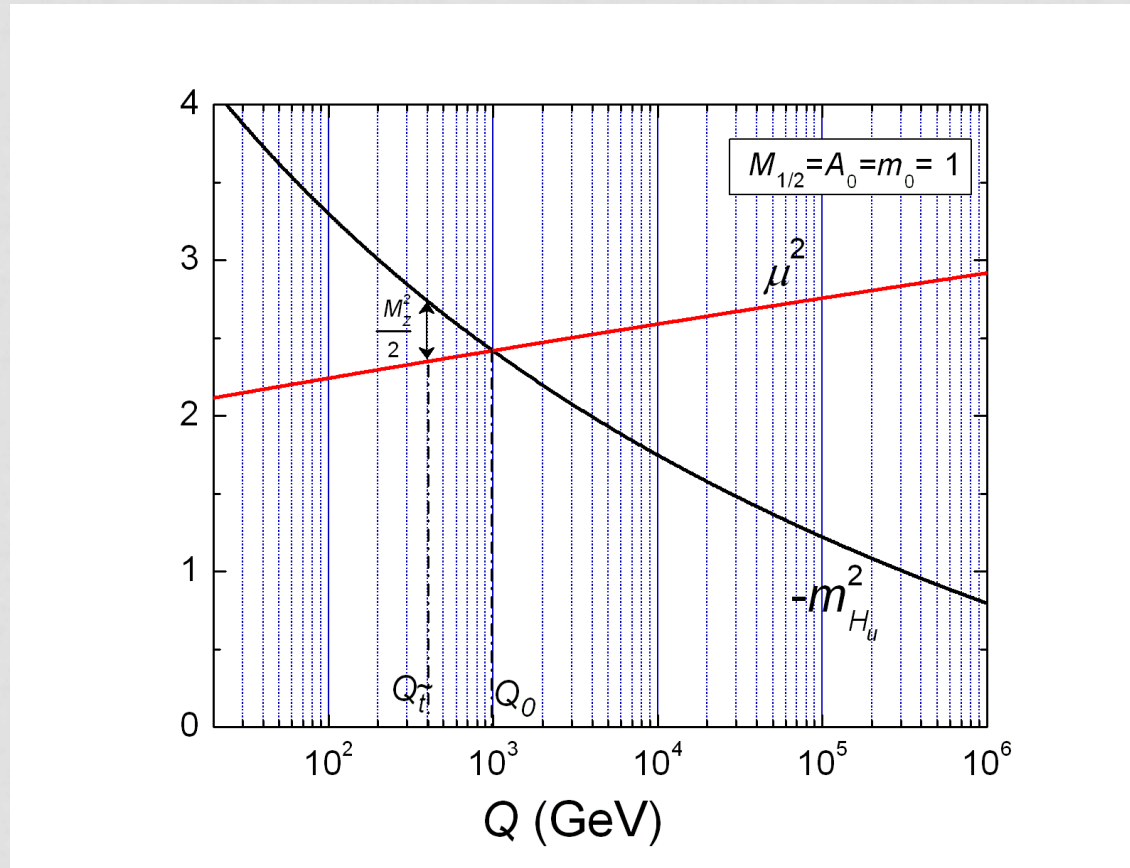


# Cancellation sensitivity

$$\frac{M_Z^2}{2\mu^2} = \frac{91^2}{2 \times 1.75 (500)^2} = 0.009464$$



However, in Radiative EWSB scenario,  
 $-m_2^2 = -m_{H_u}^2 - \mu^2$  becomes zero at a scale.



(Note : RGEs are homogeneous equation, and thus, the solution is overall-scale invariant.)

$$M_Z^2 = \underbrace{m_Z^2(Q)}_{\text{tree}} + \frac{1}{v^2 \cos 2\beta} \underbrace{\left( v_u \frac{\partial \Delta V}{\partial v_u} - v_d \frac{\partial \Delta V}{\partial v_d} \right)}_{=0}$$

$$M_Z^2 \simeq m_Z^2(Q_S)$$

at  $Q = Q_S$

$$\simeq -2\mu^2(Q_S) - 2m_{H_u}^2(Q_S)$$

$Q_S \sim Q_{\tilde{t}}$

averaged stop mass

$$M_Z^2 \simeq m_Z^2(Q_S)$$



$$M_Z^2 \simeq \ln \frac{Q_0}{Q_S} \left. \frac{dm_Z^2}{d \ln Q} \right|_{Q=Q_0}$$

$m_Z^2(Q_0) = 0$  by definition

Little hierarchy  $\longrightarrow$   $Q_S$  and  $Q_0$  are close.

(Giudice-Rattazzi, Dutta-YM)

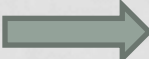
If you fix the SUSY breaking scale (or weak scale) a priori in the landscape, small  $\mu$  at the scale is preferable for mild sensitivity.

If the little hierarchy for Z boson mass/SUSY breaking is the issue (without thinking 100 GeV as a special scale a priori),  $\mu$  can be large or small, but the stop mass parameters are important for mild sensitivity.

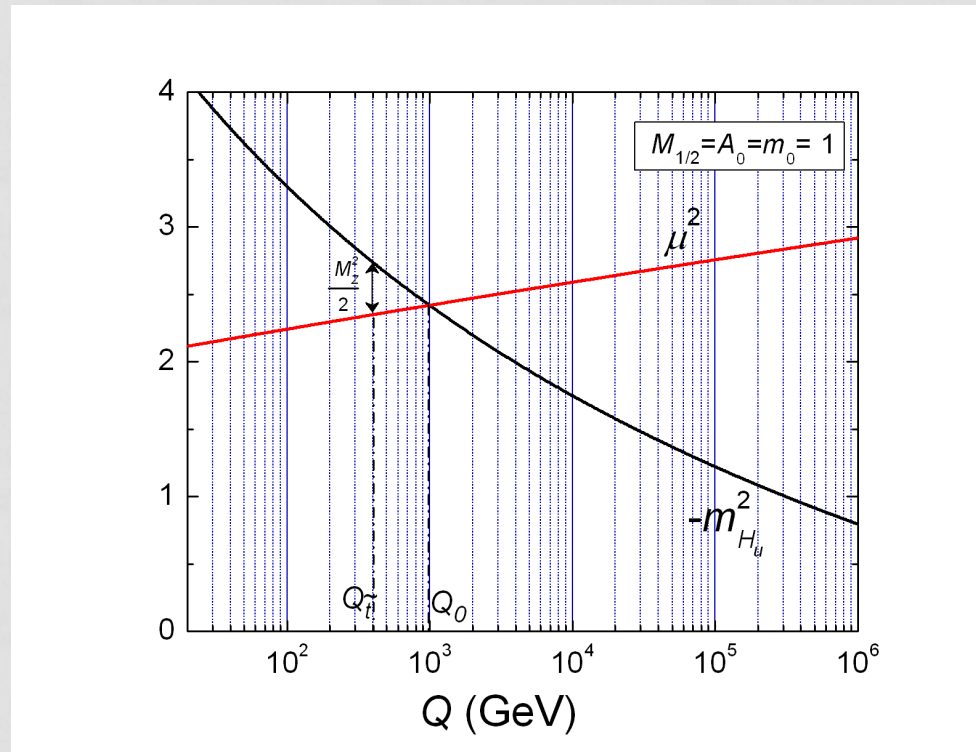
$$\Delta_S \equiv \left| \frac{\partial M_Z}{\partial M_S} \right|^{-1}$$

$$= \frac{2 \ln Q_0/Q_S}{1 - 2 \ln Q_0/Q_S} \simeq \frac{8\pi^2 M_Z^2}{3y_t^2(m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 + A_t^2) - 8\pi^2 M_Z^2}$$

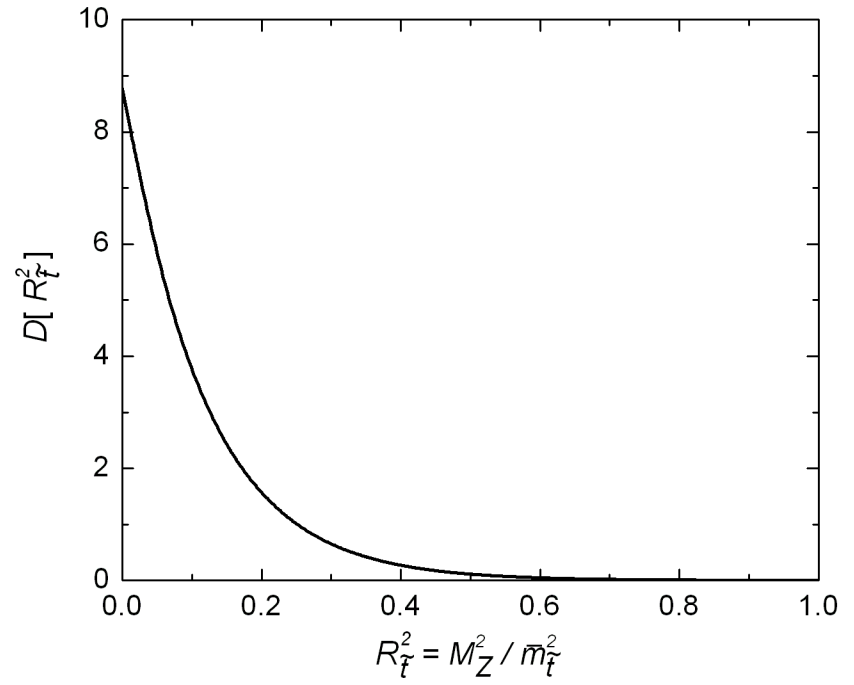
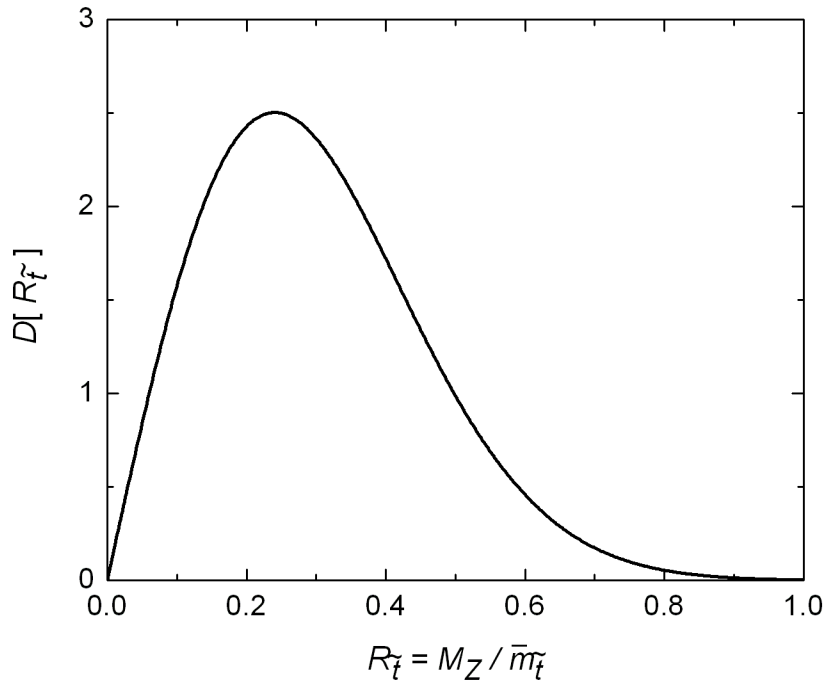
For  $m_{\tilde{t}_L} = m_{\tilde{t}_R} = A_t = 1.2 \text{ TeV}$

  $\Delta_S \simeq 0.053$

Furthermore, we do not know the parameter which is equally probable.



For example, if the overall scale is equally probable, small value of  $\ln Q_S/Q_0$  is well probable among the EWSB vacua.



## Distribution of little hierarchy

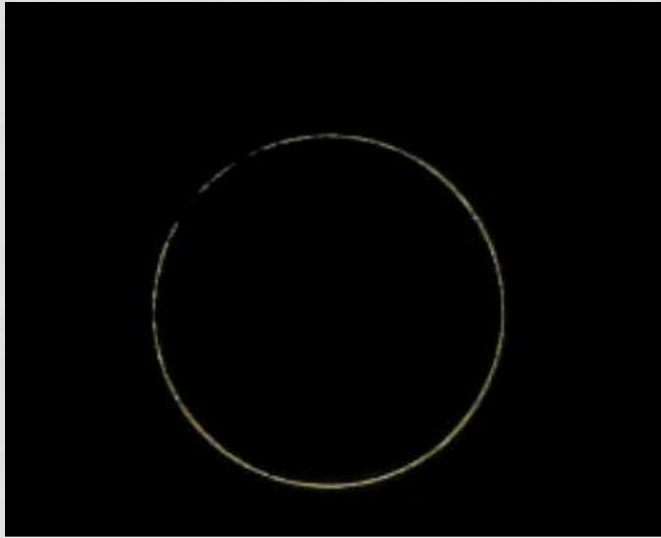
99% can be covered at  $m_{\tilde{\tau}} \sim 3$  TeV

We can live near the cliff anyway.



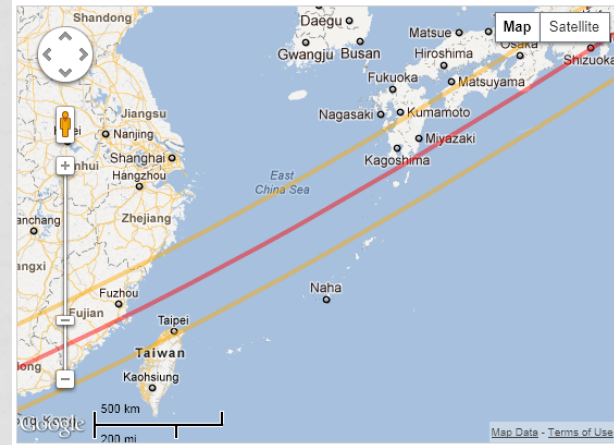


# Analogy for fun

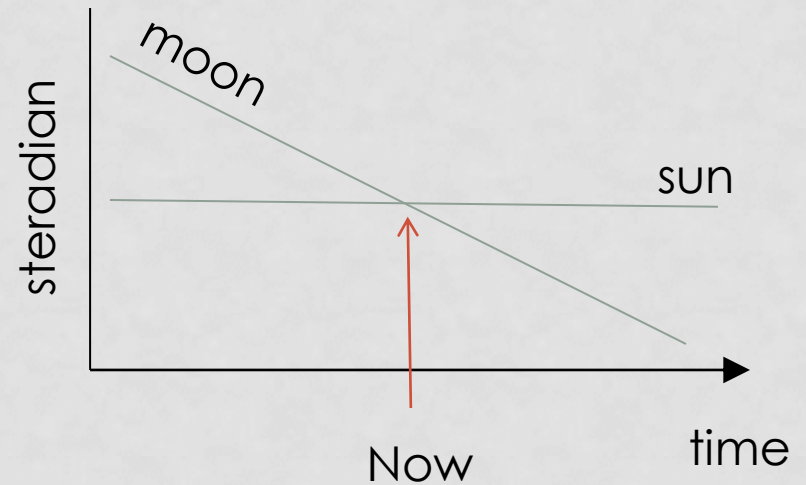


annular eclipse of the sun

Fine-tuned!  
But, the moon is moving away  
from the earth (3.8cm/year)  
because of tidal friction.



2012.5.21



# Analogy for fun

MSSM

Moon/Sun

Higgsino mass

$$\frac{\text{sun radius}}{\text{sun-earth distance}}$$

SUSY breaking  
Higgs mass



$$\frac{\text{moon radius}}{\text{moon-earth distance}}$$

Stop mass

Tidal friction

FCNC



Another  
Tuning?



Rotation period  
= Orbital period

SUSY breaking  
mechanism  
(unknown)



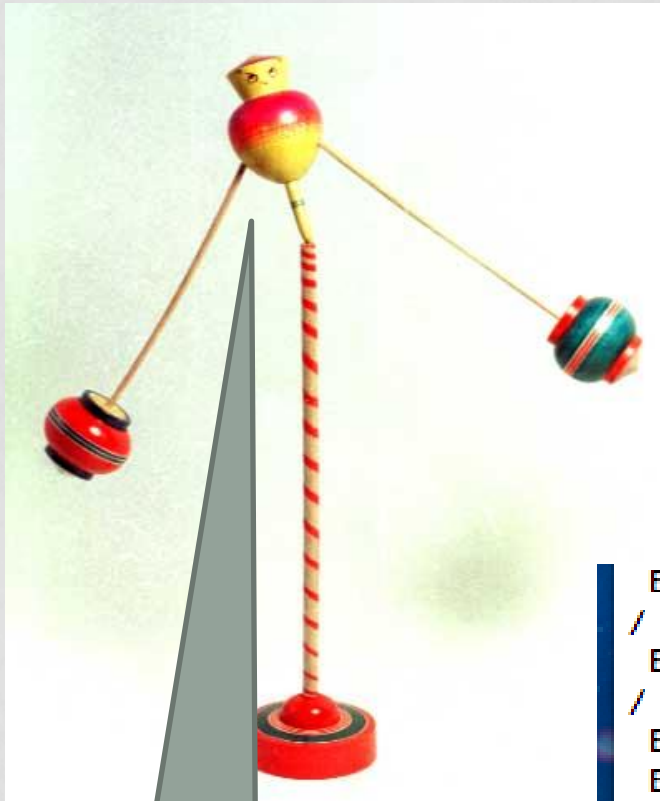
Giant impact  
hypothesis

I saw an annular  
eclipse of the sun.



Then, your moon  
should look small  
due to  
Naturalness !





I am Natural.

Anyway, it is worthwhile to chase more “comfortable” SUSY breaking scenario.

```
ENTER SUSY Les Houches Accord filename [/ for none]:  
/  
ENTER Isawig (Herwig interface) filename [/ for none]:  
/  
ENTER 1 for mSUGRA:  
ENTER 2 for mGMSB:  
ENTER 3 for non-universal SUGRA:  
ENTER 4 for SUGRA with truly unified gauge couplings:  
ENTER 5 for non-minimal GMSB:  
ENTER 6 for SUGRA+right-handed neutrino:  
ENTER 7 for minimal anomaly-mediated SUSY breaking:  
ENTER 8 for non-minimal AMSB:  
ENTER 9 for mixed moduli-AMSB:  
ENTER 10 for Hypercharged-AMSB:  
□
```

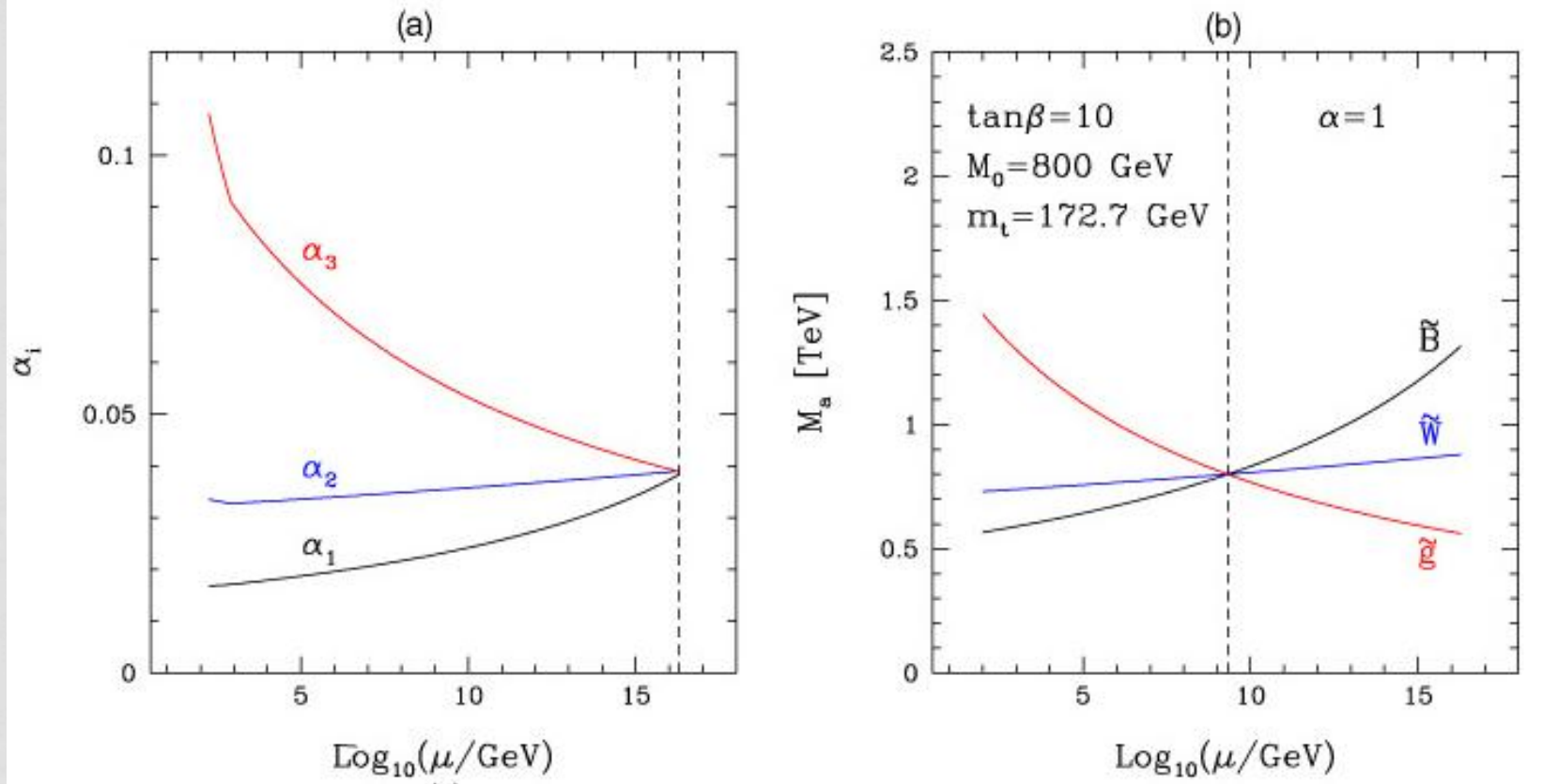
## Light stop scenario

Lighter stop  $\tilde{t}_1$  is the lightest sfermion (maybe NLSP) due to a large stop mixing.

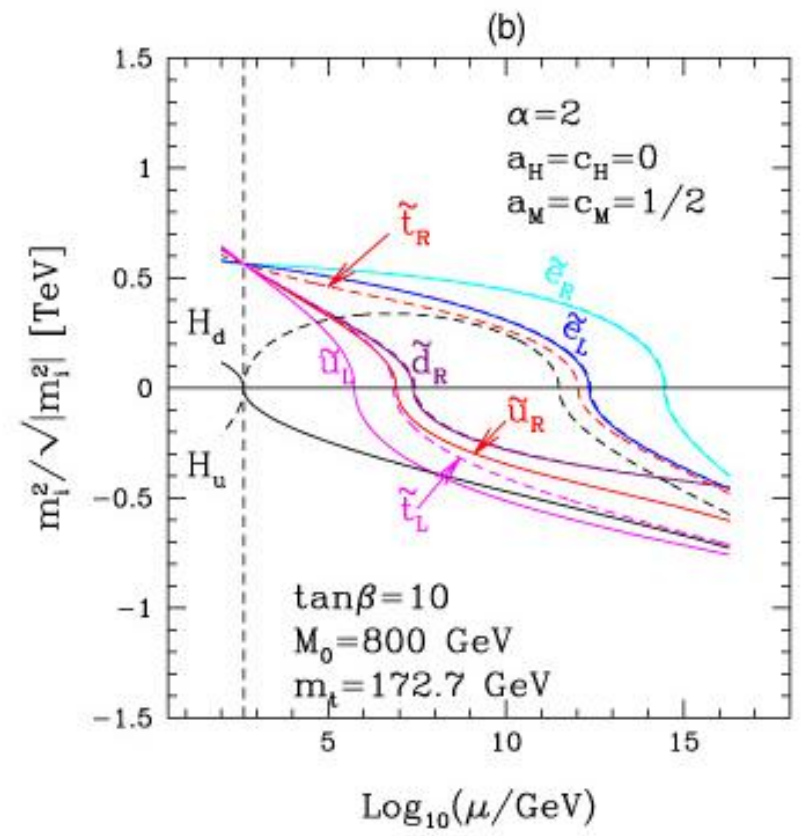
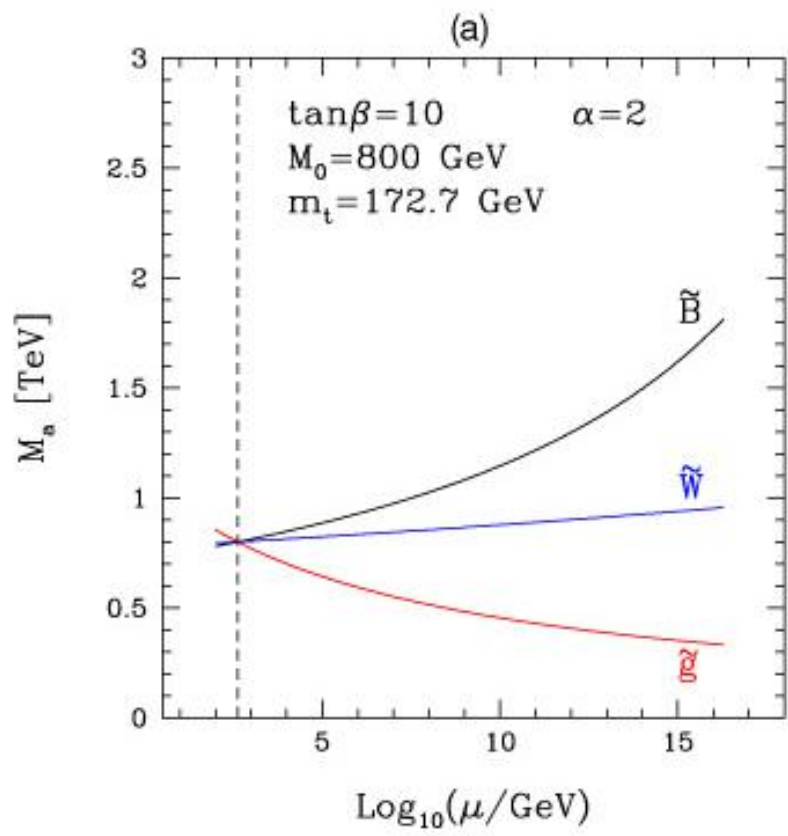
- Stop-neutralino coannihilation for relic dark matter density  
(Maybe natural in mirage mediation scenario)
- Cascade decay topology is different and less sensitive in exp.  
Light “light stop” is still allowed.
- MSSM baryogenesis (Carena-Nardini-Quiros-Wagner)

# Mirage mediation (Anomaly + mixed moduli)

(Choi-Lee-Shimizu-Kim-Okumura, Kitano-Nomura)



$$M_3 : M_2 : M_1 \simeq (1 - 0.3\alpha)g_3^2 : (1 + 0.1\alpha)g_2^2 : (1 + 0.66\alpha)g_1^2$$



# Stop mass matrix

$$(\tilde{t}_L, \tilde{t}_R) \begin{pmatrix} m_{\tilde{t}_L}^2 + m_t^2 + D_L & (A_t - \mu \cot \beta) m_t \\ (A_t - \mu \cot \beta) m_t & m_{\tilde{t}_R}^2 + m_t^2 + D_R \end{pmatrix} \begin{pmatrix} \tilde{t}_L^\dagger \\ \tilde{t}_R^\dagger \end{pmatrix}$$

$$D_L = \left( \frac{2}{3} M_W^2 - \frac{1}{6} M_Z^2 \right) \cos 2\beta$$

$$D_R = \left( -\frac{2}{3} M_W^2 + \frac{2}{3} M_Z^2 \right) \cos 2\beta$$

Stop mixing parameter :  $X_t \equiv A_t - \mu \cot \beta$

Eigenstates :  $\tilde{t}_1, \tilde{t}_2$

Large stop mixing splits the masses of eigenstates.



# The lightest Higgs mass in MSSM

Tree level :  $m_h^2 = M_Z^2 \cos^2 2\beta$

Loop corrections depend on the top quark mass and logarithm of the scale of the stop masses.

The maximal value is about 130 GeV.

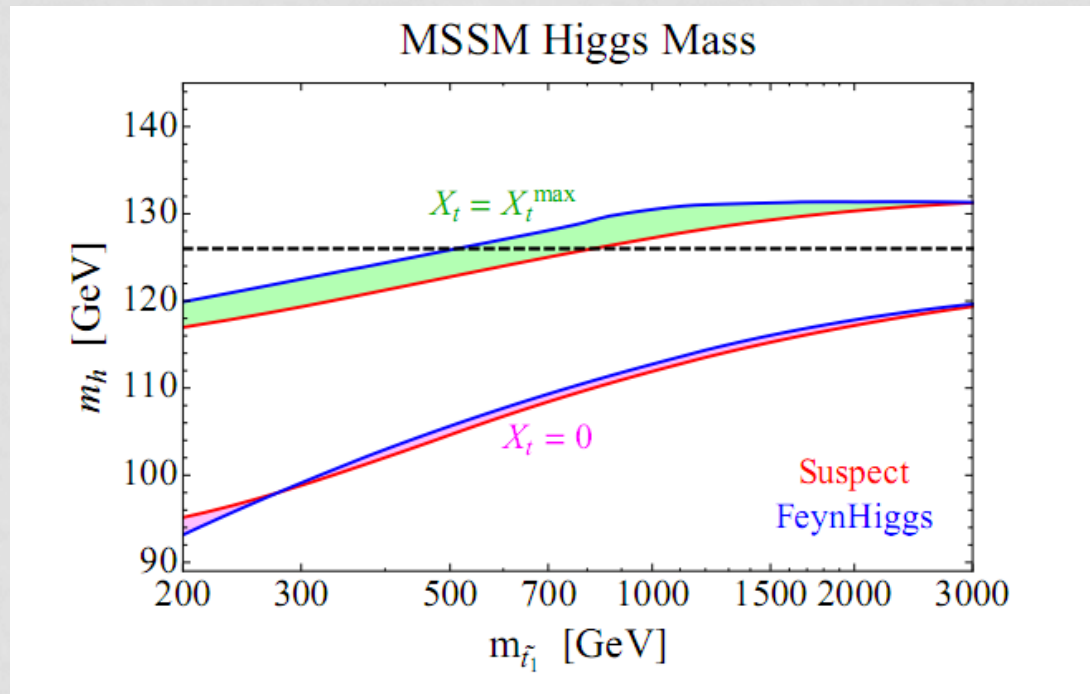
(Okada-Yamaguchi-Yanagida,  
Ellis-Ridolfi-Zwirner, Haber-Hempfling, ...)

CMS/ATLAS exclusion of SM-like Higgs (heavier than 127 GeV) is good for SUSY.

$$m_h^2 \sim M_Z^2 \cos^2 2\beta + \frac{3}{(4\pi)^2} \frac{m_t^4}{v^2} \left[ \ln \frac{m_{\tilde{t}}^2}{m_t^2} + \frac{X_t^2}{m_{\tilde{t}}^2} \left( 1 - \frac{X_t^2}{12m_{\tilde{t}}^2} \right) \right]$$

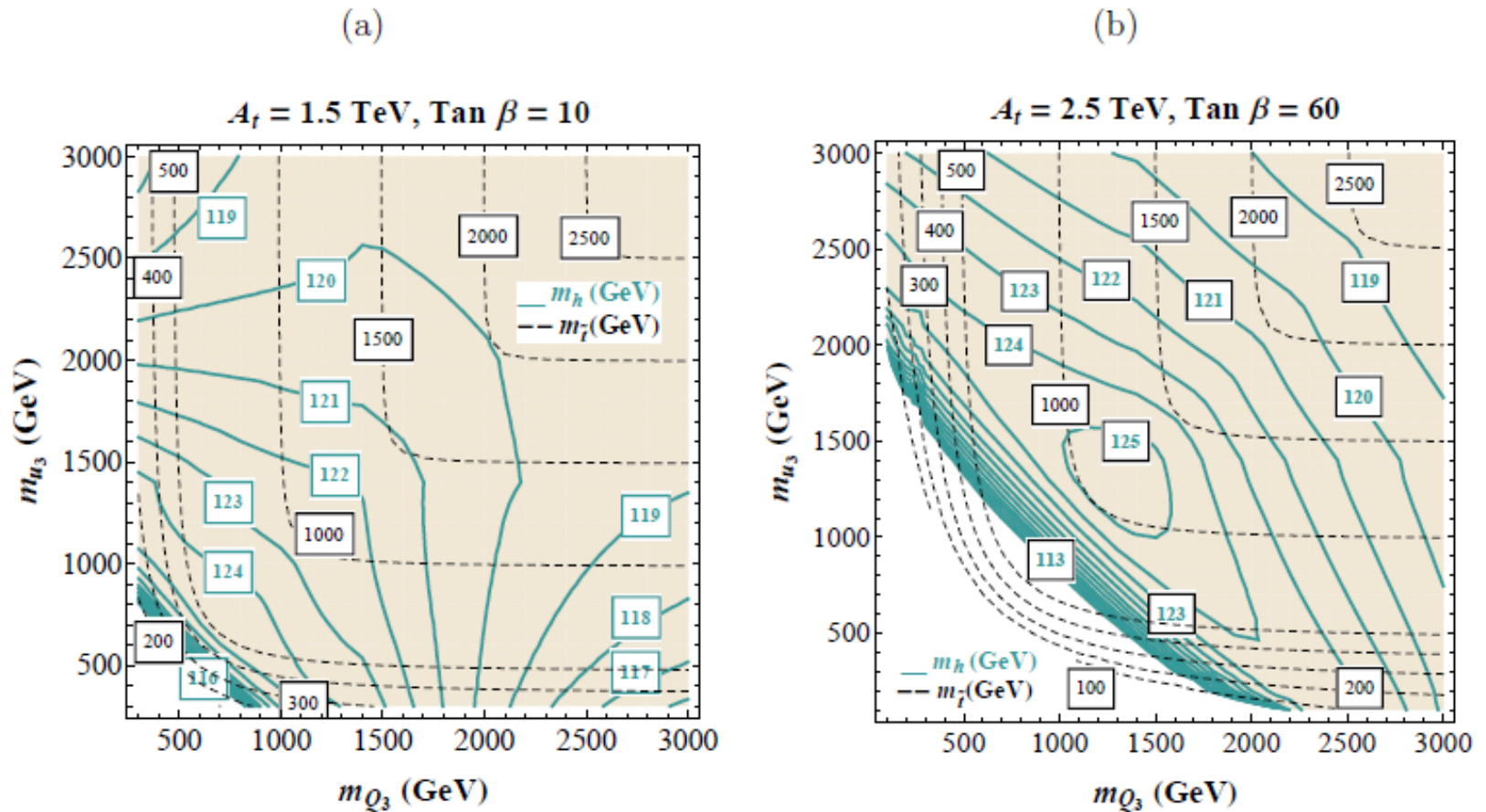
$$X_t \equiv A_t - \mu \cot \beta$$

Maximized at  $|X_t| = X_t^{\max} = \sqrt{6}m_{\tilde{t}}$



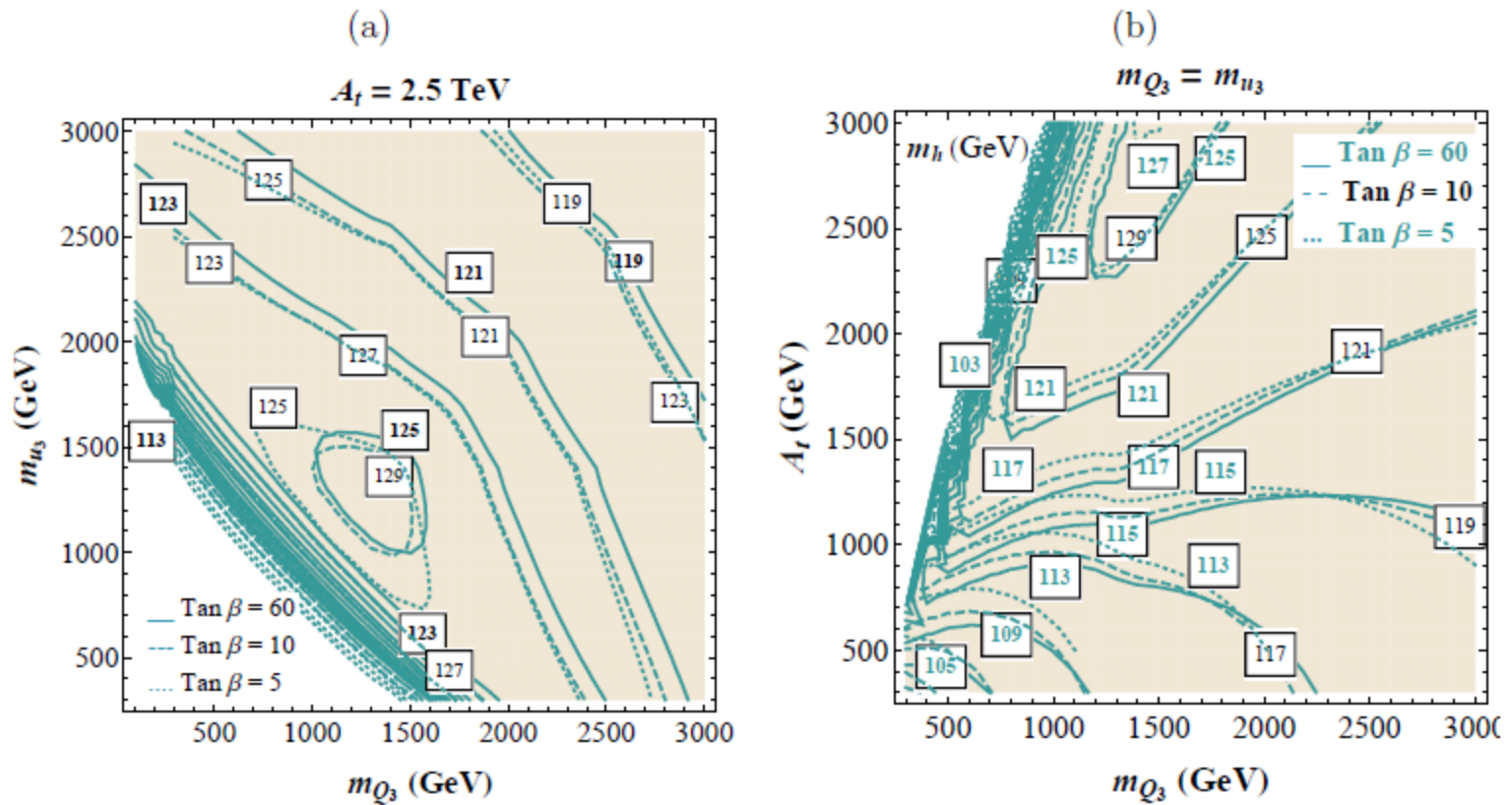
$$\tan \beta = 20$$

(Hall-Pinner-Ruderman, 1112.2703)




**Figure 1:** Contour plots of the Higgs mass in the  $m_{Q_3} - m_{u_3}$  plane, for different values of  $A_t$  and  $\tan \beta$ . The stau soft masses have been fixed at  $m_{L_3}^2 = m_{e_3}^2 = (350 \text{ GeV})^2$ , while  $\mu = 1030 \text{ GeV}$  and  $A_\tau = 500 \text{ GeV}$ , leading to a lightest stau mass of about 135 GeV for  $\tan \beta = 60$ . The lightest stop masses are overlaid in dashed black lines.

(Carena-Gori-Shah-Wagner, 1112.3336)



**Figure 2:** Contour plots of the Higgs mass in the plane of soft supersymmetry breaking parameters in the stop sector. In (a), we show the Higgs masses for  $A_t = 2.5$  TeV for three different values of  $\tan \beta$ ,  $\tan \beta = 5$  (dotted lines, green (grey) labels),  $\tan \beta = 10$  (dashed lines, black labels) and  $\tan \beta = 60$  (solid lines, green (grey) labels). The masses for  $\tan \beta = 60$  shown are smaller than the ones for  $\tan \beta = 10$  mostly due to the negative effects from the staus (see Eq. (6)), and closer to the  $\tan \beta = 5$  ones. In (b), the Higgs mass contours are shown for  $m_{Q_3} = m_{u_3}$ , varying the stop mixing parameter  $A_t$ . The stau supersymmetry breaking parameters have been kept at  $m_{L_3}^2 = m_{e_3}^2 = (350\text{GeV})^2$  and  $A_\tau = 500$  GeV, while  $\mu = 1030$  GeV.

$$\mathcal{M}_H^2 = \begin{pmatrix} m_A^2 \sin^2 \beta + M_Z^2 \cos^2 \beta + \Delta_{11} & -(m_A^2 + M_Z^2) \sin \beta \cos \beta + \Delta_{12} \\ -(m_A^2 + M_Z^2) \sin \beta \cos \beta + \Delta_{12} & m_A^2 \cos^2 \beta + M_Z^2 \sin^2 \beta + \Delta_{22} \end{pmatrix}$$

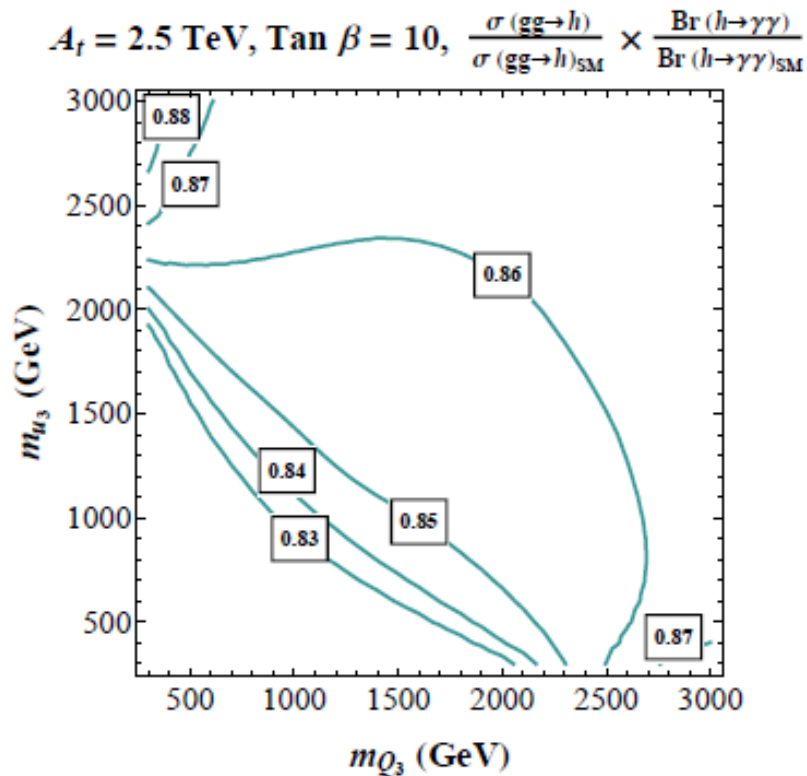
Large left-right stop mixing affects 12 elements.   $\sin \alpha$  is enhanced.

$$htt\bar{t} : \frac{\cos \alpha}{\sin \beta}$$

$$hbb\bar{b} : -\frac{\sin \alpha}{\cos \beta} (1 + (\text{finite correction term}))$$

Large left-right stop mixing reduces gluon fusion Higgs production rate.

Light stop (sbottom) with large mixing can increase the photon decay branching ratio.



**Figure 3:** Contour plots of the ratio of the  $\sigma(gg \rightarrow h) \times \text{BR}(h \rightarrow \gamma\gamma)$  to its SM value, in the  $m_{Q_3}$ - $m_{u_3}$  plane, for  $\mu = 1030$  GeV.

(For heavy stau, it is slightly lower than SM for 125 GeV Higgs.)

$$\frac{dm_{H_u}^2}{d \ln Q} = \frac{3y_t^2}{8\pi^2} (m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 + A_t^2) - \frac{1}{8\pi^2} (3g_2^2 M_2 + g'^2 M_1) + \frac{1}{16\pi^2} g'^2 S$$

$$M_Z^2 \simeq \frac{3y_t^2}{4\pi^2} (m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 + A_t^2) \ln \frac{Q_0}{m_{\tilde{t}}}$$

1. Large A-term is not very comfortable.
2. In gauge mediated SUSY breaking (GMSB), A-term is not very large.



MSSM+

# 1. Tree-level modification of quartic Higgs coupling

$$W = \lambda S H_u H_d \quad (\text{NMSSM})$$

$$\delta m_h^2 \propto \lambda^2 v^2 \sin 2\beta$$

$$W = \lambda \Sigma H_u H_u \quad \text{SU(2) adjoint, } Y = -1$$

$$\delta m_h^2 \propto \lambda^2 v^2 \sin^2 \beta$$

# 2. Additional loop corrections from vector-like fields

e.g.  $Q, \quad U^c, \quad E^c \quad \quad QU^c H_u + \bar{Q}\bar{U}^c H_d$

$\bar{Q}, \quad \bar{U}^c, \quad \bar{E}^c$



Gauge coupling unification  
can be maintained.



# Summary

1. 1-2 TeV squarks are still well-probable.
2. Light “light stop” is still allowed. Stop NLSP can be an interesting scenario.
3. 125 GeV Higgs possibility can be entertained.
4. Minimal or not is still a issue for 125 GeV Higgs to live in comfortably.

