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# Neutrinos and Supernovae

**Bob Bingham**

**STFC – Centre for Fundamental Physics (CfFP)**

**Rutherford Appleton Laboratory.**

**SUPA– University of Strathclyde.**



# Motivation

**Neutrinos are the most enigmatic particles in the Universe**

**Associated with some of the long standing problems in astrophysics**

**Solar neutrino deficit**

**Gamma ray bursters (GRBs)**

**Formation of structure in the Universe**

**Supernovae II (SNe II)**

**Stellar/Neutron Star core cooling**

**Dark Matter/Dark Energy**

**Intensities in excess of  $10^{30}$  W/cm<sup>2</sup> and luminosities up to  $10^{53}$  erg/s**



**Intense fluxes of neutrinos in Supernovae**

**Neutrino dynamics in dense plasmas (making the bridge with HEP)**

**Plasma Instabilities driven by neutrinos**

**Supernovae plasma heating: shock revival**

**Neutrino mode conversion (Neutrino oscillations)**

**Neutrino Landau damping**

**Neutron star cooling**

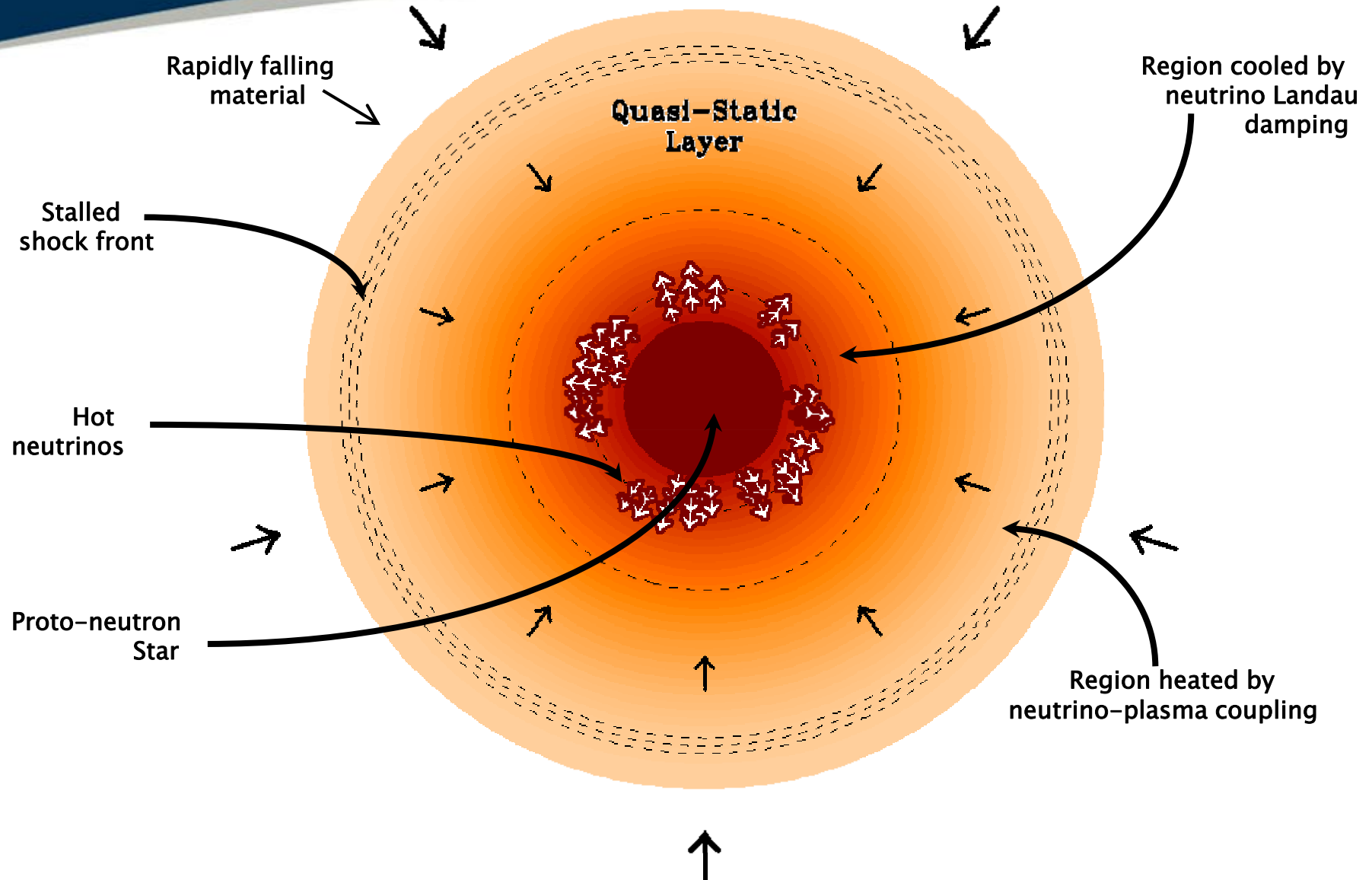
**Solar neutrino deficit**

**Gamma-ray bursters: open questions**

**Conclusions**

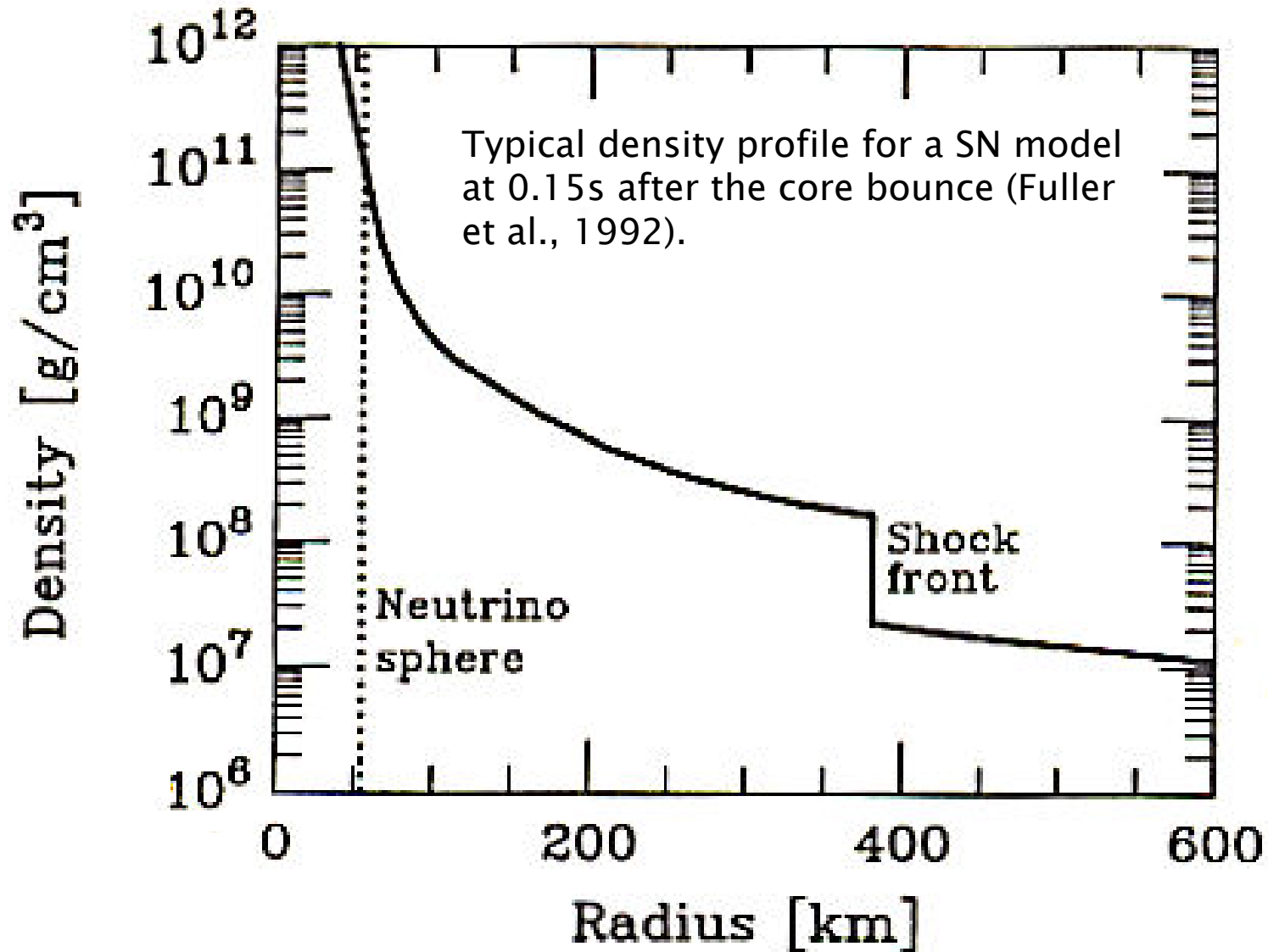


# Supernovæ Explosion





# Typical Density Profile





# Length scales

← **Compton Scale**  
**HEP**

**Hydro Scale** →  
**Shocks**

*Plasma scale*

$\lambda_D, \lambda_p, r_L$

Can intense neutrino winds drive collective and kinetic mechanisms at the *plasma scale* ?

Bingham, Bethe, Dawson, Su (1994)





# Supernova Explosion

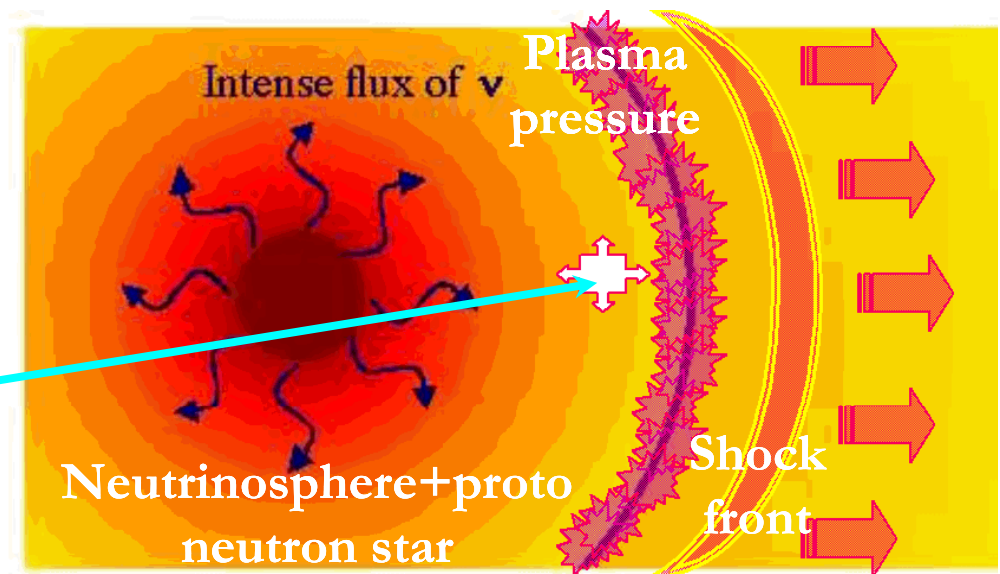
A supernova releases  $5 \times 10^{53}$  erg  
(gravitational binding energy of the original star)

- neutrinos 99 % •
- kinetic energy+light  $\sim 10^{51}$  erg •

**How to turn an implosion  
into an explosion?**

$\Rightarrow$  **Neutrino-plasma  
scattering instabilities  
in dense plasmas**

Neutrino-plasma  
heating





# Neutrino Refractive Index

The interaction can be easily represented by neutrino refractive index.

The dispersion relation:  $(E_\nu - V)^2 - p_\nu^2 c^2 - m_\nu^2 c^4 = 0$  (Bethe, 1986)

$E$  is the neutrino energy,  $p$  the momentum,  $m_\nu$  the neutrino mass.

The potential energy  $V = \sqrt{2} G_F n_e$

$G_F$  is the Fermi coupling constant,  $n_e$  the electron density

⇒ Refractive index

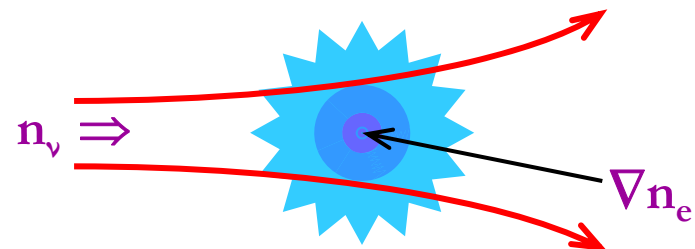
$$N_\nu = \left( \frac{ck_\nu}{\omega_\nu} \right)^2 = \left( \frac{cp_\nu}{E_\nu} \right)^2$$

$$N_\nu \cong 1 - \frac{2\sqrt{2}G_F}{\hbar k_\nu c} n_e$$

Note: cut-off density  
 $\varepsilon_\nu$  neutrino energy

$$n_{ec} > \frac{\varepsilon_\nu}{2\sqrt{2}G_F}$$

Electron neutrinos are refracted away from regions of dense plasma - similar to photons.







For intense neutrino beams, we can introduce the concept of the Ponderomotive force to describe the coupling to the plasma. This can then be obtained from the 2<sup>nd</sup> order term in the refractive index.

Definition 
$$F_{POND} = \frac{N-1}{2} \nabla \xi \quad [\text{Landau \& Lifshitz, 1960}]$$

where  $\xi$  is the energy density of the neutrino beam.

$$N = 1 - \frac{2\sqrt{2}G_F n_e}{\epsilon_\nu} \Rightarrow F_{Pond} = -\frac{\sqrt{2}G_F n_e}{\epsilon_\nu} \nabla \xi$$

$n_\nu$  is the neutrino number density.

$$F_{Pond} \equiv -\sqrt{2}G_F n_e \nabla n_\nu$$



# Neutrino Dynamics in Dense Plasma

Dynamics governed by Hamiltonian (Bethe, '86):

$$H_{eff} = \sqrt{\mathbf{p}_\nu^2 c^2 + m_\nu^2 c^4} + 2G_F n_e(\mathbf{r}, t)$$

|  $G_F$  - Fermi constant  
|  $n_e$  - electron density

$$\mathbf{F}_{pond} = -\sqrt{2}G_F \nabla n_\nu(\mathbf{r}, t)$$

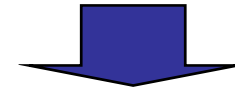
Force on a single electron  
due to neutrino distribution

**Ponderomotive force\* due to neutrinos pushes electrons to regions of lower neutrino density**

\* ponderomotive force derived from semi-classical (L.O.Silva et al, '98) or quantum formalism (Semikoz, '87)



- ¶ Effective potential due to **weak interaction** with background electrons
- ¶ Repulsive potential



$$\mathbf{F} = -\sqrt{2}G_F \nabla n_e(\mathbf{r}, t)$$

Force on a single neutrino due to electron density modulations

**Neutrinos bunch in regions of lower electron density**



## Neutrino Ponderomotive Force (2)

Force on one electron due to electron neutrino collisions  $f_{\text{coll}}$

$$f_{\text{coll}} = \sigma_{\nu_e} \xi \quad \sigma_{\nu_e} = \left( \frac{G_F k_B T_e}{2\pi \hbar^2 c^2} \right)^2 \quad \sigma_{\nu_e} \text{ is the neutrino-electron cross-section}$$

Total collisional force on all electrons is

$$F_{\text{coll}} = n_e f_{\text{coll}} = n_e \sigma_{\nu_e} \xi$$
$$\frac{F_{\text{Pond}}}{F_{\text{coll}}} = \frac{\sqrt{2\pi} \hbar^3 c^3 |k_{\text{Mod}}|}{G_F k_B^2 T^2 k_\nu}$$

$|k_{\text{mod}}|$  is the modulation wavenumber.

For a 0.5 MeV plasma  $\frac{F_{\text{Pond}}}{F_{\text{coll}}} \approx 10^{10}$

$\sigma_{\nu_e} \Rightarrow$  collisional mean free path of  $10^{16}$  cm.



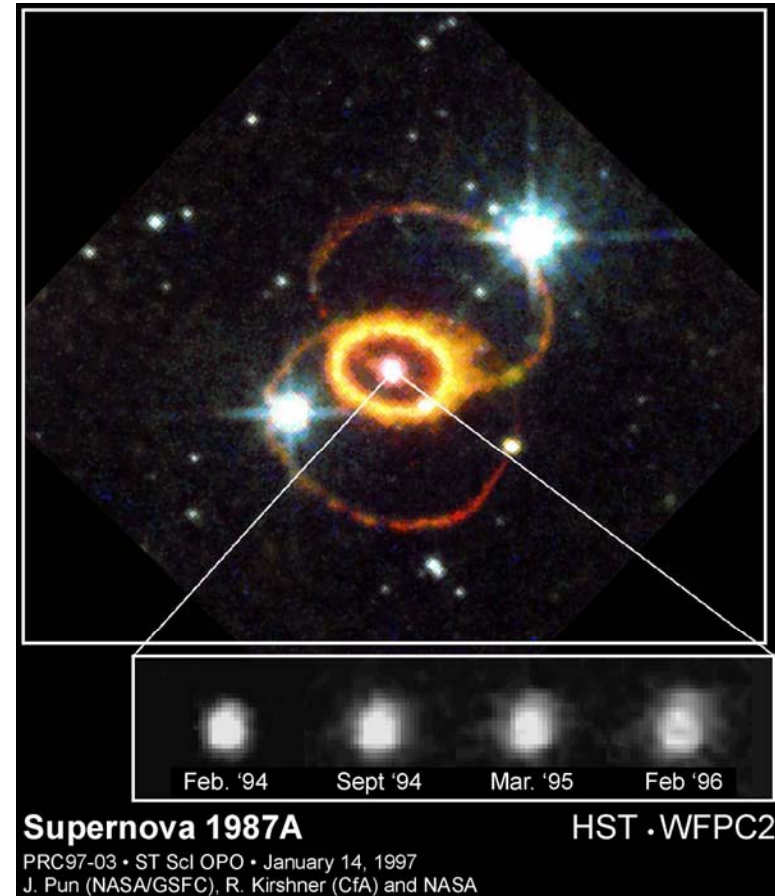
# Supernova II Physical Parameters

**To form a neutron star  $3 \times 10^{53}$  erg must be released**

(gravitational binding energy of the original star)

- **light+kinetic energy  $\sim 10^{51}$  erg •**
- **gravitational radiation  $< 1\%$  •**
- **neutrinos 99 % •**

- ¶ Electron density @ 100-300 km:  $n_{e0} \sim 10^{29} - 10^{32} \text{ cm}^{-3}$
- ¶ Electron temperature @ 100-300 km:  $T_e \sim 0.1 - 0.5 \text{ MeV}$
- ¶  $\nu_e$  luminosity @ neutrinosphere  $\sim 10^{52} - 5 \times 10^{53} \text{ erg/s}$
- ¶  $\nu_e$  intensity @ 100-300 Km  $\sim 10^{29} - 10^{30} \text{ W/cm}^2$
- ¶ Duration of intense  $\nu_e$  burst  $\sim 5 \text{ ms}$   
(resulting from  $p+e \rightarrow n+\nu_e$ )
- ¶ Duration of  $\nu$  emission of all flavors  $\sim 1 - 10 \text{ s}$





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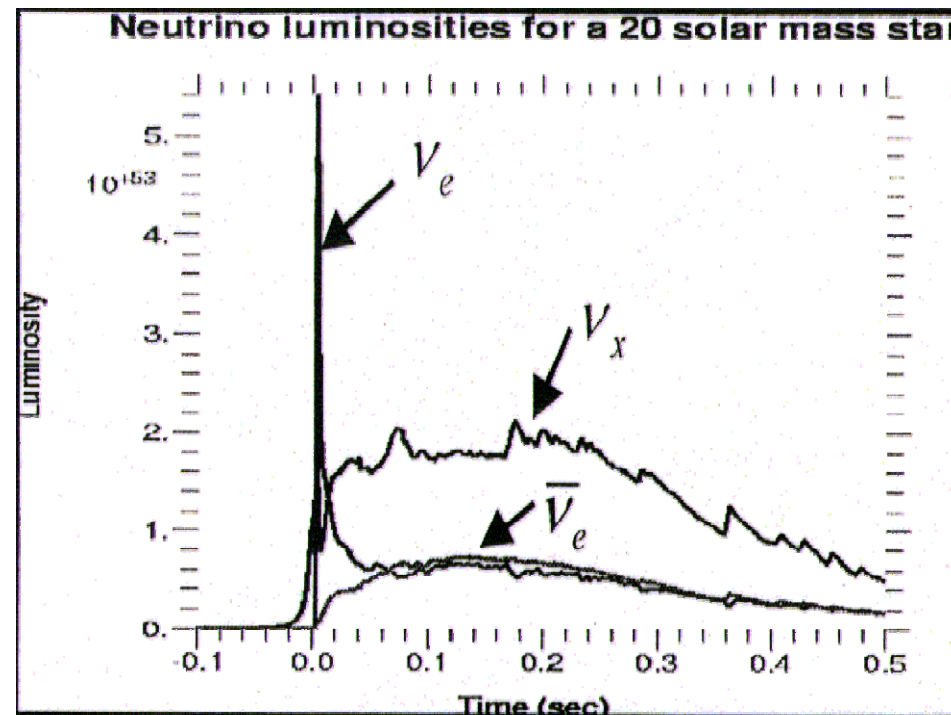
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## Neutrino heating is necessary for a strong explosion

The shock exits the surface of the proto-neutron star and begins to stall approximately 100 milliseconds after the bounce.

The initial electron neutrino pulse of  $5 \times 10^{53}$  ergs/second is followed by an “accretion” pulse of all flavours of neutrinos.

This accretion pulse of neutrinos deposits energy behind the stalled shock, increasing the matter pressure sufficiently to drive the shock completely through the mantle of the star.





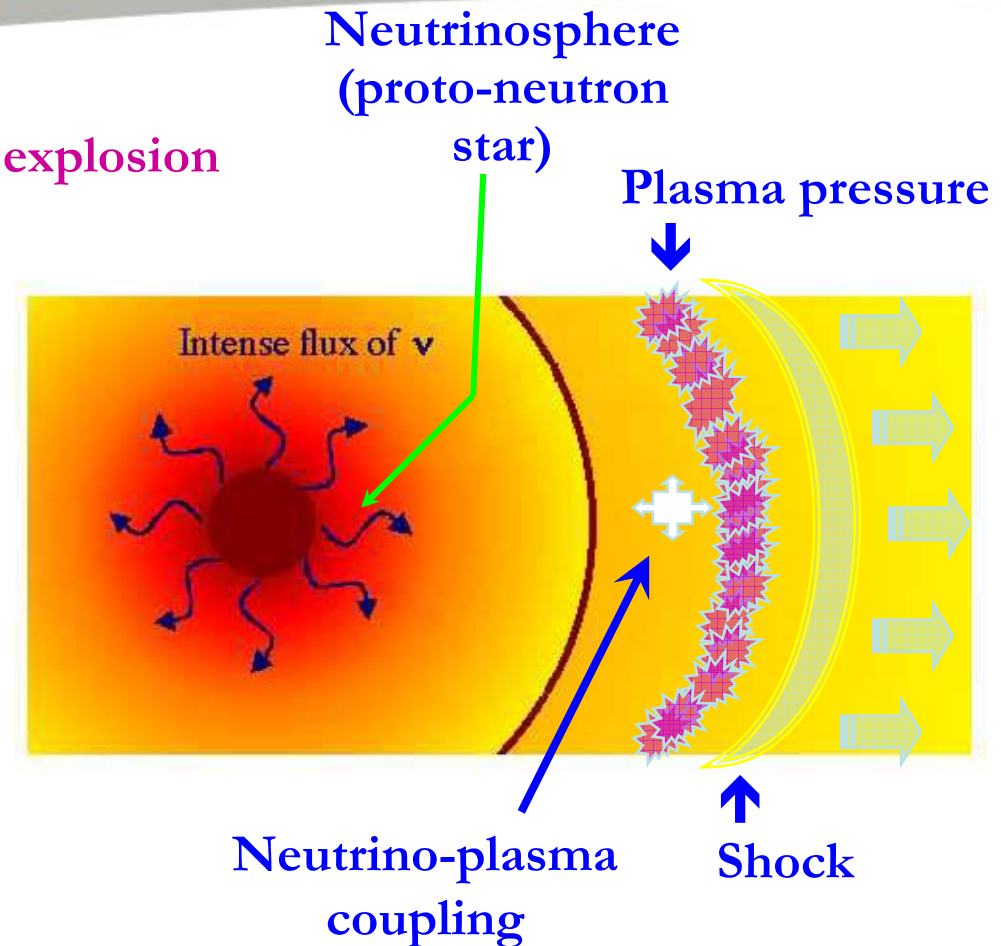
# Supernova Explosion

- **How to turn an implosion into an explosion**

- New neutrino physics
- $\lambda_{\text{mfp}}$  for  $\nu\nu$  collisions  $\sim 10^{16}$  cm in collapsed star
- $\lambda_{\text{mfp}}$  for collective plasma-neutrino coupling  $\sim 100\text{m}$

- **How?**

- New non-linear force — neutrino ponderomotive force
- For intense neutrino flux collective effects important
- Absorbs 1% of neutrino energy  
 $\Rightarrow$  sufficient to explode star



Bingham *et al.*, *Phys. Lett. A*, 220, 107 (1996)

Bingham *et al.*, *Phys. Rev. Lett.*, 88, 2703 (1999)



## Two stream instability

Neutrinos driving electron plasma waves  $v\phi \sim c$   
Anomalous heating in SNe II

## Collisionless damping of electron plasma waves

Neutrino Landau damping  
Anomalous cooling of neutron stars

## Electroweak Weibel instability

Generation of quasi-static B field  
Primordial B and structure in early Universe



# Neutrino kinetics in a dense plasma

## Kinetic equation for neutrinos

(describing neutrino number density conservation / collisionless neutrinos)

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{v}} f - \nabla_{\mathbf{v}} \cdot \left( \frac{1}{c} \mathbf{v} \nabla_{\mathbf{v}} f \right) = 0$$

## Kinetic equation for electrons driven by neutrino pond. force

(collisionless plasma)

$$\frac{\partial f_e}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{v}} f_e - \nabla_{\mathbf{v}} \cdot \left( \frac{1}{c} \mathbf{v} \nabla_{\mathbf{v}} f_e \right) = \nabla_{\mathbf{v}} \cdot \left( \frac{1}{c} \mathbf{v} f_e \right)$$

+

Maxwell's Equations





# Two stream instability driven by a neutrino beam

Usual perturbation theory over kinetic equations + Poisson's equation

$$\begin{aligned} n_e &= n_{e0} + n_{e1} & f_e &= f_{e0}(p_e) + f_{e1} \\ \mathbf{v}_e &= \mathbf{v}_e & f_\nu &= f_{\nu 0}(p_\nu) + f_{\nu 1} \\ \mathbf{v}_\nu &= \mathbf{v}_{\nu 0} + \mathbf{v}_{\nu 1} & \mathbf{E} &= \mathbf{E}_1 \end{aligned}$$

Dispersion relation for electrostatic plasma waves



Electron susceptibility

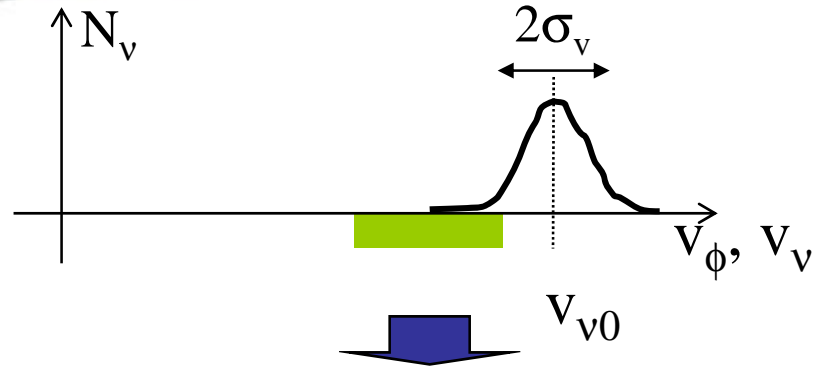
Neutrino susceptibility

$$\chi_\nu(\omega_e, k_e) = -2 G_F^2 \frac{v_e^2 n_{e0} n_{\nu 0}}{m_e \omega_{pe}^2} \left( 1 - \frac{\omega_e^2}{c^2 k_e^2} \right)^2 \chi_e \int d^3 p_\nu \frac{k_e \cdot \frac{\partial f_{\nu 0}}{\partial p_\nu}}{\omega_e - k_e \cdot \mathbf{v}_\nu}$$

(Silva et al, PRL 1999)



# Instability Regimes: hydrodynamic vs kinetic



If region of unstable PW modes **overlaps** neutrino distribution function **kinetic regime becomes important**

**Unstable PW modes** ( $\omega_L, k_L$ )

$N_{e0} = 10^{29} \text{ cm}^{-3}$     $\langle E_\nu \rangle = 10 \text{ MeV}$   
 $L_\nu = 10^{52} \text{ erg/s}$     $T_\nu = 3 \text{ MeV}$   
 $R_m = 300 \text{ Km}$     $m_\nu = 0.1 \text{ eV}$

**Kinetic instability**  $\gamma \propto G_F^2$  if  $\left| \frac{\omega_L}{k_L} - v_{v0} \right| \ll \sigma_{v_\nu}$

$\sigma_{v_\nu} / c \approx 10^{-16}$

**Hydro instability**  $\gamma \propto G_F^{2/3}$  if  $\left| \frac{\omega_L}{k_L} - v_{v0} \right| \gg \sigma_{v_\nu}$

$$\left| \frac{\omega_L}{ck_L} - \frac{v_{v0}}{c} \right| \approx \frac{\gamma_{\max}}{\omega_{pe0}} \beta_\phi \approx 10^{-14} - 10^{-11}$$

where  $v_\nu = p_\nu c^2 / E_\nu = p_\nu c^2 / (p_\nu^2 c^2 + m_\nu^2 c^4)^{1/2}$   
 - for  $m_\nu \rightarrow 0, \sigma \rightarrow 0$  **hydro regime** -



# Estimates of the Instability Growth Rate

$n_{e0} = 10^{29} \text{ cm}^{-3}$   
 $L_\nu = 10^{52} \text{ erg/s}$   
 $R_m = 300 \text{ Km}$   
 $\langle E_\nu \rangle = 10 \text{ MeV}$

Growth distance  $\sim 1 \text{ m}$   
(without collisions)

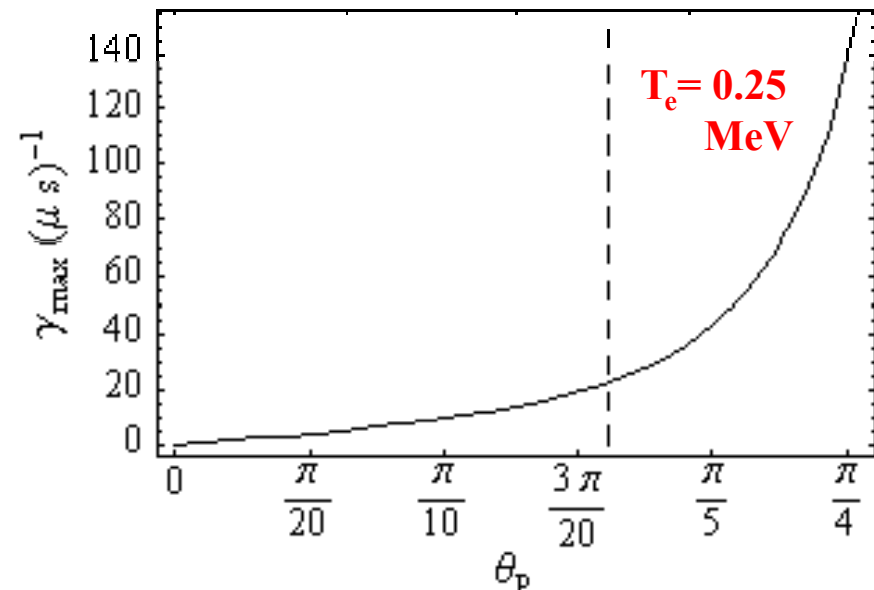
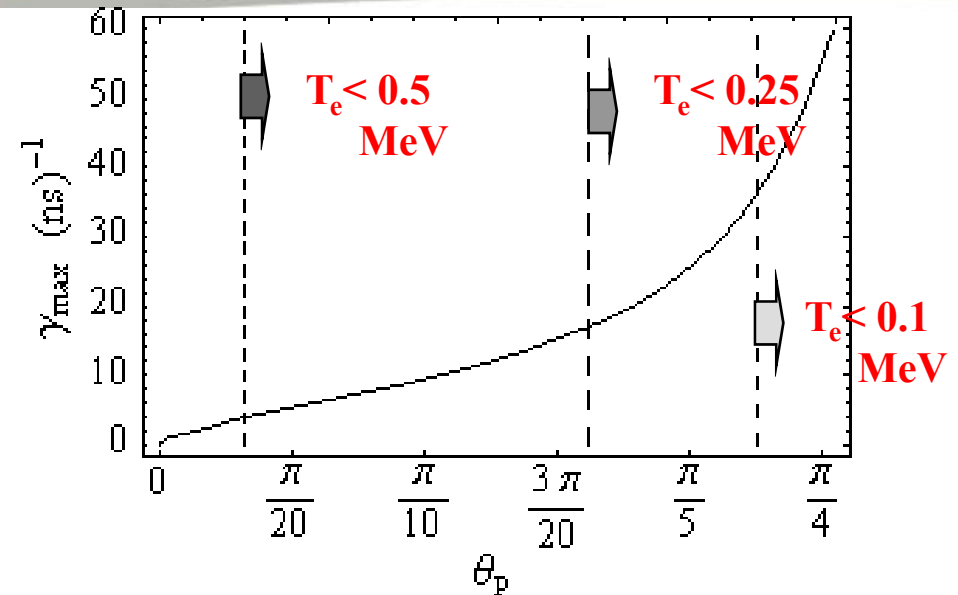
Growth distance  $\sim 300 \text{ m}$   
(with collisions)

- 6 km for 20 e-foldings -

Mean free path for  
neutrino electron single scattering  
 $\sim 10^{11} \text{ km}$

Single  $\nu$ -electron scattering  $\propto G_F^2$

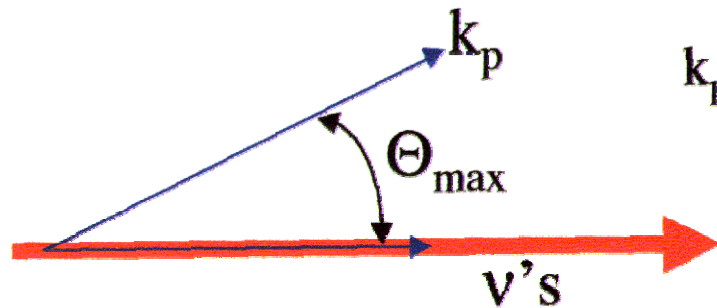
Collective mechanism much stronger  
than single particle processes





# Saturation Mechanism

Neutrino streaming instability saturates by electron Landau damping



$$k_p \sim \omega_{pe0}/c \cos \Theta \quad \Theta_{max} \sim \arccos(v_{th}/c)$$

$T_e \uparrow \Rightarrow \Theta_{max} \downarrow \Rightarrow$  **Instability Shutdown**

Modes with maximum growth rate

$$\mathbf{E}_k = E_k \delta(k_{||} - \omega_{pe0}/c) \frac{\mathbf{k}}{|\mathbf{k}|}$$



## Simplified Model

$$\frac{\partial |\mathbf{E}_k|^2}{\partial t} = 2\gamma_k |\mathbf{E}_k|^2$$

$$\gamma_k = 0 \text{ if } k > k_{max}$$

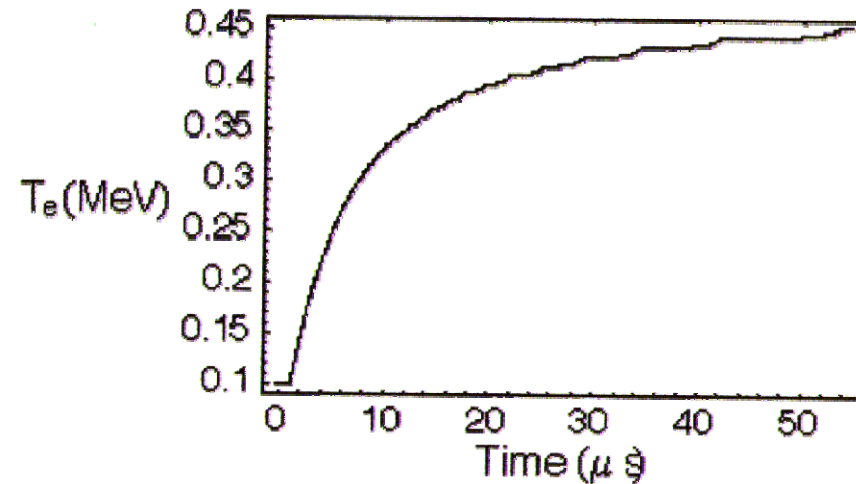
$$\frac{\partial W_{EPW}}{\partial t} = n_e \frac{\partial T_e}{\partial t} = \frac{1}{8\pi} \frac{\partial}{\partial t} \sum_{k \leq k_{max}} |\mathbf{E}_k|^2$$

$$k_{max} = \omega_{pe0}/v_{th}(T_e)$$



# Electron Heating

$n_{e0} = 10^{29} \text{ cm}^{-3}$   
 $L_\nu = 10^{52} \text{ erg/s}$   
 $R_m = 300 \text{ Km}$   
 $\langle E_\nu \rangle = 10 \text{ MeV}$   
 $T_{e0} = 0.1 \text{ MeV}$



$$\Delta W_{\text{EPW}} \sim 10^{-4} W_\nu$$

including e-i collisions

- ¶ Preliminary results indicate strong heating up to 0.5 MeV;
- ¶ Further analysis is necessary to include relativistic corrections on electron Landau damping - present model overestimates eLD;
- ¶ Initial  $v_e$  burst ( $\sim$  ms) can heat the plasma efficiently;
- ¶ Detailed quasi-linear theory for  $v$ 's and  $e$ 's will give signatures of  $v$ -driven instabilities and more accurate results  $\rightarrow$  information to be included in supernovae code
- ¶ Stimulated "Compton" scattering must also be considered



# Supernovæ explosion and neutrino driven instabilities

**e-Neutrino burst**  
 $L_\nu \sim 4 \times 10^{53} \text{ erg/s}$ ,  $\tau \sim 5 \text{ ms}$

**Neutrino emission of all flavors**  
 $L_\nu \sim 10^{52} \text{ erg/s}$ ,  $\tau \sim 1 \text{ s}$

**Due to electron Landau damping,  
plasma waves only grow in the  
lower temperature regions**

drives plasma waves through  
neutrino streaming instability

plasma waves are damped  
(collisional damping)

Plasma heating  
@ 100-300 km from center

Stimulated  
"Compton"  
scattering

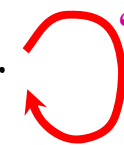
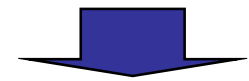
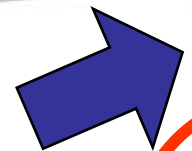
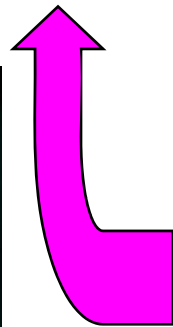
**Supernova Explosion!**

Less energy lost by  
shock to dissociate iron

Pre-heating of outer layers  
by short  $\nu_e$  burst (~ms)

Revival of stalled shock in  
supernova explosion  
(similar to Wilson mechanism)

Anomalous pressure  
increase behind shock





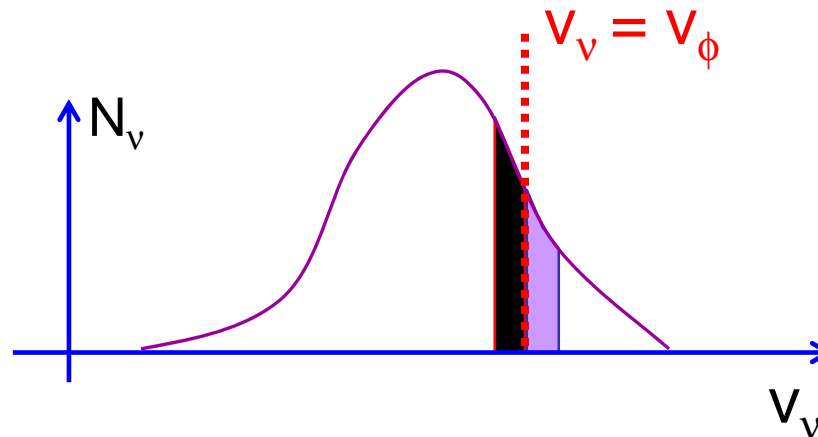
# Neutrino Landau Damping I

What if the source of free energy is in the plasma?

*Thermal spectrum of neutrinos interacting with turbulent plasma*



**Collisionless damping of EPWs by neutrinos moving resonantly with EPWs**



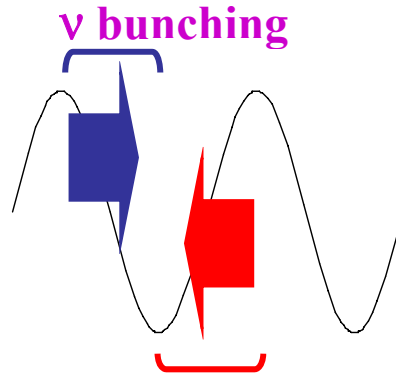
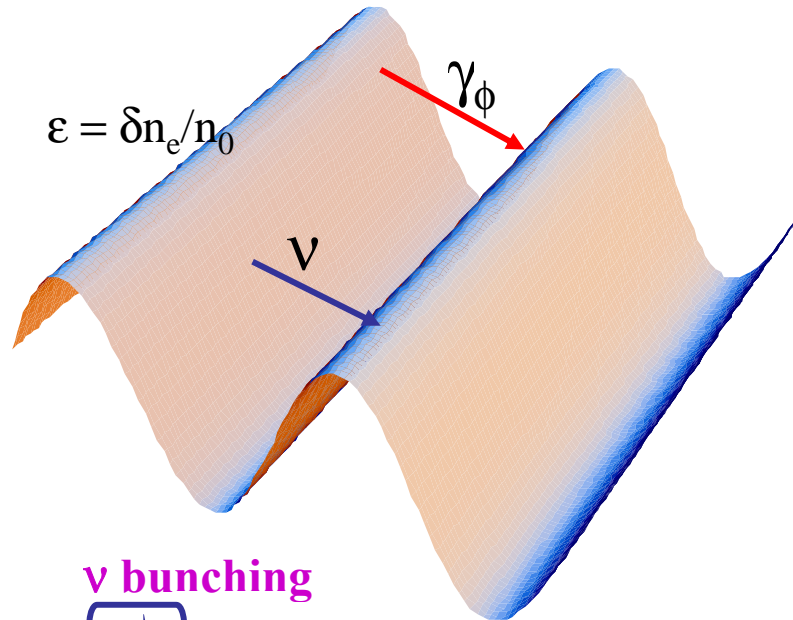
Physical picture for  
electron Landau damping  
(Dawson, '61)

**General dispersion relation describes not only the neutrino fluid instability but also the neutrino kinetic instability**

(Silva et al, PLA 2000)



# Neutrino surfing electron plasma waves



$$\gamma_\phi = 10$$

$$\varepsilon = 10^2$$

$$n_0 = 10^3 \text{ cm}^{-3}$$

$$L_\phi = \lambda \gamma_\phi^2 \approx 3 \times 10^2 \text{ cm}$$

$$dE/dL \approx 8 \sqrt{2 G_F n_0} (\lambda \gamma_\phi^2) \approx 200 \text{ eV/cm}$$

Equivalent to physical picture for RFS of photons (Mori, '98)





Neutrino Landau damping reflects contribution from the pole in neutrino susceptibility

$$\chi_L(\omega_L, \mathbf{k}_L) \propto \int d\mathbf{p}_\nu \frac{\mathbf{k}_L \cdot (\partial \hat{f}_{\nu 0} / \partial \mathbf{p}_\nu)}{\omega_L - \mathbf{k}_L \cdot \mathbf{v}_\nu} \rightarrow \int d\mathbf{p}_\perp |d\mathbf{p}_\parallel| \left\{ P \int \frac{\hat{f}_{\nu 0}(\mathbf{p}_\parallel)}{\mathbf{p}_\parallel - \mathbf{p}_{\parallel 0}} d\mathbf{p}_\parallel + i\pi \left( \frac{\hat{f}_{\nu 0}}{\mathbf{p}_\parallel - \mathbf{p}_{\parallel 0}} \right) \right\}$$

EPW wavevector  $\mathbf{k}_L = \mathbf{k}_{L\parallel}$  defines parallel direction  
neutrino momentum  $\mathbf{p}_n = \mathbf{p}_{v\parallel} + \mathbf{p}_{v\perp}$   
arbitrary neutrino distribution function  $f_{\nu 0}$   
Landau's prescription in the evaluation of  $\chi_\nu$

For a Fermi-Dirac neutrino distribution

$$\gamma_{\text{Landau}} \approx -\frac{k_L c}{2} \pi \frac{G_F^2 n_{e0} n_{\nu 0}}{m_e c^2 k_B T_\nu} \left( 1 - \frac{\omega_L^2}{c^2 k_L^2} \right)^2 \frac{\text{Li}_2(-\exp E_F / T_\nu)}{\text{Li}_3(-\exp E_F / T_\nu)}$$



## Neutrino play a critical role in Type II Supernovæ

- **Neutrino spectra and time history of the fluxes probe details of the core collapse dynamics and evolution.**
- **Neutrinos provide heating for “delayed” explosion mechanism.**
- **Sufficiently detailed and accurate simulations provide information on convection models and neutrino mass and oscillations.**