# Design Principles for Muon Colliders

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and the Muon Collider
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# Muon Collider Goals

- Reach high energy
- Achieve high luminosity
- Avoid excessive neutrino radiation
  - Neutrinos create showers in massive objects near site
  - Highly concentrated neutrino beam
  - Increases rapidly with energy

# Unique Features

- Beam isn’t stored long (decays)
  - Muon production rate determines luminosity
  - Similar to linear collider
- Beams can collide multiple times
  - Not arbitrarily large number (decays)
  - Here we beat linear colliders
  - Advantages to throwing out your beam
- Long-distance neutrino radiation

# Muon Collider Challenges

- Muons produced with large emittances
  - Requires massive amounts of cooling
- Muons decay
  - Preserving beam all the way to collider
  - Cooling and acceleration must be extremely rapid

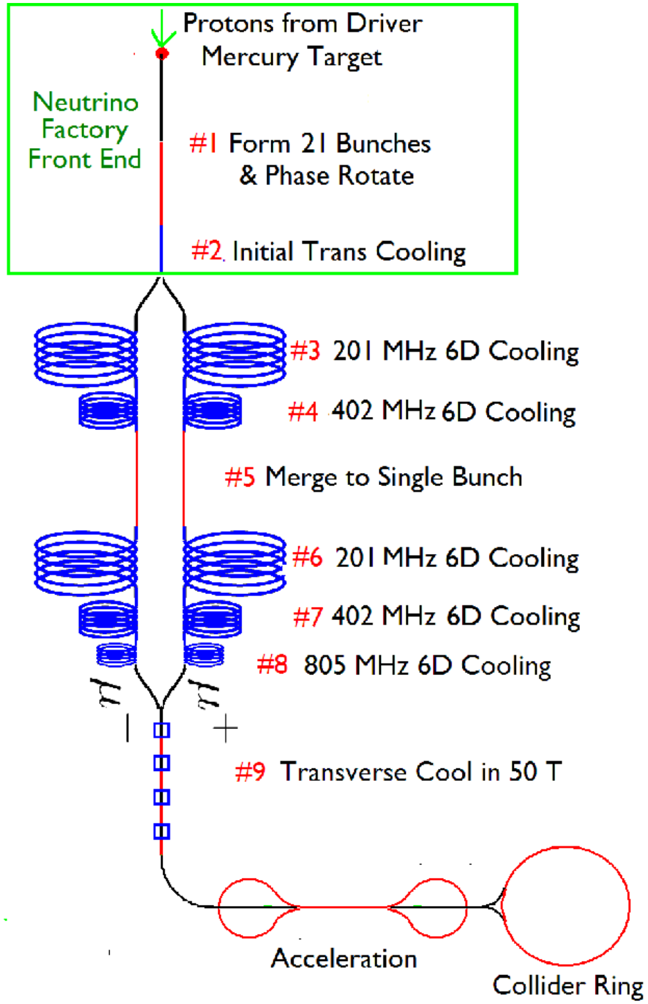
# Muon Collider Components

- High-power proton driver
- Target
- Make the beam sane
  - Capture, phase rotation, bunching
  - Could be something else…
- Cooling

# Muon Collider Components

- Rebunching: several to one
  - During cooling
  - During acceleration
  - Not at all. . .
- Acceleration
- Collider ring

# Muon Collider System



Protons from Driver
Mercury Target

Neutrino Factory Front End

#1 Form 21 Bunches & Phase Rotate

#2 Initial Trans Cooling

#3 201 MHz 6D Cooling

#4 402 MHz 6D Cooling

#5 Merge to Single Bunch

#6 201 MHz 6D Cooling

#7 402 MHz 6D Cooling

#8 805 MHz 6D Cooling

#9 Transverse Cool in 50 T

Acceleration

Collider Ring

# Luminosity
## Luminosity for Decaying Beams

- Round Gaussian beams collide, all $\mu$s decay

$$\mathcal{L} = \frac{\gamma\tau_\mu}{2T_{\text{rev}}}\frac{N^2 n_B f_P}{4\pi\sigma^2} = \frac{\tau_\mu e B_{\text{avg}}}{4\pi m_\mu}\frac{N^2 n_B f_P}{4\pi\sigma^2}$$

- $N$: particles per bunch
- $\tau_\mu$: muon rest lifetime
- $T_{\text{rev}}$: revolution time
- $f_P$: driver rep. rate
- $e$: electron charge
- $\gamma$: energy/(rest mass)
- $\sigma$: RMS size
- $n_B$: no. of bunches
- $B_{\text{avg}}$: avg. ring field
- $m_\mu$: Muon mass

# Luminosity
# Multiple-Crossing Factor

$$\mathcal{L} = \frac{\gamma\tau_\mu}{2T_{\text{rev}}}\frac{N^2 n_B f_P}{4\pi\sigma^2} = \frac{\tau_\mu e B_{\text{avg}}}{4\pi m_\mu}\frac{N^2 n_B f_P}{4\pi\sigma^2}$$

- First factor is the average number of crossings
- Always choose the highest possible magnet fields in the collider ring
  - Everything else is much harder

# Beam-Beam Tune Shift

- Maximize luminosity by running at maximum beam-beam tune shift ($\Delta\nu$)
- We only have around 1000 turns, potentially allowing large $\Delta\nu$

$$\Delta\nu = \frac{\beta^* N r_\mu}{4\pi\sigma^2\gamma} = \frac{N r_\mu}{4\pi\epsilon_n}$$

- $\beta^*$: C-S beta at IP
- $r_\mu$: classical $\mu$ radius
- $\epsilon_n$: normalized transverse emittance

# Proton Driver Power

- Power is product of
  - Particles per bunch
  - Bunches per cycle
  - Cycle frequency
  - Energy of beam
- Energy transferred to muons with efficiency $\eta_{\text{cap}}$
- Average muon energy at capture: $E_{\text{cap}}$

# Proton Driver Power

$$P_P = \frac{E_{\text{cap}}}{\eta_{\text{cap}}}\frac{N n_B f_P}{\eta_{\text{trans}}}$$

- Fraction of captured muons making it to collider: $\eta_{\text{trans}}$
- $\eta_{\text{cap}}\eta_{\text{trans}}/E_{\text{cap}}$ is the physical quantity
  - $\eta_{\text{trans}}$ depends on where “capture” ends
  - $\eta_{\text{cap}}/E_{\text{cap}}$ depends only on target/capture system

# Neutrino Radiation

- Increases strongly with energy
- Proportional to muons per second times average turns

$$C_{\mathrm{rad}} = \frac{\tau_\mu e B_{\mathrm{avg}}}{2\pi m_\mu} N n_b f_P$$

- Make ring deeper to reduce radiation
  - Reduction factor proportional to ring depth
- Straights create strong radiation

# Luminosity in Terms of Other Quantities

- Use beam-beam tune shift and proton power:

$$\mathcal{L} = \frac{\tau_\mu e B_{\text{avg}}}{2\pi m_\mu}\frac{\eta_{\text{trans}} P_P \Delta\nu}{\beta^*}\frac{\eta_{\text{cap}}\gamma}{E_{\text{cap}} r_\mu}$$

- Improve luminosity with
  - Higher $B_{\text{avg}}$
  - Reducing losses getting to collider
  - Increasing proton driver power
  - Running with larger beam-beam tune shift
  - Lower beta at IP

# Improving Luminosity

$$\mathcal{L} = \frac{\tau_\mu e B_{\text{avg}}}{2\pi m_\mu} \frac{\eta_{\text{trans}} P_P \Delta\nu}{\beta^*} \frac{\eta_{\text{cap}} \gamma}{E_{\text{cap}} r_\mu}$$

- What doesn’t directly appear
  - Beam emittance
  - Bunch structure (fewer bunches for same $P_P$)

# Improving Luminosity

$$\mathcal{L} = \frac{\tau_\mu e B_{\text{avg}}}{2\pi m_\mu} \frac{\eta_{\text{trans}} P_P \Delta\nu}{\beta^*} \frac{\eta_{\text{cap}}\gamma}{E_{\text{cap}} r_\mu}$$

- Lower emittance may allow lower $\beta^*$
  - Smaller $\sigma_z$, energy spread (longitudinal)
  - Smaller $\beta^*$ has smaller dynamic aperture
  - Improve $\beta^*$ faster than $\eta_{\text{trans}}$ reduction
- Dependence of $\beta^*$ on emittances not obvious
  - Except $\sigma_z$, may be weak

# Luminosity Constrained by Radiation

- What if we are constrained by radiation
  - Maybe we can’t go below a certain depth
  - Assuming a given energy

$$\mathcal{L} = C_{\text{rad}}\frac{\Delta\nu\gamma}{\beta^* r_\mu}$$

- Only improve with
  - Larger $\Delta\nu$
  - Smaller $\beta^*$

# Beam-Beam Tune Shift

- Reach some beam-beam tune shift

$$\Delta\nu = \frac{N r_\mu}{4\pi\epsilon_n} \qquad\qquad P_P = \frac{E_{\text{cap}}}{\eta_{\text{cap}}}\frac{N n_B f_P}{\eta_{\text{trans}}}$$

- Lower $N$ in proportion to $\epsilon_n$
- Lower $N$ permits increasing $n_B f_P$
  - Increased $f_P$ reduces p driver space charge
  - $\epsilon_n$ sufficiently low for p driver
  - Increased $n_B$, interesting. . .

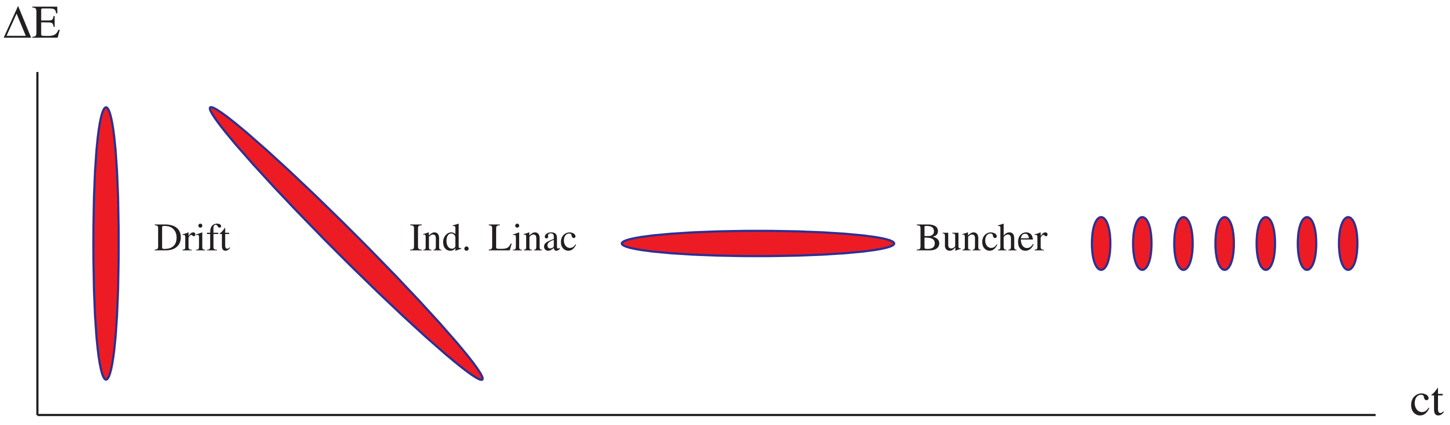
# Making the Beam Sane

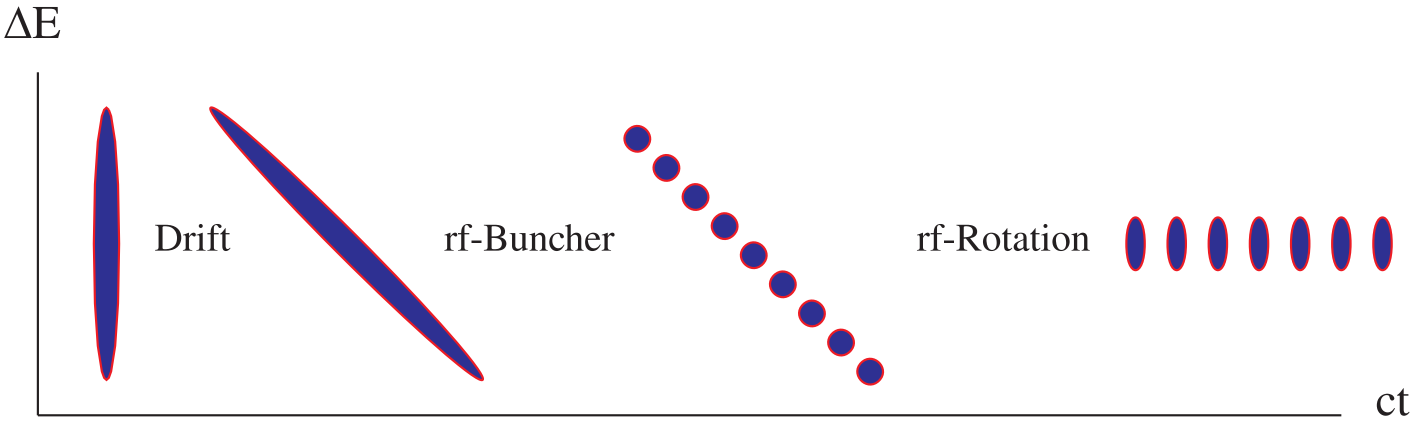
- Immediately at target
    - Energy spread too large
    - Angular spread large
- Tapered solenoid to reduce angular spread
- Reducing energy spread: phase rotation
    - Drift to introduce time spread
    - Time dep. voltage reduces energy spread
    - Bunch at 200 MHz for cooling

# Making Beam Sane
# Neuffer Phase Rotation

- Imagine beam already bunched
- Higher frequency RF than bunches
  - Early (high energy) bunches decelerated
  - Late (low energy) bunches accelerated
- Bunch and phase rotate together
- Avoids low frequency RF and/or induction linac

# Phase Rotation Scenarios





# Making Beam Sane
# Low Frequency Cooling

- Low frequency ionization cooling ring/spiral
- Reduces longitudinal emittance rapidly
- Simulated rings (Balbekov) worked well
    - Maybe not realistic. . .
- Would avoid bunch train

# Ionization Cooling Challenges

- Technology for reaching sufficiently low emittance
  - High field magnets
  - High magnetic fields on cavities
- Minimizing losses in system
- Cost of system

# Ionization Cooling
# Amount of RF Required

- Energy losses in absorber restored in RF
- Amount of RF largely determines system cost

$$\Delta E = \sum_k E_k \cos\phi_k = \frac{p\beta c}{\Lambda\eta_{\text{cool}}} \ln\frac{\epsilon_{6i}}{\epsilon_{6f}}$$

- $E_k$: RF energy gain $k$
- $p$: momentum
- $\Lambda$: partition no. sum
- $\epsilon_{6i}$: initial 6-D emit.
- $\phi_k$: RF phase $k$
- $\beta c$: velocity
- $\eta_{\text{cool}}$: efficiency
- $\epsilon_{6f}$: final 6-D emit.

# Ionization Cooling
# Amount of RF Required

$$\Delta E = \sum_k E_k \cos\phi_k = \frac{p\beta c}{\Lambda\eta_{\text{cool}}} \ln \frac{\epsilon_{6i}}{\epsilon_{6f}}$$

- What lowers $\eta_{\text{cool}}$?
  - Mismatch when entering new system
  - Approaching equilibrium
    - Maintain large angular, energy spread
    - Taper down channel $\beta$ function
    - Taper down bunch length (RF phase)

# Ionization Cooling
# Amount of RF Required

$$\Delta E = \sum_k E_k \cos\phi_k = \frac{p\beta c}{\Lambda \eta_{\text{cool}}} \ln \frac{\epsilon_{6i}}{\epsilon_{6f}}$$

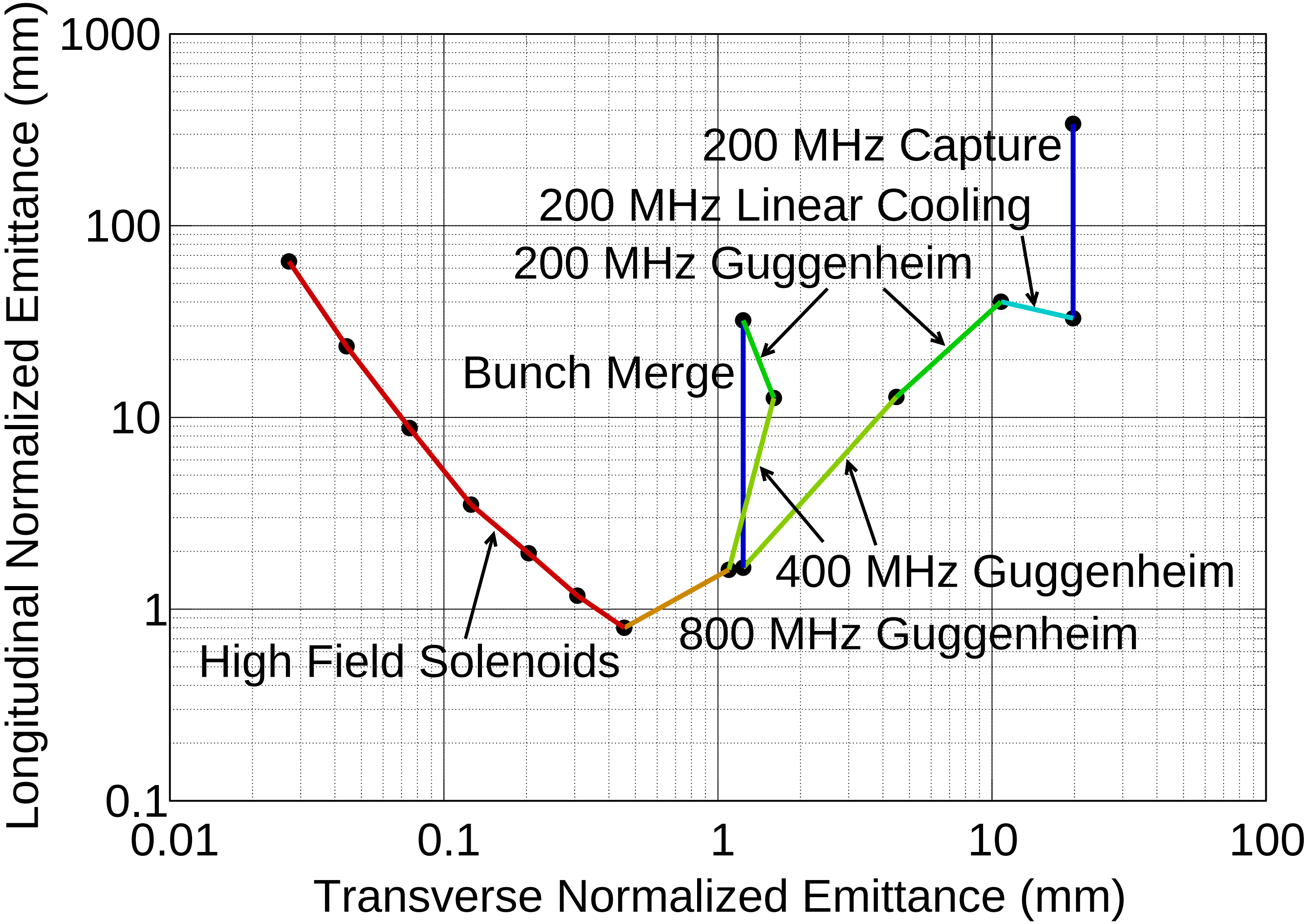
- Less voltage with smaller RF phase
  - Lower longitudinal acceptance
  - On crest, no longitudinal focusing

# Ionization Cooling Losses

$$Q \equiv \frac{d\epsilon_6/\epsilon_6}{dN/N} \quad \frac{N_f}{N_i} = \left(\frac{\epsilon_{6f}}{\epsilon_{6i}}\right)^{(1/Q)} \qquad Q = \frac{\eta_Q \Lambda \tau_\mu}{\beta m_\mu c}\frac{dE}{ds}$$

- Maximize $Q$ to minimize losses
- Keep $\eta_Q$ large: same problems as $\eta_{\text{cool}}$
- Keep average gradient large
  - Running closer to crest
  - Densely packed lattice

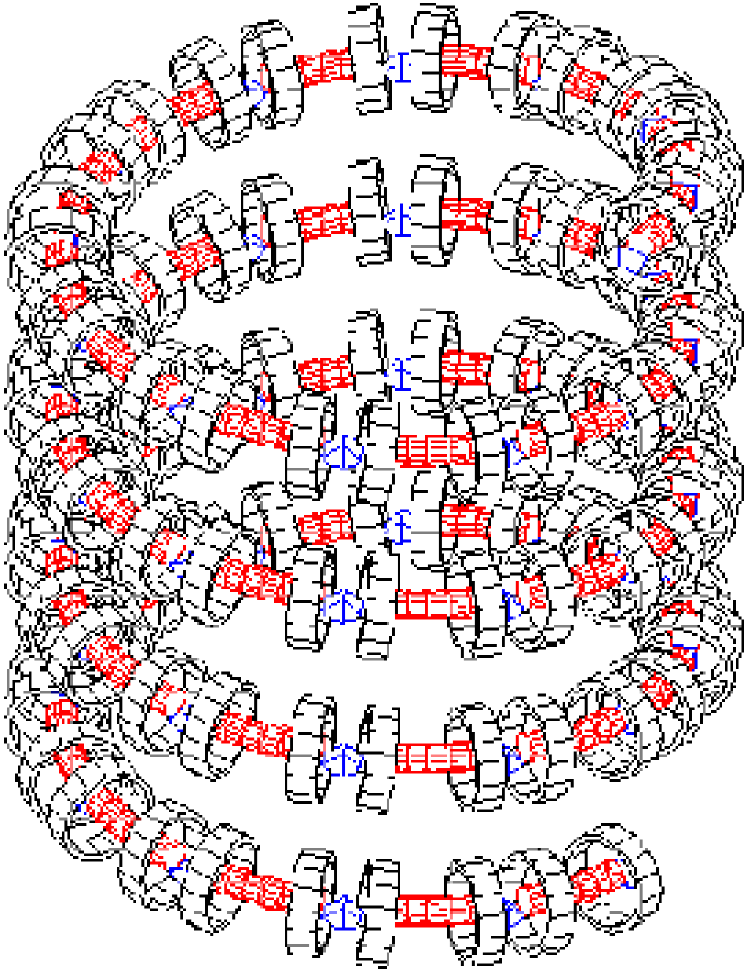
# Cooling Scheme



# Cooling Guggenheims

- Straight cooling lattice only cools transversely
- Add bend, wedge absorbers to couple to longitudinal
- Avoid injection/extraction
- Long bunch trains won't fit in ring
- Taper the channel: avoid equilibrium
- One for each sign
- Increase frequency: maintain energy spread

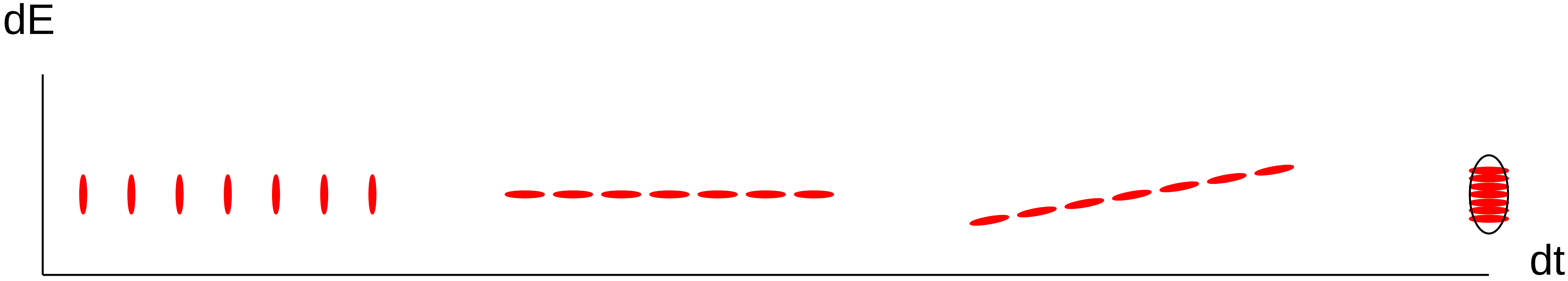
# Cooling Guggenheim



# Cooling Bunch Merge

- Prefer everything in one bunch
  - Needed to achieve beam-beam tune shift
  - Avoid if final emittance small enough
- Induce energy spread: low frequency RF
- Drift until coincide in time
  - Accelerate drift with wiggler?
- Capture in single bucket
- Significant losses: factor of 3!

# Cooling
# Bunch Merge



# Cooling
# Post Merge Guggenheims

- Longitudinal emittance now large
- Reduce longitudinal emittance
- Reduce transverse emittance also
- Space charge becomes significant
- Avoided if no bunch merge

# Cooling
# High Field Solenoids

- Need to get smaller transverse emittances
- Need large angular spreads in beam
  - Use high-field (50 T) solenoids
- Little net 6-D cooling
  - Reduces transverse emittance at expense of longitudinal

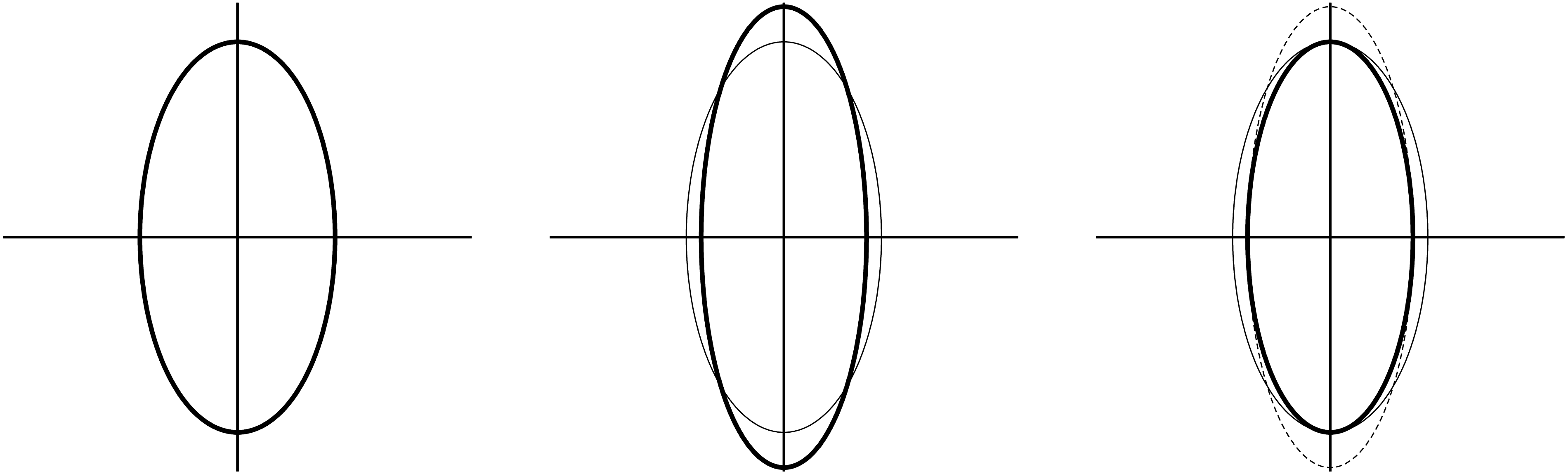
# Cooling
# Low Emittance Scheme: Why?

- Avoid the bunch merge
  - Factor of 20 cooling each transverse
  - Reduces losses by factor of 3 ($\eta_\text{trans}$)
- Allows lower $\beta*$
- Acceleration, ring beamlines less expensive
- Potentially permit high energy bunch merge
  - Maybe more efficient

# Cooling
# Low Emittance Scheme: How?

- Problem: getting small $\beta$ functions
  - High magnetic fields
- Proposal “PIC”
  - Old idea of Balbekov
  - Run on linear resonance
  - Unstable direction is transverse momentum
  - Stable position is transverse position
  - Cooling reduces transverse momentum as much as instability increases

# Cooling
# PIC Principle



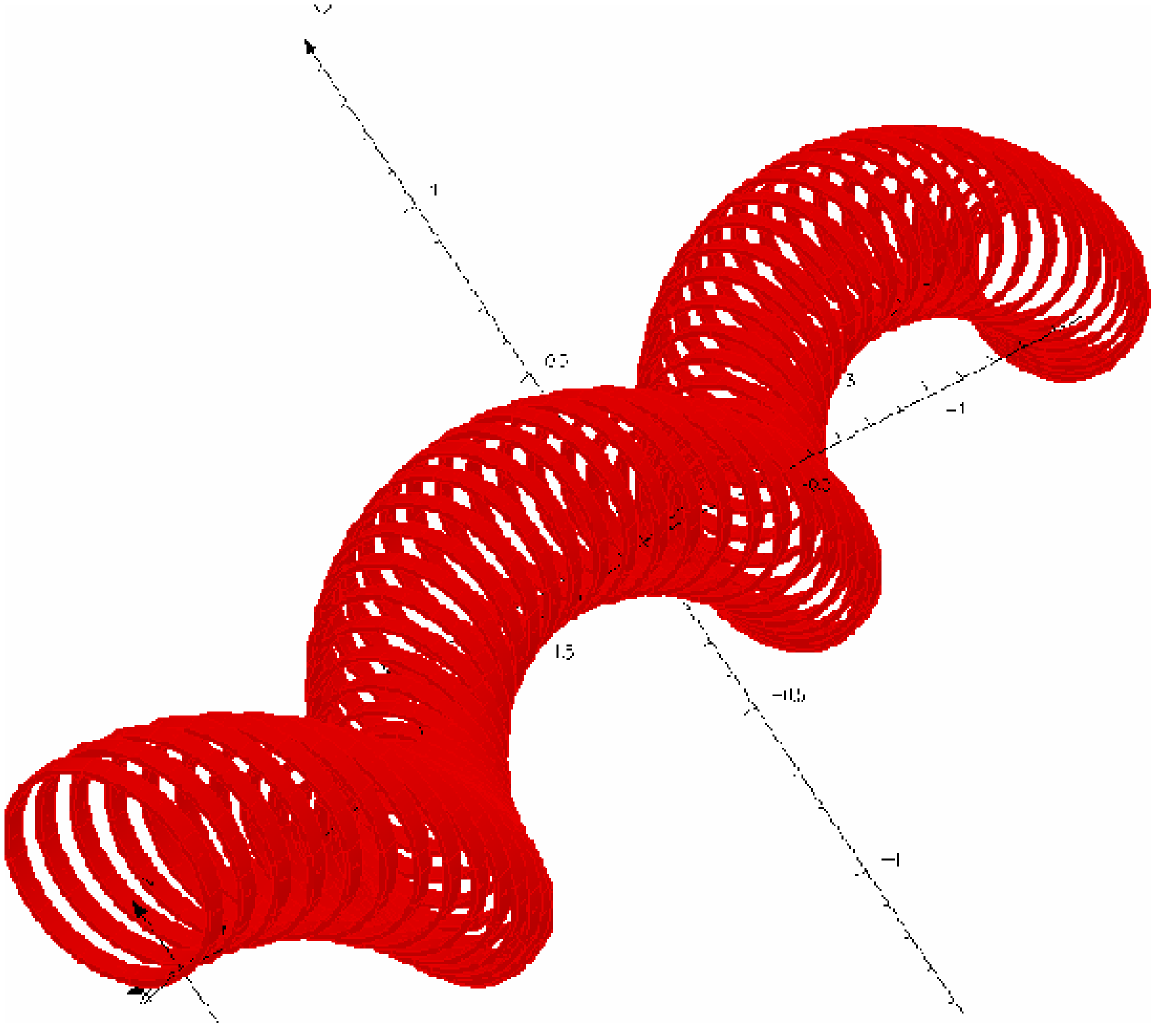
# Cooling
# Low Emittance: Issues

- No focusing
  - System becomes sensitive to perturbations
  - Space charge
  - Chromatic aberrations
- Insuring losses don't exceed benefits

# Cooling
# Other Systems

- ❍ Helical cooling channel
- ❍ Hoped to be more efficient than Guggenheim
- ❍ Still under development, various issues
  - ❑ Getting cavities in channel

# Cooling
# Helical Cooling Channel



# Acceleration

- Get to high energy without significant losses
- Constant gradient $V$,

$$\frac{N_f}{N_i} = \left(\frac{E_f + p_f c}{E_i + p_i c}\right)^{-\frac{m_\mu c}{\tau_\mu V}}$$

- Losses very modest if average gradient high

# Acceleration Techniques

- Higher energies, more time for
  - Ramping magnets
  - Varying RF frequency
- Can use higher frequency RF
- RLAs
- FFAGs, maybe adjusting RF frequency
  - More passes through RF
- Fast ramping synchrotron

# Acceleration
# Beam Loading

- Possibly more passes through RF
  - Ramping synchrotrons may use many passes
  - FFAGs can adjust RF frequency
- RF frequencies are higher
  - Less stored energy
- Power supplied to replace lost energy?

# Collider Ring

- Achieving sufficient dynamic aperture for small $\beta^*$
- Achieving larger $\Delta\nu$
- Question: to what extent do we get help from
  - Smaller longitudinal emittance
  - Smaller transverse emittance
- Highest field possible for luminosity

# Sample Parameters

| CoM energy (TeV) | 1.5 | 4 | 8 |
|---|---|---|---|
| $\mathcal{L}$ $(10^{34}$ cm$^2$s$^{-1})$ | 1 | 4 | 8 |
| $B_{\text{avg}}$ (T) | 5.2 | 5.2 | 10.4 |
| $\beta^*$ (mm) | 10 | 3 | 3 |
| $dp/p$ rms (%) | 0.09 | 0.12 | 0.06 |
| Ring depth (m) | 13 | 135 | 540 |
| $\eta_{\text{trans}}$ | 0.07 | 0.07 | 0.07 |
| $f_P$ (Hz) | 13 | 6 | 3 |
| $P_P$ (MW) | 4 | 1.8 | 0.8 |

| | | | |
|---|---|---|---|
| $\Delta\nu$ | 0.1 | $N$ $(10^{12})$ | 2 |
| $\epsilon_\perp$ (mm·mrad) | 25 | $\epsilon_\parallel$ (mm·rad) | 72 |

# Final Remarks

- Getting luminosity means
  - Increasing proton driver power
  - Increasing transmission
  - Increasing beam-beam tune shift
  - Lowering $\beta^*$
  - Increasing collider bending field
- All these increase radiation, except
  - Beam-beam tune shift
  - $\beta^*$

# Final Remarks

- Most transmission loss before acceleration
- Improving transmission requires
  - Higher average RF gradients
  - Reducing inefficiencies
    - Matching
    - Approaching equilibrium
- Best hope for improvement is eliminating rebunching

# Final Remarks

- Lowering emittance only helps indirectly
- Allows higher proton driver rep rate
    - Easier on proton driver
    - But no power reduction
- Potentially eliminate rebunching
- Maybe improve $\beta^*$
- Comes with a transmission cost