QCD

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Titles of lectures

- Lecture I: Factorization theorem
- Lecture II: Evolution and resummation
- Lecture III: PQCD for Jet physics
- Lecture IV: Hadronic heavy-quark decays

References

- Partons, Factorization and Resummation, TASI95, G. Sterman, hep-ph/9606312
- Jet Physics at the Tevatron , A. Bhatti and D. Lincoln, arXiv:1002.1708
- QCD aspects of exclusive B meson decays, H.-n. Li, Prog.Part.Nucl.Phys.51 (2003) 85, hep-ph/0303116

Lecture I
Factorization theorem

Hsiang-nan Li

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Outlines

- QCD Lagrangian and Feynman rules
- Infrared divergence and safety
- DIS and collinear factorization
- Application of factorization theorem
- kT factorization

QCD Lagrangian

See Luis Alvarez-Gaume’s lectures

Lagrangian

- SU(3) QCD Lagrangian

$$\mathcal{L}_{QCD} = \bar{\psi}(i\not{D}_a T_a - m)\psi - \frac{1}{4} F_a^{\mu\nu} F_{\mu\nu,a}$$

- Covariant derivative, gluon field tensor

$$\begin{aligned} D_a^\mu &= \partial^\mu + igA_a^\mu \\ F_a^{\mu\nu} &= \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - g f_{abc} A_b^\mu A_c^\nu \end{aligned}$$

- Color matrices and structure constants

$$[T_a^{(F)}, T_b^{(F)}] = i f_{abc} T_c^{(F)}, \quad (T_a^{(A)})_{bc} = -i f_{abc}$$

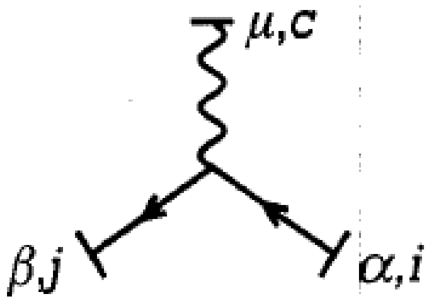
Gauge-fixing

- Add gauge-fixing term to remove spurious degrees of freedom

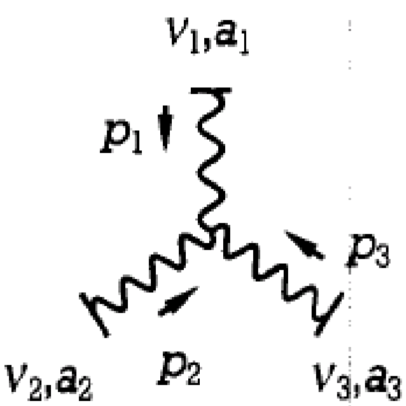
$$\mathcal{L}_{QCD} = \bar{\psi}(i \not{D}_a T_a - m)\psi - \frac{1}{4}F_a^{\mu\nu}F_{\mu\nu,a} - \frac{1}{2}\lambda(\partial_\mu A_a^\mu)^2 + \partial_\mu \eta_a^\dagger(\partial^\mu + g f_{abc} A_c^\mu)\eta_b$$

- Ghost field from Jacobian of variable change, as fixing gauge

Feynman rules

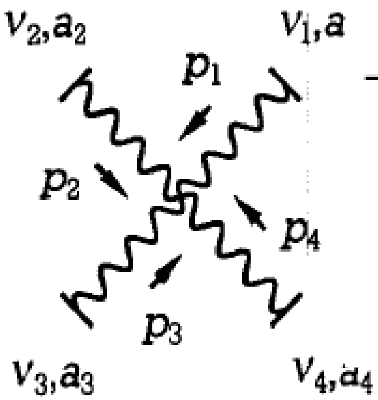


$$-ig\gamma_\mu T_c$$

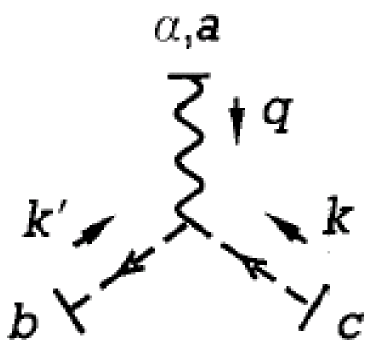


$$-gf_{a_1a_2a_3}[g^{\nu_1\nu_2}(p_1-p_2)^{\nu_3}$$
$$+g^{\nu_2\nu_3}(p_2-p_3)^{\nu_1}+g^{\nu_3\nu_1}(p_3-p_1)^{\nu_2}$$

Feynman rules



$$
\begin{aligned}
-ig^2\big[&f_{ea_1a_2}f_{ea_3a_4}(g^{\nu_1\nu_3}g^{\nu_2\nu_4}-g^{\nu_1\nu_4}g^{\nu_2\nu_3})\\
&+f_{ea_1a_3}f_{ea_4a_2}(g^{\nu_1\nu_4}g^{\nu_3\nu_2}-g^{\nu_1\nu_2}g^{\nu_3\nu_4})\\
&+f_{ea_1a_4}f_{ea_2a_3}(g^{\nu_1\nu_2}g^{\nu_4\nu_3}-g^{\nu_1\nu_3}g^{\nu_4\nu_2})\big]
\end{aligned}
$$



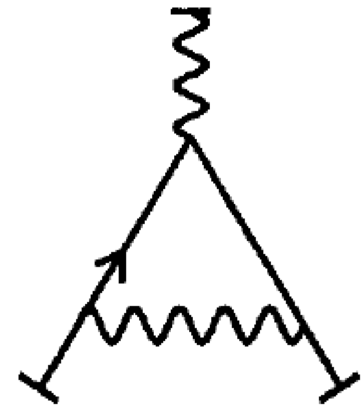
$$g f_{abc} k'_{\alpha}$$

Asymptotic freedom

- QCD confinement at low energy, hadronic bound states: pion, proton,…
- Manifested by infrared divergences in perturbative calculation of bound-state properties
- Asymptotic freedom at high energy leads to small coupling constant
- Perturbative QCD for high-energy processes

Infrared divergence and safty

Vertex correction



- Start from vertex correction as an example

$$\int \frac{d^4 l}{(2\pi)^4}(-ig\gamma^\nu T_a)\frac{i(\not{p}_1 - \not{l})}{(p_1 - l)^2 + i\epsilon}(-ie\gamma_\mu)$$
$$\times \frac{i(\not{p}_2 - \not{l})}{(p_2 - l)^2 + i\epsilon}(-ig\gamma_\nu T_a)\frac{-i}{l^2 + i\epsilon}$$

- Inclusion of counterterm is understood

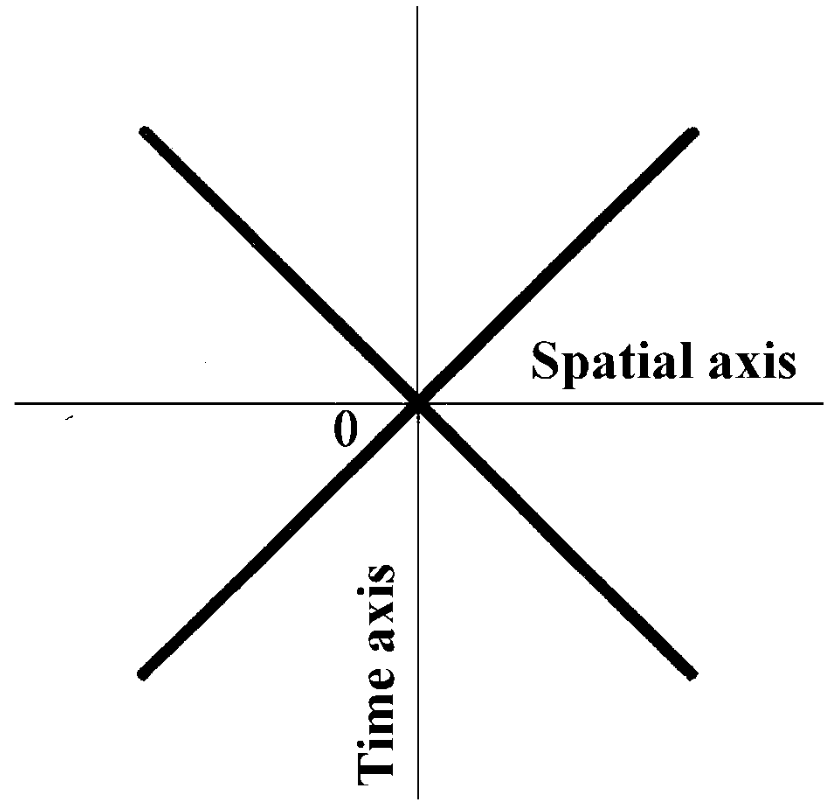
Light-cone coordinates

- Analysis of infrared divergences simplified

$$l = (l^+, l^-, l_T)$$

$$l^\pm \equiv \frac{l^0 \pm l^3}{\sqrt{2}}$$

- As particle moves along light cone, only one large component is involved



Leading regions

- Collinear region $l = (l^+, l^-, l_T) \sim (E, \Lambda^2/E, \Lambda)$
- Soft region $l \sim (\Lambda, \Lambda, \Lambda)$
- Infrared gluon $l^2 \sim \Lambda^2$
- Hard region $l \sim (E, E, E)$

- They all generate log divergences

$$\int \frac{d^4 l}{l^4} \sim \int \frac{d^4 \Lambda}{\Lambda^4} \sim \int \frac{d^4 E}{E^4} \sim \log$$

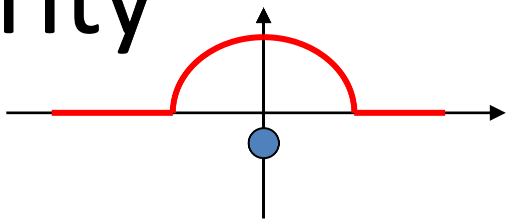
Contour integration

- In terms of light-cone coordinates, vertex correction is written as

$$\int \frac{dl^+ dl^- d^2 l_T}{(2\pi)^4} \frac{1}{2(l^+ - p_1^+)l^- - l_T^2 + i\epsilon}$$
$$\times \frac{1}{2l^+(l^- - p_2^-) - l_T^2 + i\epsilon} \frac{1}{2l^+ l^- - l_T^2 + i\epsilon}$$

- Study pole structures, since IR divergence comes from vanishing denominator

Pinched singularity



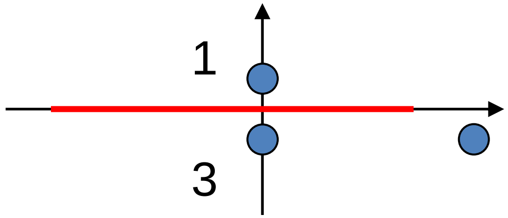
- Contour integration over l-

$$0 < l^+ < p_1^+$$

$$l^- = \frac{l_T^2}{2(l^+ - p_1^+)} + i\epsilon, \quad l^- = p_2^- + \frac{l_T^2}{2l^+} - i\epsilon \quad l^- = \frac{l_T^2}{2l^+} - i\epsilon$$

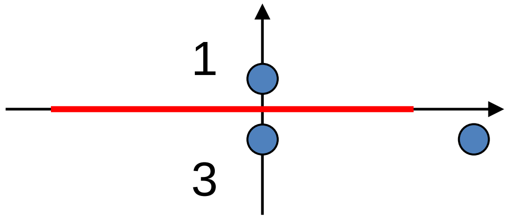
- collinear region

$$l^+ \sim O(p_1^+)$$



- Soft region

$$l^+ \sim O(l_T)$$



Double IR poles

- Contour integration over l- gives

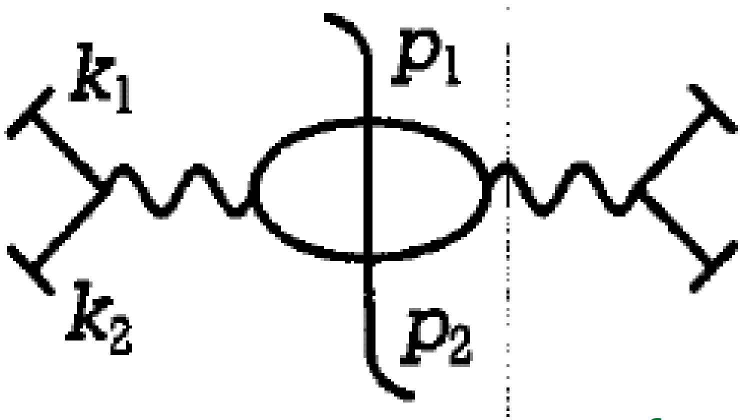
$$\frac{-i}{2p_1^+}\int\frac{dl^+d^2l_T}{(2\pi)^3}\frac{p_1^+-l^+}{2p_2^-l^+(p_1^+-l^+)+p_1^+l_T^2}\frac{1}{l_T^2}$$

$$\approx\ \frac{-i}{4p_1^+p_2^-}\frac{1}{(2\pi)^3}\int\frac{dl^+}{l^+}\int\frac{d^2l_T}{l_T^2}$$

e+e- annihilation

- calculate e+e- annihilation
- cross section = |amplitude|^2
- Born level

final-state cut



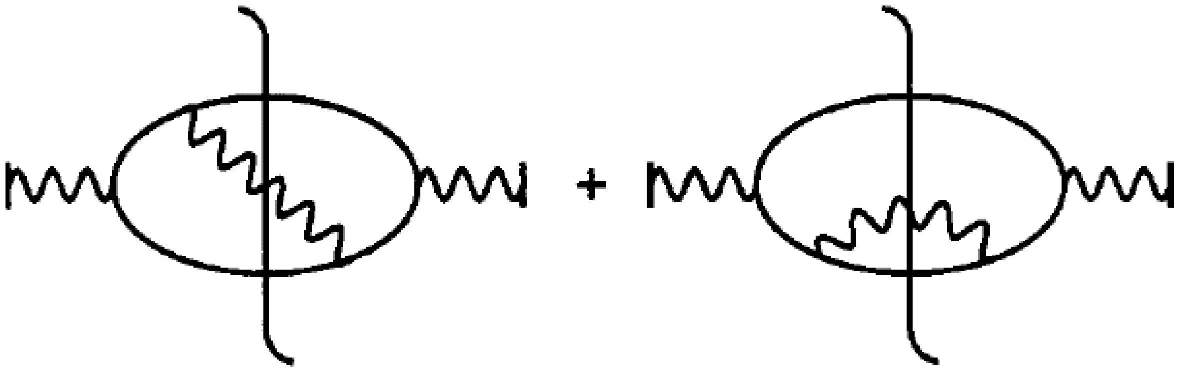
fermion charge

$$\sigma_{\text{tot}} = N(4\pi\alpha^2/3q^2)\left(\sum_f Q_f^{\,2}\right)$$

momentum transfer squared

Real corrections

- Radiative corrections reveal two types of infrared divergences from on-shell gluons
- Collinear divergence: l parallel P1, P2
- Soft divergence: l approaches zero



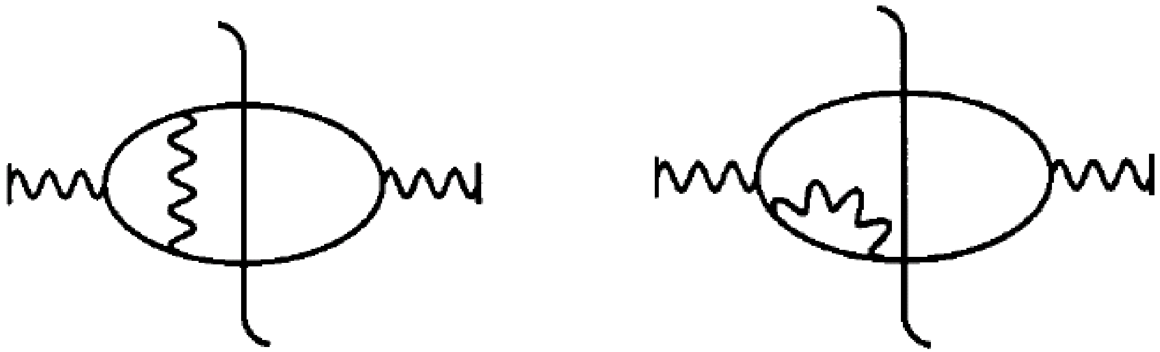
overlap of collinear and soft divergences

$$2NC_2(\mathrm{F})Q_f{}^2(\alpha\alpha_s/\pi)q^2(4\pi\mu^2/q^2)^{2\epsilon}[(1-\epsilon)/\Gamma(2-2\epsilon)]$$

$$\times\,[\epsilon^{-2}+\tfrac{3}{2}\epsilon^{-1}-\tfrac{1}{2}\pi^2+\tfrac{19}{4}+\mathrm{O}(\epsilon)].$$

Virtual corrections

- Double infrared pole also appears in virtual corrections with a minus sign



$$-2NC_2(\mathrm{F})Q_f^{\,2}(\alpha\alpha_s/\pi)q^2(4\pi\mu^2/q^2)^{2\epsilon}[(1-\epsilon)/\Gamma(2-2\epsilon)]$$

$$\times\,[\epsilon^{-2}+\tfrac{3}{2}\epsilon^{-1}-\tfrac{1}{2}\pi^2+4+\mathrm{O}(\epsilon)]$$

overlap of collinear and
soft divergences

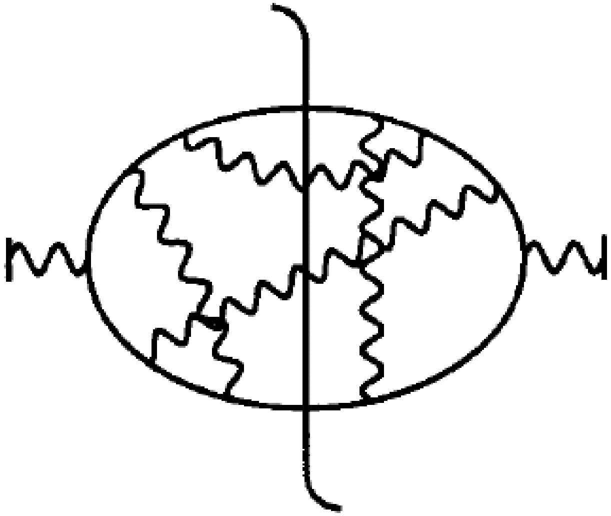
Infrared safety

- Infrared divergences cancel between real and virtual corrections
- Imaginary part of off-shell photon self-energy corrections
- Total cross section (physical quantity) of e+e- -> X is infrared safe

$$\mathrm{Im}\frac{-i}{p^2+i\varepsilon}\propto\delta(p^2)$$

propagator

on-shell
final state



KLN theorem

- Kinoshita-Lee-Neuberger theorem:
  IR cancellation occurs as integrating over all phase space of final states
- Naïve perturbation applies

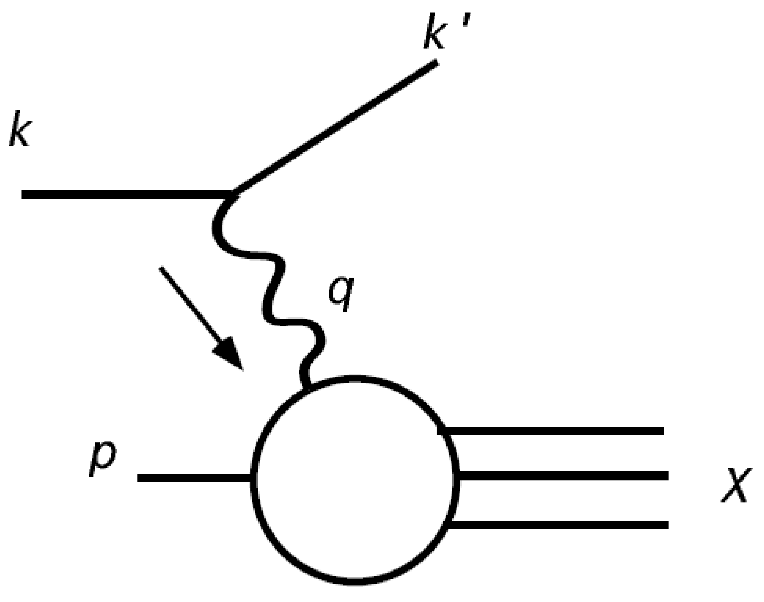
$$\sigma_{\text{tot}}(q^2) = N(4\pi\alpha^2/3q^2)\sum_f Q_f{}^2[1 + (\alpha_s/\pi)\tfrac{3}{4}C_2(\mathrm{F})]$$

- Used to determine the coupling constant

DIS and collinear factorization

Deep inelastic scattering

- Electron-proton DIS l(k)+N(p) -> l(k')+X
- Large momentum transfer $-q^2=(k-k')^2=Q^2$
- Calculation of cross section suffers IR divergence --- nonperturbative dynamics in the proton
- Factor out nonpert part from DIS, and leave it to other methods?



Structure functions for DIS

- Standard example for factorization theorem

$$d\sigma = \frac{d^3k'}{2s|\vec{k}'|}\frac{1}{(q^2)^2} L^{\mu\nu}(k,q) W^{\gamma N}_{\mu\nu}(p,q)$$

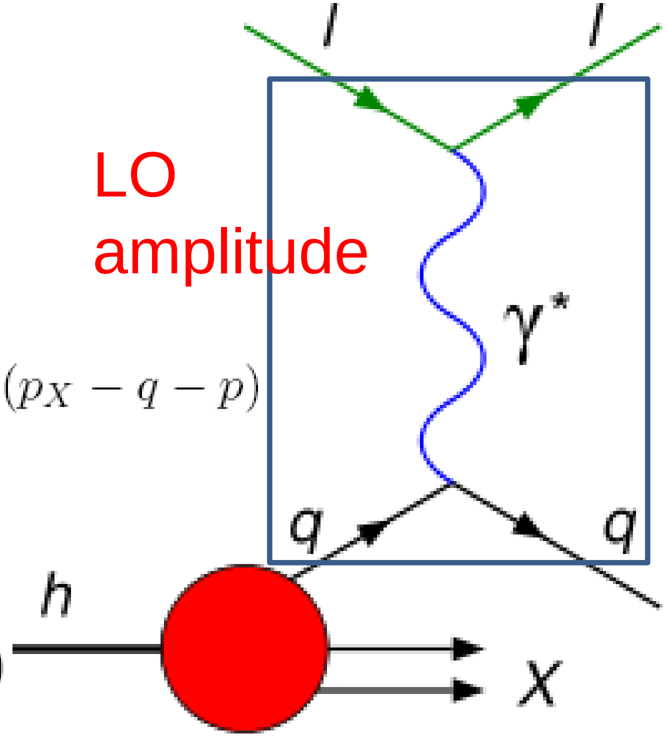
$$L^{\mu\nu} \equiv \frac{e^2}{8\pi^2} tr\left[\not k \gamma^\mu \not k' \gamma^\nu\right]$$

$$W^{\gamma N}_{\mu\nu} \equiv \frac{1}{8\pi}\sum_{\text{spins }\sigma}\sum_X \langle N(p,\sigma) \mid J_\mu(0) \mid X\rangle \times \langle X \mid J_\nu(0) \mid N(p,\sigma)\rangle (2\pi)^4 \delta^4(p_X - q - p)$$

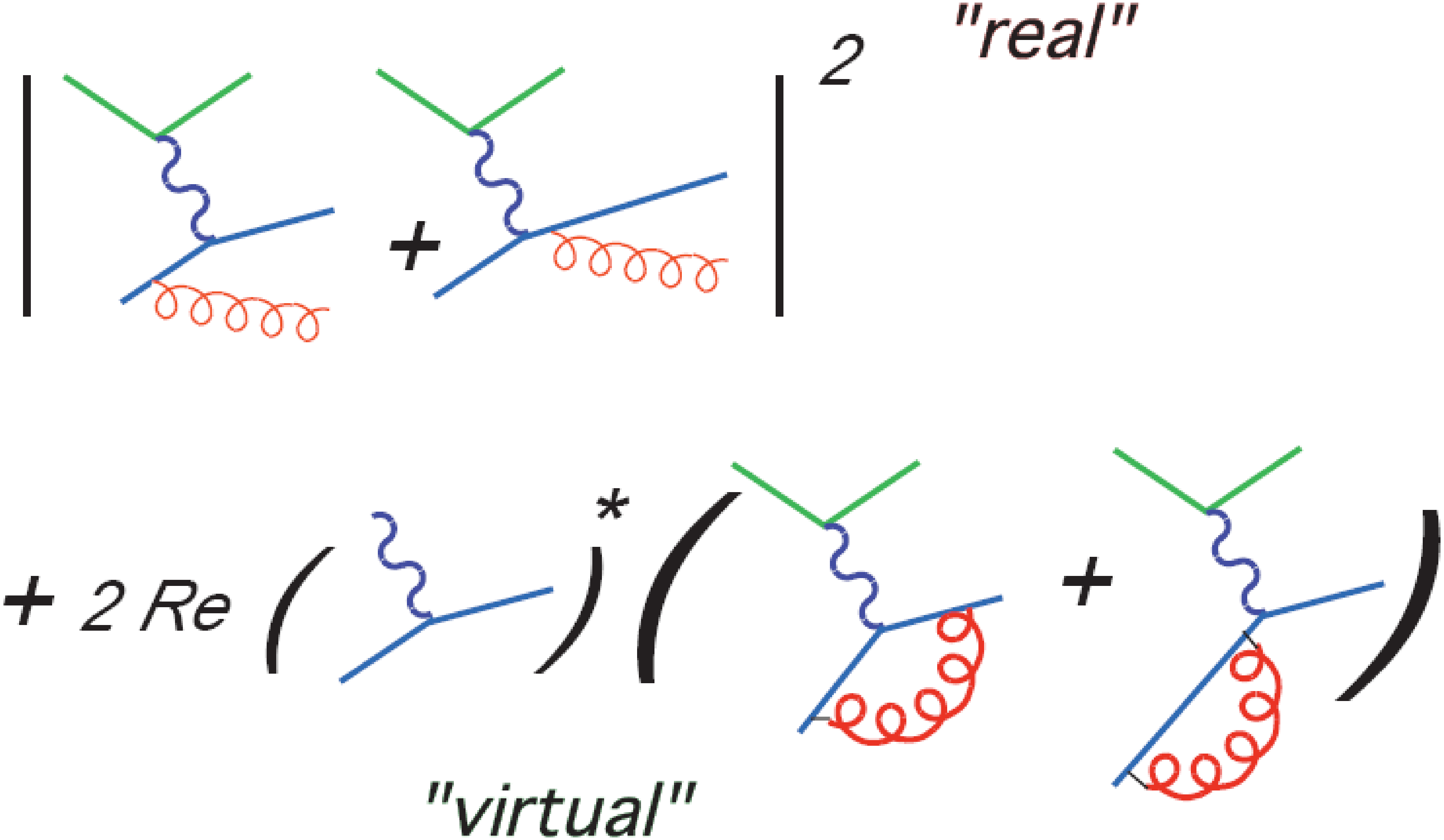
$$W^{\gamma N}_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1^{\gamma N}(x,q^2) + \left(p_\mu + q_\mu\left(\frac{1}{2x}\right)\right)\left(p_\nu + q_\nu\left(\frac{1}{2x}\right)\right) W_2^{\gamma N}(x,q^2)$$

$$x = -\frac{q^2}{2p\cdot q} \equiv \frac{Q^2}{2p\cdot q}$$

LO amplitude



NLO diagrams



NLO total cross section

$$F_1^{\gamma N}(x,Q^2) \equiv W_1^{\gamma N}, \qquad F_2^{\gamma N}(x,Q^2) = p\cdot q\, W_2^{\gamma N}$$

$$F_2^{\gamma q_f}(x,Q^2) = Q_f^2\, x \left\{ \delta(1-x) \right.$$

LO term

$$+ \frac{\alpha_s}{2\pi} C_F \left[\frac{1+x^2}{1-x} \left(\ln\frac{(1-x)}{x} - \frac{3}{4} \right) + \frac{1}{4}(9+5x) \right]_+$$

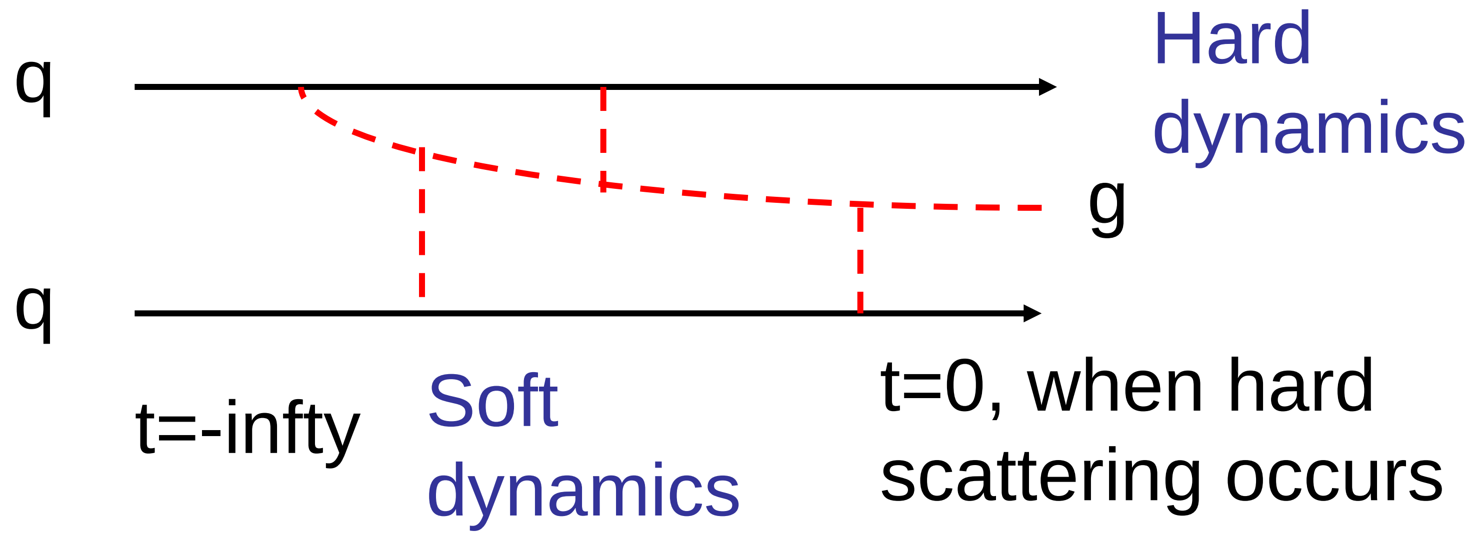
$$\left. + \frac{\alpha_s}{2\pi} C_F \left[\frac{1+x^2}{1-x} \right]_+ \left(4\pi\mu^2 e^{-\gamma_E} \right)^{\varepsilon} \int_0^{Q^2} \frac{dk_T^2}{k_T^{2+2\varepsilon}} \right\} + \cdots$$

plus function

infrared divergence

$$\int_0^1 dx\, \frac{f(x)}{(1-x)_+} \equiv \int_0^1 dx\, \frac{f(x)-f(1)}{(1-x)} \qquad \int_0^{Q^2} \frac{dk_T^2}{k_T^{2+2\varepsilon}} = \frac{1}{-\varepsilon} Q^{-2\varepsilon}$$

IR divergence is physical!



- It's a long-distance phenomenon, related to confinement.
- All physical hadronic high-energy processes involve both soft and hard dynamics.

Collinear divergence

- Integrated over final state kinematics, but not over initial state kinematics. KLN theorem does not apply
- Collinear divergence for initial state quark exists. Confinement of initial bound state
- Soft divergences cancel between virtual and real diagrams (proton is color singlet)
- Subtracted by PDF, evaluated in perturbation

hard kernel or Wilson coefficient

$$F_2^{\gamma q_f}(x, Q^2) = C_2^{(0)} \otimes \phi^{(0)} + \frac{\alpha_s}{2\pi} C_2^{(1)} \otimes \phi^{(0)} + \frac{\alpha_s}{2\pi} C_2^{(0)} \otimes \phi^{(1)} + \dots$$

Assignment of IR divergences

$$F_2^{\gamma q_f}(x,Q^2) = C_2^{(0)} \otimes \phi^{(0)} + \frac{\alpha_s}{2\pi} C_2^{(1)} \otimes \phi^{(0)} + \frac{\alpha_s}{2\pi} C_2^{(0)} \otimes \phi^{(1)} + \dots$$

$$F_2^{\gamma q_f}(x,Q^2) = Q_f^2 \, x \left\{ \delta(1-x) \right.$$

$$+ \frac{\alpha_s}{2\pi} C_F \left[\frac{1+x^2}{1-x}\left(\ln\frac{(1-x)}{x} - \frac{3}{4}\right) + \frac{1}{4}(9+5x)\right]_+$$

$$\left. + \frac{\alpha_s}{2\pi} C_F \left[\frac{1+x^2}{1-x}\right]_+ \left(4\pi\mu^2 \mathrm{e}^{-\gamma_E}\right)^{\varepsilon} \int_0^{Q^2} \frac{dk_T^2}{k_T^{2+2\varepsilon}} \right\} + \cdots$$

$$\phi_{q/q'}^{(0)}(\xi) = \delta_{qq'}\,\delta(1-\xi) \qquad C_{1,2}^{\gamma q(0)}\left(\frac{x}{\xi}\right) = Q_q^2\,\delta(1-x/\xi)$$

$$\int_x^1 d\xi\, \delta(1-x/\xi)\,\delta(1-\xi) = \delta(1-x)$$

Parton distribution function

- Assignment at one loop

$$\overline{\mathrm{MS}}: \qquad \phi^{(1)}_{q/q}(x,\mu^2) = \left(4\pi\mu^2 e^{-\gamma_E}\right)^{\varepsilon} \; P_{qq}(x) \; \int_0^{\mu^2} \frac{dk_T^2}{k_T^{2+2\varepsilon}}$$

$$C_2^{(1)}(x)_{\overline{\mathrm{MS}}} = P_{qq}(x)\ln(Q^2/\mu^2) + \mu\text{−independent}$$

- PDF in terms of hadronic matrix element reproduces IR divergence at each order

$$\phi_{f/N}(\xi,\mu^2)$$

$$= \int \frac{dy^-}{2\pi} \exp(-i\xi P^+ y^-)$$

$$\times \frac{1}{2}\sum_\sigma \langle N(P,\sigma)|\bar{q}_f(0,y^-,0_T)\frac{1}{2}\gamma^+ W(y^-,0) q_f(0,0,0_T)|N(P,\sigma)\rangle$$

Wilson links

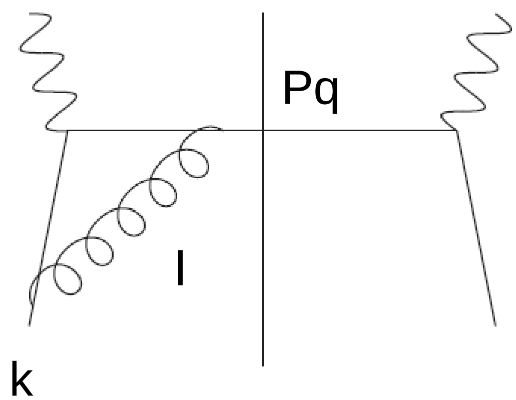
$$P^{(1)}_{qq}(x) = C_F \left[\frac{1+x^2}{1-x}\right]_+$$

splitting kernel

Factorization at diagram level
Eikonal approximation

$$\not{P}_q \gamma_\nu \frac{\not{P}_q + \not{l}}{(P_q+l)^2}\gamma^\mu \frac{\not{k}+\not{l}}{(k+l)^2}\gamma^\nu, \quad k \propto k^+, \quad P_q \propto P_q^-$$

$$\approx \not{P}_q\gamma^- \frac{\not{P}_q+\not{l}}{(P_q+l)^2}\gamma^\mu\frac{\not{k}+\not{l}}{(k+l)^2}\gamma^+, \quad l\propto l^+$$



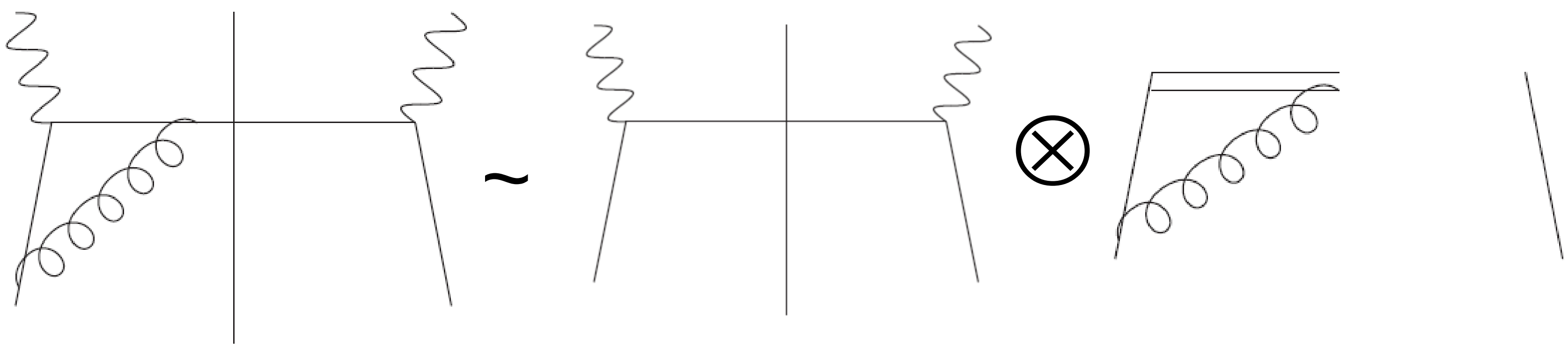
$$\approx \not{P}_q\gamma^-\frac{\not{P}_q}{2P_q\cdot l}\gamma^\mu\frac{\not{k}+\not{l}}{(k+l)^2}\gamma^+$$

$$\approx \not{P}_q\frac{2P_q^- - \not{P}_q\gamma^-}{2P_q\cdot l}\gamma^\mu \frac{\not{k}+\not{l}}{(k+l)^2}\gamma^\nu, \quad \not{P}_q\not{P}_q = P_q^2 = 0$$

$$\approx \not{P}_q\gamma^\mu\frac{\not{k}+\not{l}}{(k+l)^2}\gamma^\nu\frac{n_{-\nu}}{n_-\cdot l}$$

Effective diagrams

- Factorization of collinear gluons at leading power leads to Wilson line $W(y^-,0)$ necessary for gauge invariance
- Collinear gluons also change parton momentum

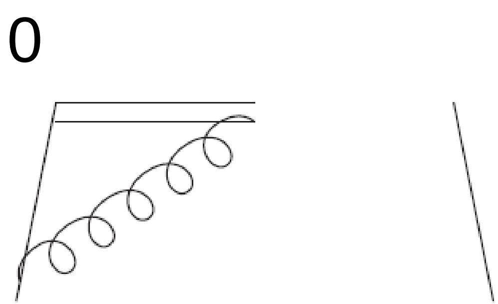


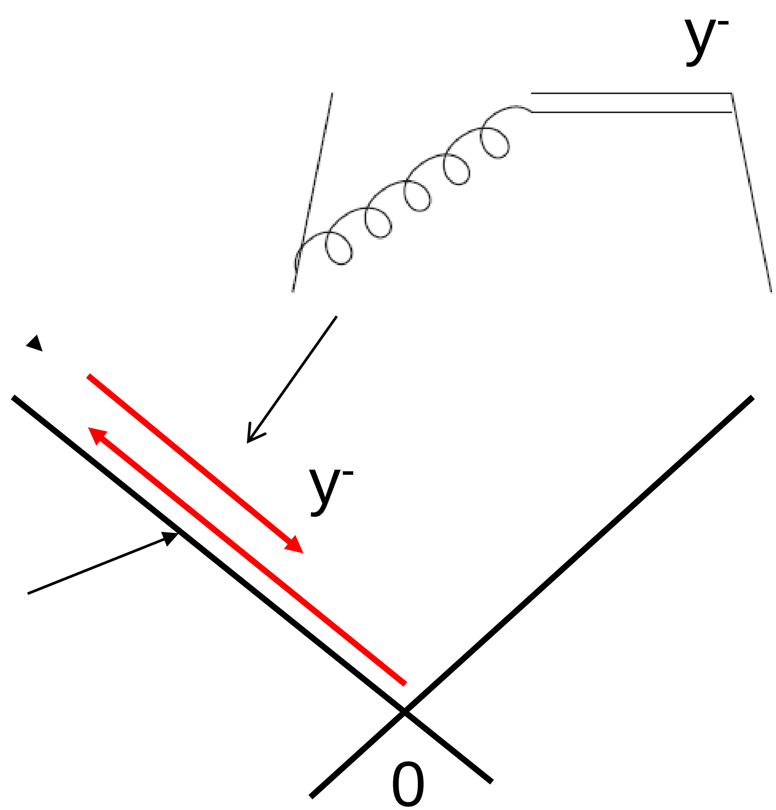
Wilson links

$$W(y^-, 0) \;=\; W(0)W^\dagger(y^-)$$

$$W(y^-) \;=\; \mathcal{P}\exp\left[-ig\int_0^\infty d\lambda n_- \cdot A(y+\lambda n_-)\right]$$

loop momentum flows through the hard kernel





loop momentum does not flow through the hard kernel

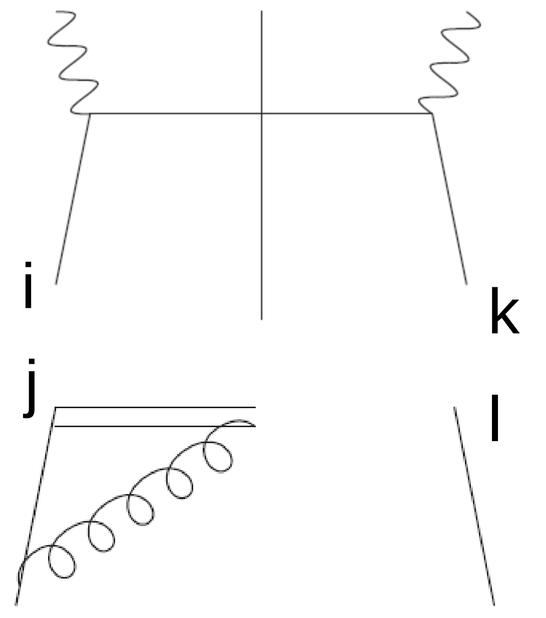
Factorization in fermion flow

- To separate fermion flows for H and for PDF, insert Fierz transformation

$$I_{ij}I_{lk} = \frac{1}{4}I_{ik}I_{lj} + \frac{1}{4}(\gamma_5)_{ik}(\gamma_5)_{lj}$$

$$+\frac{1}{4}(\gamma_\alpha)_{ik}(\gamma^\alpha)_{lj} + \frac{1}{4}(\gamma_5\gamma_\alpha)_{ik}(\gamma^\alpha\gamma_5)_{lj}$$

$$+\frac{1}{8}(\sigma_{\alpha\beta})_{ik}(\sigma^{\alpha\beta})_{lj}$$



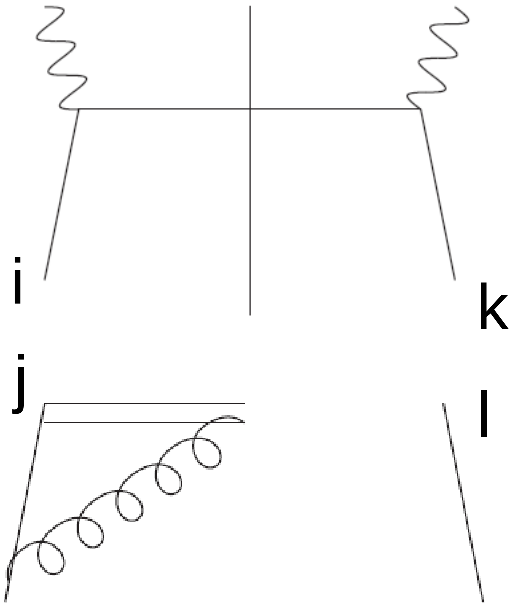
- $(\gamma^\alpha)_{lj}/2 \approx (\gamma^+)_{lj}/2$ goes into definition of PDF. Others contribute at higher powers

Factorization in color flow

- To separate color flows for H and for PDF, insert Fierz transformation

$$I_{ij} I_{lk} = \frac{1}{N_c} I_{lj} I_{ik} + 2 \sum_c (T^c)_{lj} (T^c)_{ik},$$

for color-octet state, namely for three-parton PDF

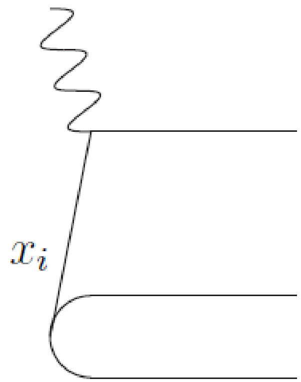
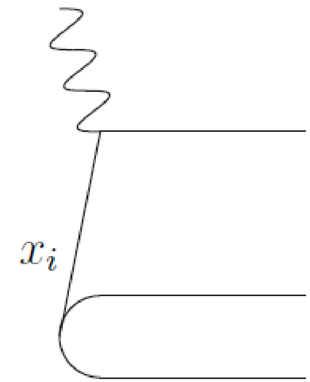


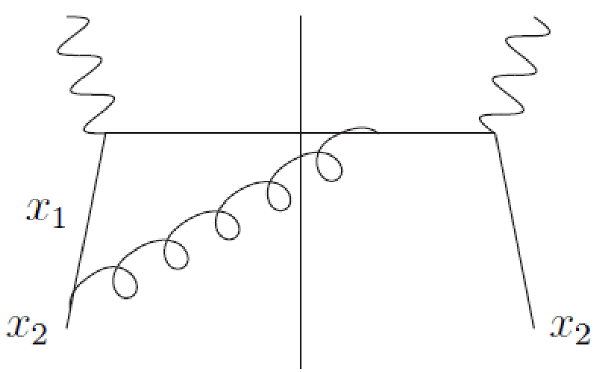
- I_{lj} / N_C goes into definition of PDF

Parton model

- The proton travels huge space-time, before hit by the virtual photon
- As Q^2 >>1, hard scattering occurs at point space-time
- The quark hit by the virtual photon behaves like a free particle
- It decouples from the rest of the proton
- Cross section is the incoherent sum of the scattered quark of different momentum

Incoherent sum

$\Big| \sum_i$  $\Big|^2 \approx \sum_i \Big|$  $\Big|^2$



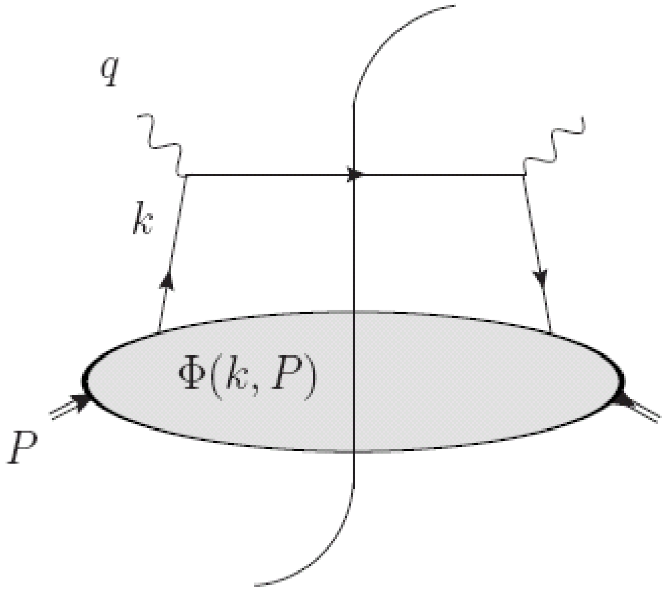
holds after collinear factorization

Factorization formula

- DIS factorized into hard kernel (infrared finite, perturbative) and PDF (nonperturbative)

$$F(x) = \Sigma_f \int_x^1 (d\xi/\xi) H_f(x/\xi) \phi_{f/N}(\xi)$$

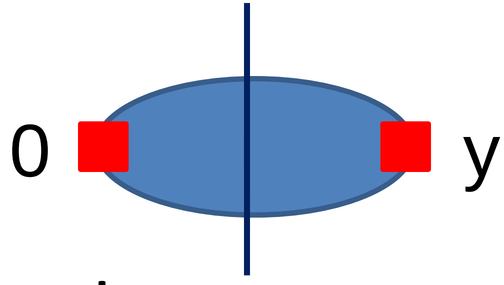
- Universal PDF describes probability of parton f carrying momentum fraction ξ in nucleon N
- PDF computed by nonpert methods, or extracted from data



$$k = (\xi P^+, 0, 0_T)$$

Expansion on light cone

- Operator product expansion (OPE): expansion in small distance y^μ
- Infrared safe $e^+e^- \to X \Rightarrow \Sigma_i C_i(y) O_i(0)$



- Factorization theorem: expansion in y^2
- Example: Deeply inelastic scattering (DIS)
- Collinear divergence in longitudinal direction exists $\Rightarrow$ (particle travels) finite y^-

Factorization scheme

- Definition of an IR regulator is arbitrary, like an UV regulator:
  $\phi^{(1)} \sim 1/\varepsilon_{IR}$+finite part
- Different inite parts shift between ϕ and H correspond to different factorization schemes
- Extraction of a PDF depends not only on powers and orders, but on schemes.
- Must stick to the same scheme. The dependence of predictions on factorization schemes would be minimized.

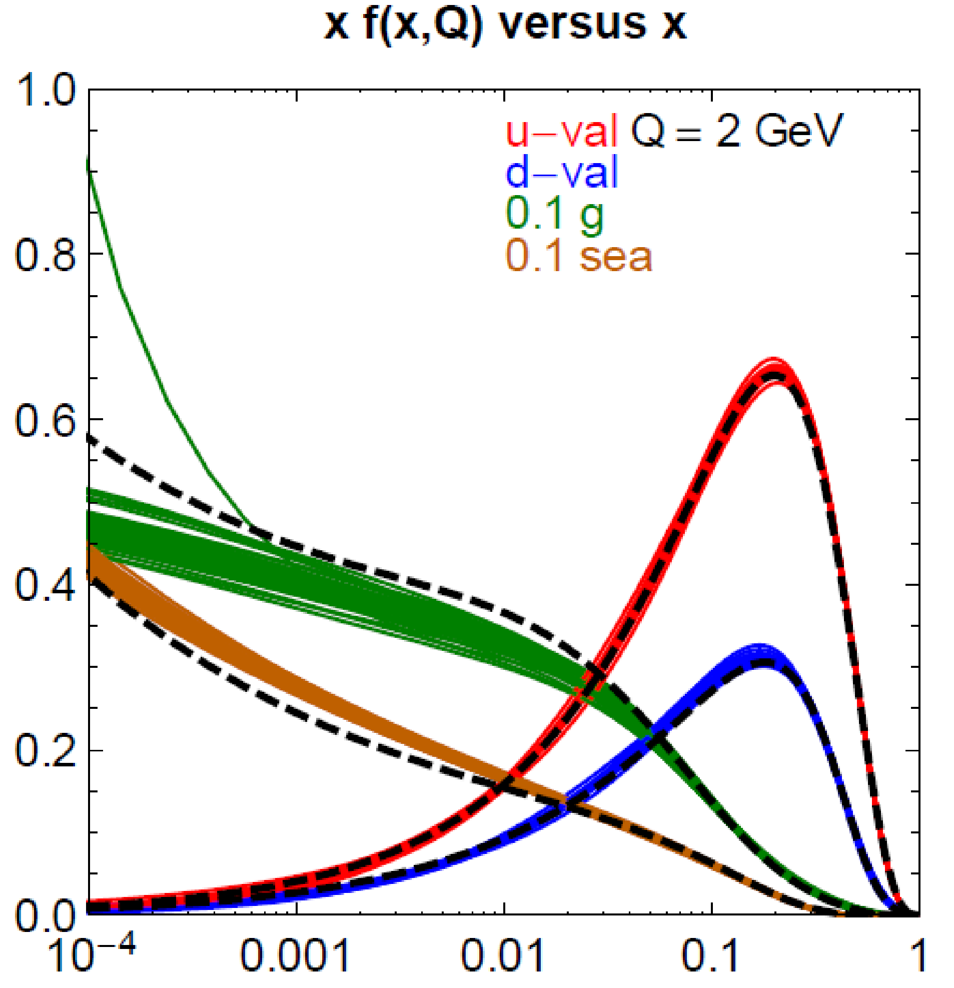
RG evolution

- $\phi(Q)$ is related to $\phi(\mu_0)$, $\mu_0 = 2$ GeV, through a RG evolution equation.
  - μ_0 is the (arbitrary) initial scale for RG evolution.
- RG improved (more reliable in perturbation) factorization formula, $F(Q)=\phi(Q,\mu_0)\,H(Q)$
- What is extracted (or derived from QCD sum rules) is the initial condition $\phi(\mu_0,\mu_0)$.
- Predictions will depend on powers, orders, factorization schemes, factorization scales, and initial scale μ_0 inevitably.

Extraction of PDF

- Fit the factorization formula $F=H^{DIS}\,\phi_{f/N}$ to data. Extract $\phi_{f/N}$ for f=u, d, g(luon), sea

CTEQ-TEA PDF
NNLO: solid color
NLO: dashed
NLO, NNLO means
Accuracy of H



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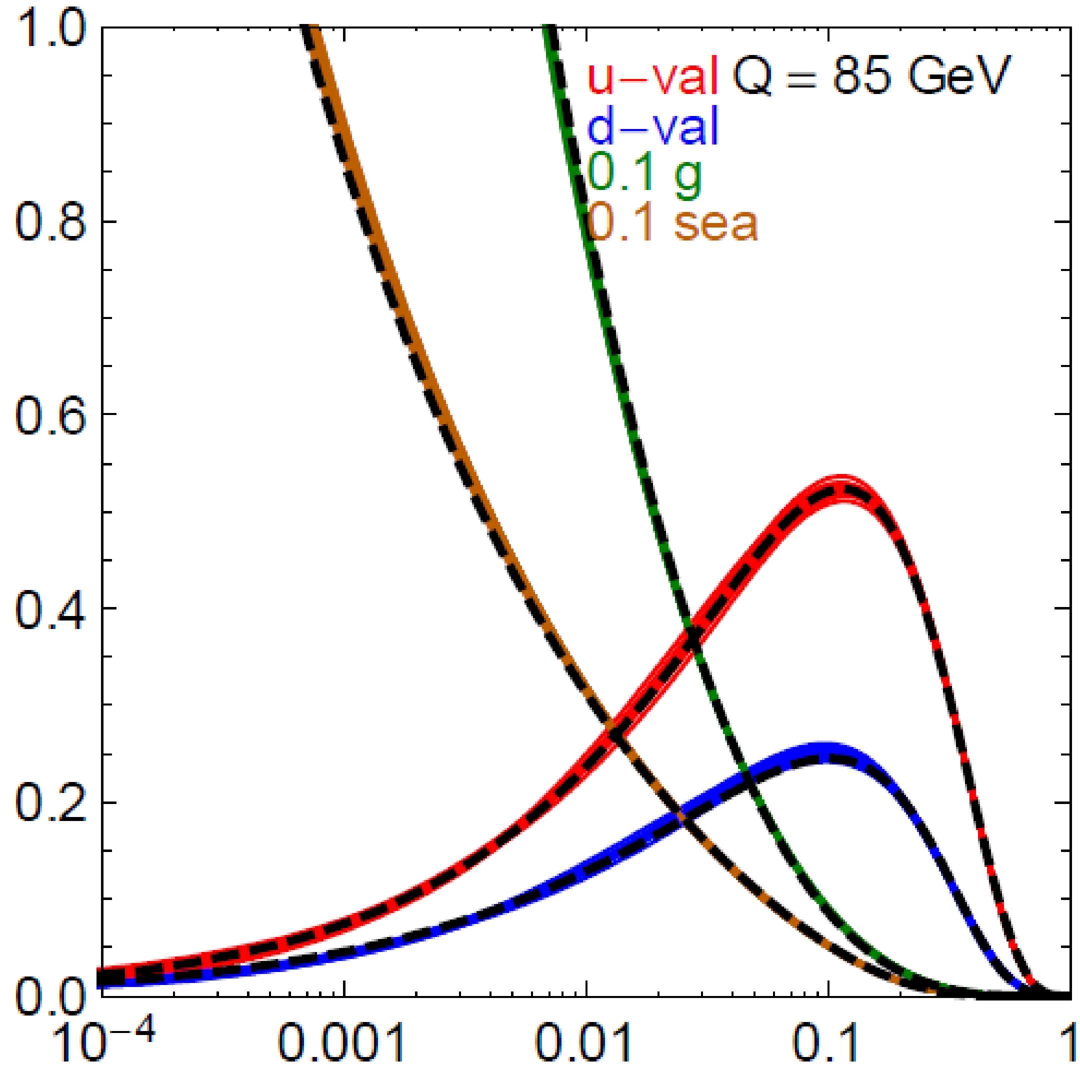
PDF with RG

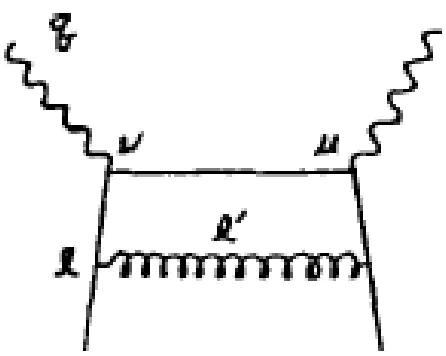
see
Lecture 2

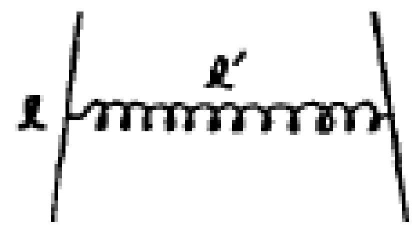


Application of factorization theorem

Hard kernel

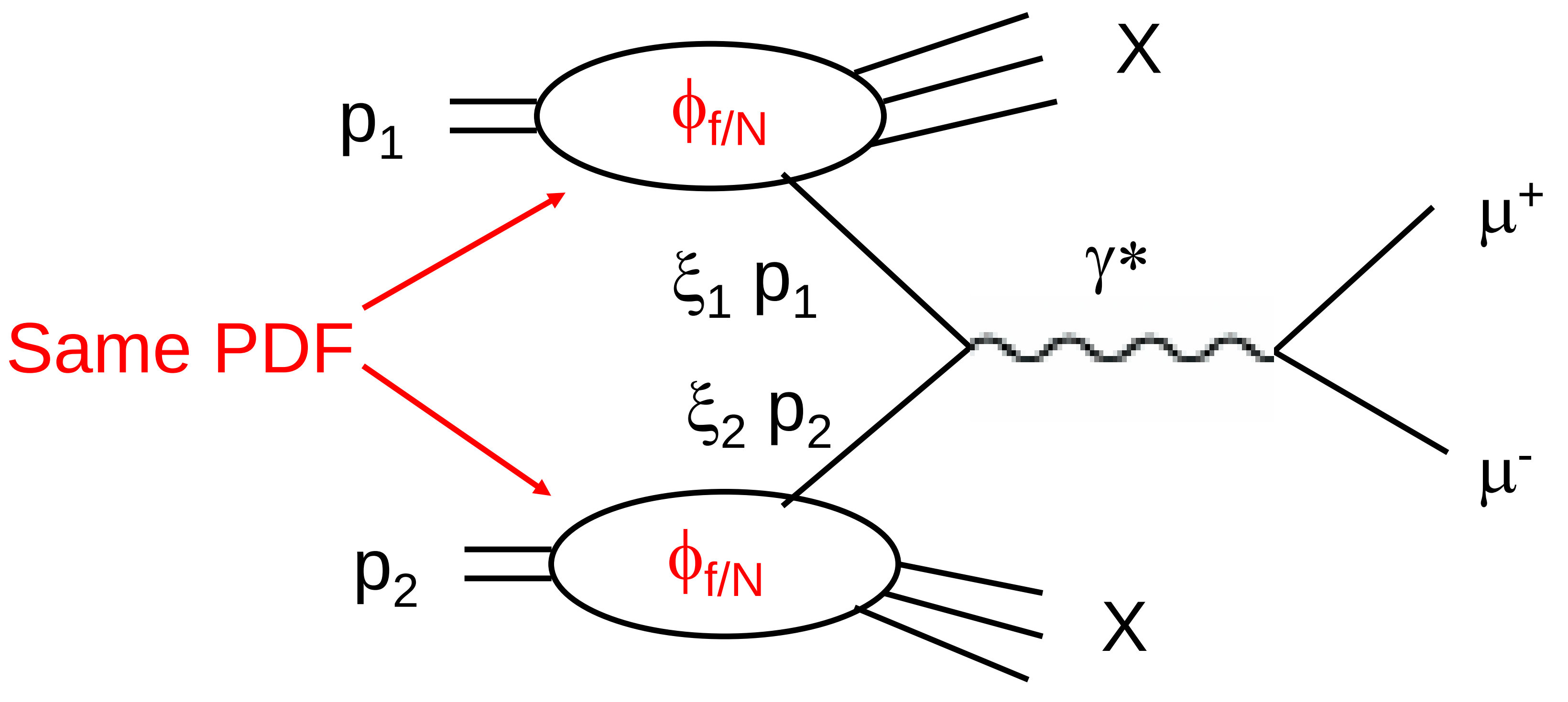
- PDF is infrared divergent, if evaluated in perturbation ⇔ confinement
- Quark diagram is also IR divergent.
- Difference between the quark diagram and PDF gives the hard kernel H^{DIS}

$H^{DIS}=$ 

$-$ 

Drell-Yan process

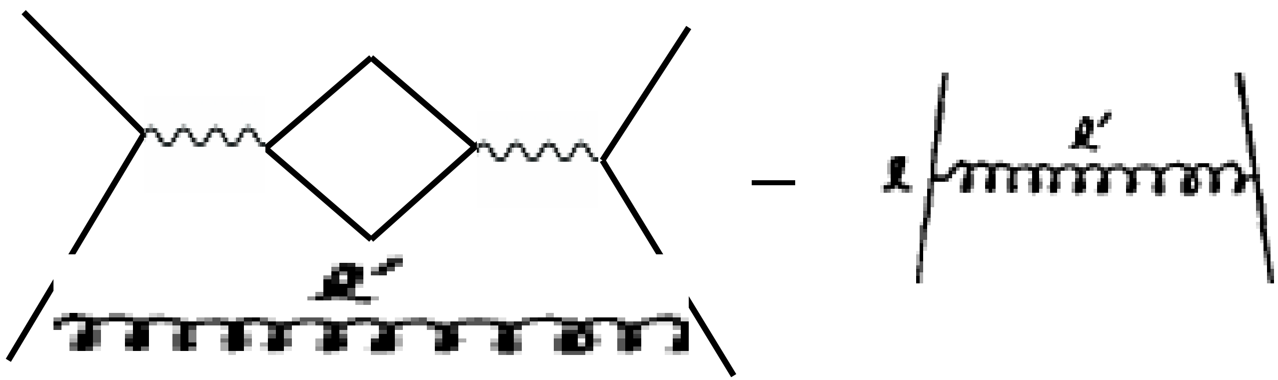
- Derive factorization theorem for Drell-Yan process $N(p_1)+N(p_2)\to\mu^+\mu^-(q)+X$



Hard kernel for DY

- Compute the hard kernel H^{DY}
- IR divergences in quark diagram and in PDF must cancel. Otherwise, factorization theorem fails

$H^{DY} =$

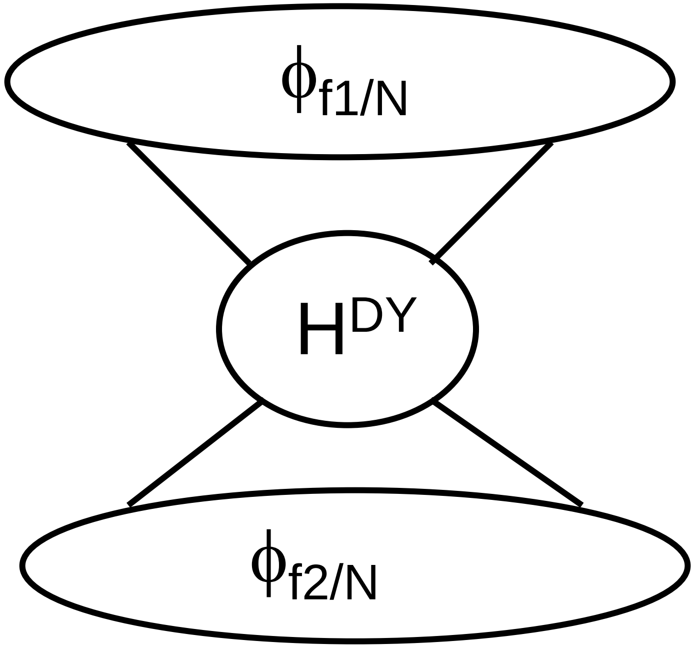


Same as in DIS

Prediction for DY

- Use $\sigma^{DY}=\phi_{f1/N} \otimes H^{DY} \otimes \phi_{f2/N}$ to make predictions for DY process

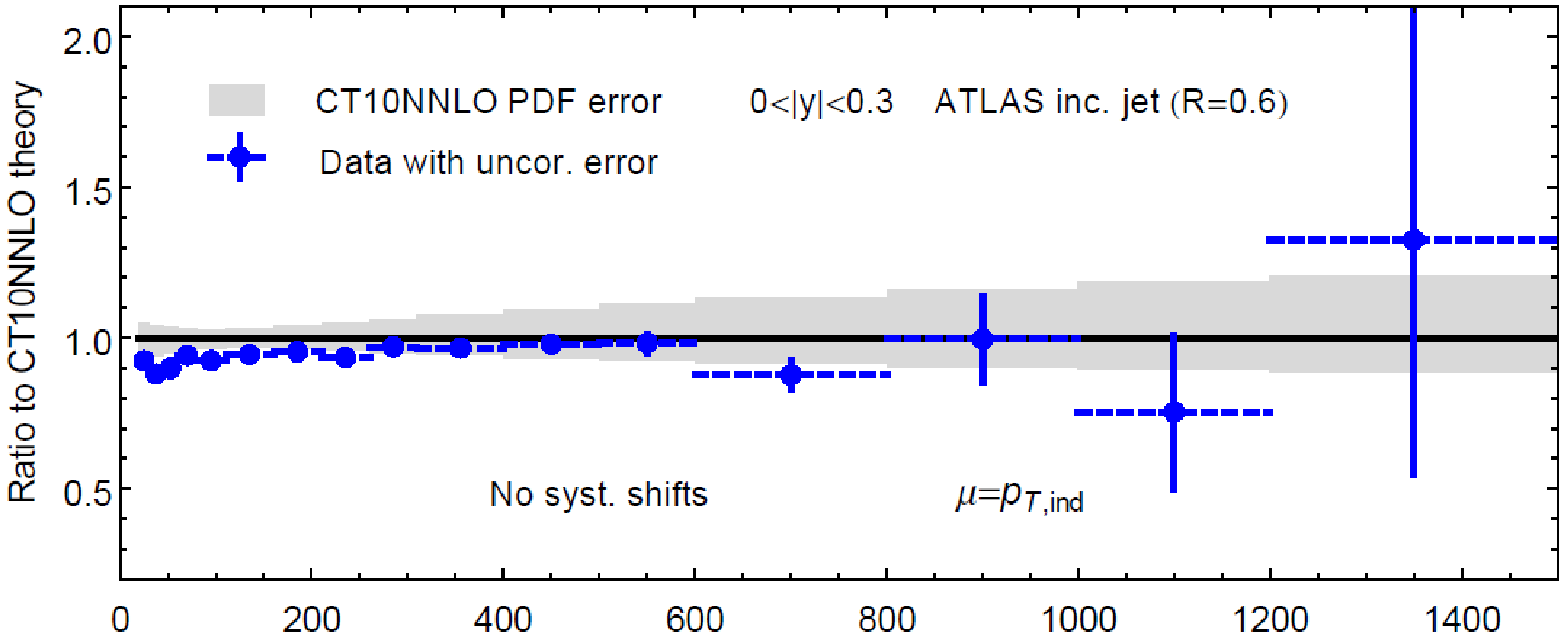
$\sigma^{DY}=$



Predictive power

- Before adopting PDFs, make sure at which power and order, and in what scheme they are defined

Nadolsky et al. 1206.3321



k_T factorization

Collinear factorization

- Factorization of many processes investigated up to higher twists
- Hard kernels calculated to higher orders
- Parton distribution function (PDF) evolution from low to high scale derived (DGLAP equation)
- PDF database constructed (CTEQ)
- Logs from extreme kinematics resummed
- Soft, jet, fragmentation functions all studied

Why k_T factorization

- k_T factorization has been developed for small x physics for some time
- As Bjorken variable $x_B=-q^2/(2p.q)$ is small, parton momentum fraction $x > x_B$ can reach $xp \sim k_T$. k_T is not negligible.
- $xp \sim k_T$ also possible in low q_T spectra, like direct photon and jet production
- In exclusive processes, x runs from 0 to 1. The end-point region is unavoidable
- But many aspects of k_T factorization not yet investigated in detail

Condition for k_T factorization

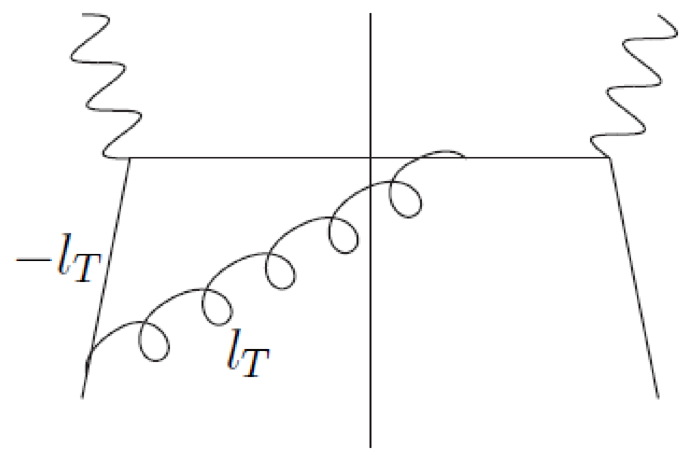
- Collinear and k_T factorizations are both fundamental tools in PQCD
- $x \neq 0$ (large fractional momentum exists) is assumed in collinear factorization.
- If small x not important, collinear factorization is self-consistent
- If small x region is important
- $x \approx 0 \Leftrightarrow y^- \approx \infty$, expansion in y^2 fails
- k_T factorization is then more appropriate

Parton transverse momentum

- Keep parton transverse momentum in H
- k_T dependence introduced by gluon emission
- Need to describe distribution in k_T

$$F(x) = \Sigma_f \int_x^1 (d\xi/\xi) \int d^2k_T H_f(x/\xi, k_T) \Phi_{f/N}(\xi, k_T)$$

$$\xi \leftrightarrow l^+, \quad k_T \leftrightarrow l_T$$



Eikonal approximation

$$\not{P}_q \gamma_\nu \frac{\not{P}_q + \not{l}}{(P_q + l)^2} \gamma^\mu \frac{\not{k} + \not{l}}{(k+l)^2} \gamma^\nu, \quad k \propto k^+, \quad P_q \propto P_q^-$$

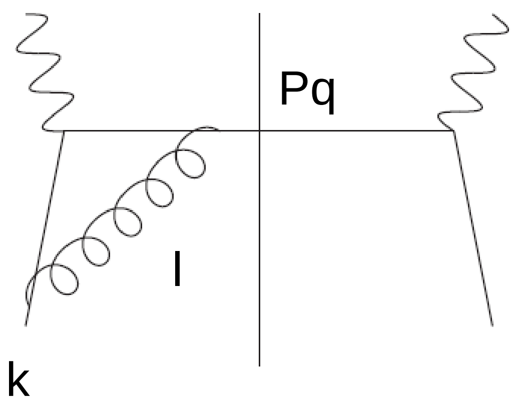
$$\approx \not{P}_q \gamma^- \frac{\not{P}_q + \not{l}}{(P_q + l)^2} \gamma^\mu \frac{\not{k} + \not{l}}{(k+l)^2} \gamma^+, \quad l \propto l^+$$

$$\approx \not{P}_q \gamma^- \frac{\not{P}_q}{2P_q \cdot l} \gamma^\mu \frac{\not{k} + \not{l}}{(k+l)^2} \gamma^+$$

drop l_T in numerator to get Wilson line

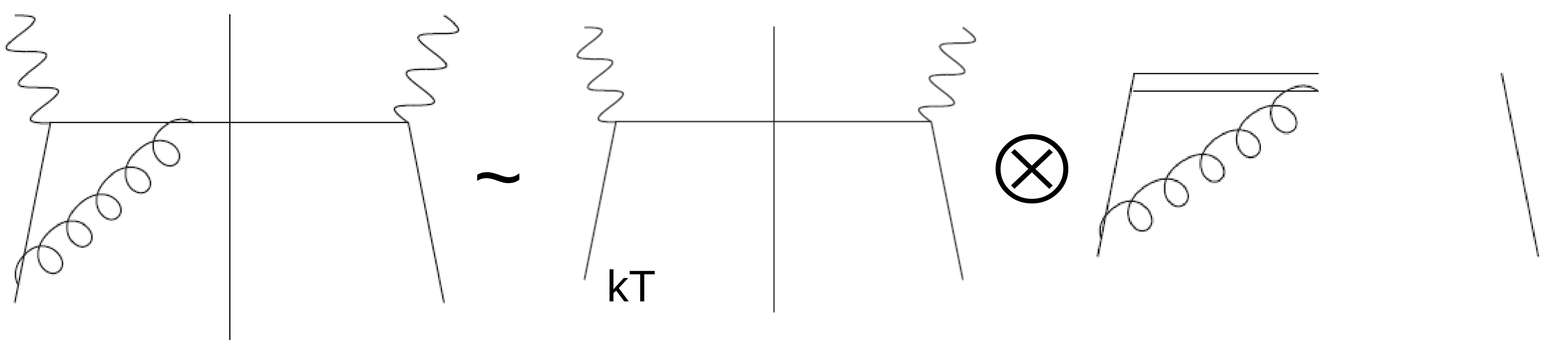
$$\approx \not{P}_q \frac{2P_q^- - \not{P}_q \gamma^-}{2P_q \cdot l} \gamma^\mu \frac{\not{k} + \not{l}}{(k+l)^2} \gamma^\nu, \quad \not{P}_q \not{P}_q = 0$$

$$\approx \not{P}_q \gamma^\mu \frac{\not{k} + \not{l}}{(k+l)^2} \gamma^\nu \frac{n_{-\nu}}{n_- \cdot l}$$



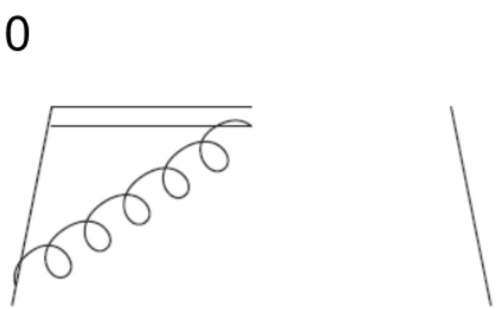
Effective diagrams

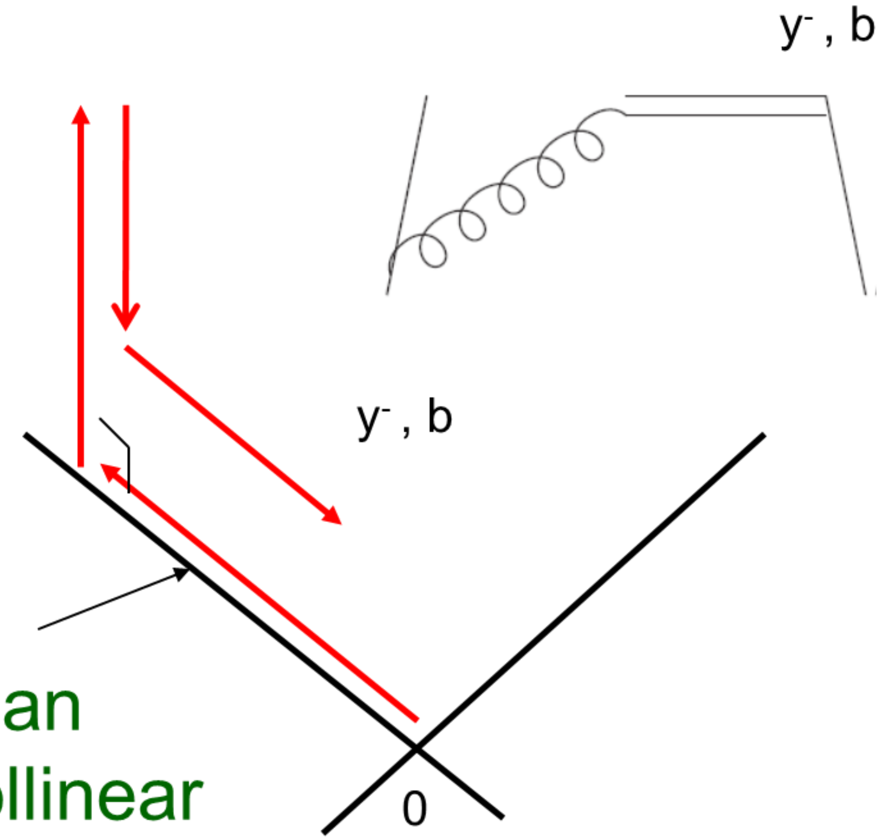
- Parton momentum $k = (\xi P^+, 0, k_T)$
- Only minus component is neglected
- k_T appears only in denominator
- Collinear divergences regularized by k_T^2
- Factorization of collinear gluons at leading power leads to Wilson links W(y⁻,0)



Wilson links

loop momentum l flows through the hard kernel





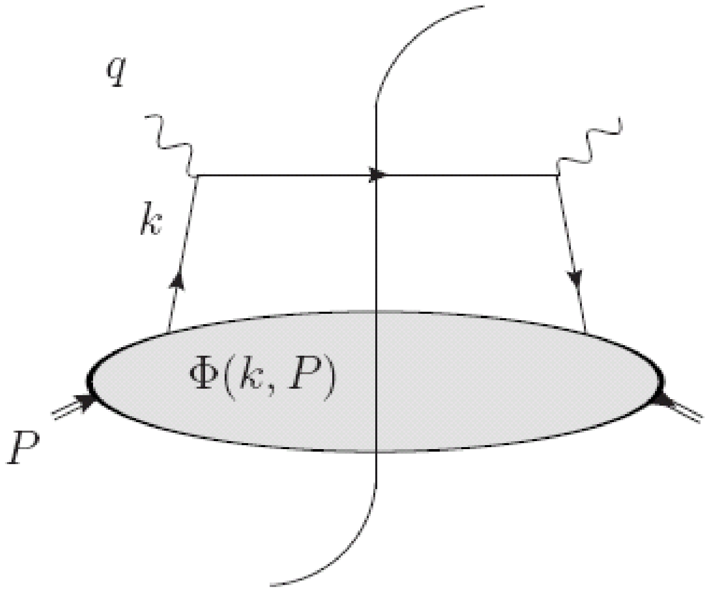
l does not flow through the hard kernel. Feynman rules the same as in collinear factctorization

Factorization in k_T space

Universal transverse-momentum-dependent (TMD) PDF $\Phi_{f/N}(\xi, k_T)$ describes probability of parton carrying momentum fraction and transverse momentum

If neglecting k_T in H, integration over k_T can be worked out, giving

$$\int d^2k_T \Phi_{f/N}(\xi, k_T) \Rightarrow \phi_{f/N}(\xi)$$



Summary

- Despite of nonperturbative nature of QCD, theoetical framework with predictive power can be developed
- It is based on factorization theorem, in which nonperturbative PDF is universal and can be extracted from data, and hard kernel can be calculated pertuebatvely
- k_T factorization is more complicated than collinear factorization, and has many difficulties