

# Measuring CKM4 at LHC

Fanrong Xu

National Taiwan University

Jan. 19-20, 2012

**Focus Workshop on Heavy Quarks at LHC**

(Based on W.-S. Hou, M. Kohda and F. Xu, PRD 84, 094027 (2011) )

# Outline

## Introduction

## 3 new measurements

$$B_s \rightarrow \mu^+ \mu^-$$

$$B_s^0 \rightarrow J/\psi \phi(f_0)$$

$$B \rightarrow K^* \mu^+ \mu^-$$

## Constraint from $b \rightarrow s$ decay

## Constraint from $b'$ and $t'$ decay

## Constraint from $b \rightarrow d$ decay

## Summary

# Introduction

## ► Why SM4

Mini-review

### Four statements about the fourth generation

Bob Holdom<sup>1</sup>, WS Hou<sup>2</sup>, Tobias Hurth<sup>3</sup>, Michelangelo L Mangano<sup>\*3</sup>,  
 Saleh Sultansoy<sup>4</sup> and Gokhan Ünel<sup>5</sup>

Published: 28 September 2009

PMC Physics A 2009, 3:4 doi:10.1186/1754-0410-3-4

Received: 19 August 2009

Accepted: 28 September 2009

famous

(100 ~ 499 cites)

#### Abstract

This summary of the Workshop "Beyond the 3-generation SM in the LHC era" presents a brief discussion of the following four statements about the fourth generation: 1) It is not excluded by EW precision data; 2) It addresses some of the currently open questions; 3) It can accommodate emerging possible hints of new physics; 4) LHC has the potential to discover or fully exclude it.

## ► Prof. Yuan's talk ("From Top to Heavy Quark") in the morning

# Introduction

- ▶ Direct search at LHC – Prof. Chen's talk

- ▶  $t'$  quark

- $m_{t'} > 450\text{GeV @95\%CL}$ , "CMS PAS EXO-11-051", 2011/07/22

- $m_{t'} > 490\text{GeV @95\%CL}$ ,  $1.1\text{fb}^{-1}$ , "CMS PAS EXO-11-054", 2011/10/04

- ▶  $b'$  quark

- $m_{b'} > 495\text{GeV @95\%CL}$ ,  $1.14\text{fb}^{-1}$  "CMS PAS EXO-11-036", 2011/08/27



**3 new measurements**

- Constraint from  $b \rightarrow s$  decay
- Constraint from  $b'$  and  $t'$  decay
- Constraint from  $b \rightarrow d$  decay
- Summary

$$B_s \rightarrow \mu^+ \mu^-$$

$$B_s^0 \rightarrow J/\psi \phi(f_0)$$

$$B \rightarrow K^* \mu^+ \mu^-$$

## 3 New Measurements

$$\begin{aligned}
 B_s &\rightarrow \mu^+ \mu^- \\
 B_s^0 &\rightarrow J/\psi \phi(f_0) \\
 B &\rightarrow K^* \mu^+ \mu^-
 \end{aligned}$$

$$B_s \rightarrow \mu^+ \mu^-$$

PRL 107, 191801 (2011)

PHYSICAL REVIEW LETTERS

week ending  
4 NOVEMBER 2011Search for  $B_s^0 \rightarrow \mu^+ \mu^-$  and  $B^0 \rightarrow \mu^+ \mu^-$  Decays with CDF II

(CDF Collaboration)

$$\mathcal{B}^{SM}(B_s \rightarrow \mu^+ \mu^-) = (3.2 \pm 0.2) \times 10^{-9}$$

(Received 10 July 2011; published 1 November 2011; corrected 17 November 2011)

A.J. Buras 1012.1447

A search has been performed for  $B_s^0 \rightarrow \mu^+ \mu^-$  and  $B^0 \rightarrow \mu^+ \mu^-$  decays using  $7 \text{ fb}^{-1}$  of integrated luminosity collected by the CDF II detector at the Fermilab Tevatron collider. The observed number of  $B^0$  candidates is consistent with background-only expectations and yields an upper limit on the branching fraction of  $\mathcal{B}(B^0 \rightarrow \mu^+ \mu^-) < 6.0 \times 10^{-9}$  at 95% confidence level. We observe an excess of  $B_s^0$  candidates. The probability that the background processes alone could produce such an excess or larger is 0.27%. The probability that the combination of background and the expected standard model rate of  $B_s^0 \rightarrow \mu^+ \mu^-$  could produce such an excess or larger is 1.9%. These data are used to determine  $\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^-) = (1.8^{+1.1}_{-0.6}) \times 10^{-8}$  and provide an upper limit of  $\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^-) < 4.0 \times 10^{-8}$  at 95% confidence level.

PRL 107, 191802 (2011)

PHYSICAL REVIEW LETTERS

week ending  
4 NOVEMBER 2011Search for  $B_s^0 \rightarrow \mu^+ \mu^-$  and  $B^0 \rightarrow \mu^+ \mu^-$  Decays in  $pp$  Collisions at  $\sqrt{s} = 7 \text{ TeV}$ S. Chatrchyan *et al.*<sup>\*</sup>

(CMS Collaboration)

(Received 29 July 2011; published 1 November 2011)

$$\mathcal{B}(B_s \rightarrow \mu^+ \mu^-) < 1.2(1.5) \times 10^{-8} @ 90(95)\% CL$$

300 pb<sup>-1</sup>, LHCb@EPS-HEP2011

A search for the rare decays  $B_s^0 \rightarrow \mu^+ \mu^-$  and  $B^0 \rightarrow \mu^+ \mu^-$  is performed in  $pp$  collisions at  $\sqrt{s} = 7 \text{ TeV}$ , with a data sample corresponding to an integrated luminosity of  $1.14 \text{ fb}^{-1}$ , collected by the CMS experiment at the LHC. In both cases, the number of events observed after all selection requirements is consistent with expectations from background and standard-model signal predictions. The resulting upper limits on the branching fractions are  $\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^-) < 1.9 \times 10^{-8}$  and  $\mathcal{B}(B^0 \rightarrow \mu^+ \mu^-) < 4.6 \times 10^{-9}$ , at 95% confidence level.

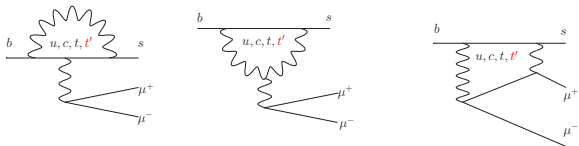
$$B_s \rightarrow \mu^+ \mu^-$$

$$B_s^0 \rightarrow J/\psi \phi(f_0)$$

$$B \rightarrow K^* \mu^+ \mu^-$$

# $B_s \rightarrow \mu^+ \mu^-$

$bs \rightarrow \ell^+ \ell^-$



- ▶ large uncertainty:  $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-) \propto f_{B_s}^2$
- ▶ note  $\Delta m_{B_s} \propto f_{B_s}^2 \hat{B}_{B_s}$
- ▶ reduce the uncertainty to bag parameter:  $\frac{\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)}{\Delta m_{B_s}^{|th.} / \Delta m_{B_s}^{|exp.}}$

$$\mathcal{B}(B_s \rightarrow \mu^+ \mu^-) = C \frac{\tau_{B_s} \eta_Y^2}{\hat{B}_{B_s} \eta_B} \frac{|\lambda_t^{SM} Y_0(x_t) + \lambda_{t'} \Delta Y_0|^2}{|\Delta_{12}^s| / \Delta m_{B_s}^{|exp}}$$

with  $\lambda_{t'} = V_{t'b} V_{t's}^*$





- Constraint from  $b \rightarrow s$  decay
- Constraint from  $b'$  and  $t'$  decay
- Constraint from  $b \rightarrow d$  decay
- Summary

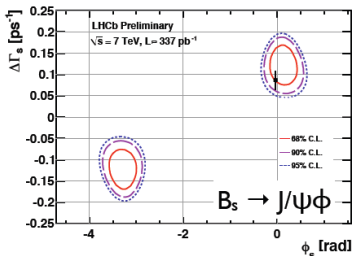
$$B_s \rightarrow \mu^+ \mu^-$$

$$B_s^0 \rightarrow J/\psi \phi(f_0)$$

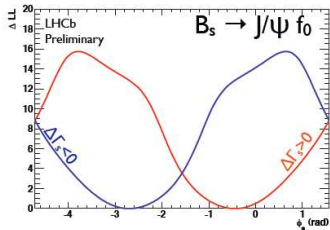
$$B \rightarrow K^* \mu^+ \mu^-$$

## $B_s^0 \rightarrow J/\psi \phi(f_0)$

### ▶ LHCb@LP2011 (G. Raven)



$$\phi_s = 0.13 \pm 0.18 \pm 0.07 \text{ rad} \quad (337 \text{ pb}^{-1})$$



$$\phi_s = -0.44 \pm 0.44 \pm 0.02 \text{ rad} \quad (378 \text{ pb}^{-1})$$



$$\phi_s = 0.03 \pm 0.16 \pm 0.07 \text{ rad}$$

### ▶ SM

$$\phi_s = -2\Phi_{B_c} = 0.0038 \pm 0.0010 \quad (\text{A. Lenz 1102.4274v1})$$

Consistent

# $\sin 2\Phi_{B_s}$

- ▶ The time-dependent CPV (TCPV) asymmetry for  
 $B_s \rightarrow J/\psi \phi(f_0)$

$$A(\Delta t) = -(S_{J/\psi \phi} \sin(\Delta M_{B_s} \Delta t) + \mathcal{A}_{J/\psi \phi} \cos(\Delta M_{B_s} \Delta t))$$

$$S_{J/\psi \phi} \cong \sin 2\Phi_{B_s}$$

free of hadronic uncertainty

$$B_s \rightarrow \mu^+ \mu^-$$

$$B_s^0 \rightarrow J/\psi \phi(f_0)$$

$$B \rightarrow K^* \mu^+ \mu^-$$

# $\sin 2\Phi_{B_s}$

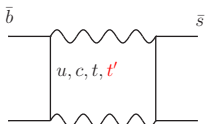
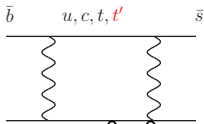
- The time-dependent CPV (TCPV) asymmetry for  $B_s \rightarrow J/\psi \phi(f_0)$

$$A(\Delta t) = -(\mathcal{S}_{J/\psi \phi} \sin(\Delta M_{B_s} \Delta t) + \mathcal{A}_{J/\psi \phi} \cos(\Delta M_{B_s} \Delta t))$$

$$\mathcal{S}_{J/\psi \phi} \cong \sin 2\Phi_{B_s}$$

free of hadronic uncertainty

- $2\Phi_{B_s} = \text{Arg}(M_{12}^s)$



$$M_{12}^s = \frac{G_F^2 M_W^2}{12\pi^2} m_{B_s} f_{B_s}^2 \hat{B}_{B_s} \eta_B \left[ (\lambda_t^{SM})^2 S_0 + 2\lambda_t^{SM} \lambda_{t'} \Delta S_0^{(1)} + \lambda_{t'}^2 \Delta S_0^{(2)} \right]$$

with  $\lambda_t^{SM} = V_{ts}^* V_{tb}$ ,  $\lambda_{t'} = V_{t's}^* V_{t'b}$

## 3 new measurements

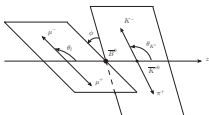
- Constraint from  $b \rightarrow s$  decay
- Constraint from  $b'$  and  $t'$  decay
- Constraint from  $b \rightarrow d$  decay
- Summary

$$B_s \rightarrow \mu^+ \mu^-$$

$$B_s^0 \rightarrow J/\psi \phi(f_0)$$

$$B \rightarrow K^* \mu^+ \mu^-$$

# $A_{FB}$ of $B \rightarrow K^* \mu^+ \mu^-$



## 3 new measurements

Constraint from  $b \rightarrow s$  decayConstraint from  $b'$  and  $t'$  decayConstraint from  $b \rightarrow d$  decay

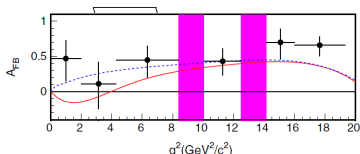
Summary

$$B_s \rightarrow \mu^+ \mu^-$$

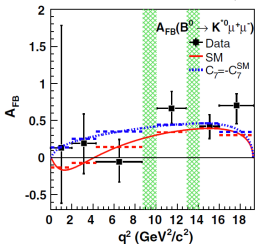
$$B_s^0 \rightarrow J/\psi \phi(f_0)$$

$$B \rightarrow K^* \mu^+ \mu^-$$

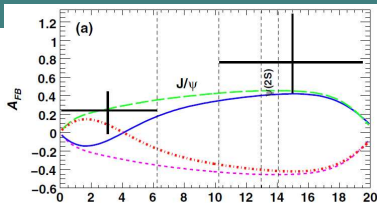
# $A_{FB}$ of $B \rightarrow K^* \mu^+ \mu^-$



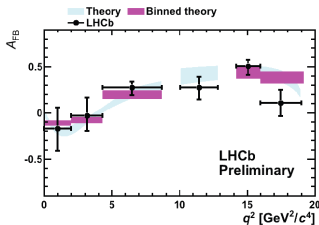
Belle, PRL 103, 171801 (2009)



CDF, PRL 106, 161801(2011)



BaBar, PRD 79, 031102(R) (2009)

309pb<sup>-1</sup> data, M. Patel, LHCb@EPS-HEP2011

# $A_{FB}$ of $B \rightarrow K^* \mu^+ \mu^-$

## Summary of Theoretical status

- ▶ The contribution of heavy quark to  $b \rightarrow s \ell^+ \ell^-$

W.-S. Hou, R.S. Willey and A. Soni, Phys. Rev. Lett. 58, 1608 (1987)

- ▶  $B \rightarrow K^* \ell^+ \ell^-$  @ NLL , naive factorization

A. Ali, P. Ball, L. T. Handoko, and G. Hiller, PRD 61, 074024 (2000)

A. J. Buras, M. Münz, PRD 52, 186 (1995), ...

- ▶  $B \rightarrow K^* \ell^+ \ell^-$  @ NNLL , QCDF/SCET

- ▶ Formulation

U. Egede, T. Hurth, J. Matias, M. Ramon, W. Reece, JHEP11 (2008) 032

- ▶ Wilson coefficient

C. Bobeth, M. Misiak, J. Urban, NPB 574 (2000) 291;

C. Bobeth, P. Gambino, M. Gorbahn and U. Haisch, JHEP 04 (2004) 071

H. Asatryan, H. Asatrian, C. Greub, M. Walker, PLB 507 (2001) 162

- ▶ Form factor:

M. Beneke, Th. Feldmann, NPB 592 (2001) 3

# $A_{FB}$ of $B \rightarrow K^* \mu^+ \mu^-$

## ► Decay amplitude

$$\mathcal{M}_{b \rightarrow s \ell^+ \ell^-} = -\frac{G_F \alpha}{\sqrt{2} \pi} \lambda_t \left[ \tilde{C}_9^{\text{eff}}(\bar{s} \gamma_\mu \mathbb{P}_L b)(\bar{\ell} \gamma^\mu \ell) + \tilde{C}_{10}^{\text{eff}}(\bar{s} \gamma_\mu \mathbb{P}_L b)(\bar{\ell} \gamma^\mu \gamma_5 \ell) \right. \\ \left. - 2 \frac{m_b}{q^2} \tilde{C}_7^{\text{eff}}(\bar{s} i \sigma^{\mu\nu} q_\nu \mathbb{P}_R b)(\bar{\ell} \gamma^\mu \ell) \right]$$



$$B_s \rightarrow \mu^+ \mu^-$$

$$B_s^0 \rightarrow J/\psi \phi(f_0)$$

$$B \rightarrow K^* \mu^+ \mu^-$$

## $A_{FB}$ of $B \rightarrow K^* \mu^+ \mu^-$

- Decay amplitude

$$\mathcal{M}_{b \rightarrow s \ell^+ \ell^-} = -\frac{G_F \alpha}{\sqrt{2} \pi} \lambda_t \left[ \tilde{C}_9^{\text{eff}} (\bar{s} \gamma_\mu \mathbb{P}_L b) (\bar{\ell} \gamma^\mu \ell) + \tilde{C}_{10}^{\text{eff}} (\bar{s} \gamma_\mu \mathbb{P}_L b) (\bar{\ell} \gamma^\mu \gamma_5 \ell) \right. \\ \left. - 2 \frac{m_b}{q^2} \tilde{C}_7^{\text{eff}} (\bar{s} i \sigma^{\mu\nu} q_\nu \mathbb{P}_R b) (\bar{\ell} \gamma^\mu \ell) \right]$$

- take naive factorization as the example

$$\frac{dA_{FB}}{dq^2} \propto \text{Re}(\tilde{C}_9^{\text{eff}} \tilde{C}_{10}^*) + F \text{Re}(\tilde{C}_7^{\text{eff}} \tilde{C}_{10}^*)$$

with  $\tilde{C}_i = \tilde{C}_i^t + \frac{\lambda_{t'}}{\lambda_t} \Delta C_i$  in SM4.

# $A_{FB}$ of $B \rightarrow K^* \mu^+ \mu^-$

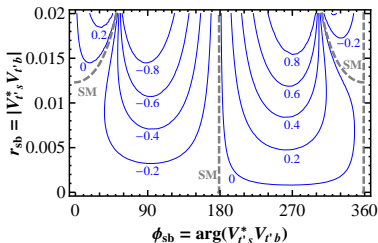
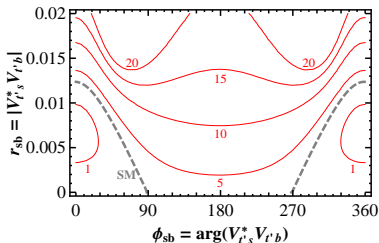
## Some remarks

- ▶  $C_{10}$  is general complex
- ▶ In realistic calculation, NLO Wilson coefficients,  $O(\alpha_s)$  order soft form factors, and non-factorization contribution are included
- ▶ QCDF/SCET approach is applicable only at  $q^2 < 6\text{GeV}^2$
- ▶ **The  $A_{FB}$  zero crossing point  $s_0$  is free of hadronic uncertainty**
- ▶ Fourth generation information:  $\lambda_{t'} = V_{t's}^* V_{tb}$  and  $m_{t'}$  can be read from  $A_{FB}$  and  $s_0$

# Constraint of $V_{t'b} V_{t's}^*$

# Contours in $(r_{sb}, \phi_{sb})$

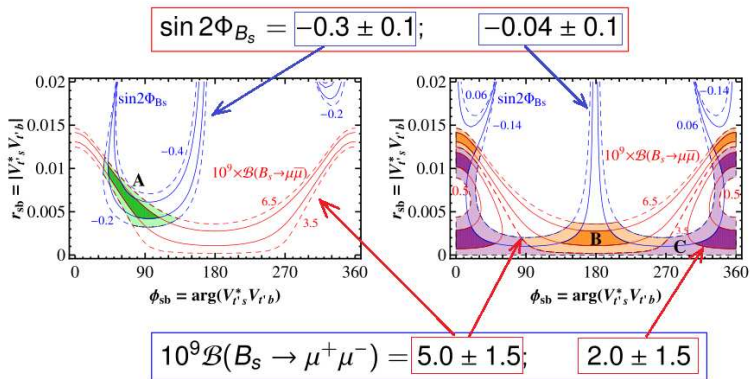
$$\lambda_{t'} = V_{t's}^* V_{t'b} = r_{sb} e^{i\phi_{sb}}, \quad m_{t'} = 550 \text{ GeV}$$


 $\sin 2\Phi_{B_s}$ 

 $10^9 \times \mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$ 

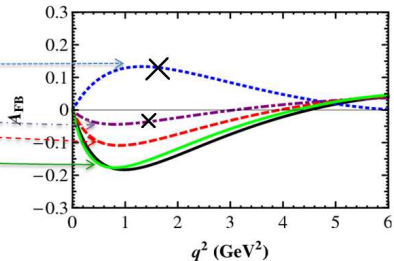
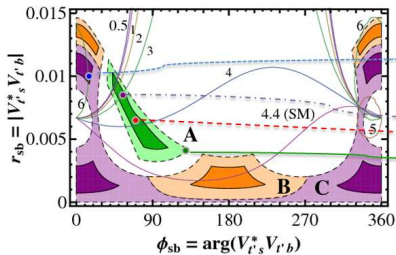
$\sin 2\Phi_{B_s}$  and  $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$  contours are combined to determine the possible regions.

# Allowed $(r_{sb}, \phi_{sb})$ region

$m_{t'} = 550 \text{ GeV}$ , LHCb  $> 1 \text{ fb}^{-1}$  (2012)

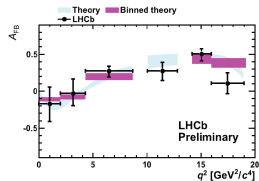


# Allowed region and $A_{FB}$

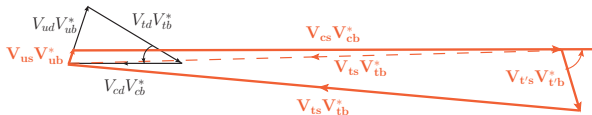


- ▶ further discrimination from  $A_{FB}$  and  $s_0$
- ▶ different behaviors of  $A_{FB}$
- ▶ some parameter space can be excluded by LHCb

$$|V_{t'b} V_{t's}^*| \leq 0.008 \quad \text{for } m_{t'} = 550 \text{ GeV}$$



# $b \rightarrow s$ quadrangle



- ▶ “ $b \rightarrow d$ ” triangle has been examined a lot
- ▶  $b \rightarrow s$  relevant to BAU
- ▶ once  $m_{t'}$  is known,  $b \rightarrow s$  quadrangle can be determined

Introduction

3 new measurements

Constraint from  $b \rightarrow s$  decay

**Constraint from  $b'$  and  $t'$  decay**

Constraint from  $b \rightarrow d$  decay

Summary

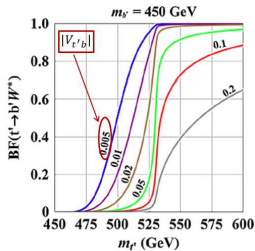
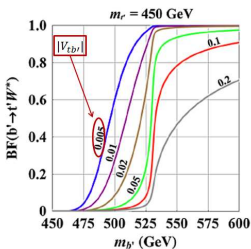
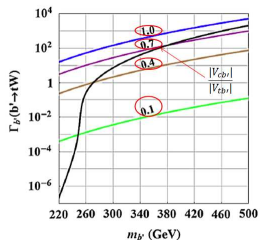
## Constraint of $|V_{t'b}|$ and $|V_{tb'}|$



# The decay of $b'$ and $t'$

## More than $b' \rightarrow tW$

Y. Chao, K. Chen, S. Chen, W. Hou, B. Huang, Y. Lei, PRD 84, 014029 (2011)



$b' \rightarrow cW$

- ▶ When  $\left| \frac{V_{cb'}}{V_{tb'}} \right| > 0.7$ ,  $b' \rightarrow cW$  will dominate.
- ▶ Assuming  $b' \rightarrow t'W^*$  and  $b' \rightarrow tW$  to be two dominant modes,  $\mathcal{B}(b' \rightarrow t'W^*)$  is plotted above, which depends on  $V_{tb'}$ .
- ▶ Once  $m_{b'}$  and relevant branching ratios are measured, mixing matrix can be determined.

Introduction

3 new measurements

Constraint from  $b \rightarrow s$  decay

Constraint from  $b'$  and  $t'$  decay

Constraint from  $b \rightarrow d$  decay

Summary

Future constraint of  $V_{t'b} V_{t'd}^*$

$b \rightarrow d$ 

- ▶ In  $b \rightarrow s$ ,  $\lambda_u$  is neglected,  $\lambda_c = -\lambda_t(-\lambda_{t'})$

$b \rightarrow d$ 

- ▶ In  $b \rightarrow s$ ,  $\lambda_U$  is neglected,  $\lambda_c = -\lambda_t(-\lambda_{t'})$
- ▶ But in  $b \rightarrow d$ ,  $\lambda_U$  is the same order of  $\lambda_t$  and  $\lambda_c$  ( $\lambda_{t'}$ ), should be kept

$b \rightarrow d$ 

- ▶ In  $b \rightarrow s$ ,  $\lambda_u$  is neglected,  $\lambda_c = -\lambda_t(-\lambda_{t'})$
- ▶ But in  $b \rightarrow d$ ,  $\lambda_u$  is the same order of  $\lambda_t$  and  $\lambda_c$  ( $\lambda_{t'}$ ), should be kept
- ▶ The  $\lambda_u$  contribution can be interpreted by exclusive process

$$B \rightarrow \rho/\pi \ell^+ \ell^-$$

## $b \rightarrow d$

- ▶ In  $b \rightarrow s$ ,  $\lambda_u$  is neglected,  $\lambda_c = -\lambda_t(-\lambda_{t'})$
- ▶ But in  $b \rightarrow d$ ,  $\lambda_u$  is the same order of  $\lambda_t$  and  $\lambda_c$  ( $\lambda_{t'}$ ), should be kept
- ▶ The  $\lambda_u$  contribution can be interpreted by exclusive process  
 $B \rightarrow \rho/\pi \ell^+ \ell^-$
- ▶ Experimentally,  $B \rightarrow \pi \ell^+ \ell^-$  has been measured

$$\mathcal{B}(B \rightarrow \pi \ell^+ \ell^-) < 9.1 \times 10^{-8} \text{ @90\% (BaBar 2007)}$$

$$\mathcal{B}(B \rightarrow \pi \ell^+ \ell^-) < 6.2 \times 10^{-8} \text{ @90\% (Belle 2008)}$$

## $b \rightarrow d$

- ▶ In  $b \rightarrow s$ ,  $\lambda_u$  is neglected,  $\lambda_c = -\lambda_t(-\lambda_{t'})$
- ▶ But in  $b \rightarrow d$ ,  $\lambda_u$  is the same order of  $\lambda_t$  and  $\lambda_c$  ( $\lambda_{t'}$ ), should be kept
- ▶ The  $\lambda_u$  contribution can be interpreted by exclusive process

$$B \rightarrow \rho/\pi \ell^+ \ell^-$$

- ▶ Experimentally,  $B \rightarrow \pi \ell^+ \ell^-$  has been measured

$$\mathcal{B}(B \rightarrow \pi \ell^+ \ell^-) < 9.1 \times 10^{-8} \text{ @90\% (BaBar 2007)}$$

$$\mathcal{B}(B \rightarrow \pi \ell^+ \ell^-) < 6.2 \times 10^{-8} \text{ @90\% (Belle 2008)}$$

- ▶ How about  $\mathcal{B}(B \rightarrow \rho \ell^+ \ell^-)$  and  $A_{FB}$ ?

## $b \rightarrow d$

- ▶ In  $b \rightarrow s$ ,  $\lambda_u$  is neglected,  $\lambda_c = -\lambda_t(-\lambda_{t'})$
- ▶ But in  $b \rightarrow d$ ,  $\lambda_u$  is the same order of  $\lambda_t$  and  $\lambda_c$  ( $\lambda_{t'}$ ), should be kept
- ▶ The  $\lambda_u$  contribution can be interpreted by exclusive process

$$B \rightarrow \rho/\pi \ell^+ \ell^-$$

- ▶ Experimentally,  $B \rightarrow \pi \ell^+ \ell^-$  has been measured

$$\mathcal{B}(B \rightarrow \pi \ell^+ \ell^-) < 9.1 \times 10^{-8} \text{ @90\% (BaBar 2007)}$$

$$\mathcal{B}(B \rightarrow \pi \ell^+ \ell^-) < 6.2 \times 10^{-8} \text{ @90\% (Belle 2008)}$$

- ▶ How about  $\mathcal{B}(B \rightarrow \rho \ell^+ \ell^-)$  and  $A_{FB}$ ?
- ▶ In progress ...



# Summary

- ▶ The 3 new measurements are studied in SM4.

# Summary

- ▶ The 3 new measurements are studied in SM4.
- ▶ Combining the latest experimental results, with the 2012 expectation, we give new constraints of  $V_{t'b} V_{t's}^*$ , which helps to determine CKM4.

## Summary

- ▶ The 3 new measurements are studied in SM4.
- ▶ Combining the latest experimental results, with the 2012 expectation, we give new constraints of  $V_{t'b} V_{t's}^*$ , which helps to determine CKM4.
- ▶  $b \rightarrow s$  quadrangle could be determined.

## Summary

- ▶ The 3 new measurements are studied in SM4.
- ▶ Combining the latest experimental results, with the 2012 expectation, we give new constraints of  $V_{t'b} V_{t's}^*$ , which helps to determine CKM4.
- ▶  $b \rightarrow s$  quadrangle could be determined.
- ▶ From  $t'$  and  $b'$  decay, some of CKM matrix elements can be determined separately.

## Summary

- ▶ The 3 new measurements are studied in SM4.
- ▶ Combining the latest experimental results, with the 2012 expectation, we give new constraints of  $V_{t'b} V_{t's}^*$ , which helps to determine CKM4.
- ▶  $b \rightarrow s$  quadrangle could be determined.
- ▶ From  $t'$  and  $b'$  decay, some of CKM matrix elements can be determined separately.
- ▶  $b \rightarrow d \dots$