Instituto Superior Técnico
GoLP/CFP

Direct acceleration of muons with variable-frequency lasers

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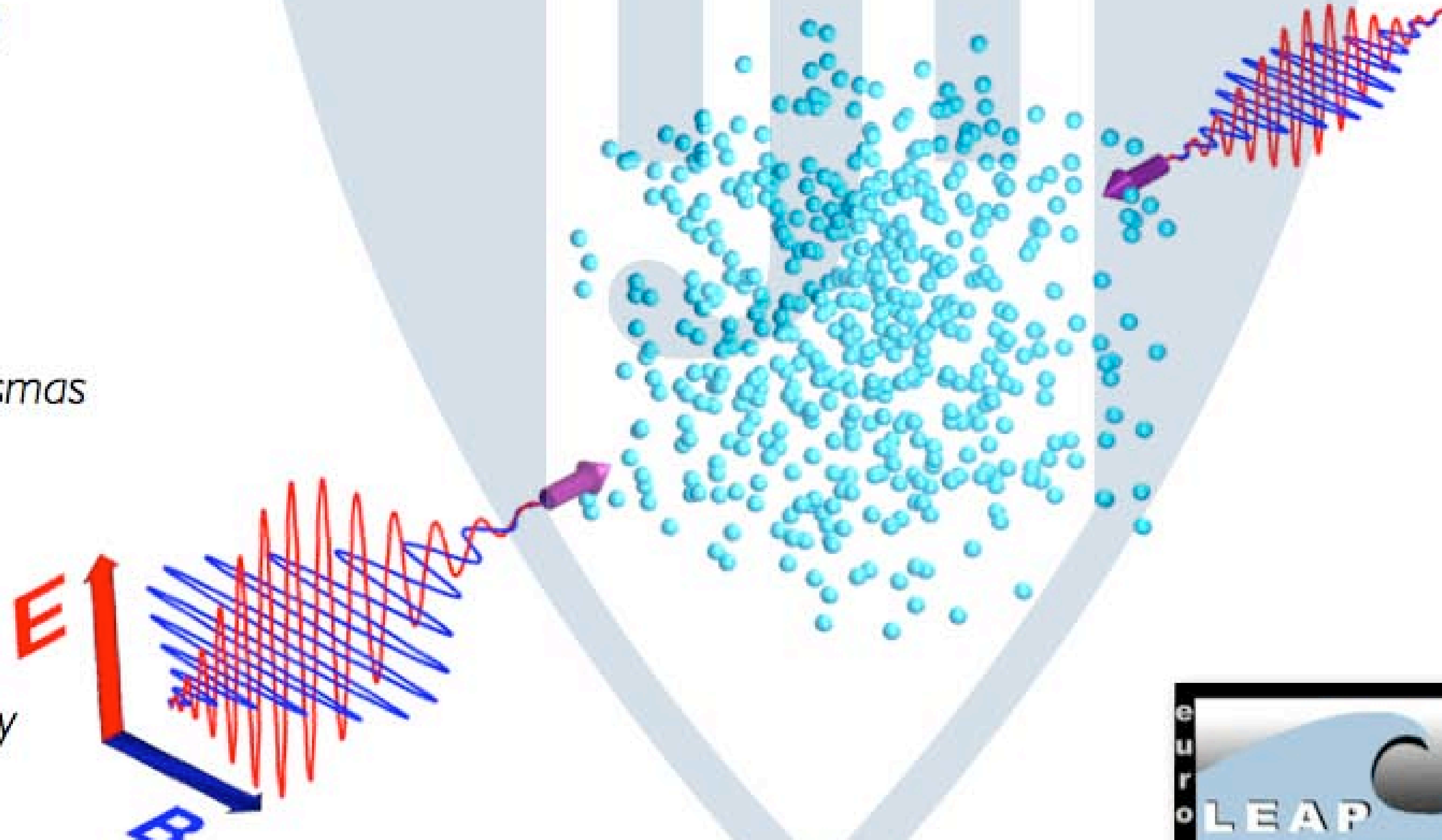

Outline

- Introduction
- 1D theory for direct particle acceleration
  - working principle & basic equations
  - resonant solutions & trapping
  - acceleration with linearly-chirped lasers
- Estimates for key beam features
  - general scaling laws
  - possible application to muon acceleration?
- Testing: 2D PIC simulations with protons
- Conclusions & perspectives

Next topic

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Laser-based particle acceleration

Method \ Particles	**Electrons** relativistic intensities ($>10^{18}$ W/cm^2) achievable with modern IR lasers	**Ions** relativistic intensities ($>10^{24}$ W/cm^2) beyond present technology
Direct	Several schemes with one- or multi-pulse configurations	MOSTLY UNEXPLORED
Indirect	Several schemes exploiting electrostatic plasma waves	Several schemes exploiting electrostatic fields in solid targets

Laser-based particle acceleration

Particles / Method	**Electrons** relativistic intensities ($>10^{18}$ W/cm^2) achievable with modern IR lasers	**Ions** relativistic intensities ($>10^{24}$ W/cm^2) beyond present technology
Direct	Several schemes with one- or multi-pulse configurations	Acceleration in slow EM beat-wave with variable-frequency lasers @ nonrelativistic intensities
Indirect	Several schemes exploiting electrostatic plasma waves	Several schemes exploiting electrostatic fields in solid targets

Laser-based particle acceleration

Method \ Particles	**Electrons** relativistic intensities ($>10^{18}$ W/cm^2) achievable with modern IR lasers	**Muons?** relativistic intensities ($\sim10^{22}$ W/cm^2) achievable with next-generation lasers
Direct	Several schemes with one- or multi-pulse configurations	Acceleration in slow EM beat-wave with variable-frequency lasers @ nonrelativistic intensities
Indirect	Several schemes exploiting electrostatic plasma waves	Several schemes exploiting electrostatic fields in solid targets

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Acceleration principle

Particles

- charge: q
- mass: M

Acceleration principle

EM wave 1

- coherent
- polarized
- nonrelativistic $\left(\frac{qA_1}{Mc^2} \ll 1\right)$
- chirped

Particles

- charge: q
- mass: M

EM wave 2

- coherent
- polarized
- nonrelativistic $\left(\frac{qA_2}{Mc^2} \ll 1\right)$
- chirped

Acceleration principle

- Counterpropagating lasers $\Longrightarrow$ slow ponderomotive beat wave
- Frequency variation $\Longrightarrow$ variation of beat-wave phase velocity
- Particles are trapped and continuously accelerated

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Acceleration principle

- laser 1 (chirped)
- laser 2 (fixed frequency)
- beat wave
- ponderomotive beat wave

Acceleration principle

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- laser 2 (fixed frequency)
- beat wave
- ponderomotive beat wave

Field configuration

EM wave 1

$$\mathbf{A}_1 = A_1 \cos(\theta) \sin[\Phi_1(\xi_1)]\,\hat{\mathbf{e}}_y + A_1 \sin(\theta) \cos[\Phi_1(\xi_1)]\,\hat{\mathbf{e}}_z$$

$$\xi_1 = x - ct$$

$$k_1(x,t) = \Phi_1', \quad \omega_1(x,t) = \Phi_1'$$

+

EM wave 2

$$\mathbf{A}_2 = A_2 \cos(\theta) \sin[\Phi_2(\xi_2)]\,\hat{\mathbf{e}}_y + A_2 \sin(\theta) \cos[\Phi_2(\xi_2)]\,\hat{\mathbf{e}}_z$$

$$\xi_2 = -x - ct$$

$$k_2(x,t) = -\Phi_2', \quad \omega_2(x,t) = \Phi_2'$$

=

Slow beat wave

$$k(x,t) = \tfrac{1}{2}(\Phi_1' + \Phi_2')$$

$$\omega(x,t) = \tfrac{c}{2}(\Phi_1' - \Phi_2')$$

$$v_\phi(x,t) = \tfrac{\omega}{k} = c\frac{\Phi_1' - \Phi_2'}{\Phi_1' + \Phi_2'}$$

+

Fast beat wave

$$K(x,t) = \tfrac{1}{2}(\Phi_1' - \Phi_2')$$

$$\Omega(x,t) = \tfrac{c}{2}(\Phi_1' + \Phi_2')$$

$$V_\phi(x,t) = \tfrac{\Omega}{K} = c\frac{\Phi_1' + \Phi_2'}{\Phi_1' - \Phi_2'}$$

initial beat wave velocity = initial ion velocity

$$v_\phi(0,0) = c\beta_0 \quad \Rightarrow \quad \frac{\omega_{02}}{\omega_{01}} = \frac{1-\beta_0}{1+\beta_0}$$

Basic equations

Equation of motion

$$\frac{\mathrm{d}p_x}{\mathrm{d}t} = -\frac{q^2}{2\gamma M c^2}\frac{\partial}{\partial x}\left(\mathbf{A}_1 + \mathbf{A}_2\right)^2$$

- average over fast oscillations
- normalization

$$\hat{A}_1 = \frac{qA_1}{Mc^2} \ll 1 \qquad \hat{A}_2 = \frac{qA_2}{Mc^2} \ll 1$$

$$\hat{x} = k_0 x \qquad \hat{t} = k_0 ct \qquad k_0 = k\,(0,0)$$

$$\hat{p}_x = \frac{p_x}{Mc} \qquad \gamma^2 = 1 + \hat{p}_x^2 + \hat{A}_1{}^2/2 + \hat{A}_2{}^2/2 + \hat{A}_1\hat{A}_2\cos\left(\Phi_1 - \Phi_2\right)$$

Ponderomotive equation

$$\frac{\mathrm{d}\hat{p}_x}{\mathrm{d}\hat{t}} = -\frac{\hat{\mathbf{A}}_1\hat{\mathbf{A}}_2}{2\gamma}\frac{\partial}{\partial \hat{x}}\cos\left(\Phi_1 - \Phi_2\right)$$

Energy equation

$$\frac{\mathrm{d}\gamma}{\mathrm{d}\hat{t}} = \frac{\hat{\mathbf{A}}_1\hat{\mathbf{A}}_2}{2\gamma}\frac{\partial}{\partial \hat{t}}\cos\left(\Phi_1 - \Phi_2\right)$$

$\Phi_1' - \Phi_2' \neq$ constant

$\Downarrow$

Wave-particle energy transfer

Trapping

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}\tau^2} = -\frac{\partial}{\partial\psi} U(\psi,\tau)$$

beat-wave trajectory: $\hat{x}_{\phi_0}(\hat{t})$ such that

$$\Phi_1\left(\hat{x}_{\phi_0} - \hat{t}\right) - \Phi_2\left(-\hat{x}_{\phi_0} - \hat{t}\right) = \phi_0$$

phase difference: $\psi = 2\left[\hat{x} - \hat{x}_{\phi_0}(\hat{t})\right]$

proper time: $\mathrm{d}\tau = \mathrm{d}\hat{t}/\gamma$

inertial force: $\alpha_{\phi_0}(\tau) = \dfrac{\mathrm{d}^2\hat{x}_{\phi_0}}{\mathrm{d}\tau^2} = \gamma\dfrac{\mathrm{d}}{\mathrm{d}\hat{t}}\left(\gamma\dfrac{\mathrm{d}}{\mathrm{d}\hat{t}}\hat{x}_{\phi_0}\right)$

Effective potential

$$U(\psi,\tau) = 2\hat{\mathbf{A}}_1\hat{\mathbf{A}}_2\cos\left[\Phi_1\left(\hat{\xi}_{1\phi_0} + \frac{\psi}{2}\right) - \Phi_2\left(\hat{\xi}_{2\phi_0} - \frac{\psi}{2}\right)\right] + 2\alpha_{\phi_0}\psi$$

trapped particles $\Rightarrow |\psi| \ll 2|\hat{\xi}_{j\phi_0}|$

$\hat{\xi}_{1\phi_0} = \hat{x}_{\phi_0} - \hat{t}$

$\hat{\xi}_{2\phi_0} = -\hat{x}_{\phi_0} - \hat{t}$

$$U(\psi,\tau) \approx 2\mathbf{A}_1\mathbf{A}_2\cos\left(\hat{k}\psi + \phi_0\right) + 2\alpha_{\phi_0}\psi$$

necessary condition for trapping

$$\underbrace{\left|\alpha_{\phi_0}(\hat{t})\right|}_{\text{inertial force}} < \underbrace{\hat{\mathbf{A}}_1\hat{\mathbf{A}}_2\hat{k}\left[\hat{x}_{\phi_0}(\hat{t}), \hat{t}\right]}_{\text{max ponderomotive force}}$$

Resonant solutions

Resonant solutions with exact phase-locking are defined by

$$\Phi_1\left[\hat{X}\left(\hat{t}\right)-\hat{t}\right]-\Phi_2\left[-\hat{X}\left(\hat{t}\right)-\hat{t}\right]=\phi_0$$

$\Phi_1'=0$ or $\Phi_2'=0$ $\Rightarrow$ analytical solution

resonance criterion

$$\Phi_1\left(\hat{\xi}_1\right)=\phi_0+\frac{1}{2\mu_0}\log\left[1+2\mu_0\left(1+\beta_0\right)\hat{\xi}_1\right]$$

$$\Phi_2\left(\hat{\xi}_2\right)=\left(1-\beta_0\right)\xi_2$$

resonant trajectory

$$\hat{X}\left(\tau_\parallel\right)=\frac{1}{4\mu c_0}\left[c_0^2\left(\mu\tau_\parallel-2\right)\mu\tau_\parallel-2\log\left(1-\mu\tau_\parallel\right)\right]$$

$$\hat{t}\left(\tau_\parallel\right)=-\frac{1}{4\mu c_0}\left[c_0^2\left(\mu\tau_\parallel-2\right)\mu\tau_\parallel+2\log\left(1-\mu\tau_\parallel\right)\right]$$

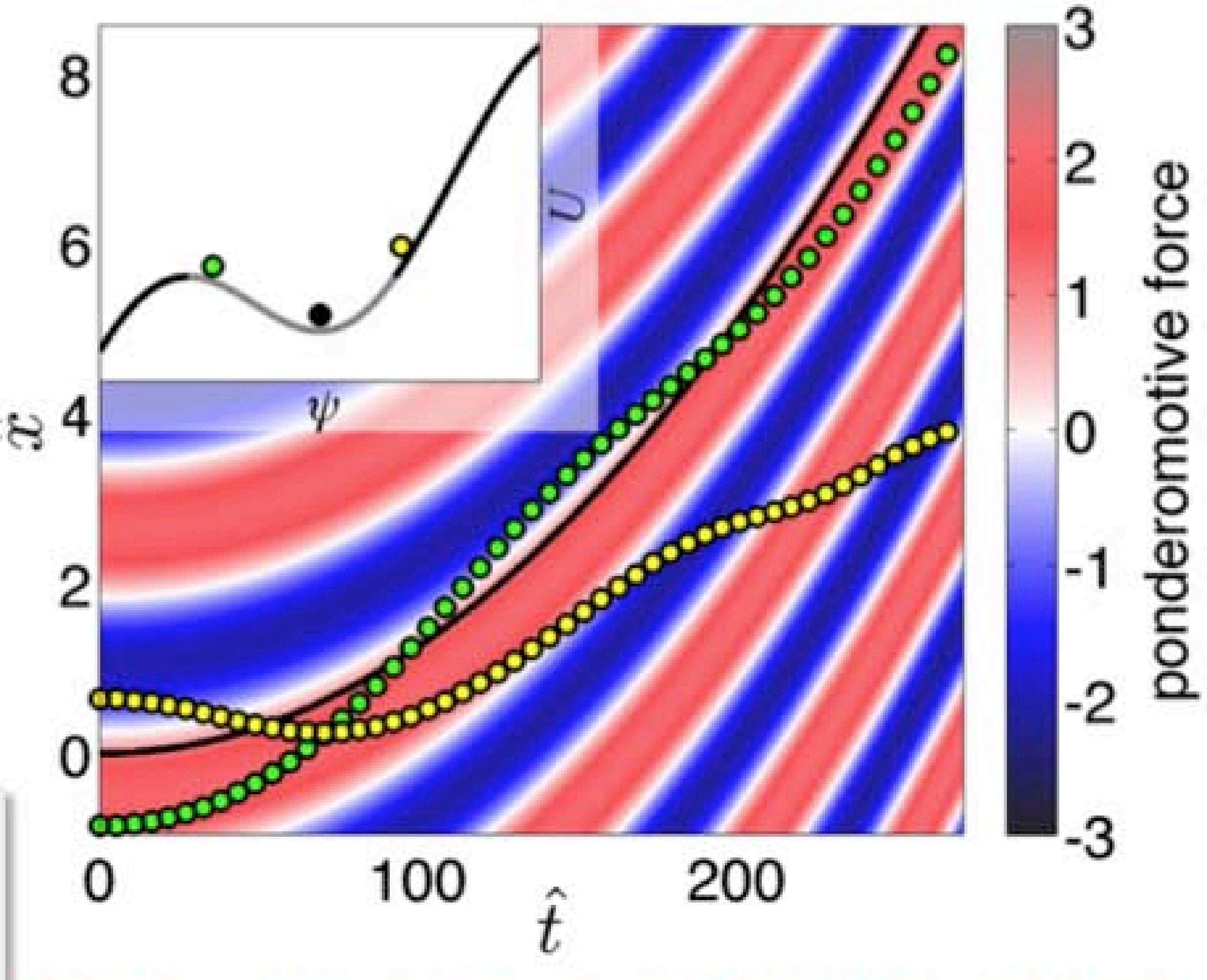
stable for $\cos\left(\phi_0\right)<0$

stable for $\cos\left(\phi_0\right)>0$

$\mu=\mu_0/\gamma_{0\parallel}$ $\quad\mu_0=\hat{A}_1\hat{A}_2\sin\left(\phi_0\right)/\gamma_{0\perp}^2$

$\gamma_{0\perp}^2=1+\hat{A}_1^2/2+\hat{A}_2^2/2+\hat{A}_1\hat{A}_2\cos\left(\phi_0\right)$

$\gamma_{0\parallel}^2=1-\beta_0^2$ $\quad\tau_\parallel=\gamma_{0\perp}\tau$ $\quad d\tau=d\hat{t}/\gamma$



Linearly chirped lasers

Laser 1

$$\Phi_1 = \phi_{01} + (1+\beta_0)\,\hat{\xi}_1 + \sigma_1\hat{\xi}_1^2$$

$$\hat{\omega}_1 = \hat{k}_1 = 1 + \beta_0 + 2\sigma_1\hat{\xi}_1$$

Laser 2

$$\Phi_2 = \phi_{02} + (1-\beta_0)\,\hat{\xi}_2 + \sigma_2\hat{\xi}_2^2$$

$$\hat{\omega}_2 = -\hat{k}_2 = 1 - \beta_0 + \sigma_2\hat{\xi}_2^2$$

$$\sigma_+ = \sigma_1 + \sigma_2 \qquad \sigma_- = \sigma_1 - \sigma_2$$

Beat wave trajectory

$$\hat{x}_{\phi_0}\left(\hat{t}\right) = \frac{\sigma_+}{\sigma_-}\hat{t} - \frac{1}{\sigma_-}\left[1 - \sqrt{1 - 2\left(\sigma_+ - \beta_0\sigma_-\right)\hat{t} + \left(\sigma_+^2 - \sigma_-^2\right)\hat{t}^2}\right]$$

chirped laser and particle copropagating

$\Downarrow$

trapping regions get wider

chirped laser and particle counterpropagating

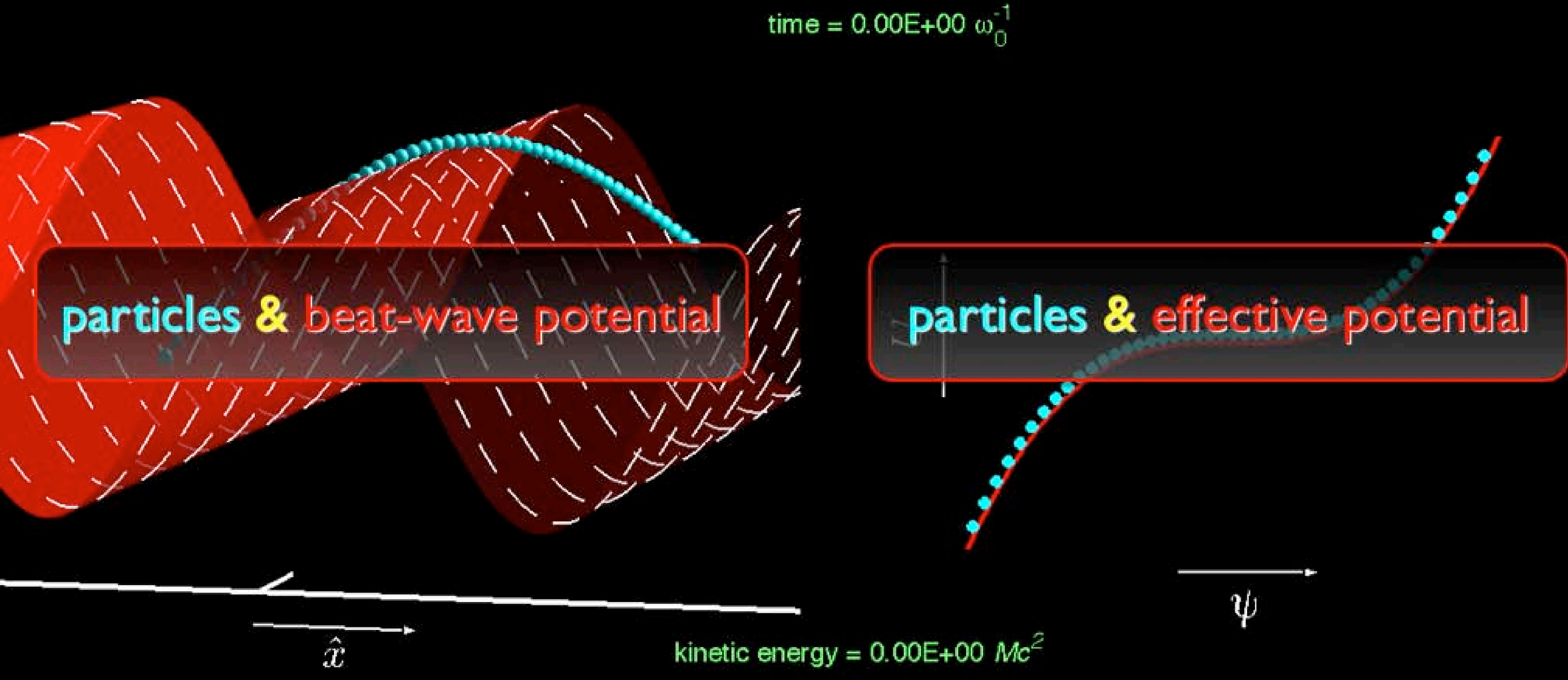
$\Downarrow$

trapping regions get narrower

Energy gain can be maximized using two linearly chirped lasers with $\sigma_1\sigma_2 < 0$

Trapping dynamics and early stages

laser 1 linearly chirped: $\sigma_1 = -\hat{\mathbf{A}}_1 \hat{\mathbf{A}}_2 = -3 \times 10^{-5}$



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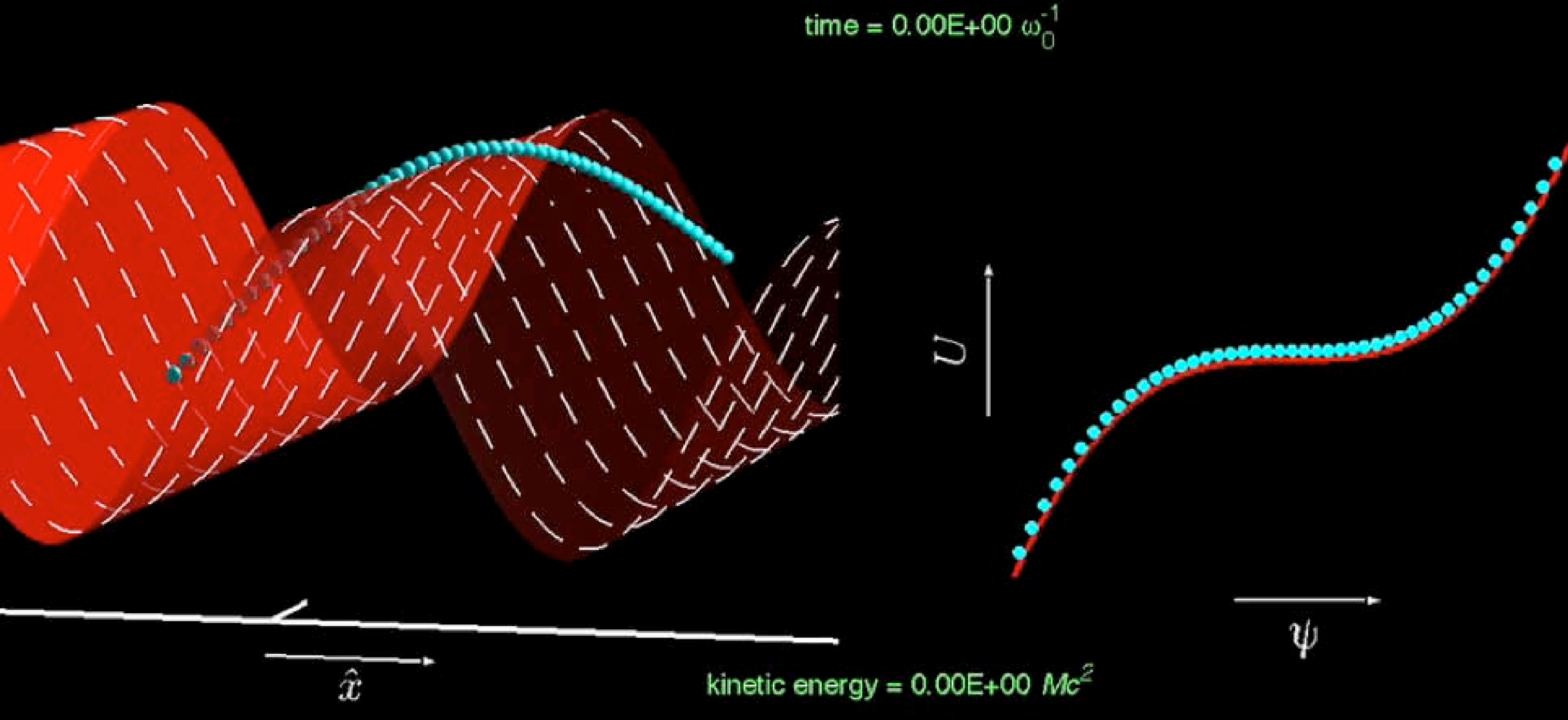

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Space-charge & pulse-shape effects

Space charge must not perturb the beat-wave structure

Constraint on particle density

$$n_{\mathrm{p}}[10^{19}\mathrm{cm}^{-3}] \ll Z_{\mathrm{p}}^{-1} I_2^{1/2}[10^{20}\mathrm{W/cm^2}] I_2^{1/2}[10^{20}\mathrm{W/cm^2}]$$

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laser intensities has slow longitudinal dependence

$\partial A_j/\partial \xi_j$ negligible for long pulses but acceleration effective only where $A_1 A_2$ is high enough

transverse ponderomotive force pushes particles aside

gain in transverse momentum

$$\frac{|p_\perp|}{|p_\parallel|} \sim \frac{\lambda_0}{2\pi w_0} \ll 1$$

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Acceleration region limited by spot size and Rayleigh length

Maximum trapped charge

$$Q[\mathrm{pC}] \approx 16\eta_\mathrm{tr} Z_\mathrm{p} n_\mathrm{p}[10^{19}\mathrm{cm}^{-3}] w_0^4[\mu\mathrm{m}]/\lambda_0[\mu\mathrm{m}]$$

Scalings and laser requirements

Scaling laws for nonrelativistic regime (useful particularly for heavy ions)

Acceleration distance vs. time

$$\Delta x[\mu\mathrm{m}] \approx 6 Z_\mathrm{p}^2 A_\mathrm{p}^{-2} I_1^{1/2}[10^{20}\mathrm{W/cm^2}] I_2^{1/2}[10^{20}\mathrm{W/cm^2}] \lambda_{01}^{1/2}[\mu\mathrm{m}] \lambda_{02}^{1/2}[\mu\mathrm{m}] \Delta T^2[\mathrm{ps}]$$

Maximum energy gain vs. laser intensity & time

$$\Delta\mathcal{E}_\mathrm{M}[\mathrm{MeV}] \approx 0.8\ Z_\mathrm{p}^4 A_\mathrm{p}^{-3} I_1[10^{20}\mathrm{W/cm^2}] I_2[10^{20}\mathrm{W/cm^2}] \lambda_{01}[\mu\mathrm{m}] \lambda_{02}[\mu\mathrm{m}] \Delta T^2[\mathrm{ps}]$$

Scaling laws for relativistic regime depend on the specific chirp laws

Maximum energy gain vs. laser parameters

$$\begin{aligned}\Delta\mathcal{E}_\mathrm{M}[\mathrm{MeV}] &\approx 0.08\ Z_\mathrm{p}^4 A_\mathrm{p}^{-3} \mathcal{E}_1[\mathrm{J}] \mathcal{E}_2[\mathrm{J}] w_{01}^{-2}[\mu\mathrm{m}] w_{02}^{-2}[\mu\mathrm{m}] \lambda_{01}[\mu\mathrm{m}] \lambda_{02}[\mu\mathrm{m}] \\ &= 8\times 10^{-7}\ Z_\mathrm{p}^4 A_\mathrm{p}^{-3} \mathcal{E}_1[\mathrm{J}] \mathcal{E}_2[\mathrm{J}] Z_\mathrm{R1}^{-1}[\mu\mathrm{m}] Z_\mathrm{R2}^{-1}[\mu\mathrm{m}]\end{aligned}$$

Large variations in velocity require large excursions in frequency

⇓

Acceleration could require multi-stage processes involving different laser technologies

Frequency excursion vs. velocity

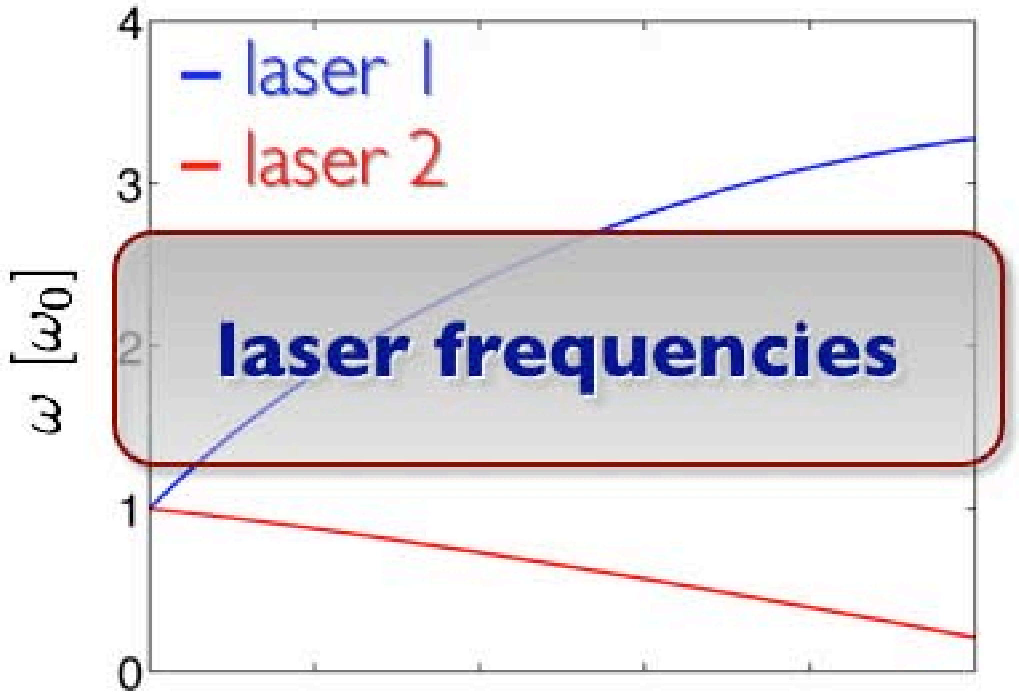
$$\frac{2\beta_0 + \Delta\omega_1 - \Delta\omega_2}{2 + \Delta\omega_1 + \Delta\omega_2} = \beta$$

$\Delta\omega_2 = 0$: $$\Delta\omega_1 = \frac{2(\beta - \beta_0)}{1-\beta}$$

$\Delta\omega_2 = -\Delta\omega$, $\Delta\omega_1 = \Delta\omega$: $$\Delta\omega = \beta - \beta_0$$

$\Delta\omega_1 = 0$: $$\Delta\omega_2 = \frac{2(\beta_0 - \beta)}{1+\beta}$$

Muon acceleration (example)

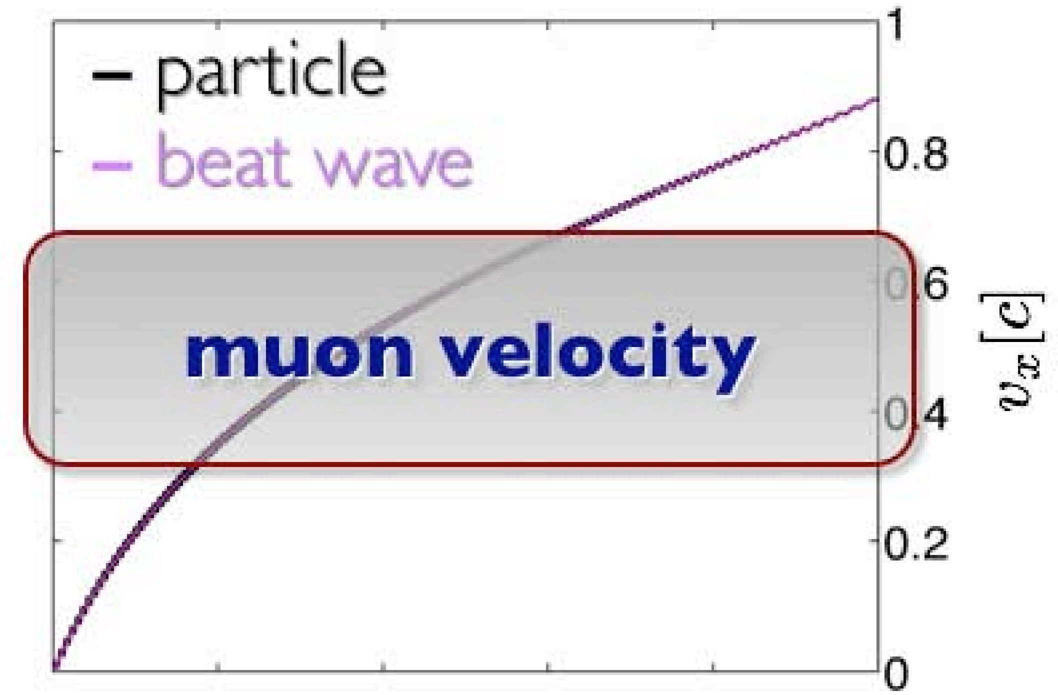


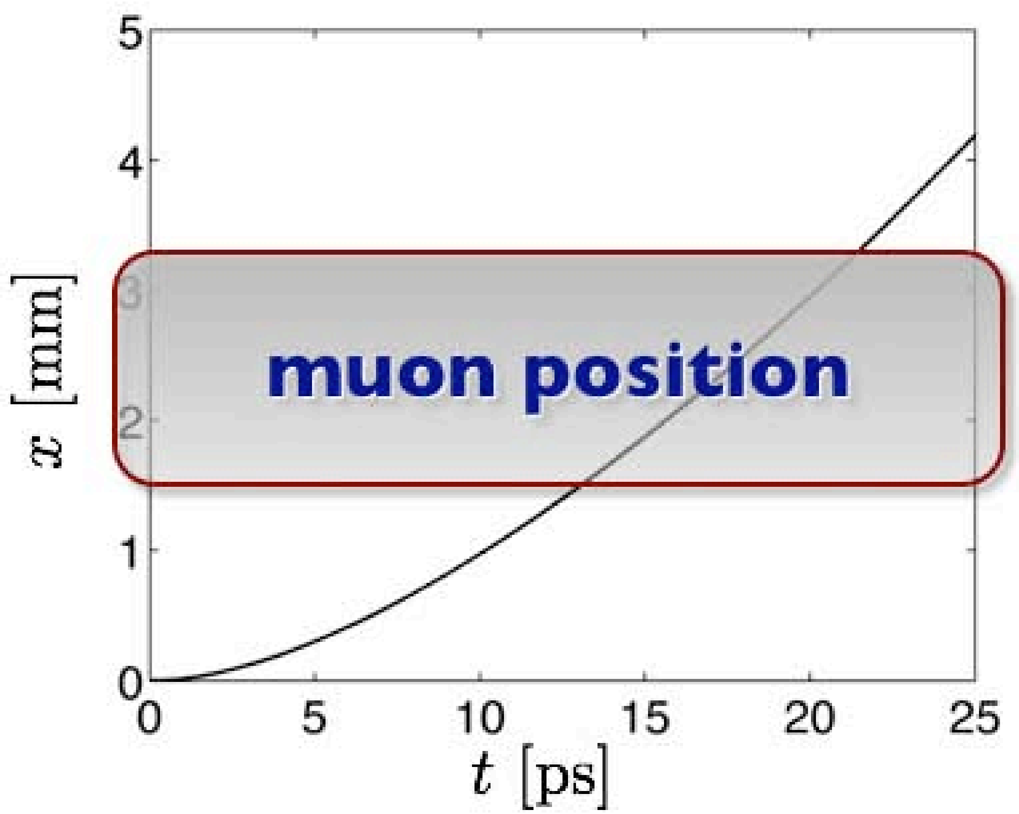
kJ lasers
linear chirp

$\hat{A}_1 \hat{A}_2 = 1.5 \times 10^{-4}$

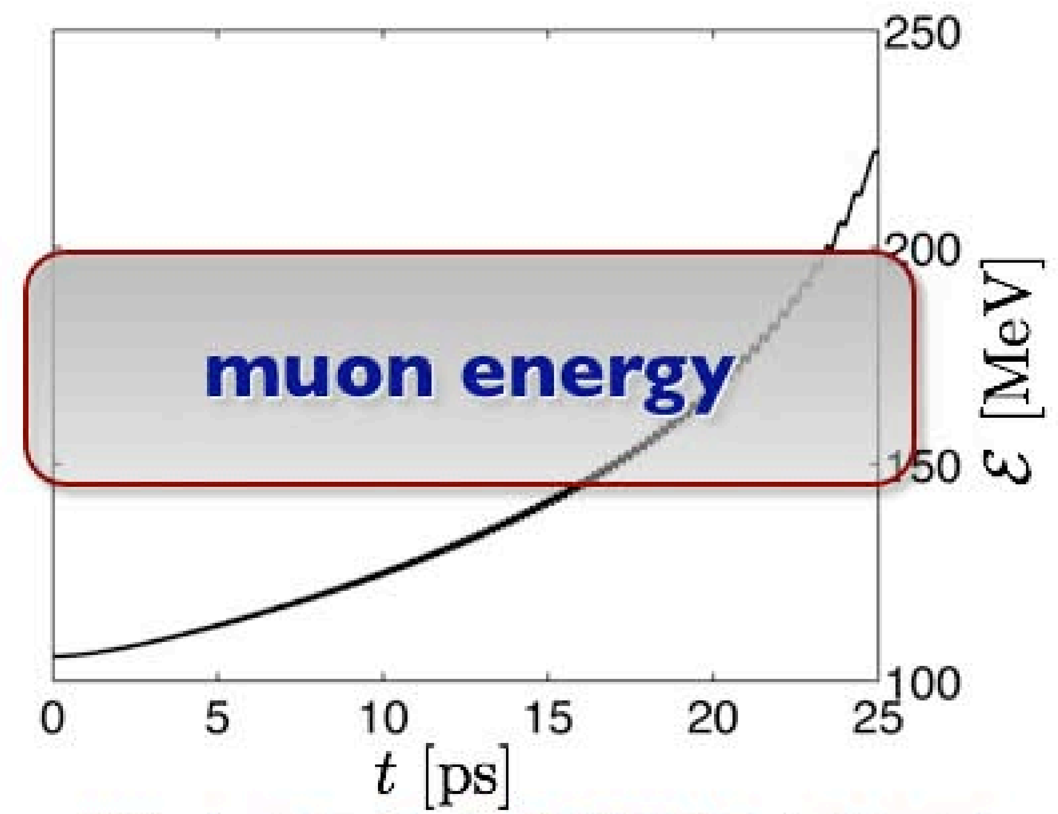
$\sigma_1 = -4.4 \times 10^{-5}$

$\sigma_2 = 4.4 \times 10^{-6}$





120 MeV gain
in 25 ps
over 4 mm



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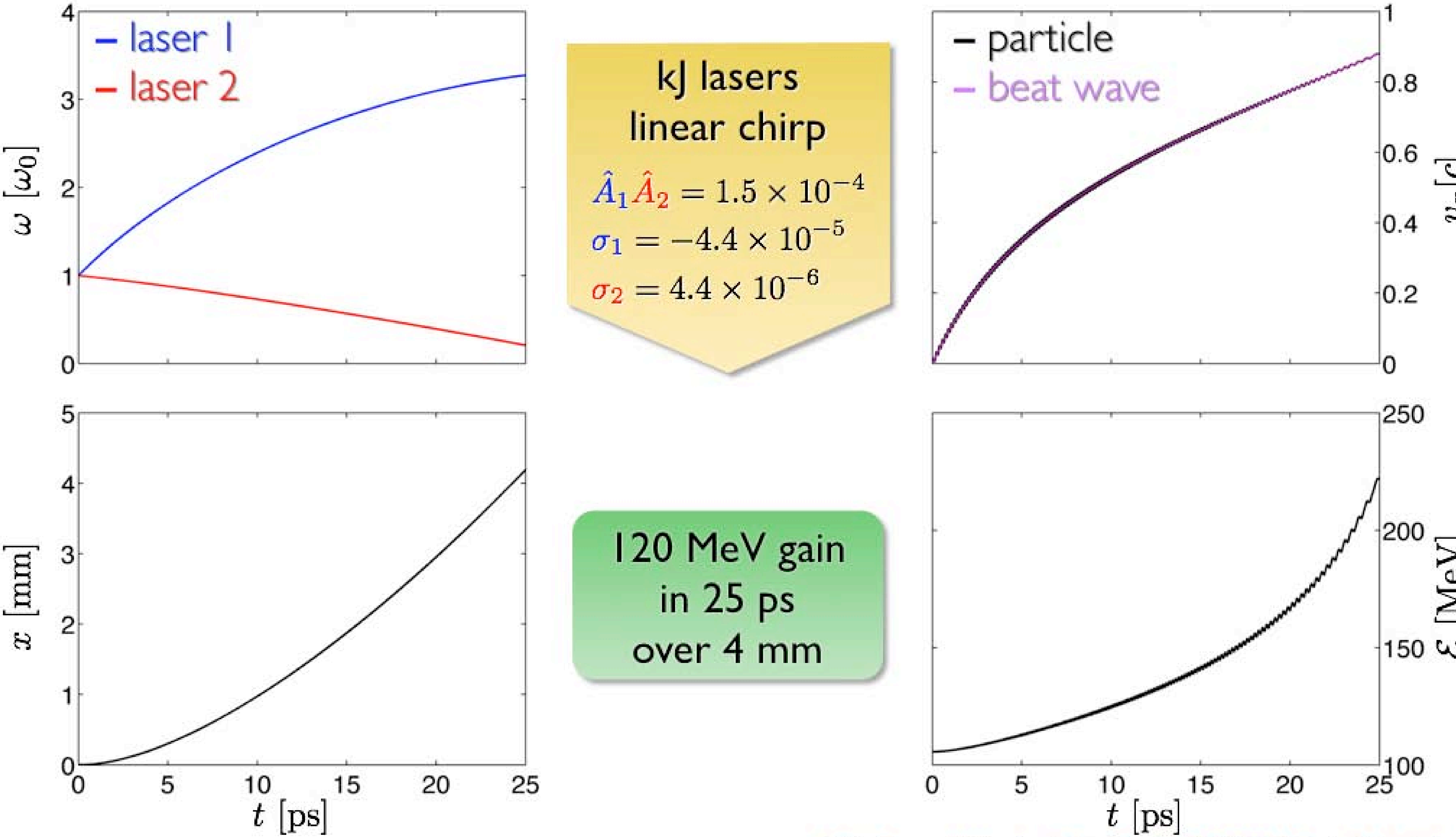

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Particle-in-cell simulations: osiris 2.0

osiris
v2.0

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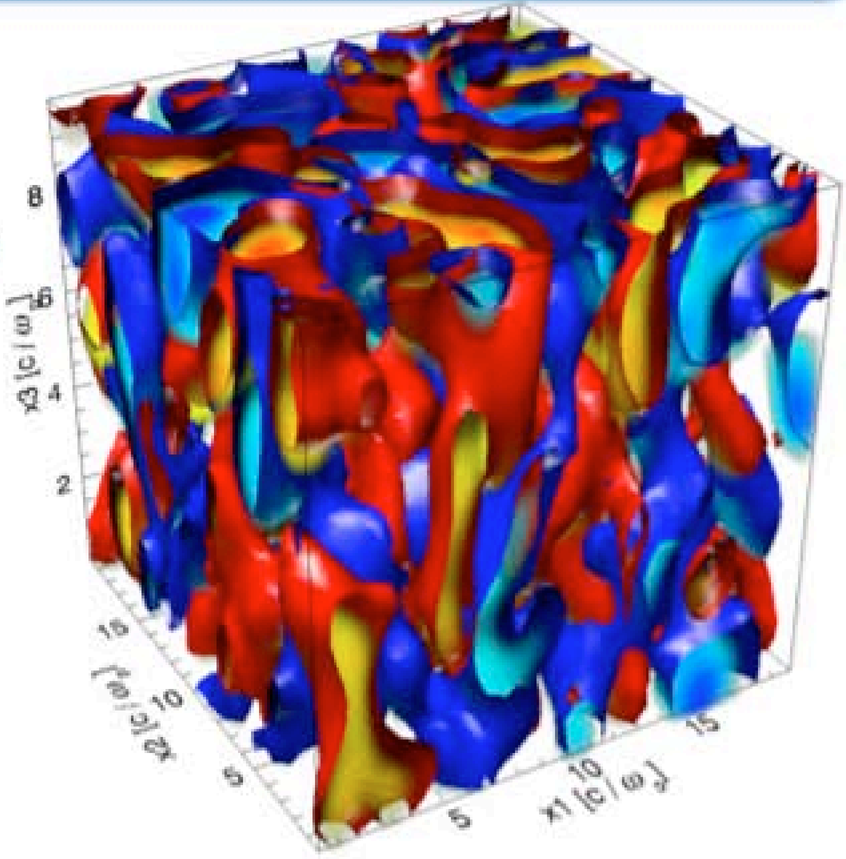
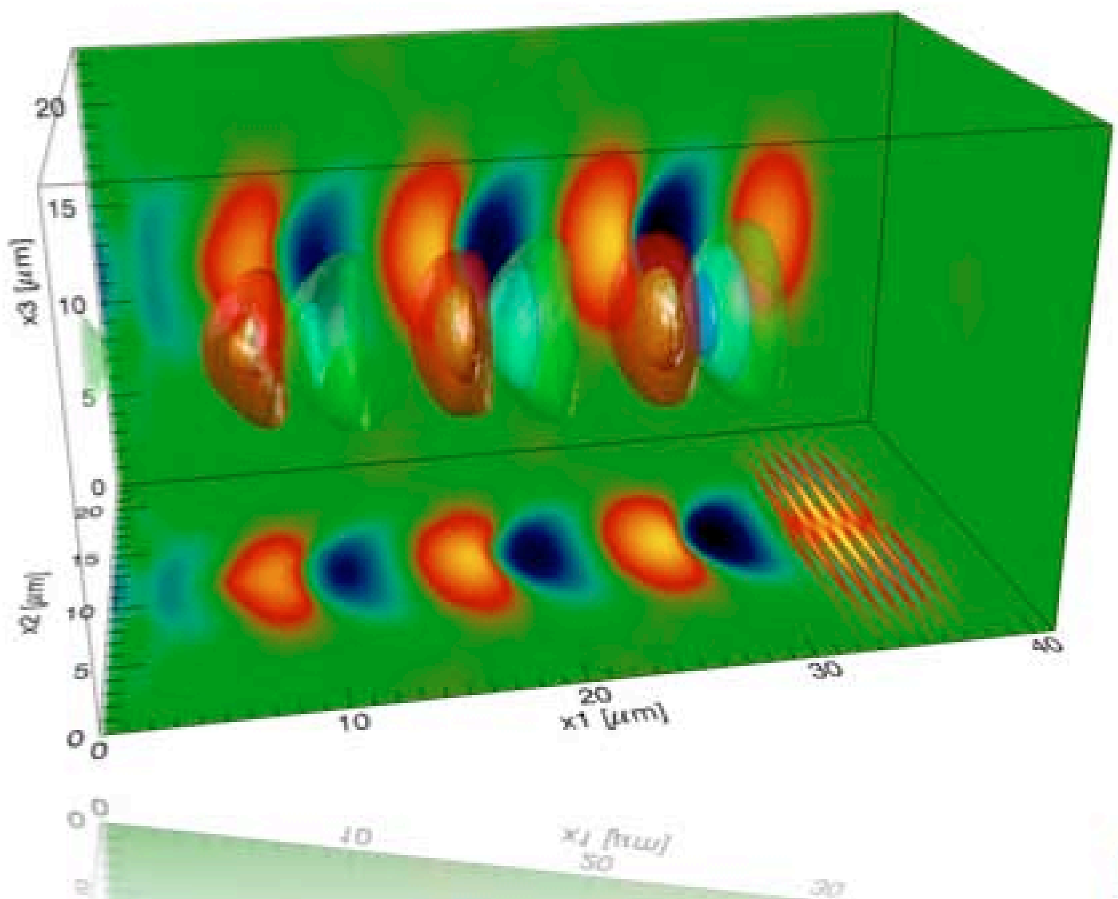
osiris framework

- Massively Parallel, Fully Relativistic Particle-in-Cell (PIC) Code
- Visualization and Data Analysis Infrastructure
- Developed by the osiris.consortium
  - ⇒ UCLA + IST + USC





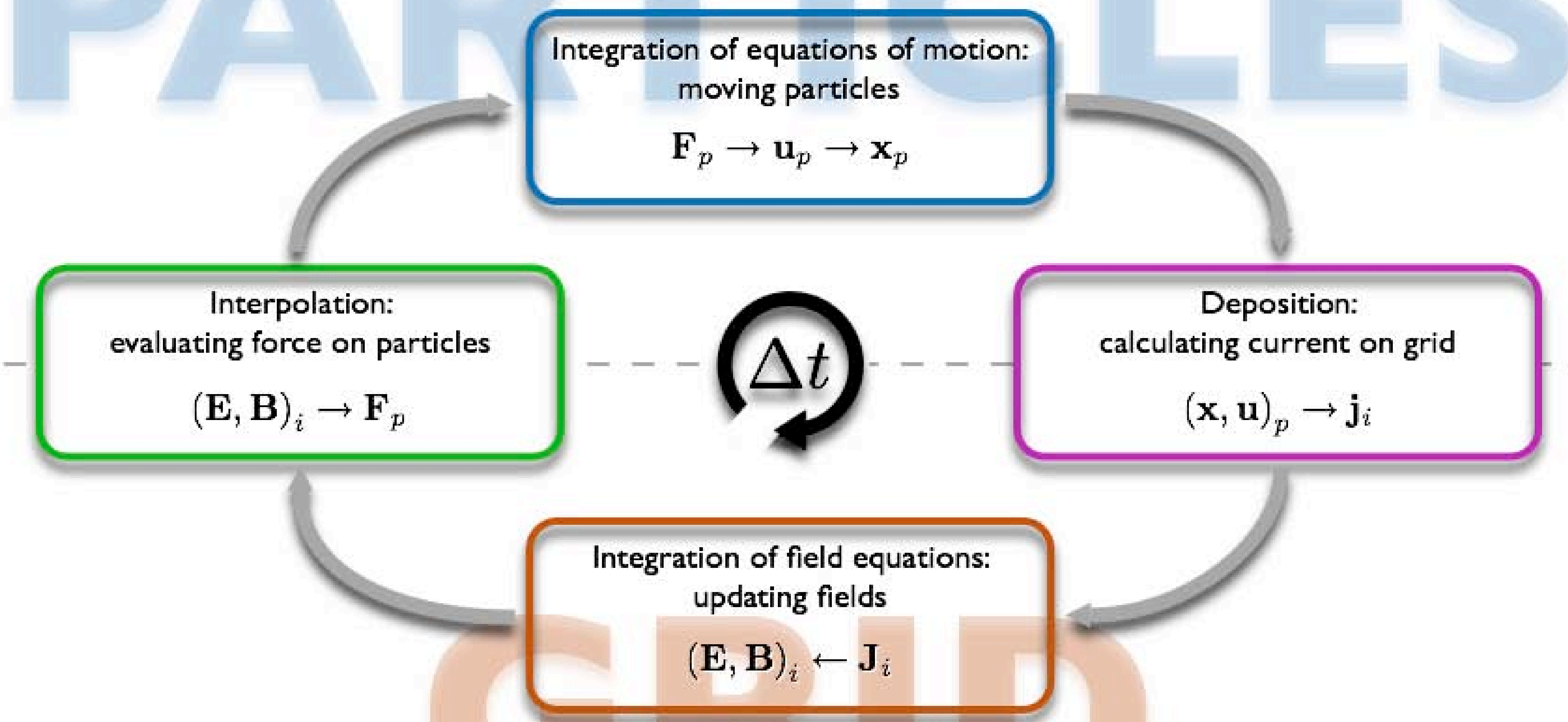
New Features in v2.0

- Bessel Beams
- Binary Collision Module
- Tunnel (ADK) and Impact Ionization
- Dynamic Load Balancing
- Parallel I/O

Particle-in-cell loop in osiris 2.0

osiris v2.0

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = q\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right)$$

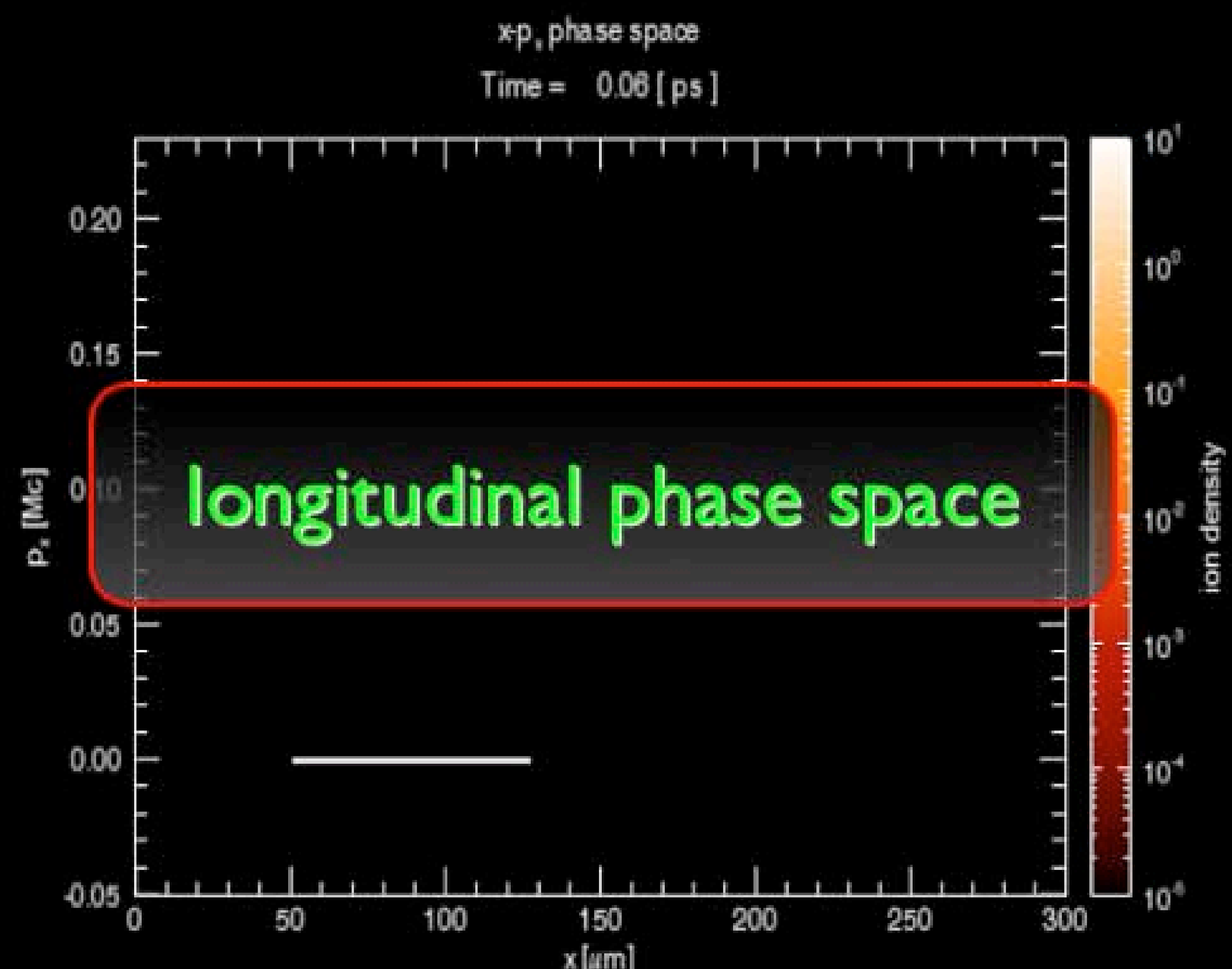


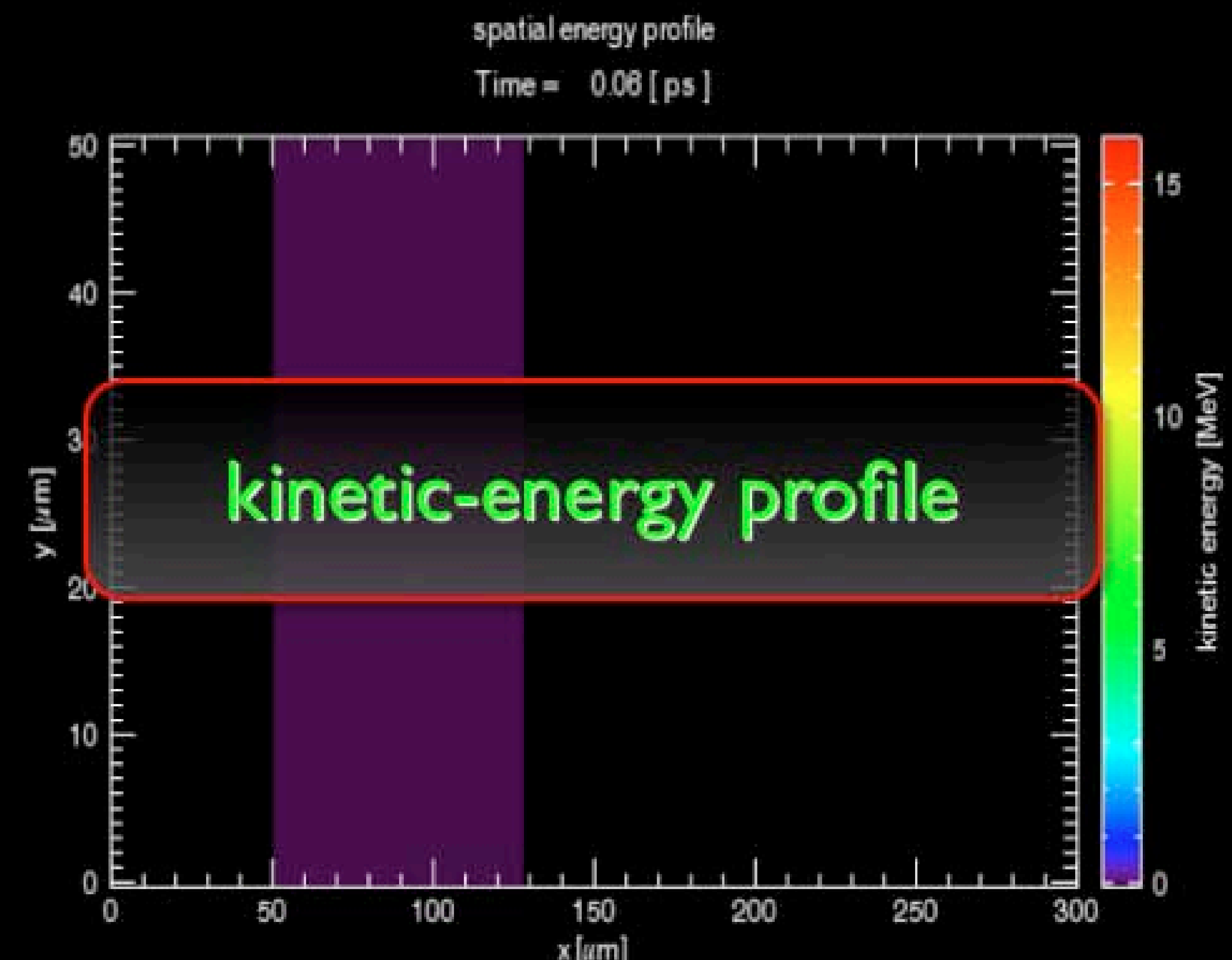
$$\frac{\partial \mathbf{E}}{\partial t} = c\nabla \times \mathbf{B} - 4\pi\mathbf{j}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla \times \mathbf{E}$$

Proton acceleration in a plasma

peak intensities:	$I_1 = 1.3 \times 10^{21}\ \mathrm{W/cm^2}$	proton density:	$n_\mathrm{i} = 5 \times 10^{16}\ \mathrm{cm^{-3}}$
	$I_2 = 8.5 \times 10^{20}\ \mathrm{W/cm^2}$	slab thickness:	$75\ \mu\mathrm{m}$
chirp coefficient:	$\sigma = -2 \times 10^{-5}\ k_0^2$	pulse duration:	$4.2\ \mathrm{ps}$
ref. wavelength:	$\lambda_0 = 820\ \mathrm{nm}$	spot size:	$10\ \mu\mathrm{m}$





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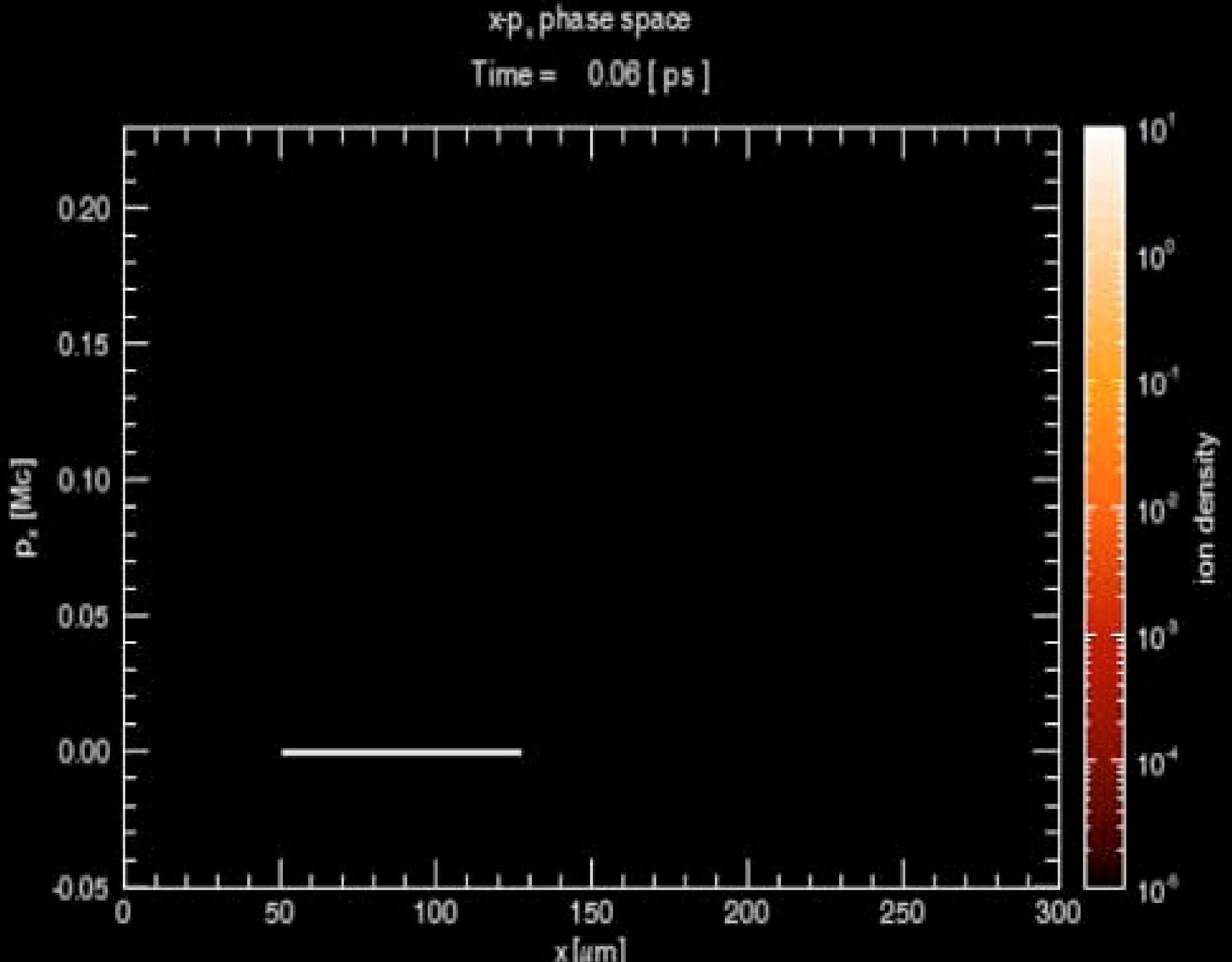
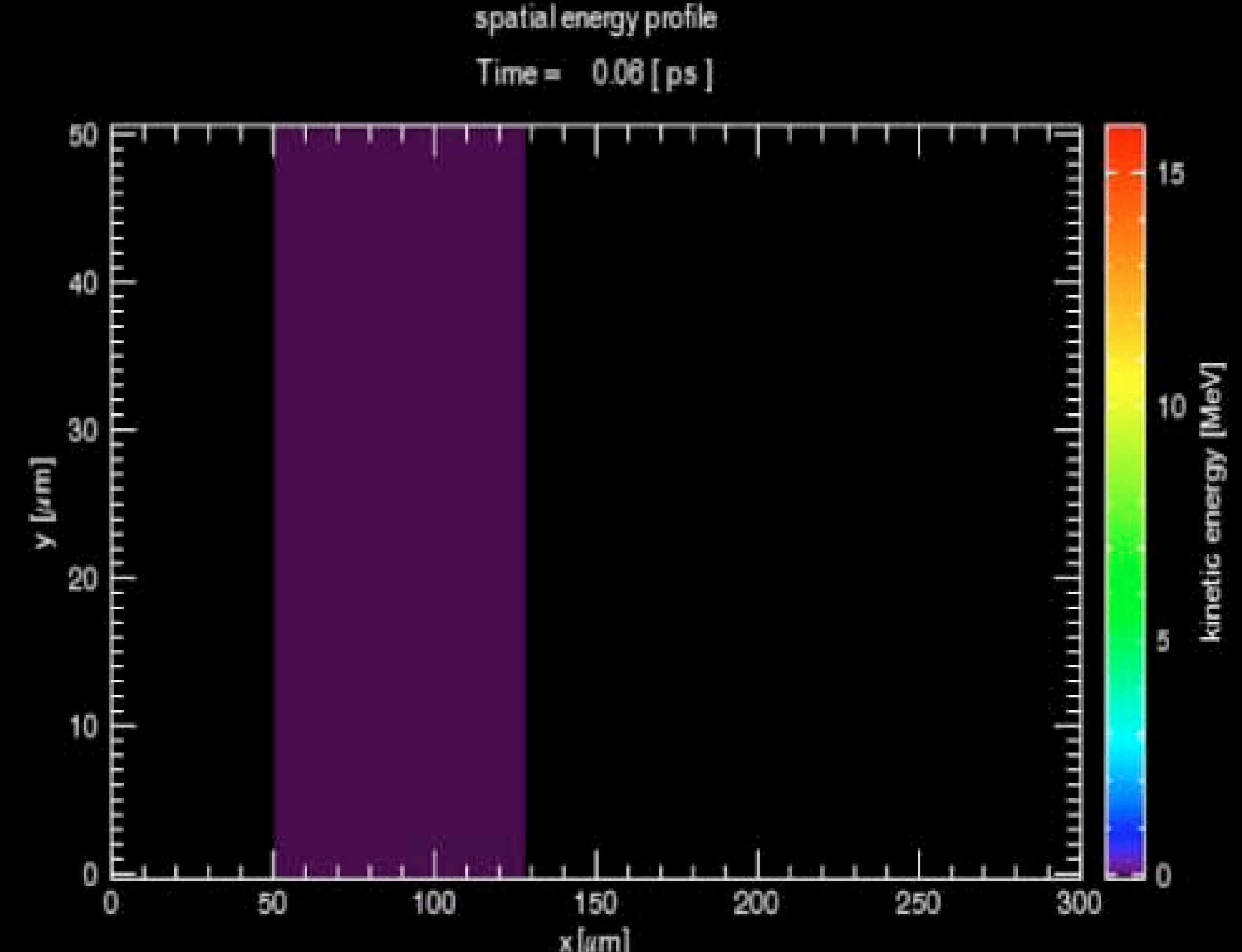
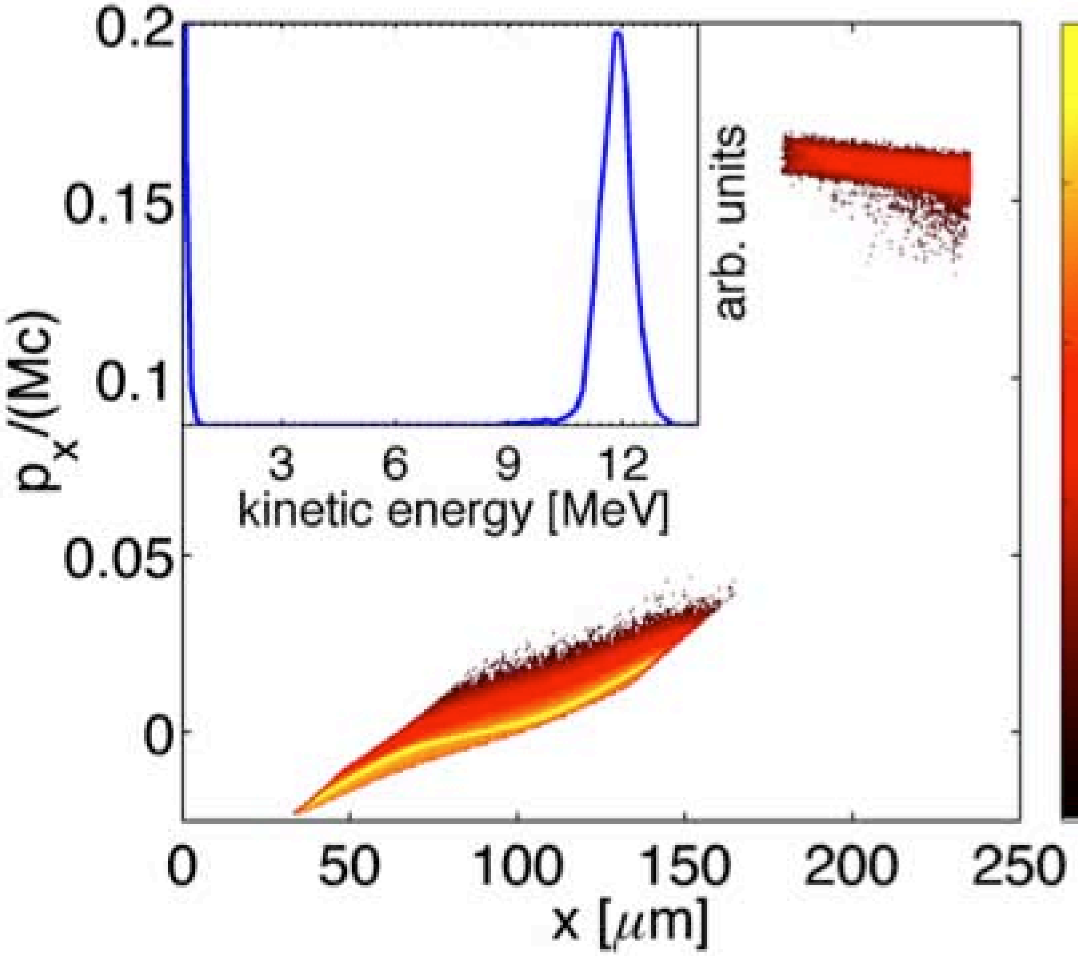
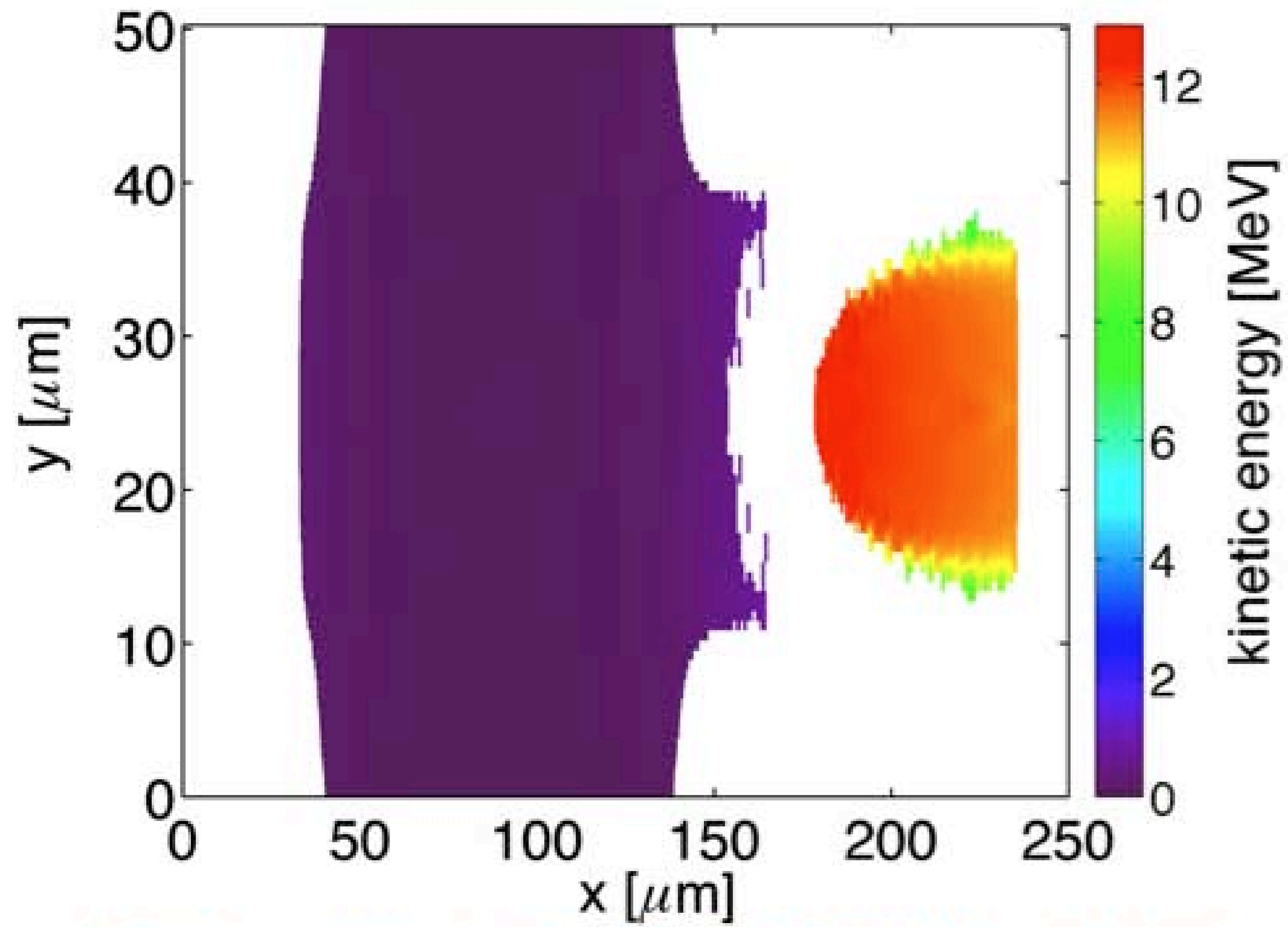




Proton beam features after 6 ps

- monoenergetic beam @ 12 MeV with 7.5% energy spread
- bunch is 60 μm and 20 μm wide
- transverse momentum spread < 0.01 Mc
- 4.2% of charge in focal region trapped (~8 pC in 3D)





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Conclusions & perspectives

- ☑ Variable-frequency lasers allow for direct acceleration of charged particles @ nonrelativistic intensities: particularly suitable for ions
- ☑ Physical mechanism is robust: works with any source of charged particles (e.g., external beams, tenuous plasmas)
- ☑ Production of monoenergetic charged-particle beams achievable, energy distribution tunable by regulating the laser chirp laws
- ☑ Method works in a test-particle regime: excellent controllability, but very demanding in terms of required laser energy

Open problems

- Transverse focusing is needed: could be provided by suitably shaped lasers (e.g., using annular transverse profiles)
- Diffraction limits the acceleration distance: wide-spot size or guiding may be necessary to reach high energy gain

Application to muon acceleration

- Technique could be employed to extract muons from background plasmas and accelerate them to relativistic energies
- High energy gain in a single stage is extremely challenging from the technological point of view: multi-stage approach is needed