Lecture 3
PQCD for jet physics

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Outlines

• Jet in experiment
• Jet in theory
• Jet substructures
Introduction

• Jets are abundantly produced at colliders
• Jets carry information of hard scattering and parent particles
• Study of jets is crucial
• Usually use event generators
• How much can be done in PQCD
• A lot!
Dijet in e+e- annihilation

- Dijet production is part of total cross section
- Born cross section is the same as total cross section

\[ \sigma_{2j}^{(0)}(Q, \epsilon, \delta) = N \left( \sum_f Q_f^2 \right) \frac{4\pi \alpha^2}{3Q^2} \]

half angle of jet cone

energy resolution for dijet production

constrained phase space for real gluons
NLO corrections

• Isotropic soft gluons within energy resolution
\[2 \ln^2(2\epsilon E / \mu) - \pi^2/6\]

• Collinear gluons in cone with energy higher than resolution
\[-3 \ln(E \delta / \mu) - 2 \ln^2 2\epsilon - 4 \ln(E \delta / \mu) \ln(2\epsilon) + \frac{17}{4} - \pi^2/3\]

• Virtual corrections
\[-2 \ln^2(E / \mu) + 3 \ln(E / \mu) - \frac{7}{4} + \pi^2/6\]

• Dijet cross section is infrared finite, but logarithmically enhanced
\[(3 \ln \delta + 4 \ln \delta \ln 2\epsilon + \pi^2/3 - \frac{5}{2})\]

overlap of collinear and soft logs
Jet phenomenology

Jets from Quantum Chromodynamics

George Sterman
Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11790

and

Steven Weinberg
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 26 July 1977)

The properties of hadronic jets in $e^+e^-$ annihilation are examined in quantum chromodynamics, without using the assumptions of the parton model. We find that two-jet events dominate the cross section at high energy, and have the experimentally observed angular distribution. Estimates are given for the jet angular radius and its energy dependence. We argue that the detailed results of perturbation theory for production of arbitrary numbers of quarks and gluons can be reinterpreted in quantum chromodynamics as predictions for the production of jets.

jet as an observable (jet physics)
not quarks and gluons
double-log enhancement
jet substructures
Jet in experiment
Jets
Coordinates for jets

pseudorapidity

\[ \eta = + \ln \left[ \cot \left( \frac{\theta}{2} \right) \right] \]

\[ \theta = 0 \Rightarrow \eta = \infty, \quad \theta = 90^\circ \Rightarrow \eta = 0, \quad \theta = 180^\circ \Rightarrow \eta = -\infty \]
Jet algorithms

• Comparison of theory with experiment is nontrivial
• Need jet algorithms
• Algorithms should be well-defined so that they map experimental measurements with theoretical calculations as close as possible
• Infrared safety is important guideline, because Sterman-Weinberg jet is infrared finite
Types of algorithms

• Two main classes of jet algorithms
• Cone algorithms: stamp out jets as with a cookie cutter
  Geometrical method
• Sequential algorithms: combine parton four-momenta one by one
  Depend on particle kinematics
Seeded cone algorithm

- Find stable cones via iterative-cone procedure
- Start from seed particle $i$ and consider set of particles $j$ with separations smaller than jet cone
  \[
  \Delta R_{ij} \equiv (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2 < R
  \]
  - If the cone is stable, procedure stops. Otherwise the cone center $J$ is taken as a new seed, and repeat the above procedure
- A stable cone is a set of particles $i$ satisfying
  \[
  \Delta R_{ij} < R
  \]
- Examples:
  \[
  R < R_{12} < 2R
  \]
Iterative step 1
Iterative step 2
Iterative step 3
Iterative step 4

![Graph showing cone iteration]
Problem of seeded cone

- Geometrical algorithm does not differentiate infrared gluons from ordinary gluons.
- Final results (split-merge) depend on soft radiation and collinear splitting.
- Virtual (real) soft gluon contributes to two (single) jet cross section, no cancellation.
Not infrared safe

• How about starting from the hardest particle?
• Collinear splitting change final results

• Virtual (real) gluon contributes to single (two) jet cross section, no cancellation
• Seeded cone algorithm is not infrared safe
Sequential algorithms

• Take kT algorithm as an example.
• For any pair of particles i and j, find the minimum of

\[ d_{ij} = \frac{\min\{k_{ti}^2, k_{tj}^2\}}{R^2} \Delta R_{ij}^2 \approx k_{t,ij}^2, \quad d_{iB} = k_{ti}^2, \quad d_{jB} = k_{tj}^2 \]

• If it is diB or djB, i or j is a jet, removed from the list of particles. Otherwise, i and j merged
• Repeat procedure until no particles are left
• Differentiate infrared and ordinary gluons
Step 1

\[ d_{12} < d_{13} < d_{23} < d_{(1,2,3)B} < d_{i4} \]
Step 2
Step 3

\[ d_{12} < d_{(1,2)B} < d_{i4} \]
Step 4
Step 5
Recombination Algorithms

- **k_T algorithm**  start with softer particles

\[ d_{ij} = \min(p_{T_i}^2, p_{T_j}^2) \left( \frac{\Delta R}{R_0} \right)^2, \quad d_{iB} = p_{T_i}^2 \]

- **C/A algorithm**

\[ d_{ij} = \left( \frac{\Delta R}{R_0} \right)^2, \quad d_{iB} = 1 \]

- **anti-k_T algorithm**  start with harder particles

\[ d_{ij} = \min(p_{-T_i}^2, p_{-T_j}^2) \left( \frac{\Delta R}{R_0} \right)^2, \quad d_{iB} = p_{-T_i}^2 \]

\[ (\Delta R)^2 \equiv (\Delta \eta)^2 + (\Delta \phi)^2 \]
Infrared safety

• In seeded cone algorithm

• In kt algorithm, remain two jets—-infrared safety
Jet in theory
Factorization of DIS

• More sophisticated factorization is needed for jet production in DIS
• Cross section = H convoluted with PDF and Jet
• H is defined as contribution with collinear piece for initial state and collinear piece for final state being subtracted
• Basis for applying PQCD to jet physics
Jet production in DIS

• Restrict phase space of final-state quark and gluon in small angular separation

• Jet production enhanced by collinear dynamics

\[ I_{3,IR} = (2\pi) \int_0^1 \frac{dk}{k} \int_0^{\frac{\alpha}{2\pi}} \frac{d\theta}{\theta} \]
Wilson link

• Feynman rules with $\xi$ are from Wilson link

$$\Phi_\xi^{(f)}(\infty, 0; 0) = \mathcal{P} \left\{ e^{-ig \int_0^\infty d\eta \xi \cdot A^{(f)}(\eta \xi^\mu)} \right\}$$

• Represented by double lines

collinear gluon detached and factorized

go into jet function
Quark Jet function

• Eikonalization leads to factorization

\[ J_i^q(m_j^2, p_0, J_i, R) = \frac{(2\pi)^3}{2\sqrt{2}(p_0, J_i)^2} \frac{\xi_\mu}{N_c} \sum_{N_{J_i}} \text{Tr} \left\{ \gamma^\mu \langle 0| q(0) \Phi^{(\bar{q})\dagger}_{\xi}(\infty, 0)| N_{J_i} \rangle \right\} \]

\[ \times \langle N_{J_i}| \Phi_{\xi}(\infty, 0) \bar{q}(0)| 0 \rangle \left\{ \delta(m_j^2 - \tilde{m}_j^2(N_{J_i}, R)) \right\} \]

\[ \times \delta^{(2)}(\tilde{n} - \tilde{n}(N_{J_i})) \delta(p_0, J_i - \omega(N_{J_i})) \]

• Define jet axis, jet energy, jet invariant mass

• Wilson links are needed for gauge invariance of nonlocal matrix elements

• LO jet \( J_i^{(0)}(m_{J_i}^2, p_0, J_i, R) = \delta(m_{J_i}^2) \)

Almeida et al. 08
Gluon jet function

• Similar definition for gluon jet function

\[ J^{g}_{i}(m_{J}^{2}, p_{0,J_{i}}, R) = \frac{(2\pi)^{3}}{2(p_{0,J_{i}})^{3}} \sum_{N_{J_{i}}} \langle 0|\xi_{\sigma}F^{\sigma \nu}(0)\Phi_{\xi}^{(g)\dagger}(0, \infty)|N_{J_{i}}\rangle \]
\[ \times \langle N_{J_{i}}|\Phi_{\xi}^{(g)}(0, \infty)F_{\nu}^{\rho}(0)\xi_{\rho}|0\rangle \delta(m_{J}^{2} - \tilde{m}_{J}^{2}(N_{J_{i}}, R)) \]
\[ \times \delta^{(2)}(\hat{n} - \tilde{n}(N_{J_{i}}))\delta(p_{0,J_{i}} - \omega(N_{J_{c}})) \]
NLO diagrams

- quark jet

- gluon jet
Underlying events

• Everything but hard scattering
• Initial-state radiation, final-state radiation, multi-parton interaction
Power counting

- ISR, FSR are leading power, and should be included in jet definition

- MPI are sub-leading power: chance of involving more partons in scattering is low. They should be excluded
Pile-up events should be excluded

4 pile-up vertices
NLO jet distribution

- Divergence of NLO quark jet distribution at small MJ
Soft/collinear gluons vs jet mass

• Small jet mass

\[ p \rightarrow k_1 \rightarrow k_2 \]

large jet mass

\[ p \rightarrow k_1 \]

\[ p \rightarrow k_2 \]
Double logarithm

• Total NLO in Mellin space.

\[ \int_0^1 dx (1-x)^{N-1} J_q^{(1)} = \frac{\alpha_s(\mu^2) C_F}{\pi R^2 P_T^2} \left[ -\frac{1}{2} \ln^2 \tilde{N} - \left( \ln \nu^2 - \frac{3}{4} \right) \ln \tilde{N} \right] \]

\[ x \equiv M_J^2 / (R P_T)^2 \]

\[ \ln M_J \rightarrow \ln N \]

• Double log hints resummation

• Angular resolution is related to jet mass. When \( M_J \) is not zero, particles in a jet can not be completely collimated.

• Energy resolution is also related to jet mass. When \( M_J \) is not zero, the jet must have finite minimal energy
Resummation

• Recall low $p_T$ spectra of direct photon dominated by soft/collinear radiations

![Diagram showing particle interactions]

• Require $kT$ resummation
• Jet mass arises from soft/collinear radiations
• Can be described by resummation!

• Anti-$kT$ algorithm is preferred in view point of resummation
Jet substructures
Boosted heavy particles

• Large Hadron Collider (LHC) provide a chance to search new physics
• New physics involve heavy particles decaying possibly through cascade to SM light particles
• New particles, if not too heavy, may be produced with sufficient boost \( \rightarrow \) a single jet
• How to differentiate heavy-particle jets from ordinary QCD jets?
• Similar challenge of identifying energetic top quark at LHC
Fat QCD jet fakes top jet at high pT

Thaler & Wang
0806.0023
Pythia 8.108
Jet substructure

- Make use of jet internal structure in addition to standard event selection criteria
- Energy fraction in cone size of $r$, $\Psi(r)$, $\Psi(R) = 1$
- Quark jet is narrower than gluon jet
- Heavy quark jet energy profile should be different
Jet substructures are fingerprint prints of particles crucial for particle identification
Various approaches

• Monte Carlo: leading log radiation, hadronization, underlying events

• Fixed order: finite number of collinear/soft radiations

• Resummation: all-order collinear/soft radiations
Why resummation?

• Monte Carlo may have ambiguities from tuning scales for coupling constant
• NLO is not reliable at small jet mass
• Predictions from QCD resummation are necessary

Tevatron data vs MC predictions

N. Varelas 2009
Resummation equation

• Up to leading logs, resummation equation

\[- \frac{n^2}{P_j \cdot n} P_J^\alpha \frac{d}{dn^\alpha} J = [G^{(1)} + K_v^{(1)} + K_r^{(1)}] \otimes J\]

• See Lecture 2

• Regarded as associating soft gluon in Kr in single-log kernel into jet function J

• This is anti-\(kT\) algorithm!
Predictions for jet mass distribution

NLO in initial condition

CTEQ6L PDFs

Li, Li, Yuan, 2011
Energy profiles

• If can calculate jet mass in arbitrary jet cone size $R$, can certainly calculate jet energy in arbitrary jet cone $\Psi(r)$
• It is still attributed to soft/collinear radiations
• Resummation applies
Jet energy functions

- Jet energy function for quark

\[
\frac{(2\pi)^3}{2\sqrt{2}(P^0_J)^2 N_c} \sum_{\sigma, \lambda} \int \frac{d^3p}{(2\pi)^3 2p^0} \frac{d^3k}{(2\pi)^3 2k^0} [p^0 \Theta(r - \theta_p) + k^0 \Theta(r - \theta_k)]
\times \text{Tr} \left\{ \xi \langle 0| q(0) W^{(q)}_\xi(\infty, 0)|p, \sigma; k, \lambda \rangle \langle k, \lambda; p, \sigma | W^{(\bar{q})}_\xi(\infty, 0)\bar{q}(0)|0 \rangle \right\}
\times \delta(M^2_J - (p + k)^2) \delta(\hat{n} - \hat{n}_{p+k}) \delta(P^0_J - p^0 - k^0),
\]

- Jet energy function for gluon

\[
\frac{(2\pi)^3}{2(P^0_J)^3 N_c} \sum_{\sigma, \lambda} \int \frac{d^3p}{(2\pi)^3 2p^0} \frac{d^3k}{(2\pi)^3 2k^0} [p^0 \Theta(r - \theta_p) + k^0 \Theta(r - \theta_k)]
\times \langle 0| \xi \sigma F^{\sigma\nu}(0) W^{(g)}_\xi(\infty, 0)|p, \sigma; k, \lambda \rangle \langle k, \lambda; p, \sigma | W^{(g)}_\xi(\infty, 0) F^\nu_\rho(0)\xi_\rho|0 \rangle
\times \delta(M^2_J - (p + k)^2) \delta(\hat{n} - \hat{n}_{p+k}) \delta(P^0_J - p^0 - k^0),
\]

insert step functions
Resummation equation

- Resummation equation for jet profile

\[
\bar{K}_r^{(1)}(1) = g^2 C_F \int \frac{d^4 l}{(2\pi)^3} \frac{n^2}{(n \cdot l + i\epsilon)^2} \delta(l^2 - a^2) \Theta \left( r - \frac{|l| \sin \theta}{P_j^0} \right)
\]

\[
- \frac{n^2}{v \cdot n} \frac{d}{dn_\alpha} \hat{J}_q^E(1, P_T, \nu^2, R, r)
\]

\[
= 2[G^{(1)} + K^{(1)}(1)] \hat{J}_q^E(1, P_T, \nu^2, R, r)
\]

- Have considered N=1 here, corresponding to integration over jet mass (insensitive to nonperturbative physics)

- Resum \( \alpha_s \ln^2 r, \alpha_s \ln r \) from phase space constraint for real gluons
Soft gluon effect

- Soft real gluon in Kr renders jet axis of other particles inclined by small angle $l^0 \sin \theta / P_j^0$
- This jet axis cannot go outside of the subcone
- This is how real gluons affect $r$ dependence

$$l^0 \sin \theta / P_j^0 < r$$
Quark jet or gluon jet?

- It is a quark jet!
Opportunities at LHC

- It is a gluon jet!
- Test new physics models from composition of observed jets, e.g., CDF “W+jj” anomaly
Comparison with CDF data

\[ \Psi(r) = \frac{1}{N_{\text{jet}}} \sum_{\text{jets}} \frac{P_T(0, r)}{P_T(0, R)}, \quad 0 \leq r \leq R \]

quark, gluon jets, convoluted with LO hard scattering, PDFs
Compasasion with CMS data

$\Psi(r)$

20 GeV < $P_T$ < 30 GeV

$\Psi(r)$

40 GeV < $P_T$ < 50 GeV

$\Psi(r)$

60 GeV < $P_T$ < 80 GeV

$\Psi(r)$

80 GeV < $P_T$ < 100 GeV
Higgs jet

• One of major Higgs decay modes $H \to bb$ with Higgs mass $\sim 125$ GeV

• Important background $g \to bb$

• Analyze substructure of Higgs jet improves its identification

• For instance, color pull made of soft gluons

Gallicchio, Schwartz, 2010
Color pull

- Higgs is colorless, bb forms a color dipole
- Soft gluons exchanged between them
- Gluon has color, b forms color dipole with other particles, such as beam particles
Summary

• Jet substructures can be studied in PQCD
• Start with Sterman-Weinberg definition, apply factorization and resummation, predict observables consistent with data
• Fixed-order calculation not reliable at small MJ
• Event generators have ambiguities
• Can improve jet identification and new particle search