Fermilab

http://www.fnal.gov

STUDIES FOR A MUON COLLIDER OPTICS

Eliana GIANFELICE

eliana@fnal.gov

INTRODUCTION

Muons

First proposed by Budker (1967), the idea of a Muon Collider has been relaunched by recent progress on new ideas for small emittance muon beams.

Muons are

- point-like as leptons → the whole beam energy is carried by the interacting particles.
- but heavy (207 times heavier than leptons) → no radiation, in practice ($U_{turn} = q^2\beta^3\gamma^4/3\epsilon_0 R$)

The small lifetime ($\tau = \gamma$ 2.2 μs) requires

- large number of muons be produced
- and 6D cooled and quickly accelerated.

Machine Parameters

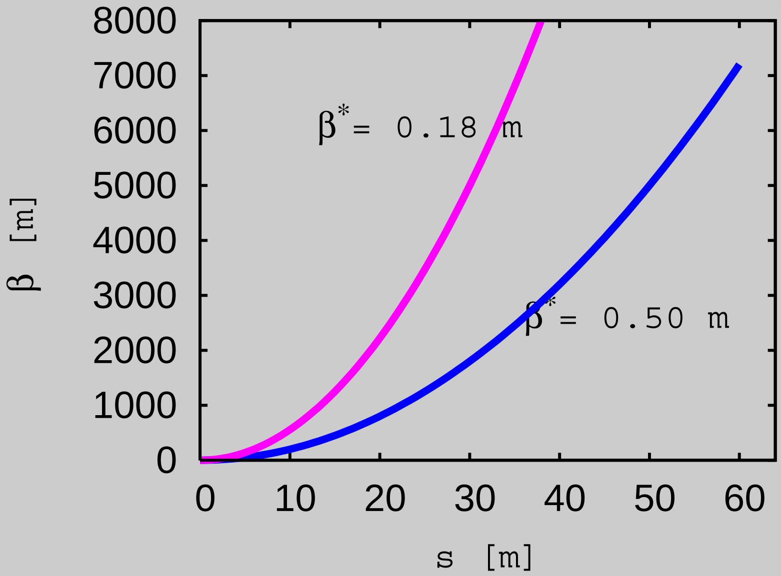
Tentative Design Parameters (high transv. emittance scenario)	
E_{beam}	750 GeV
N_b × Num. of muons/bunch	$1 \times 11.3 \cdot 10^{11}$
ϵ_N	12.3 μm
Δp/p	0.2 %
Bunch length	10 mm
RF Voltage @ 800 MHz	$5.6 \times 10^3 \alpha_p$ GV
Lenght	3141 m
Average Luminosity	2×10^{34} cm^{-2} s^{-1}
β_x^*, β_y^*	10 mm
Number of IPs	2
beam-beam tune shift/IP	0.100

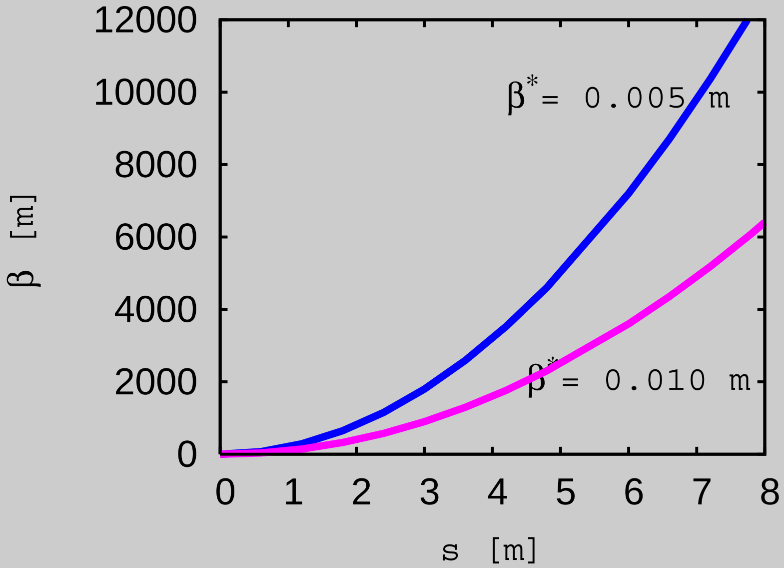
Comparisons

Assuming we are able to accelerate enough muons to 750 GeV, the design of the collider ring itself is not trivial either:

$$\beta(s) = \beta^* + \frac{s^2}{\beta^*}$$

Compare to other colliders:





HERA-p: $\beta_y^* = 0.18$ m, $\hat{\beta}_x = 1600$ m (first quadrupole @11 m from IP).

LHC: $\beta_x^* = \beta_y^* = 0.50$ m, $\hat{\beta}_x = \hat{\beta}_y = 4718$ m (first quadrupole @23 m).

Design Issues

Large β at strong quadrupoles:

- large chromatic effects → strong sextupoles → reduced DA
- sensitivity to misalignments and field errors
- ....

The only "advantage" (wrt. hadron machines): long term stability is not required !

Attempts to design a Muon Collider have recently resumed at Fermilab, with the primary goal of addressing the energy acceptance and DA issues.

The "first generation" of Muon Collider designs has proven that it is possible to design a ring with $\beta^* = 3$ mm and $\alpha_p \simeq$ -9×10^{-5} ($\hat{\beta} \geq 145$ km!), but for most of them the momentum acceptance was too small (-0.12 ‰ $\div$ 0.16 ‰). More work is needed to get the desired energy acceptance and DA (in presence of realistic errors).

Design constraints

Design constraints	
β^*_x, β^*_y $(\epsilon_x = \epsilon_y)$	10 mm
free space around IP	± 6 m
$\lvert\alpha_p\rvert$	$\leq 1 \times 10^{-4}$
$\hat{g}$	$\leq$ 220 Tm^{-1}
	↓
	$\boxed{\text{k} \leq 0.09\ \text{m}^{-2}\ @\ 750\ \text{GeV}}$
$\hat{B}$	10 T

Why 6 m free space?

“Bogacz law”:

$$s = f = \frac{1}{K\ell} \simeq \sqrt{\hat{\beta}\beta^*}$$

Larger gradient, if available (and if experiments can cope with smaller space), would help reducing chromaticity

$$K\ell\,\hat{\beta} \simeq \sqrt{\frac{\hat{\beta}}{\beta^*}}$$

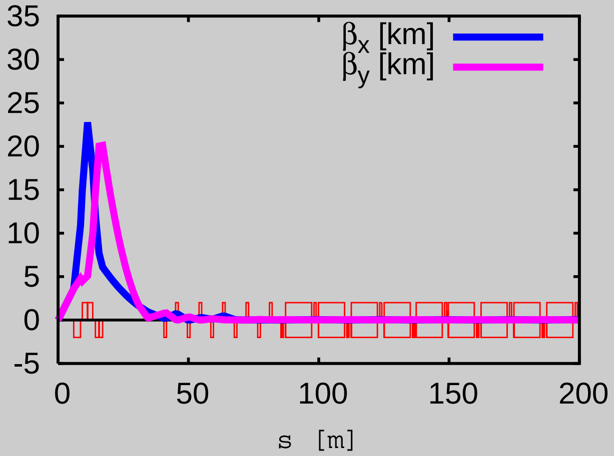
A "CONVENTIONAL" DESIGN

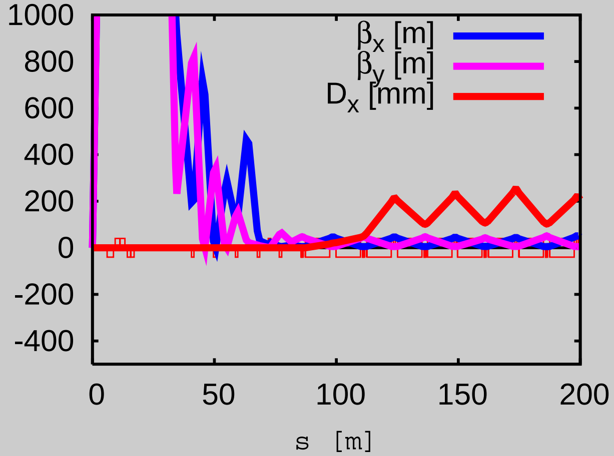
The Lattice

Dispersion free IR, "classic" dispersion suppressor and a FODO structure for the arcs

$$1 \times 10^{-4} = \alpha_p = \frac{L_{arcs}}{L} \frac{\pi^2}{N_{cell}^2 \sin^2\left(\frac{\mu_{cell}}{2}\right)}$$

With $\mu_{cell} = 108°$ it is $N_{cell} \simeq 390$ $(L \simeq L_{arcs})$.





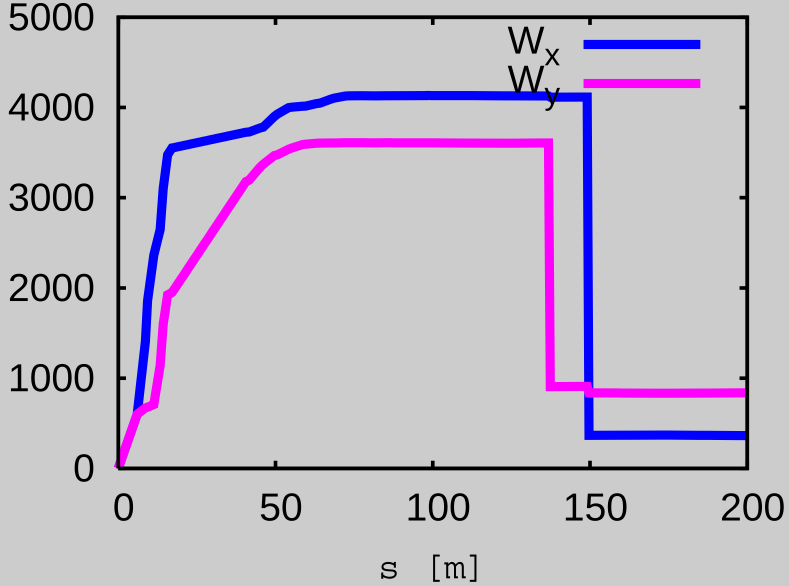
- **L = 9500 m (25 m FODO cells, B_{dip} = 2 T)**
- **Q_x = 115.20 ξ_x^{nat} = -1475**
- **Q_y = 117.65 ξ_y^{nat} = -1312**

Chromaticity Correction

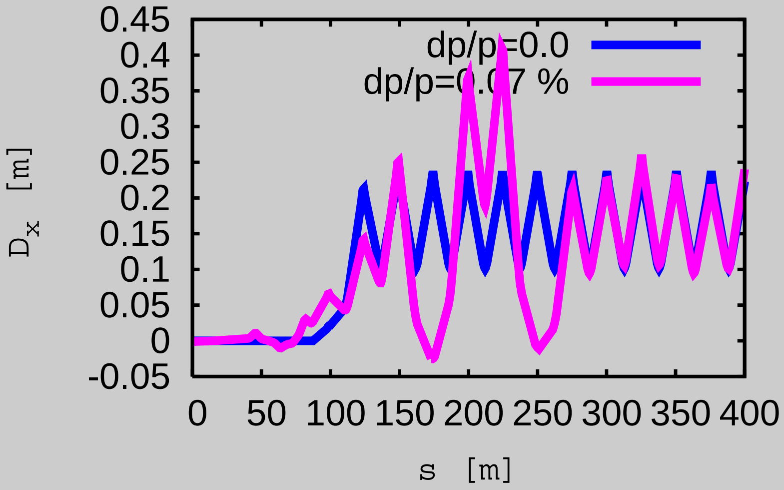
We must correct the linear and 2^{th} order chromaticity. The main contribution comes from the IR qudrupoles, but it cannot be locally corrected with sextupoles as the IR is dispersion free! The phase advance is adjusted so that the first sextupole is in a knot of the IR chromatic beta-wave; a second sextupole at $\Delta\mu = (2n + 1)\,\pi$ corrects the resulting large dispersion perturbation.

cell1	cell2	cell3	cell4	cell5	cell6	cell7	cell8	cell9	cell10
A	-	-	-	-	A	-	-	-	-

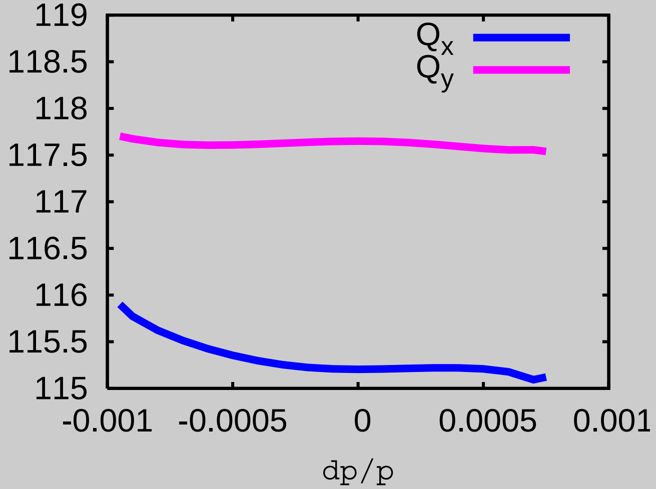
The linear chromaticity is corrected with one sextupole family (per plane) in the remaining cells so that the lowest order of the 3^{th} order resonance driving terms vanish.



W_x and W_y vs. s (MAD chromatic functions)



dispersion



tunes vs. dp/p

→ $\Delta p/p \simeq \pm 0.08$ **%.**

AN ALTERNATIVE DESIGN ATTEMPT

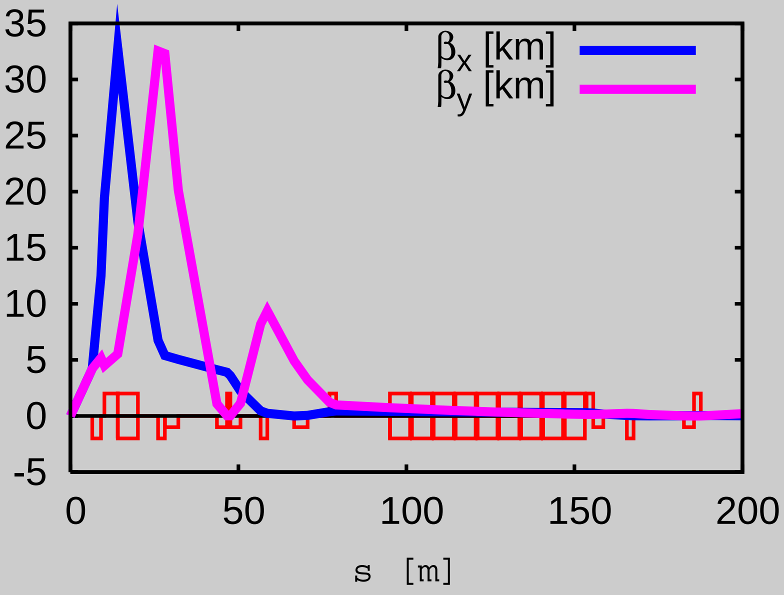
Montague chromatic functions describing the change of the twiss parameters with momentum $\delta \equiv \Delta p/p$

$$B \equiv \frac{\Delta\beta}{\beta} \qquad \text{and} \qquad A \equiv \beta\Delta\left(\frac{\alpha}{\beta}\right)$$

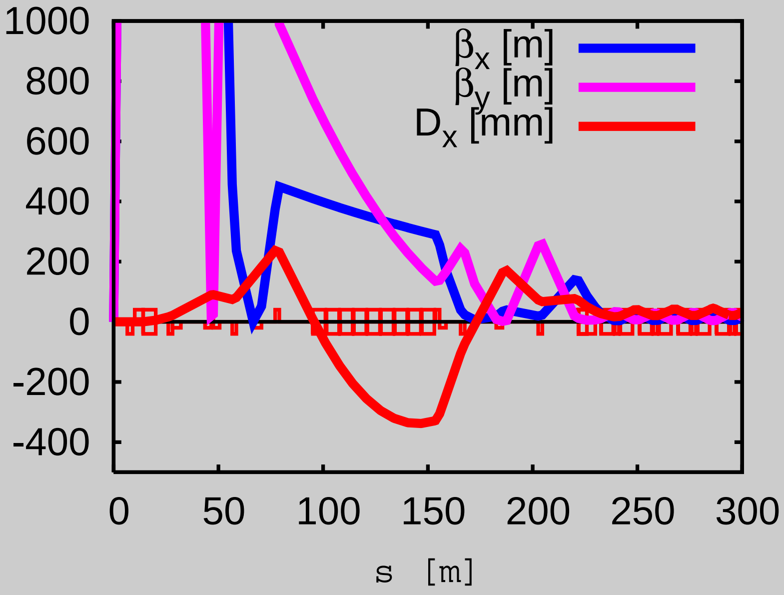
$$\frac{dB}{ds} = -2A\frac{d\mu}{ds} \quad \text{and} \quad \frac{dA}{ds} = 2B\frac{d\mu}{ds} + \sqrt{\beta(0)\beta(\delta)}\Delta K$$

⇒ Introduce bending magnet close to the IP and compensate chromatic beta wave locally, that is before the phase advance changes after the first quadrupole.

In addition, by introducing large bending angles (with $D_x = D'_x = 0$ at the IP), it is possible to get a negative α_p section which allows to decrease the arc length.



IR



Matching section

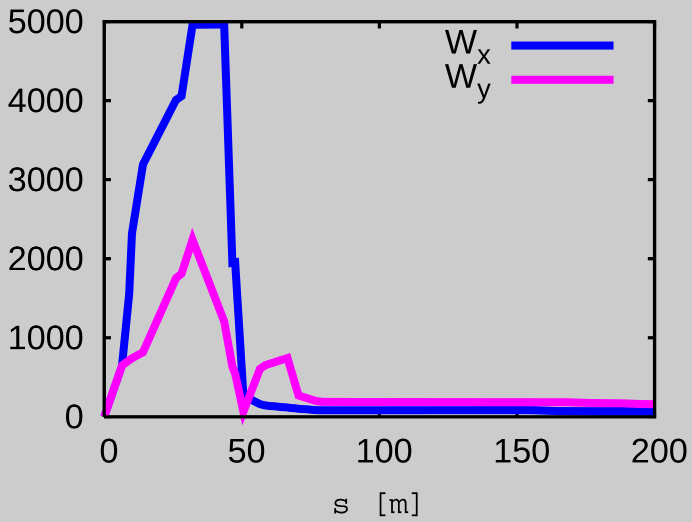
$L = 3131.8$ m (with 18 m long 108° FODO cells, $B_{dip} = 9.5$ T)

$\alpha_p = 4.9 \times 10^{-5}$

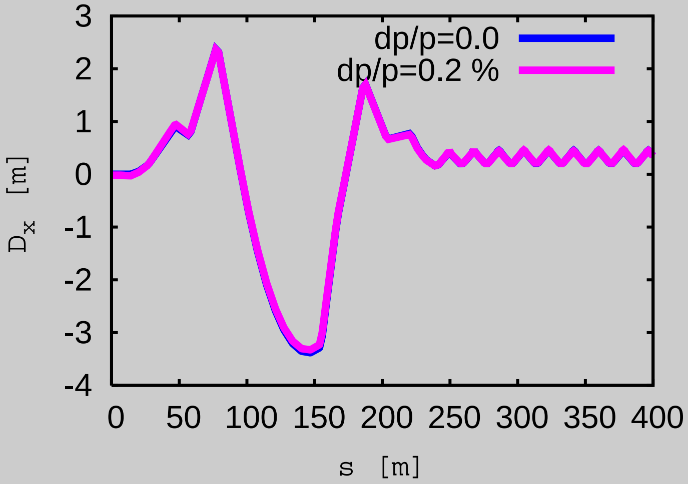
$Q_x = 41.13 \qquad \xi_x^{nat} = -1435$

$Q_y = 42.20 \qquad \xi_y^{nat} = -1540$

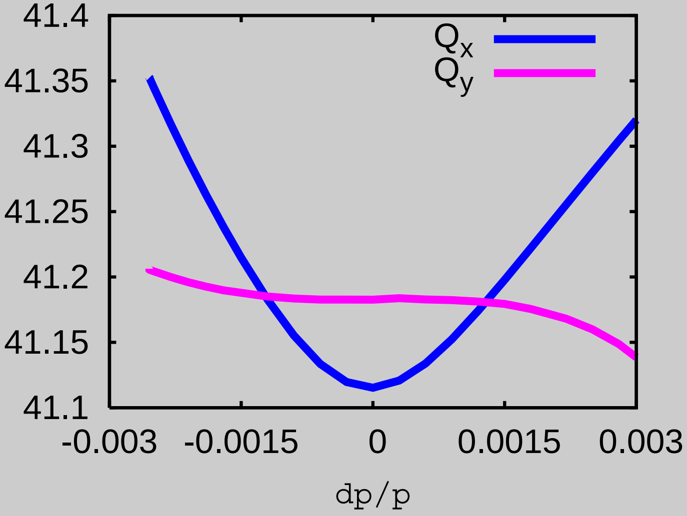
3 sextupoles located at 32, 47 and 71 m correct the chromatic beta and 2 sexupoles at 158 and 185 m correct the 2th order dispersion.



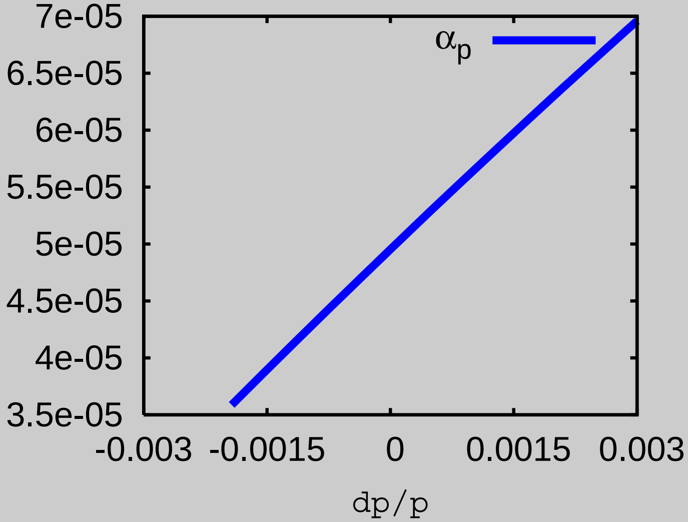
W_x and W_y vs. s



Dispersion



Tunes vs. dp/p



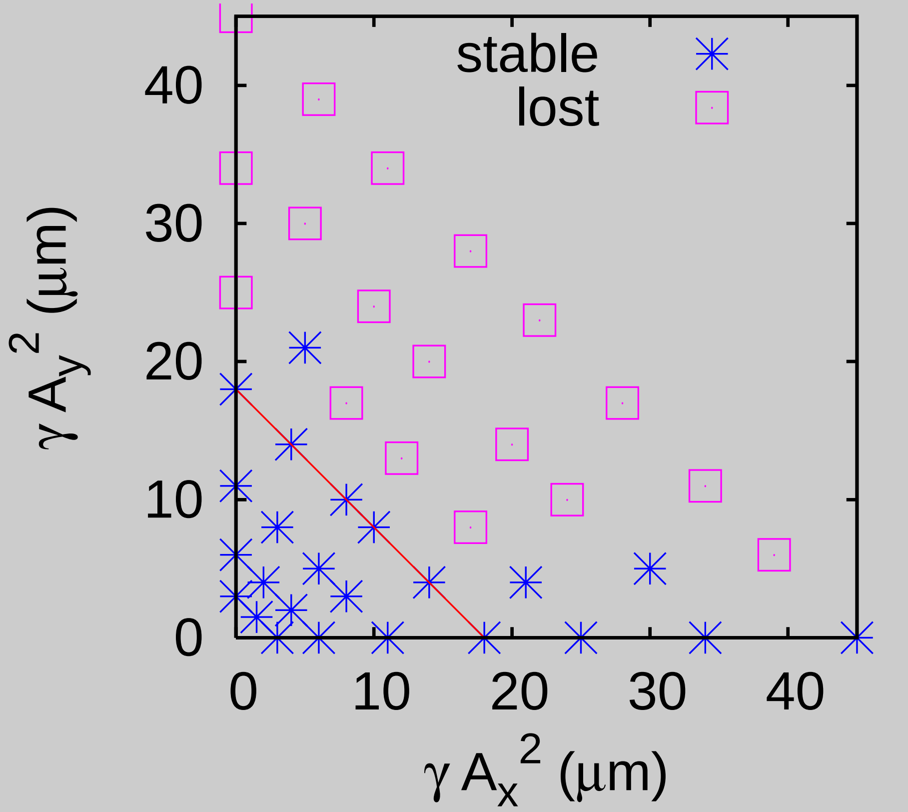
α_p vs. dp/p

$\Rightarrow$ **Sufficient for the "high transverse emittance" case!**

Tune Dependence
on Amplitude (w/o octupoles)
(MAD8 STATIC)

dQ_1/dE_1	0.5×10^8
dQ_1/dE_2	2.0×10^8
dQ_2/dE_2	0.6×10^8

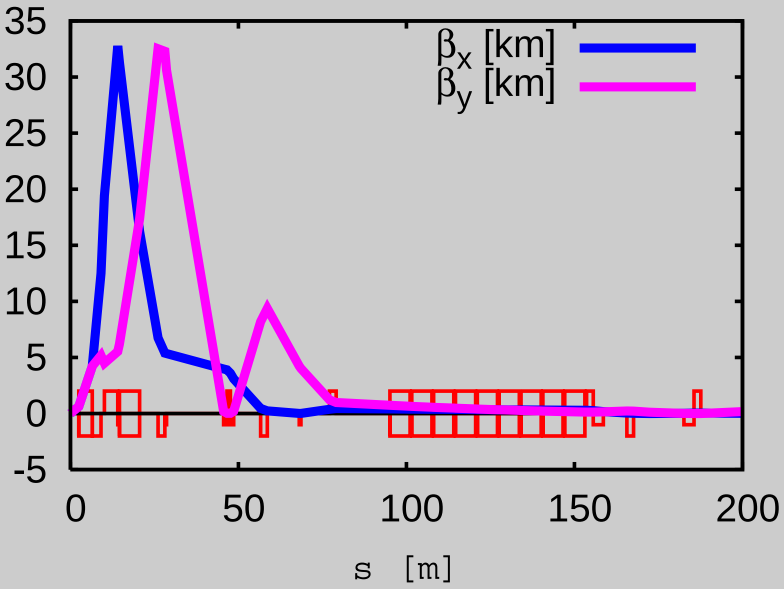
Dynamic Aperture (with octupoles)



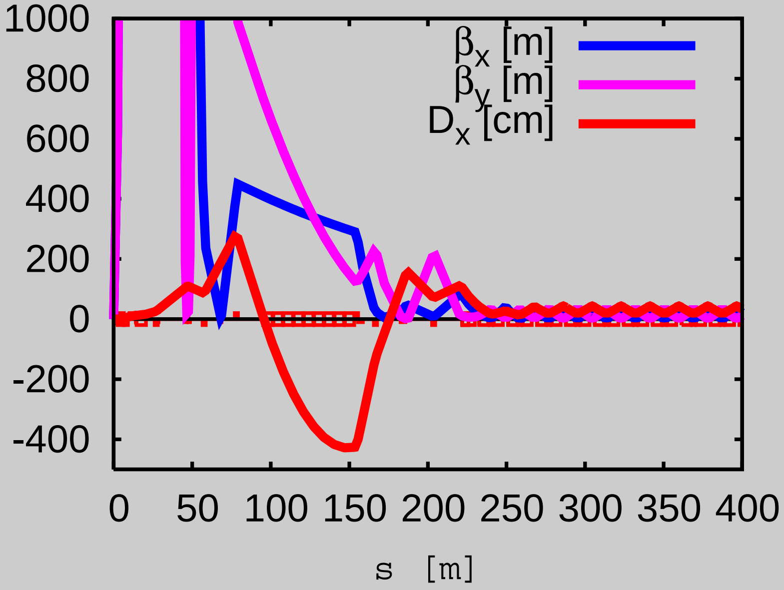
Octupoles are used to correct the large detuning with amplitude, but the DA ($\simeq 1.2\ \sigma$) is still too small!

“Dipole First” Optics

Introduce a dipole (B=7.5 T, ℓ=4 m) before the first quadrupole to increase the dispersion at the IR sextupoles. Free space: ± 2.5 m



IR

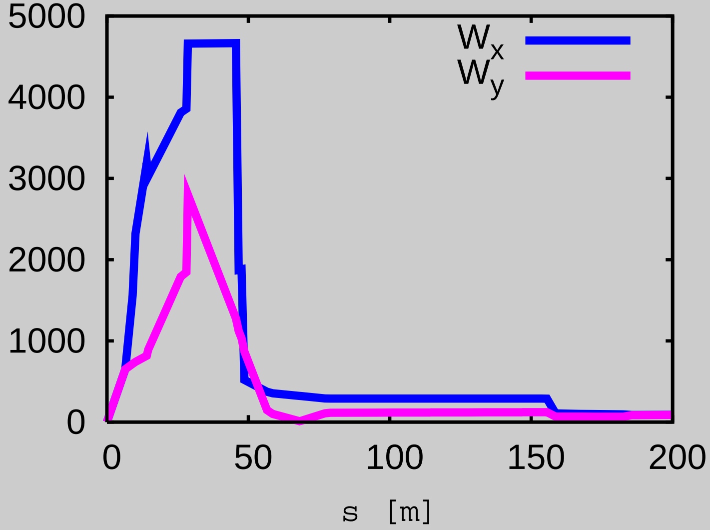


Matching section

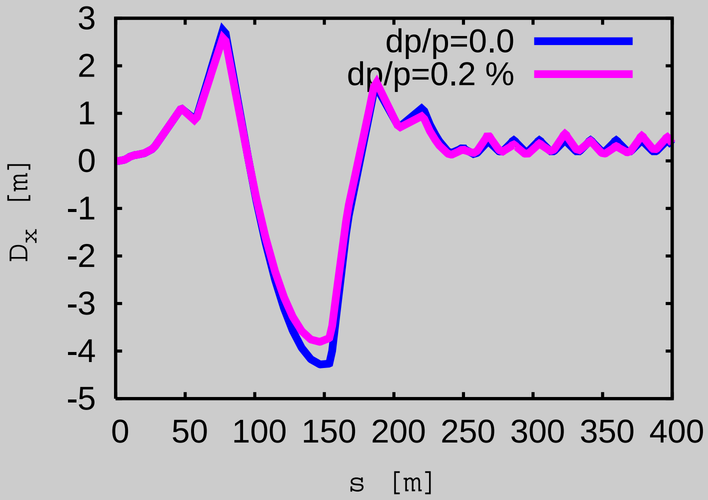
4 sextupoles located at 15, 29, 47 and 69 m correct the chromatic beta.

2 sextupoles at 158 and 185 m correct the 2th order dispersion.

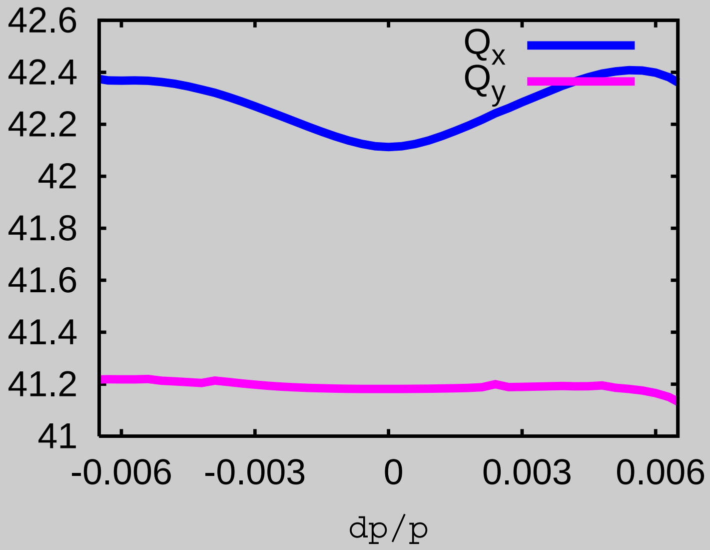
Octupoles are located at 10, 21 and 26 m (detuning correction) and 154 m (2th order chromaticity).



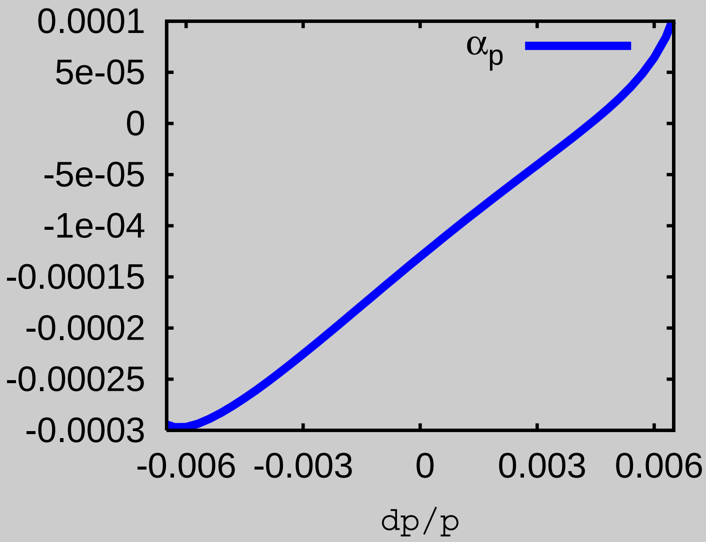
W_x and W_y vs. s



Dispersion



Tunes vs. dp/p



α_p vs. dp/p

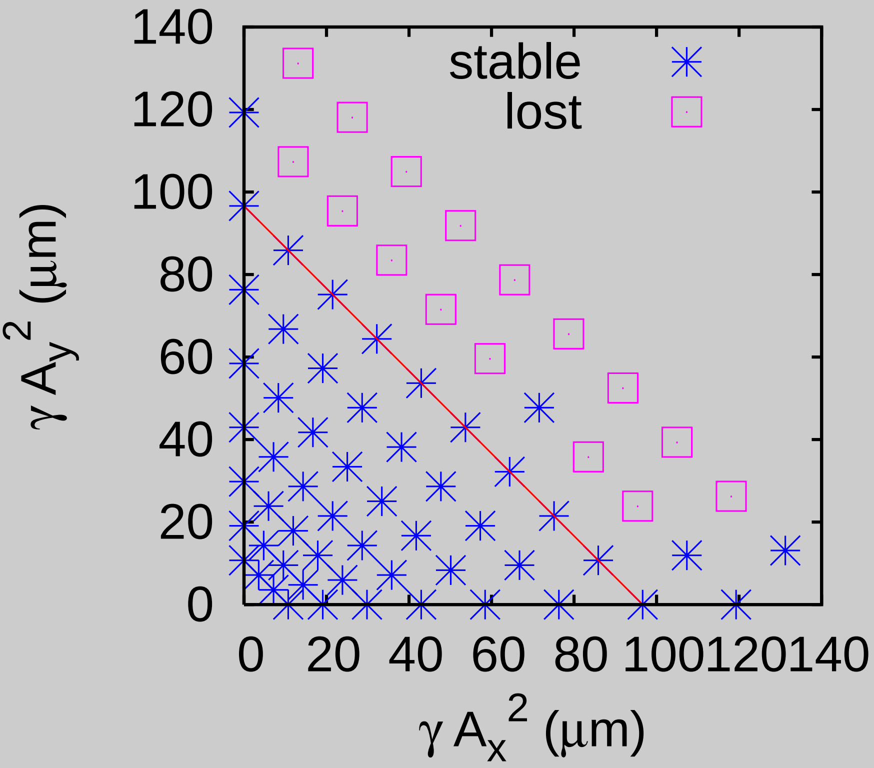
(adjustements needed!)

Tune Dependence on Ampiltude (with octupoles)

(MAD8 STATIC)

dQ_1/dE_1	0.4×10^8
dQ_1/dE_2	0.2×10^8
dQ_2/dE_2	0.1×10^8

Dynamic Aperture (with octupoles)



$\simeq 2.8\,\sigma$

Oide design for a 3 mm β^* optics (1996?).

It uses KEK-B Factory modules for the arcs: 2.5 π phase advance cells

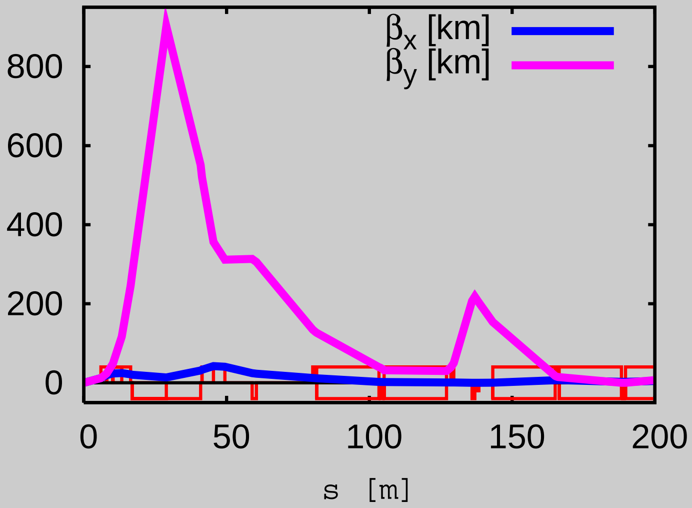
- dispersion (and thus α_p) tunability
- one IP
- *no local* IR chromaticity correction (!)
- non-interleaved sextupole correction :convenient for DA
- 10 families of sextupoles per plane
- octupoles and decapoles included in the optimization

$$L = 5700 \text{ m}$$

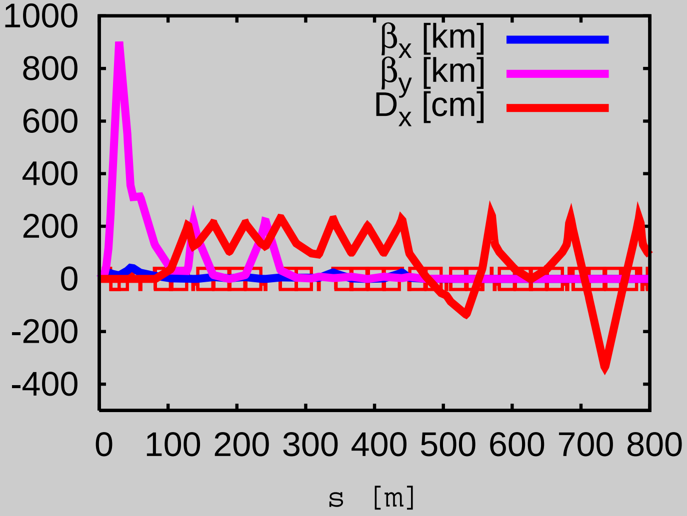
$$\alpha_p = 5 \times 10^{-5}$$

$$Q_x = 31.55 \qquad \xi_x^{nat} = -1237$$

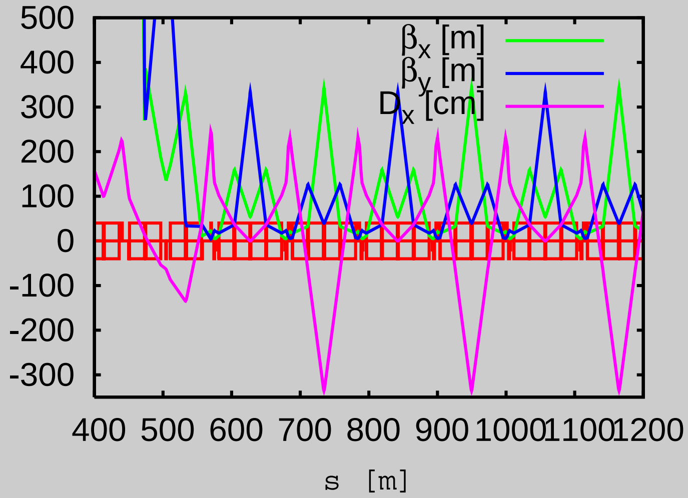
$$Q_y = 31.56 \qquad \xi_y^{nat} = -13249 \;(\hat{\beta}_y = 900 \;\text{ km!})$$



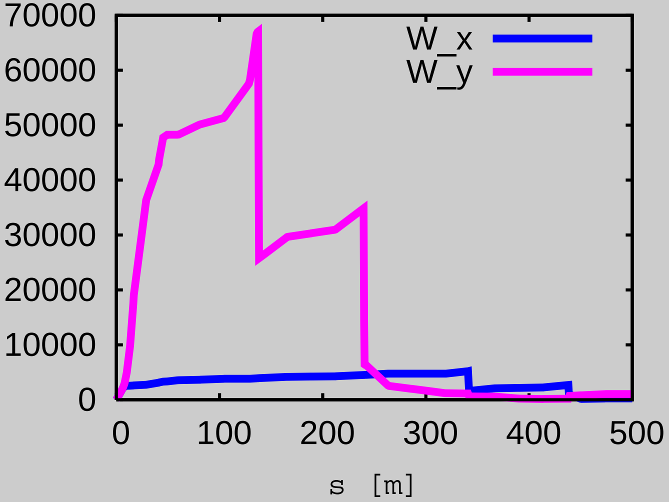
IR



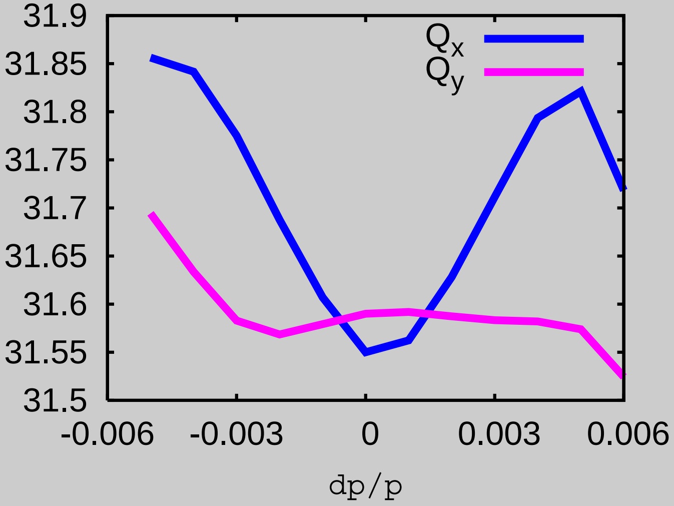
Matching section



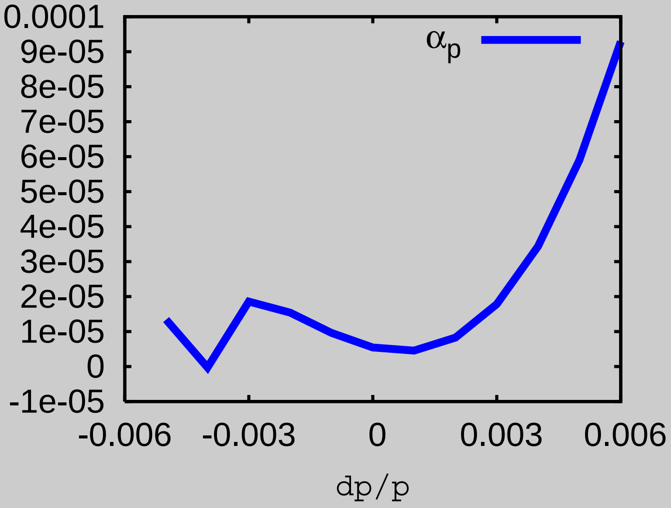
Arc



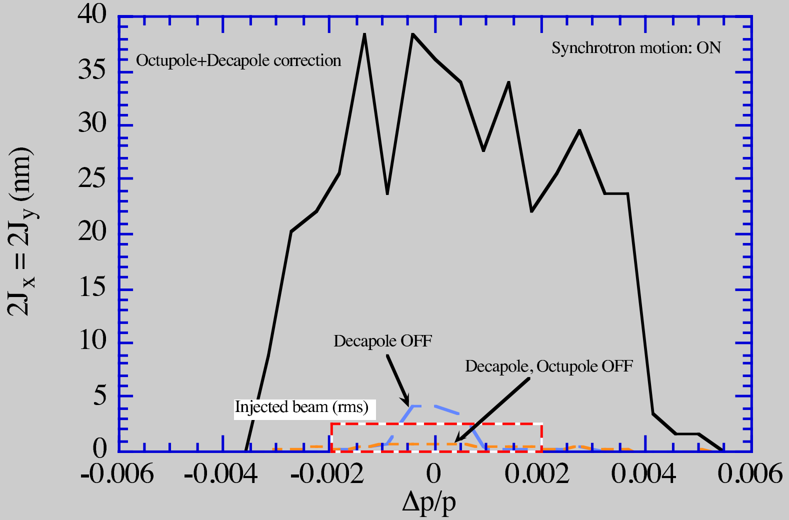
W_x and W_y vs. s



Tunes vs. dp/p



α_p vs. dp/p



⇐

Dynamic aperture with synchrotron oscillations, SAD calculation including quadrupole fringe fields: 4.5 σ at $\Delta p/p = 0$

(K. Oide courtesy)

SUMMARY AND OUTLOOK

- Progress has been done in the design of a Muon Collider: the IR has been optimised so to get the required energy acceptance and in addition reduce the machine length
  - The Dipole first optics gives a hope to obtain the required DA (by further optimization).
  - Consider 1 IP only option.
  - The issue, however, of the synchrotron radiation in the strong dipole must be still addressed.

There is still a very long way to reach the final goal of a realistic design! Although not quite feasible, what do we learn from Oide design?

- increase sextupole family number to improve dp/p range
- try non-interleaved scheme to improve DA: likely it requires a different optics in the arcs !

Acknowledgements

Yuri Alexahin