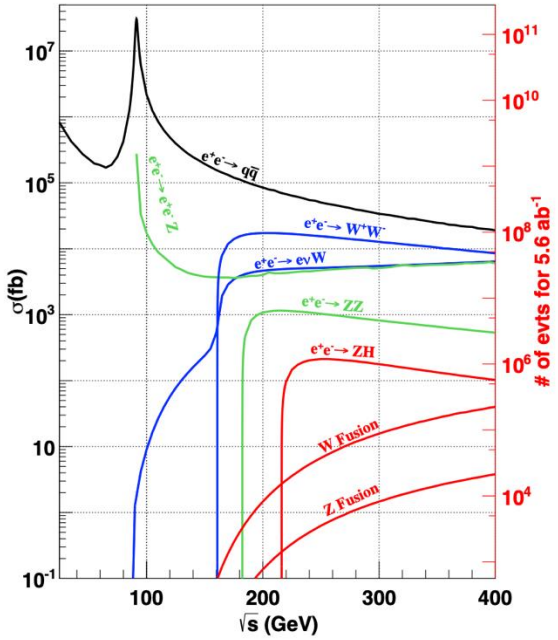
W precision measurements at future e+e- colliders

Paolo Azzurri – INFN Pisa

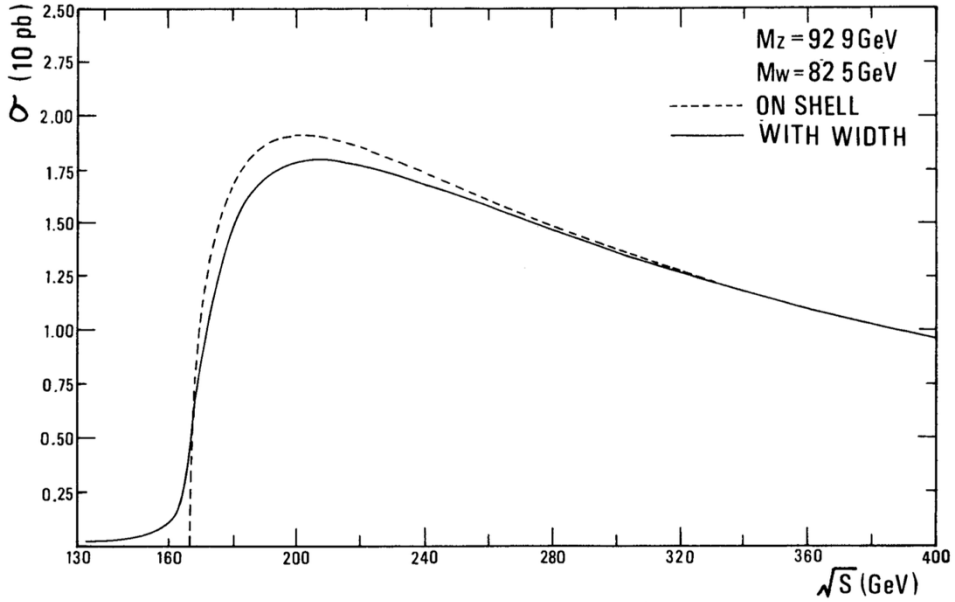
Gearing up for Future Colliders

CERN June 29 2026

WW production



arXiv:1810.09037 (CEPC)



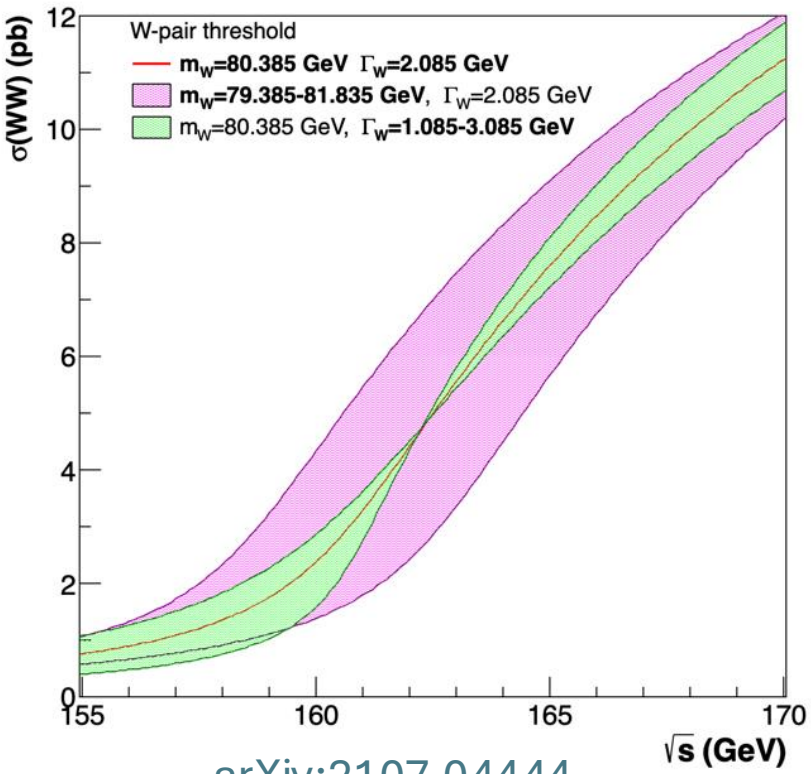
Mod.Phys.Lett.A 1 (1986) 203

WW samples	155-170 GeV	230-250 GeV	350-365 GeV
FCCee	50-80 M	180 M	40 M
CEPC	3-5 M	170 M	
LEP3	15-25 M	43 M	
LCF	0.4 M (?)	48 M	5 M

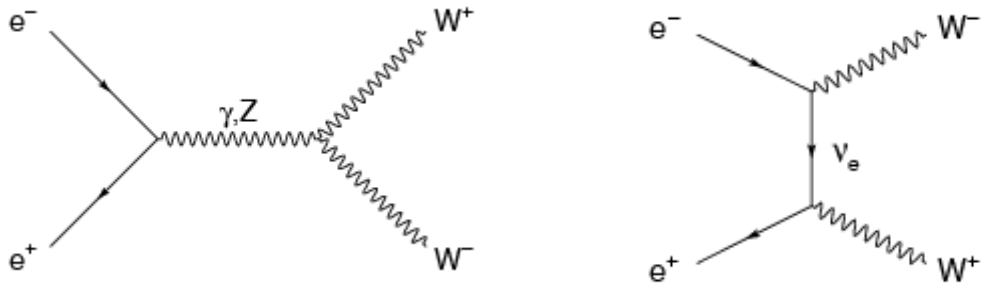
LHC WW samples Run2+Run2 : 900 M (from tt→WWbb) + 100 M (excl WW), up to multi TeV $\sqrt{s'}$

FCC-ee EW Precision group is working on refining current estimates

m_W and Γ_W from WW threshold production



arXiv:2107.04444

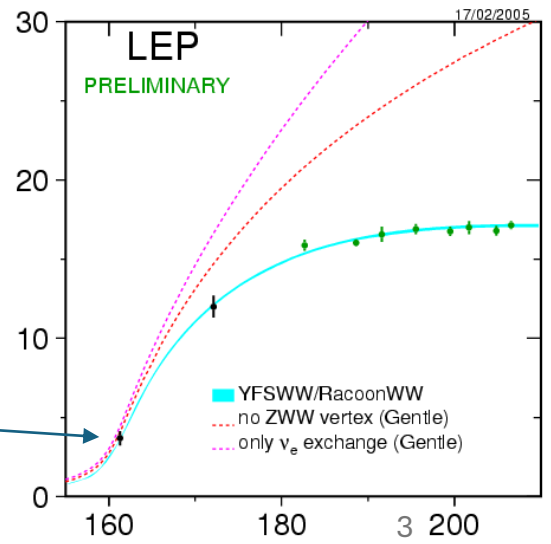


WW cross section rise $\beta = \sqrt{1 - 4m_W^2/s}$ driven by t-channel production

Extract the W mass inverting the m_W dependence

$$\sigma(m_W, E) \qquad m_W = \sigma^{-1}(E)$$

$$\Delta m_W = \left(\frac{d\sigma}{dm_W}\right)^{-1} \Delta\sigma$$



ALEPH Phys.Lett.B 401 (1997) 347 with 10/pb $m_W = 80.14 \pm 0.34$ GeV

stat extrapolation to 10/ab $\Longrightarrow$ $\Delta m_W = 0.34$ MeV

The WW threshold : W mass uncertainties

$$\sigma = \frac{N}{L} - \sigma_B \frac{1}{\varepsilon}$$

$$\Delta m_W(stat) = \left(\frac{d\sigma}{dm_W}\right)^{-1} \frac{\sqrt{\sigma}}{\sqrt{L}} \frac{1}{\sqrt{\varepsilon p}}$$

Statistical

$$\Delta\sigma_{WW} = \frac{\Delta\sigma_B}{\varepsilon}$$

$$\Delta m_W(B) = \left(\frac{d\sigma}{dm_W}\right)^{-1} \left(\frac{\Delta\sigma_B}{\varepsilon} \oplus \Delta\sigma_{TH}\right)$$

Background and Theory

$$\Delta\sigma_{WW} = \sigma\left(\frac{\Delta\varepsilon}{\varepsilon} \oplus \frac{\Delta L}{L}\right)$$

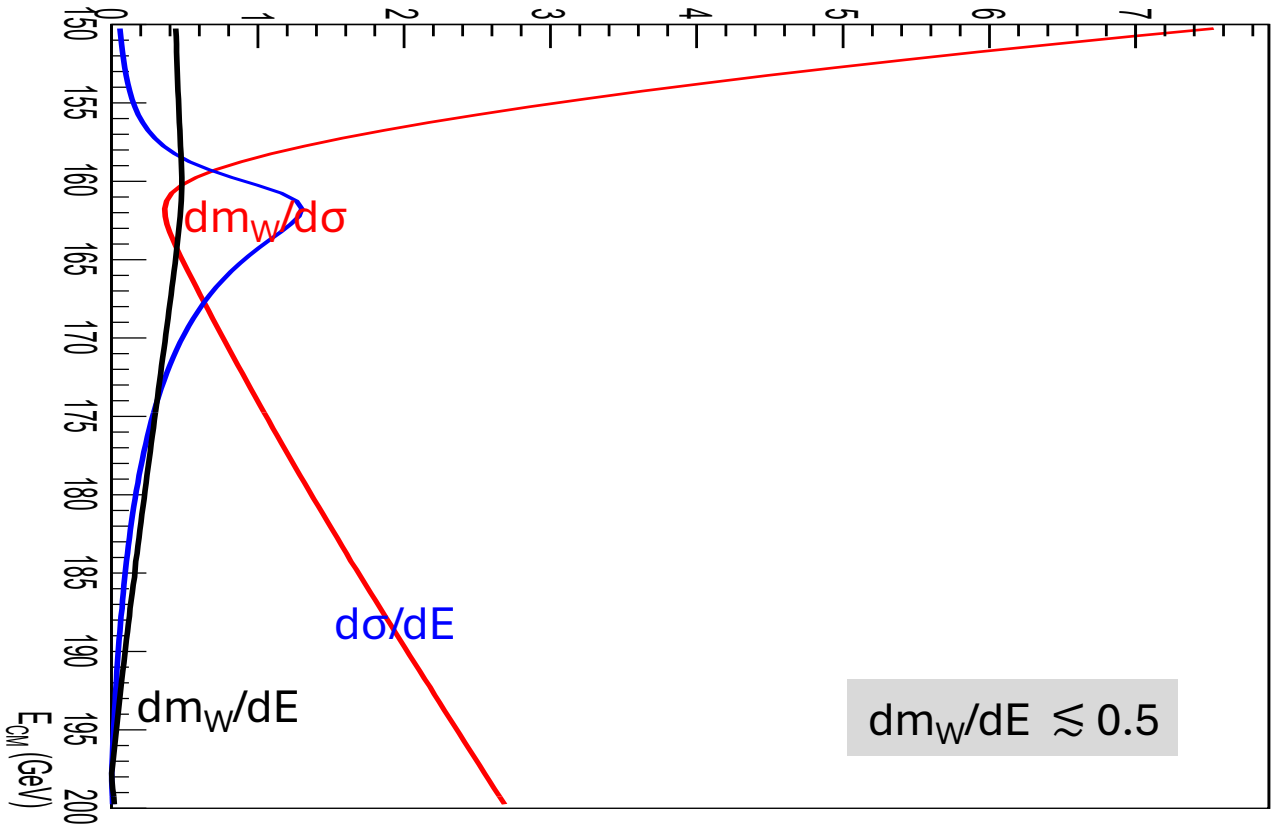
$$\Delta m_W(\varepsilon) = \sigma\left(\frac{d\sigma}{dm_W}\right)^{-1} \left(\frac{\Delta\varepsilon}{\varepsilon} + \frac{\Delta L}{L}\right)$$

Acceptance and Luminosity

$$\Delta m_W(E) = \left(\frac{d\sigma}{dm_W}\right)^{-1} \left(\frac{d\sigma}{dE}\right) \Delta E \leq \frac{1}{2}\Delta E$$

Collision energy

The WW threshold W mass : beam energy



$dm_W/dE \lesssim 0.5$

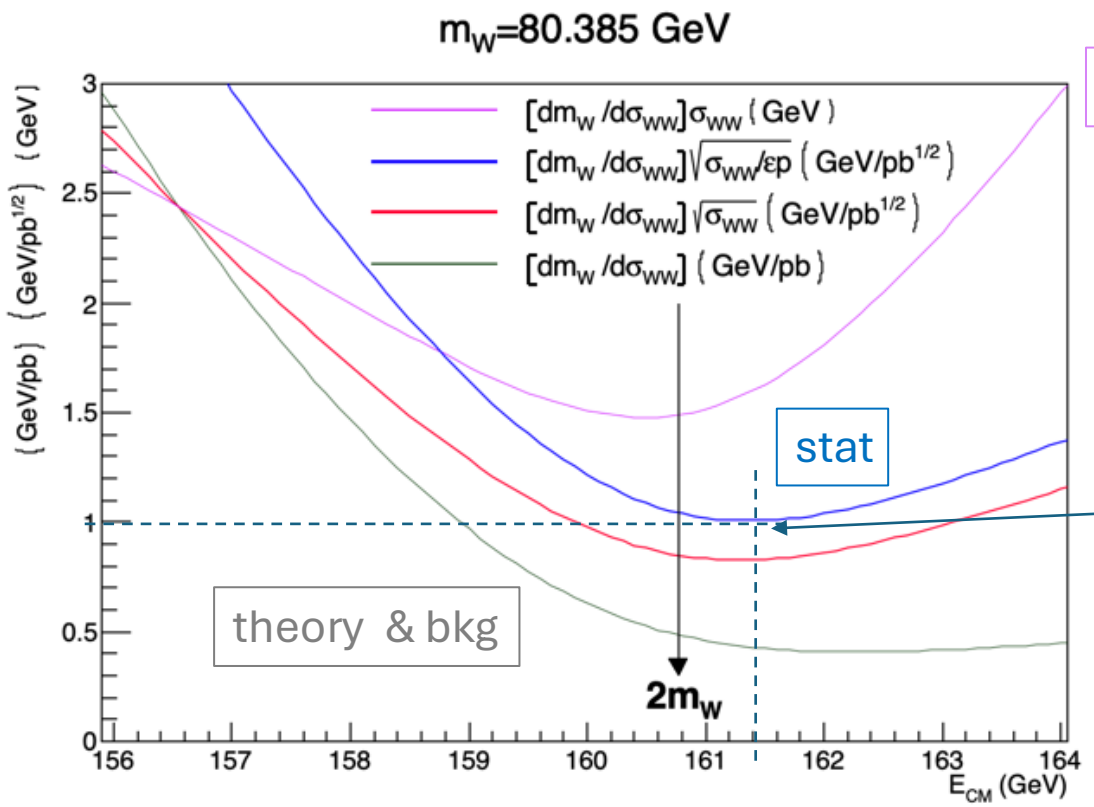
$$\Delta m_W(E) = \left(\frac{d\sigma}{dm_W}\right)^{-1}\left(\frac{d\sigma}{dE}\right)\Delta E \leq \frac{1}{2}\Delta E$$

Uncertainty on beam energy $\Delta E_b = \frac{1}{2}\Delta E$ translates directly to m_W

$$\Delta \boldsymbol{E_b} \cong \Delta \boldsymbol{m_W}$$

Very limited variations of the dm_W/dE coefficient with E_{CM} in the threshold region

The WW threshold : W mass optimal E_{CM}



acceptance & lumi

optimal for stat is also close to optimal for syst contributions

stat uncertainty assuming event selection quality
$Q=\sqrt{\varepsilon p}$ with fixed $\varepsilon=0.75$ and $\sigma_B=0.3$pb

Max stat sensitivity at $E_{CM}\sim 2m_W + 0.6$ GeV

$$\left[\left(\frac{d\sigma}{dm_W}\right)^{-1}\frac{\sqrt{\sigma}}{\sqrt{\varepsilon p}}\right]_{min} \cong 1\,\frac{GeV}{pb^{1/2}} = 1\,\frac{MeV}{ab^{1/2}}$$

With L=12/ab $\Longrightarrow$ Δm_W(stat) = 0.3 MeV

WW threshold : W mass precision requirements

Conditions to achieve $\Delta m_W(\text{syst}) < \Delta m_W(\text{stat}) =$ **0.3 MeV**
with a single point WW threshold measurement

current theory precision
$\Rightarrow \Delta \boldsymbol{m_W}$ = 3 MeV

$$\Delta m_W(B) = \left(\frac{d\sigma}{dm_W}\right)^{-1} \left(\frac{\Delta\sigma_B}{\varepsilon} \oplus \Delta\sigma_{TH}\right)$$

Background and Theory $\Delta\sigma_{TH} < \mathbf{1fb}$ $(\Delta\sigma_{TH}/\sigma_{TH} < 2 \cdot 10^{-4})$
$\Delta\sigma_B/\varepsilon < \mathbf{1fb}$ $(\Delta\sigma_B/\sigma_B < 4 \cdot 10^{-3})$

$$\Delta m_W(\varepsilon) = \sigma\left(\frac{d\sigma}{dm_W}\right)^{-1} \left(\frac{\Delta\varepsilon}{\varepsilon} + \frac{\Delta L}{L}\right)$$

Acceptance and Luminosity $\left(\frac{\Delta\varepsilon}{\varepsilon} \oplus \frac{\Delta L}{L}\right) < \mathbf{2 \cdot 10^{-4}}$

$$\Delta m_W(E) = \left(\frac{d\sigma}{dm_W}\right)^{-1} \left(\frac{d\sigma}{dE}\right) \Delta E \leq \frac{1}{2}\Delta E$$

Collision energy $\Delta E_b < 0.3\ MeV$ $(\Delta E_b/E_b < 4 \cdot 10^{-6})$

The WW threshold : background syst

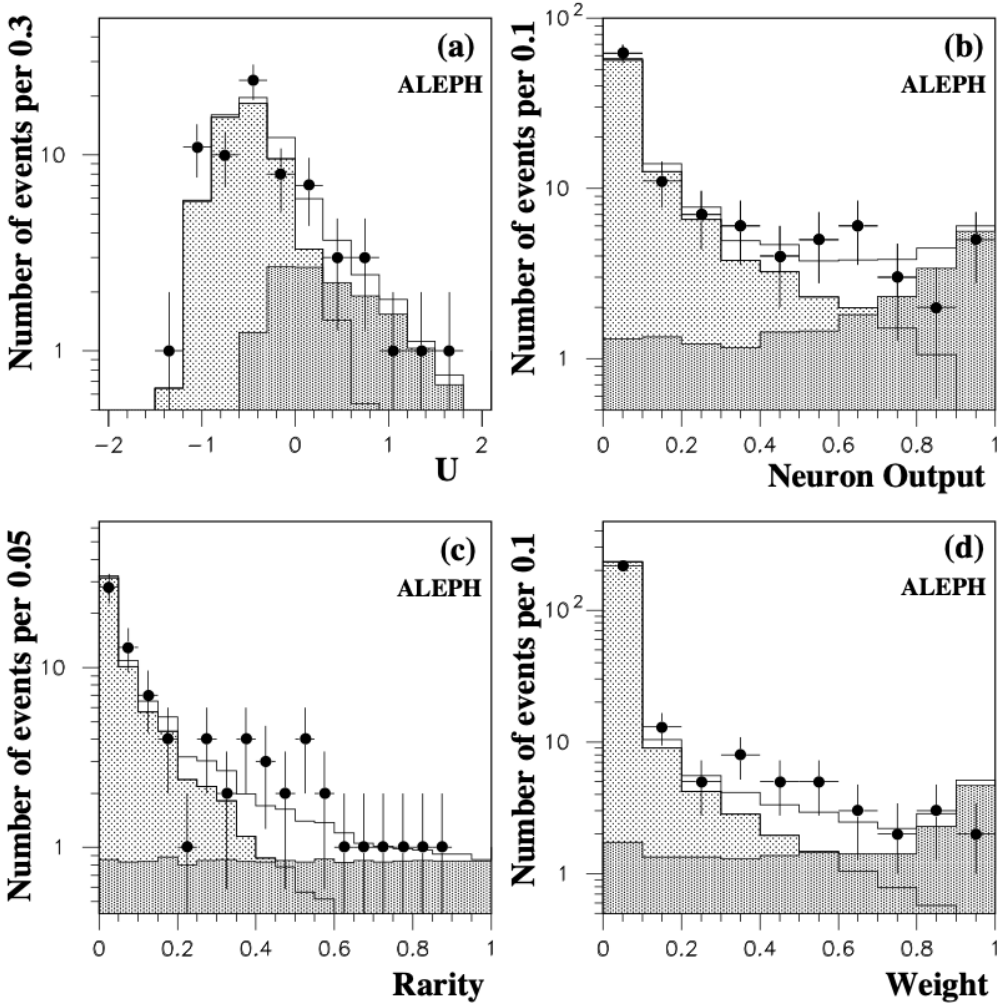
almost all bkg in the 4q channel

Selection	Expected signal	Expected background	Observed
$W^+W^- \to q\bar{q}q\bar{q}$	9.6 ± 1.0	3.44 ± 0.39	14
$W^+W^- \to q\bar{q}e\bar{\nu}_e$	3.89 ± 0.44	0.18 ± 0.27	3
$W^+W^- \to q\bar{q}\mu\bar{\nu}_\mu$	4.19 ± 0.46	0.27 ± 0.15	2
$W^+W^- \to q\bar{q}\tau\bar{\nu}_\tau$	2.32 ± 0.28	0.96 ± 0.34	7
$W^+W^- \to \ell^+\nu_\ell\ell'^-\bar{\nu}_{\ell'}$	2.58 ± 0.28	$0.19^{+0.12}_{-0.04}$	2
Combined	22.6 ± 2.4	5.0 ± 0.6	28

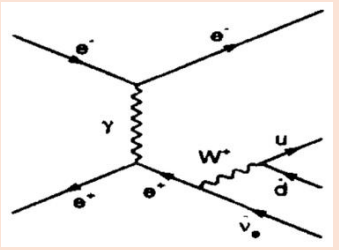
OPAL Phys. Lett. B 389 (1996) 416.

Phys.Lett.B 401 (1997) 347

purity ~95% achieved in the last bins



4-fermion-CC03 interference effects



positive & negative effects (10-50 fb) reported in the various channels, within the LEP analyses acceptance

WW threshold : acceptance (and bkg) syst

Syst unc at higher E_{CM} (207 GeV) on σ_{WW} (~16pb)

Source	uncertainty (fb)			
	$\ell\nu\ell\nu$	$\ell\nu$qq	qqqq	total
Tracking	4	19	31	5
Simulation of calorimeters	-	9	26	31
Hadronization models	-	27	8	35
Z peak $q\bar{q}$ fragmentation	-	-	20	20
Inter-W final state interaction	-	-	28	28
Background contamination	9	5	31	35
Lepton identification	1	2	-	3
Beam-related background	10	17	37	22
$\mathcal{O}(\alpha)$ corrections DPA	2	9	12	6
Luminosity	8	35	44	87
Simulation statistics	6	20	14	25
Total	17	57	87	126

ALEPH Eur.Phys.J.C 38 (2004) 147

Source	$\sigma_{WW}^{q\bar{q}q\bar{q}}$ (pb)	$\sigma_{WW}^{q\bar{q}l\nu}$ (pb)	$\sigma_{WW}^{l\nu l\nu}$ (pb)
Four-jet modelling	±0.051	±0.014	-
Background cross-sections	±0.009	±0.016	±0.006
Fragmentation	±0.045	±0.038	-
Final state interactions	±0.025	-	-
Radiative corrections	±0.008	±0.008	±0.002
Luminosity (theor)	±0.011	±0.010	±0.002
Luminosity (exp)	±0.045	±0.043	±0.011
Detector effects	±0.045	±0.053	±0.033
Monte Carlo statistics	±0.005	±0.014	±0.033

DELPHI Eur.Phys.J.C 34 (2004) 127

NP QCD effects have important impacts on both qqqq and qq$\ell\nu$

improvements in fragmentation and hadronization modeling plus constraints from control data (Z→qq)

much less worrisome than using jet properties for kin reco

can roughly scale/4 for equivalent ε effects at threshold σ_{WW} (~4pb)

target : bring table items below 4fb(/4=1fb)

20-30fb reported for LEP2 ⇒ Δm_W = 1.5-2 MeV

WW threshold @ ILC

arXiv:1603.06016 & arXiv:1908.11299

ILC polarised collisions : enhance (x4) t-channel WW production or suppress it to control

Channel	Efficiency (%)	$\sigma^U_{\rm bkgd}$ (fb)	$A^B_{\rm LR}$	Eff. syst. (%)	Bkgd syst.	$A^B_{\rm LR}$ syst.
lvlv	87.5	10	0.15	0.1	free	0.025
qqlv	87.5	40	0.30	0.1	free	0.012
qqqq	83.5	200	0.48	0.1	free	0.005

Table 3: Experimental assumptions for the WW event selection near threshold using a polarized scan

$\sqrt{s}$ (GeV)	L (fb^{-1})	f	$\lambda_{e^-}\lambda_{e^+}$	N_{ll}	N_{lh}	N_{hh}	N_{RR}
160.6	4.348	0.7789	$-+$	2752	11279	12321	926968
		0.1704	$+-$	20	67	158	139932
		0.0254	$++$	2	19	27	6661
		0.0254	$--$	21	100	102	8455
161.2	21.739	0.7789	$-+$	16096	67610	73538	4635245
		0.1704	$+-$	98	354	820	697141
		0.0254	$++$	37	134	130	33202
		0.0254	$--$	145	574	622	42832
161.4	21.739	0.7789	$-+$	17334	72012	77991	4639495
		0.1704	$+-$	100	376	770	697459
		0.0254	$++$	28	104	133	33556
		0.0254	$--$	135	553	661	42979
161.6	21.739	0.7789	$-+$	18364	76393	82169	4636591
		0.1704	$+-$	81	369	803	697851
		0.0254	$++$	43	135	174	33271
		0.0254	$--$	146	618	681	42689
162.2	4.348	0.7789	$-+$	4159	17814	19145	927793
		0.1704	$+-$	16	62	173	138837
		0.0254	$++$	10	28	43	6633
		0.0254	$--$	46	135	141	8463
170.0	26.087	0.7789	$-+$	63621	264869	270577	5560286
		0.1704	$+-$	244	957	1447	838233
		0.0254	$++$	106	451	466	40196
		0.0254	$--$	508	2215	2282	50979

Table 1: Illustrative example of the numbers of events in each channel for the standard 100 fb^{-1} 6-point ILC scan with 4 helicity configurations. Columns give the center-of-mass energy, $\sqrt{s}$, the apportioned integrated luminosity, the fraction for each helicity configuration, $\lambda_{e^-}\lambda_{e^+}$, and the numbers of events observed in each channel.

with 100 fb−1

Fit type	Uncertainty source	ΔM_W [MeV]	ΔM_W (syst.) [MeV]
fixbkg	Background	3.20	2.30
fixpol	Polarization	3.73	1.27
fixeff	Efficiency	3.86	1.18
fixlum	Luminosity	3.76	0.78
fixALRB	$A^B_{\rm LR}$	3.86	0.80
fixall	Statistical	2.43	
	Systematic		3.10
standard	Total Error	3.94	

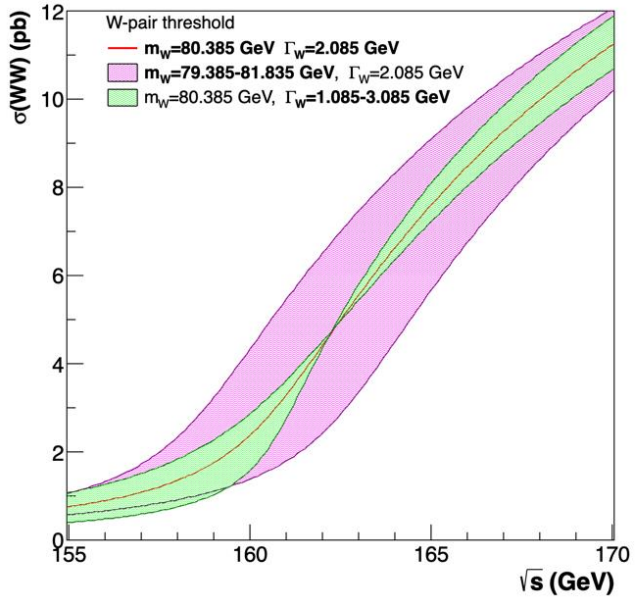
$$\Delta m_W(\text{MeV}) = 2.4\ (\text{stat}) \oplus 3.1\ (\text{syst}) \oplus 0.8\ (\sqrt{\text{s}}) \oplus \text{theory}$$

fitted $\Delta\varepsilon \sim 10^{-3}$ and $\Delta\sigma_B$ ~6 fb
additional impact of pol uncertainty syst

Δm_W = 4 MeV

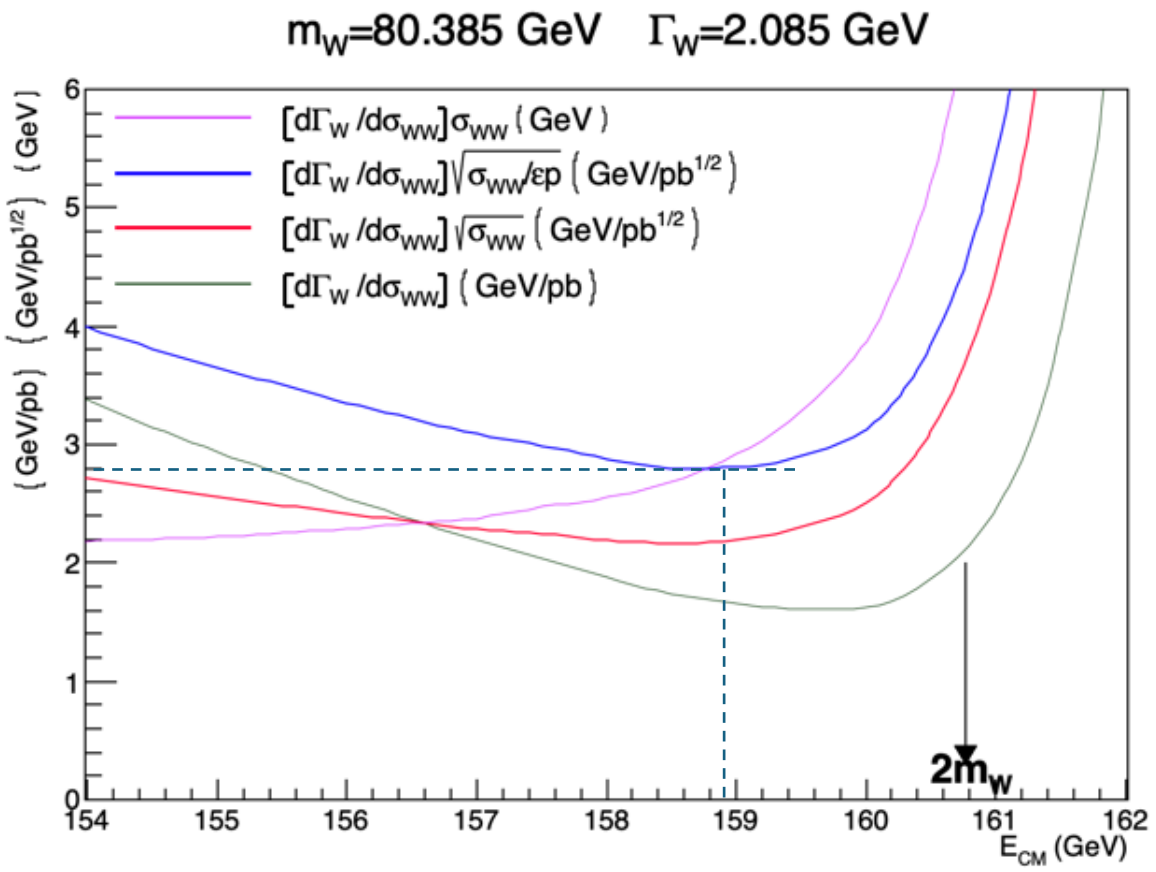
WW threshold : W mass and width

arXiv:1703.01626
arXiv:2107.04444



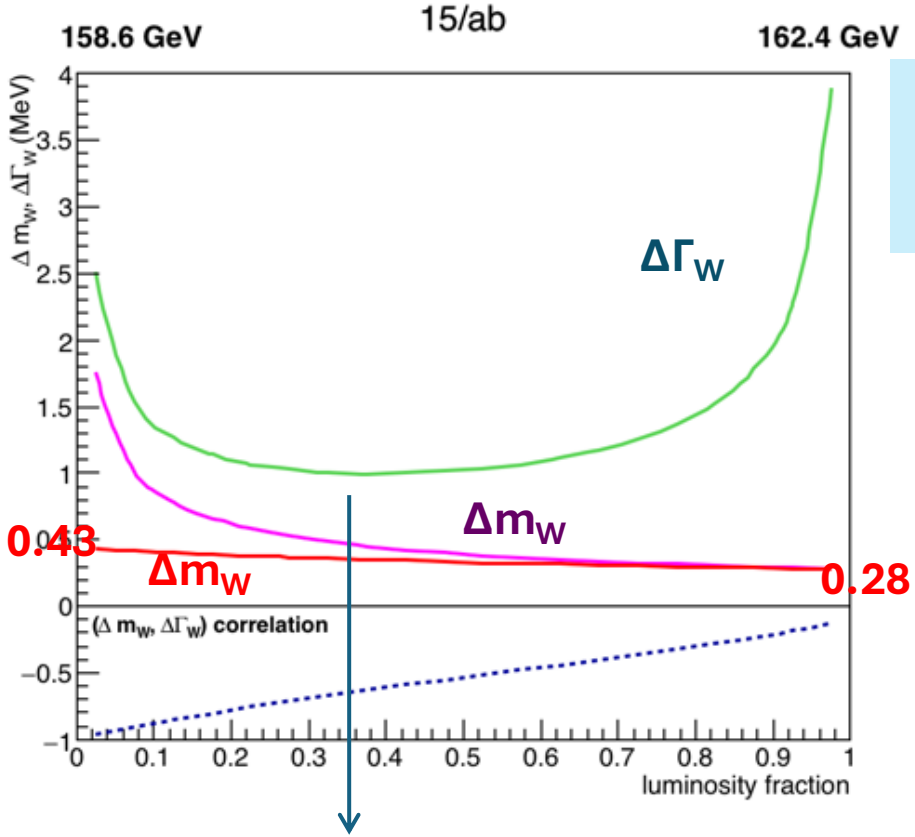
Max stat sensitivity at E_{CM}~ $2m_W$ - Γ_W

$$\left[\left(\frac{d\sigma}{d\Gamma_W}\right)^{-1}\frac{\sqrt{\sigma}}{\sqrt{\varepsilon p}}\right]_{min} \cong 2.8\ \frac{GeV}{pb^{1/2}} = 2.8\ \frac{MeV}{ab^{1/2}}$$



WW threshold : W mass and width

Scans of possible E_1 E_2 data taking energies and luminosity fractions f (at the E_2 point)



Δm_W=0.45 MeV , $\Delta\Gamma_W$=1 MeV (r=-0.6)
Δm_W=0.35 MeV

A -minimum of $\Delta\Gamma_W$=0.91 MeV with Δm_W=0.55 MeV
taking data at E_1=**156.6** GeV E_2=**162.4** GeV f=**0.25**
yields Δm_W=0.47 MeV (as single par)

B- minimum of Δm_W=0.28 MeV $\Delta\Gamma_W$=3.3 MeV with
E_1=**155.5** GeV E_2=**162.4** GeV f=**0.95**
yields Δm_W=0.28 MeV (as single par)

C- minimum of $\Delta\Gamma_W$=0.96 MeV +Δm_W=0.41 MeV with
E_1=**157.5** GeV E_2=**162.4** GeV f=**0.45**
yields and Δm_W=0.37 MeV (as single par)

Δm_W , $\Delta\Gamma_W$: error on W mass and width from fitting both
Δm_W : error on W mass from fitting only m_W

W mass and width : uncertainty propagation

With cross section $\sigma_1\sigma_2$ measurements at two energies E_1 E_2 : uncertainty propagation

$$\begin{cases}\sigma_1 = \sigma_{WW}(E_1, m_W, \Gamma_W)\\ \sigma_2 = \sigma_{WW}(E_2, m_W, \Gamma_W)\end{cases}$$

$$\begin{cases}\Delta\sigma_1 = a_1\Delta m + b_1\Delta\Gamma\\ \Delta\sigma_2 = a_2\Delta m + b_2\Delta\Gamma\end{cases}$$

$$a_1 = \frac{d\sigma_1}{dm} \qquad b_1 = \frac{d\sigma_1}{d\Gamma}$$

$$a_2 = \frac{d\sigma_2}{dm} \qquad b_2 = \frac{d\sigma_2}{d\Gamma}$$

$$\Delta m = -\frac{b_2\Delta\sigma_1 - b_1\Delta\sigma_2}{a_2b_1 - a_1b_2}$$

$$\Delta\Gamma = \frac{a_2\Delta\sigma_1 - a_1\Delta\sigma_2}{a_2b_1 - a_1b_2}$$

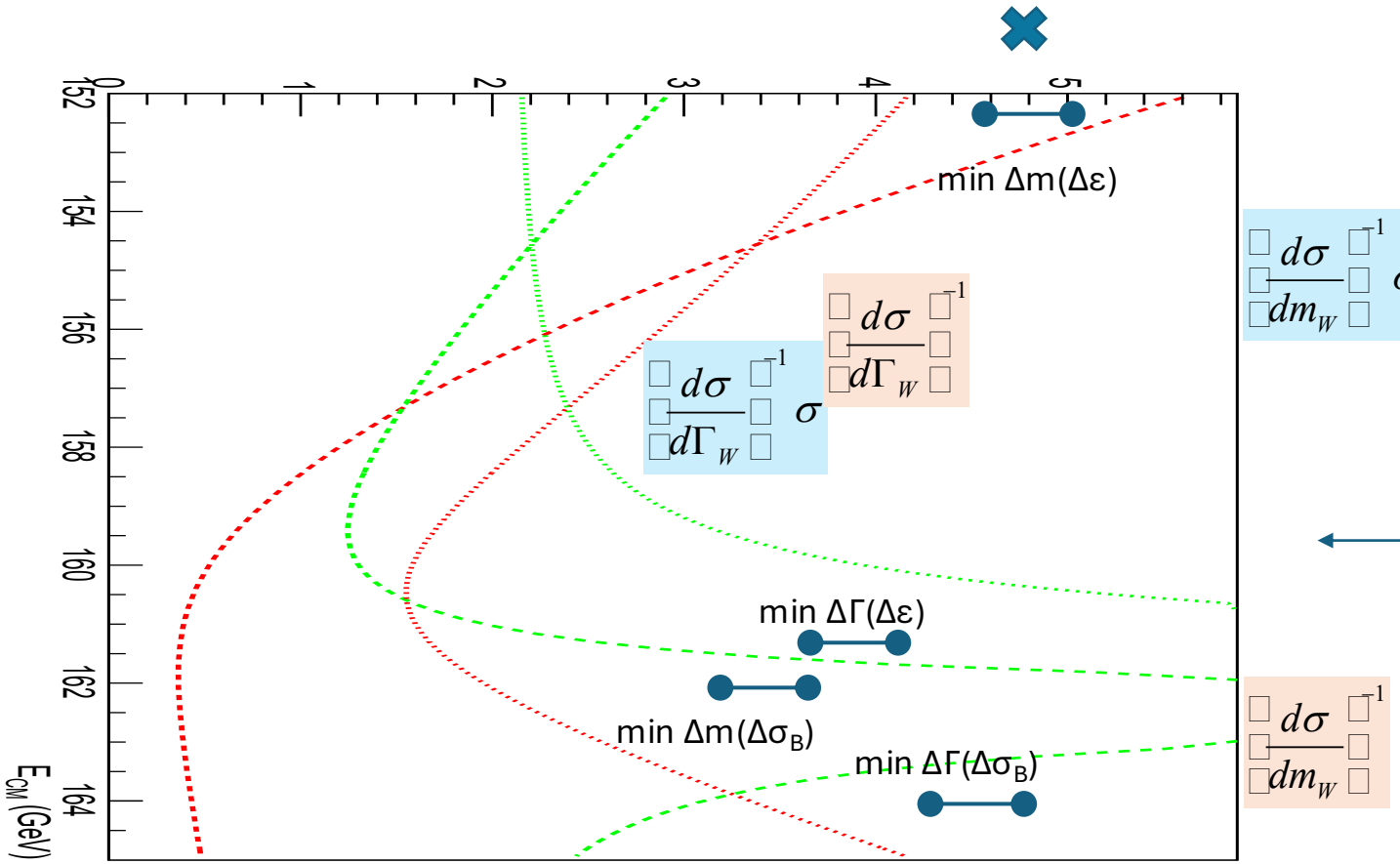
$$r = -\frac{1}{\Delta m\Delta\Gamma}\frac{a_2b_2\Delta\sigma_1^2 + a_1b_1\Delta\sigma_2^2}{\left(a_2b_1 - a_1b_2\right)^2}$$

Δm,ΔΓ linear correlation with uncorrelated $\Delta\sigma_1$,$\Delta\sigma_2$

with E_{CM}=**157.3 162.6** GeV the (syst) uncertainties on $\mathbf{m_W}$ are the same as for the single E_{CM} case

$$\Delta m = \frac{\Delta\sigma_2}{a_2}$$

W mass and width : control of systematics



optimal E points with limiting correlated systs

Scans of (E_1, E_2, f) data taking **assuming limiting syst uncertainties**, either $\Delta\varepsilon+\Delta L$ or $\Delta\sigma_B+\Delta\sigma_{TH}$

More complex situation, depends very much on the correlation of uncertainties between the energy points (that can be quite large)

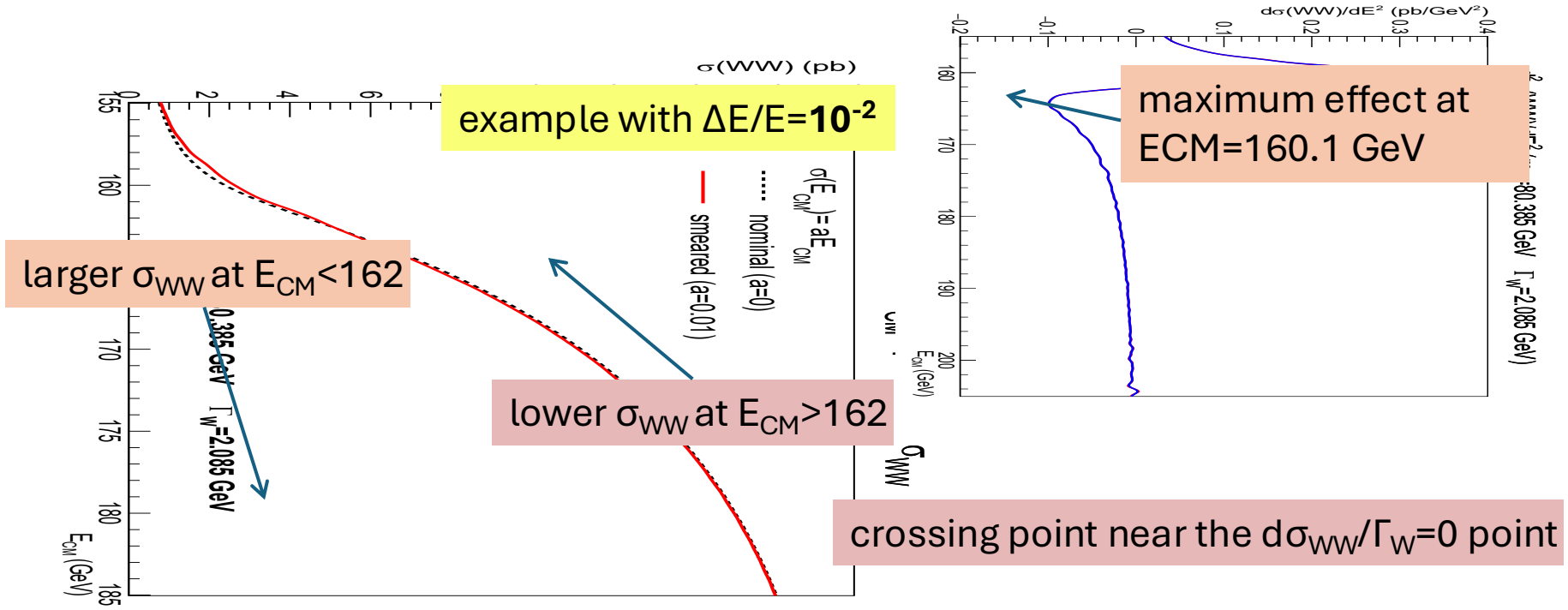
Correlated syst can cancel taking data at different E_{CM} points where the relevant differential factors are equal (around their minima)

>2 energy points will be beneficial to reduce the impact of (correlated) systematic uncertainties
careful choice of additional points recommended

partially explored in [Eur. Phys. J. C 80 no. 1, (2020) 66]

WW threshold : energy spread effects

arXiv:1909.12245



$\sigma(E_{CM})$=**(0.47-1.10) 10^{-3}** E_{CM}

Optimal m_W & Γ_W points @E_{CM}=**157.3 & 162.6** GeV

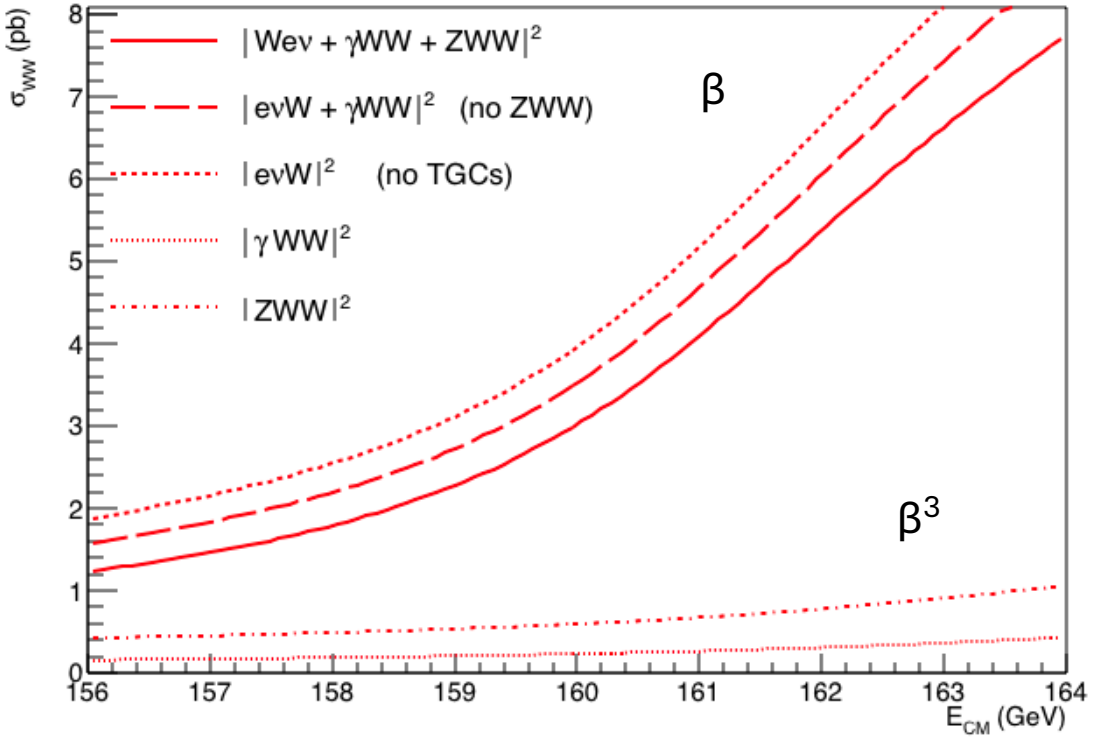
➔ $\Delta\sigma_{WW}$ = +(0.24-1.3) fb & = -(0.18-1.0) fb

➔ **Δm_W = -(0.09-0.48) MeV**

➔ **$\Delta\Gamma_W$ = +(0.6-3.3) MeV**

Maximum effects are at the level of Δm_W(stat) and 2x $\Delta\Gamma_W$ (stat) so that control on the beam energy RMS <50% is required to avoid additional syst contributions from this source

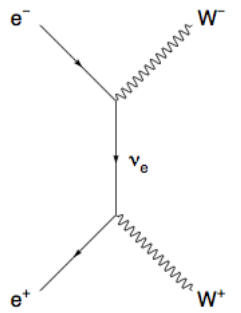
TGCs at threshold

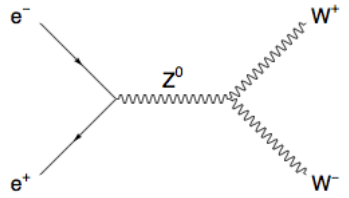


SU(2)⊗U(1) Gauge Cancellations

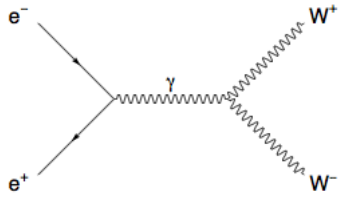
without TGCs
σ_{WW} +40% @157GeV +25%@162GeV

Wev





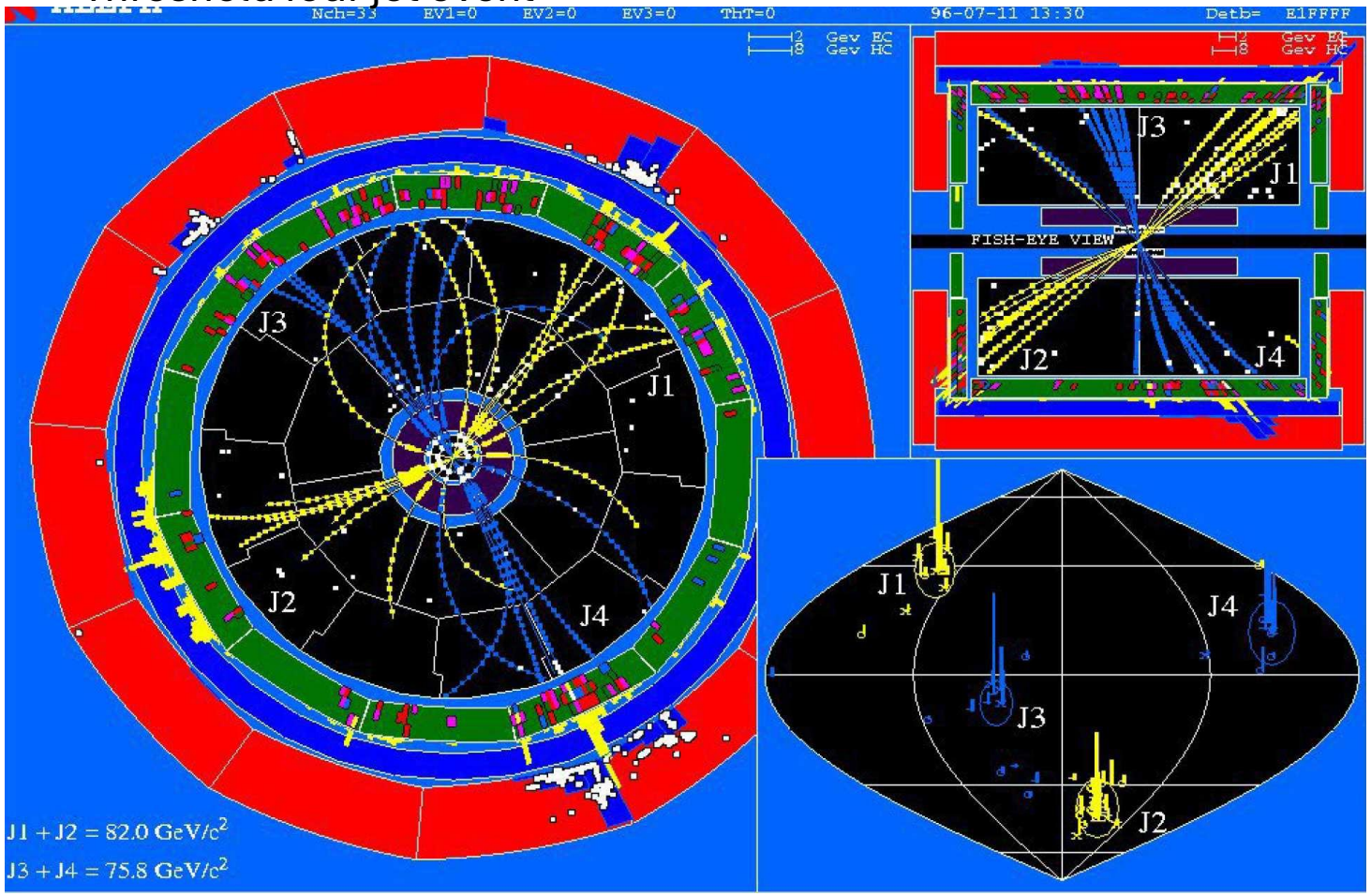
WWZ

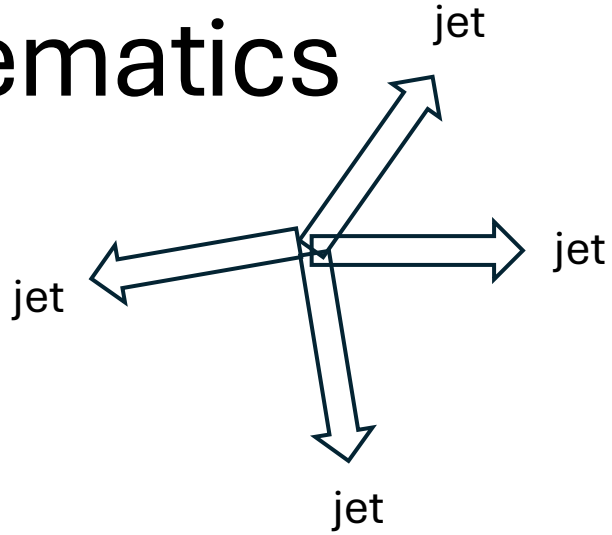


WWγ

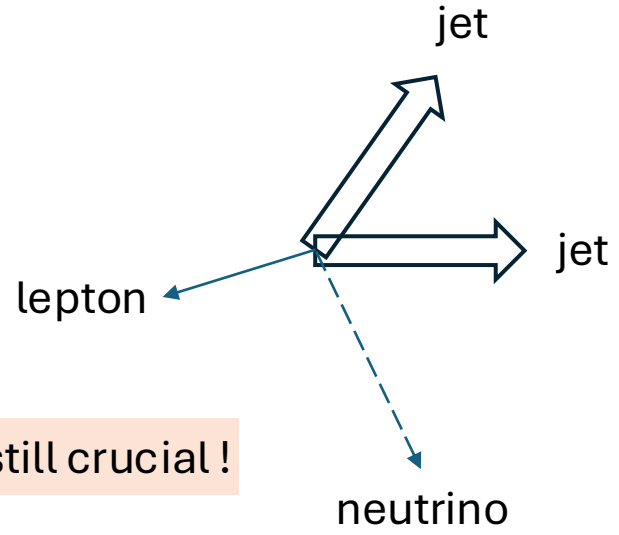
W mass from decay kinematics

Threshold four jet event





Kinematic fit : 4P conservation (+M equality) with jet energy-momentum rescaling



E_{CM} is still crucial !

kinematic fit improves mass resolution from 8-9 GeV to 2-3 GeV ! (arXiv:1302.3415)

W mass from kinematics with 4P fit (LEP2)

Formula for 2-jets final state from ee→Zγ→qqγ

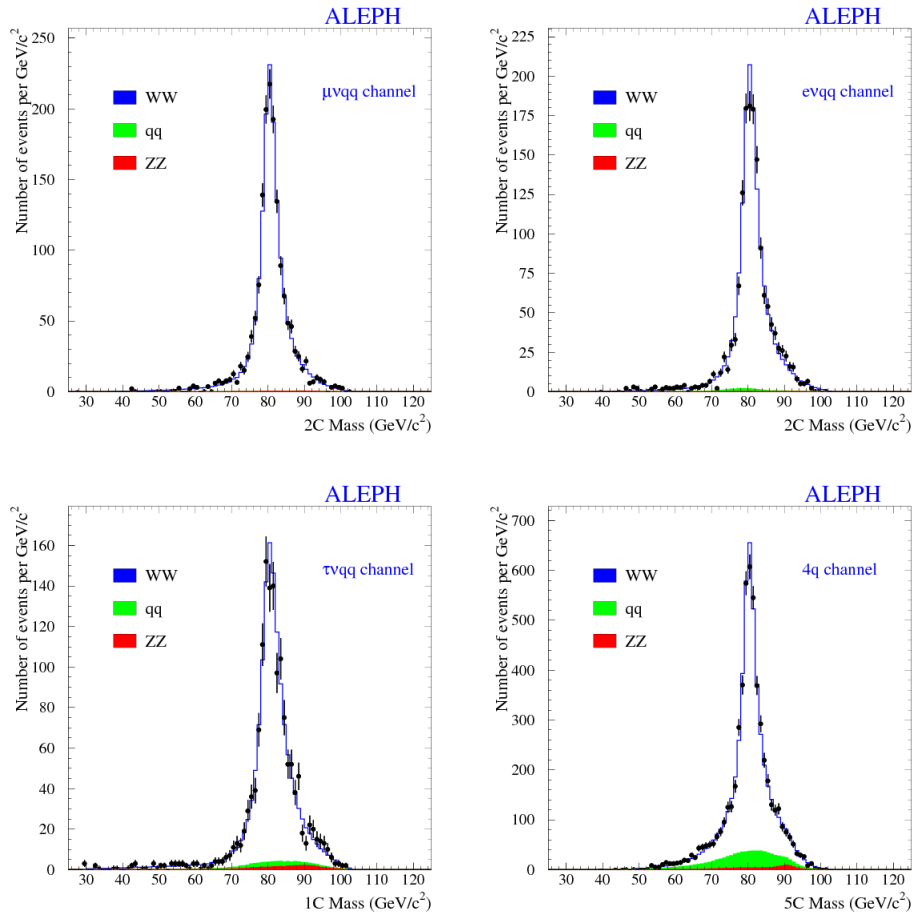
$$M_Z^2 = s\frac{\beta_1 \sin\theta_1 + \beta_2 \sin\theta_2 - \beta_1\beta_2|\sin(\theta_1+\theta_2)|}{\beta_1 \sin\theta_1 + \beta_2 \sin\theta_2 + \beta_1\beta_2|\sin(\theta_1+\theta_2)|}$$

E_{CM} is again a main ingredient: sets jet energy scale
other main ingredients are the jets (and lepton) **angles**
secondary ingredients are the **jet velocities** (β=p/E)

statistical uncertainties ALEPH LEP2→ FCCee extrapolated

Stat uncertainty	Δm_W	$\Delta\Gamma_W$
e**ν**qq	87 MeV → 0.9 MeV	200 MeV → 2 MeV
μνqq	82 MeV → 0.8 MeV	200 MeV → 2 MeV
τνqq	121 MeV → 1.2 MeV	320 MeV → 3.2 MeV
qqqq	70 MeV → 0.7 MeV	120 MeV → 1.2 MeV
combined	43 MeV → 0.4 MeV	90 MeV → 0.9 MeV

LEP2 (ALEPH) from ~10k WW @ E_{CM}=183-209 GeV



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W kinematic fit : systematics

ΔE_{CM}=0.3 MeV at E_{CM}=162.6 GeV
[with Δm_W (stat)(162)~1 MeV]

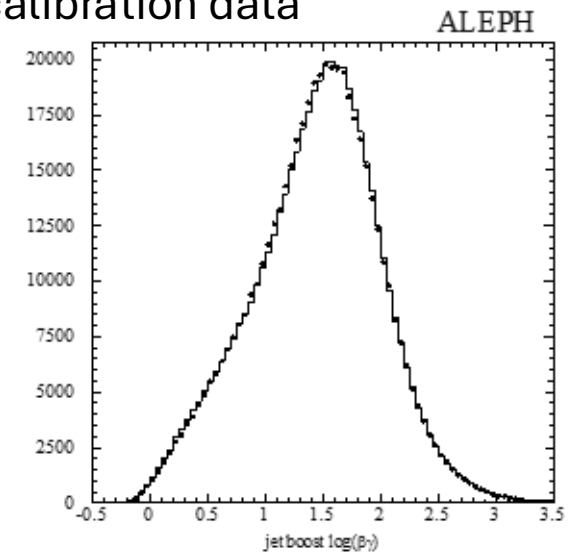
For larger E_{beam} at E_{CM}=240-365 GeV can make use of radiative Z-returns (Zγ) and ZZ events
ΔE_{CM}(240GeV)~2 MeV & ΔE_{CM}=(365 GeV) ~10 MeV

Table 9: Summary of the systematic errors on m_W and Γ_W in the standard analysis averaged ove 183-209 GeV for all semileptonic channels. The column labelled $\ell\nu q\bar{q}$ lists the uncertainties in m_W use in combining the semileptonic channels.

	Δm_W (MeV/c^2)				$\Delta\Gamma_W$ (MeV)			
Source	$e\nu q\bar{q}$	$\mu\nu q\bar{q}$	$\tau\nu q\bar{q}$	$\ell\nu q\bar{q}$	$e\nu q\bar{q}$	$\mu\nu q\bar{q}$	$\tau\nu q\bar{q}$	$\ell\nu q\bar{q}$
e+μ momentum	3	8	-	4	5	4	-	4
e+μ momentum resoln	7	4	-	4	65	55	-	50
Jet energy scale/linearity	5	5	9	6	4	4	16	6
Jet energy resoln	4	2	8	4	20	18	36	22
Jet angle	5	5	4	5	2	2	3	2
Jet angle resoln								7
Jet boost	17	17	20	17	3	3	3	3
Fragmentation	10	10	15	11	22	23	37	25
Radiative corrections						2	2	2
LEP energy	9	9	10	9	7	7	10	8
Calibration ($e\nu q\bar{q}$ only)	10	-	-	4	20	-	-	9
Ref MC Statistics	3	3	5	2	7	7	10	5
Bkgnd contamination	3	1	6	2	5	4	19	7

total 54(stat)+25(syst) = **60** MeV

lepton and jet uncertainties from (Z) calibration data

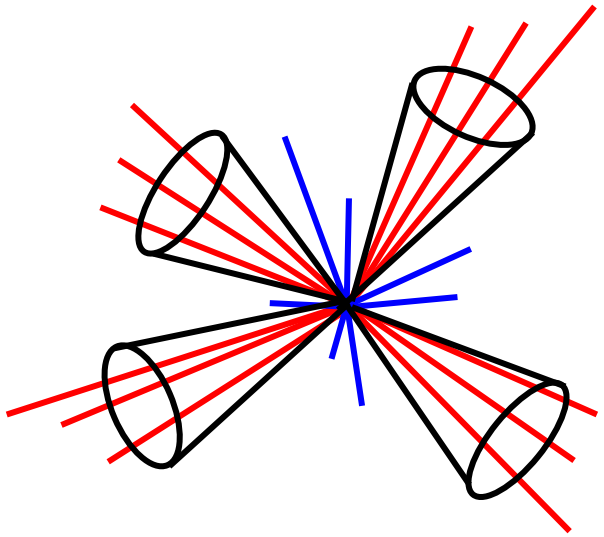


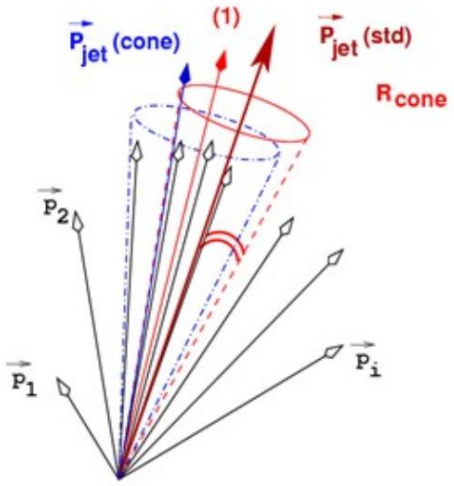
W kinematic fit : systematics in 4q

Table 8: Summary of the systematic errors on m_W and Γ_W averaged over 183-209 GeV in the $q\bar{q}q\bar{q}$ channel for the standard, PCUT (= 3.0 GeV/c) and CONE (R=0.4) reconstructions.

	Δm_W (MeV/c^2)			$\Delta\Gamma_W$ (MeV)		
Source	standard	PCUT	CONE	standard	PCUT	CONE
Jet energy scale/linearity	2	2	3	2	12	4
Jet energy resoln	0	1	0	7	9	10
Jet angle	6	6	6	1	3	3
Jet angle resoln	1	3	2	15	18	9
Jet boost	14	15	11	5	5	4
Fragmentation	10	20	20	20	40	40
Radiative Corrections	2	2	2	5	7	7
LEP energy	9	10	10	7	7	7
Ref MC Statistics	2	3	3	5	7	7
Bkgnd contamination	8	5	5	29	31	32
Colour reconnection	79	28	36	104	24	45
Bose-Einstein effects	6	2	3	20	10	10

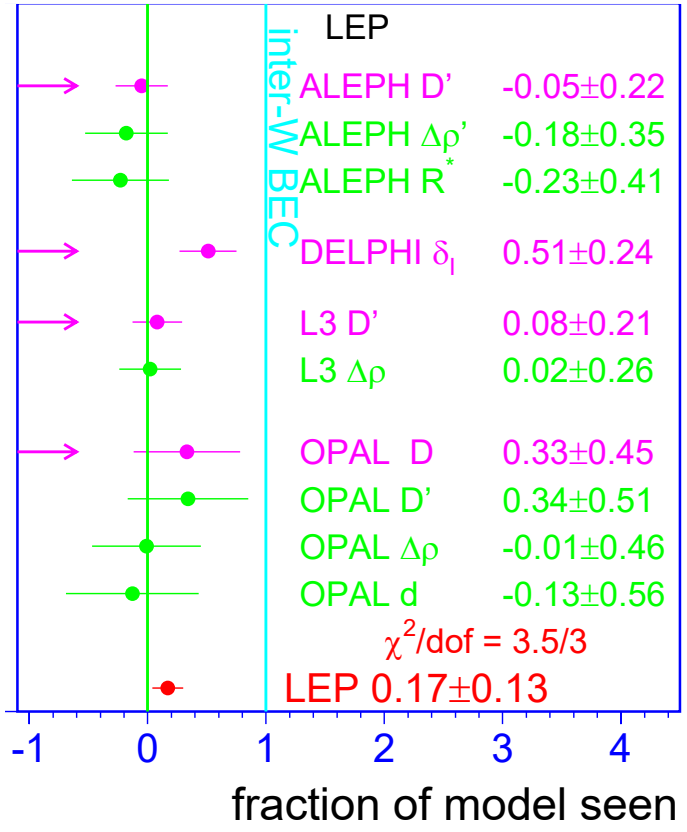
total 70(stat)+28(syst))+28(FSI) = **80** MeV



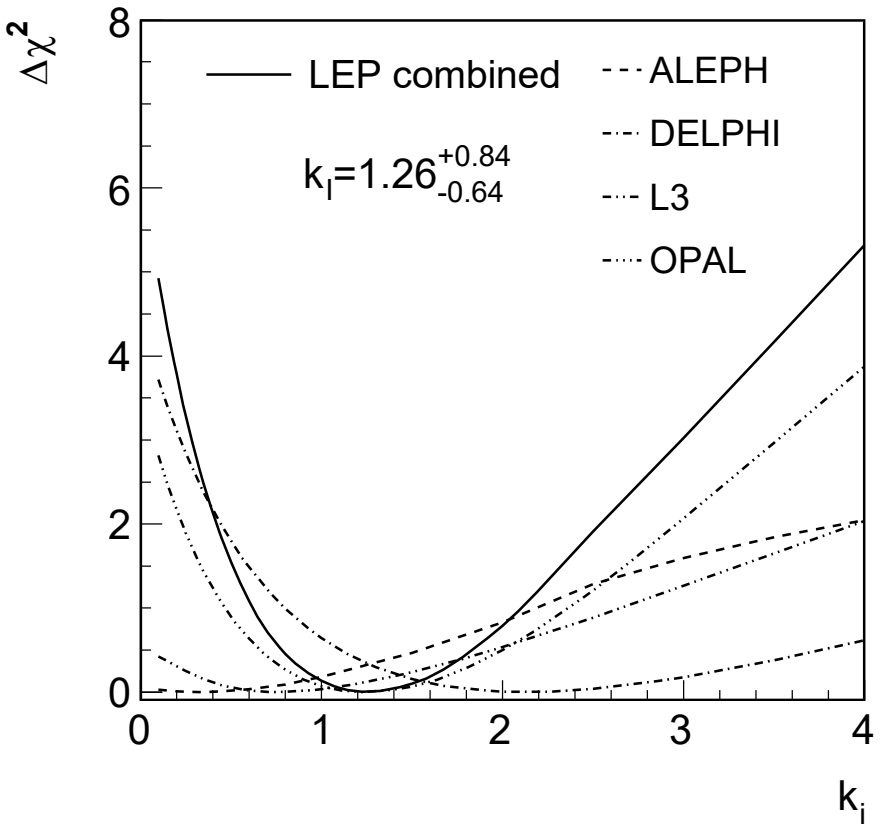


arXiv:1302.3415)

W kinematic fit : FSI systematics in 4q



Measured BEC expressed as the relative fraction of the model with inter-W correlations.



Individual and LEP combined Δχ2 curves for the measurement of the CR parameter kI in the SK1 model.

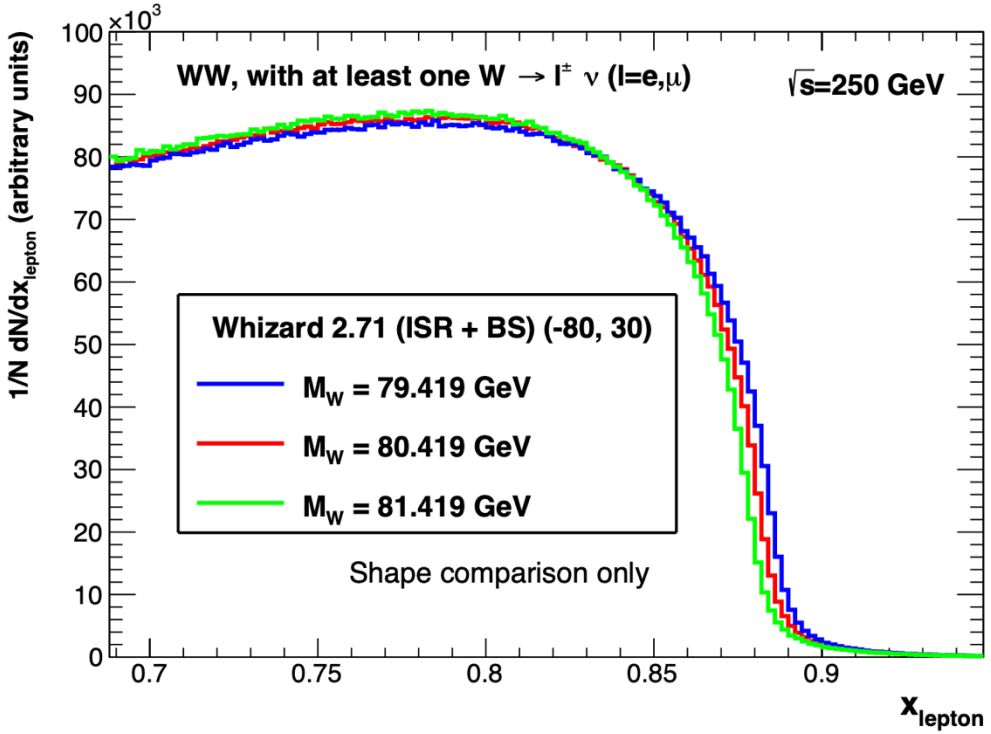
LEP combined results

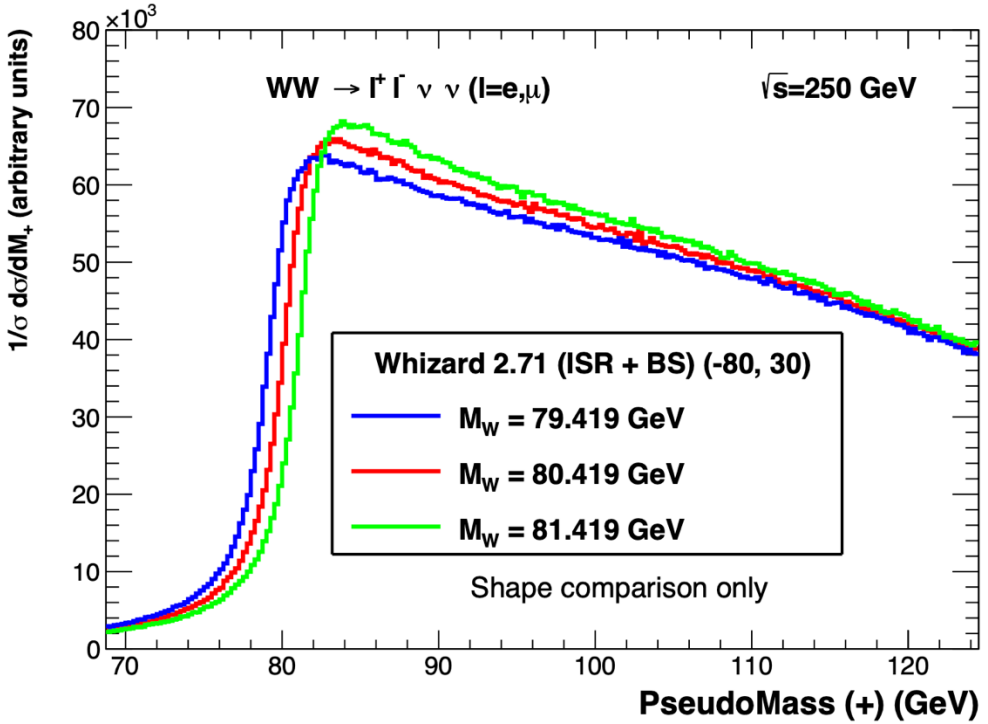
Direct Reconstruction			
Experiment	$W^+W^- \to q\bar{q}\ell\nu_\ell$ m_W[GeV]	$W^+W^- \to q\bar{q}q\bar{q}$ m_W[GeV]	Combined m_W[GeV]
Published			
ALEPH	80.429 ± 0.060	80.475 ± 0.080	80.444 ± 0.051
DELPHI	80.339 ± 0.075	80.311 ± 0.137	80.336 ± 0.067
L3	80.212 ± 0.071	80.325 ± 0.080	80.270 ± 0.055
OPAL	80.449 ± 0.063	80.353 ± 0.083	80.416 ± 0.053
LEP combination			
ALEPH	80.429 ± 0.059	80.477 ± 0.082	80.444 ± 0.051
DELPHI	80.339 ± 0.076	80.310 ± 0.101	80.330 ± 0.064
L3	80.217 ± 0.071	80.324 ± 0.090	80.254 ± 0.058
OPAL	80.449 ± 0.062	80.353 ± 0.081	80.415 ± 0.052

Source	Systematic Uncertainty in MeV			
	on m_W			on Γ_W
	$q\bar{q}\ell\nu_\ell$	$q\bar{q}q\bar{q}$	Combined	
ISR/FSR	8	5	7	6
Hadronisation	13	19	14	40
Detector effects	10	8	9	23
LEP energy	9	9	9	5
Colour reconnection	–	35	8	27
Bose-Einstein Correlations	–	7	2	3
Other	3	10	3	12
Total systematic	21	44	22	55
Statistical	30	40	25	63
Statistical in absence of systematics	30	31	22	48
Total	36	59	34	83

W mass from lepton Energy and Pseudomass

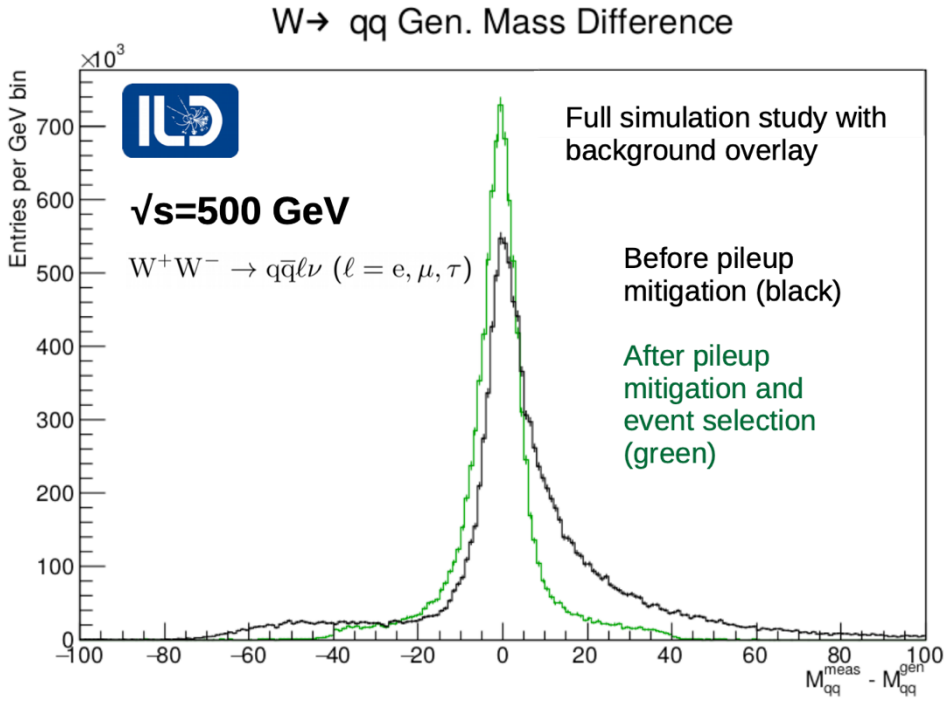
Endpoints in the lepton (or jet) energy a
$E\ell = E_{CM}(1 \pm \beta)$ where β is the W velocity





expected statistical Δm_W =4.4 MeV with 2/ab@250 GeV
experimental syst from lepton energy calibration

W mass from the hadronic mass



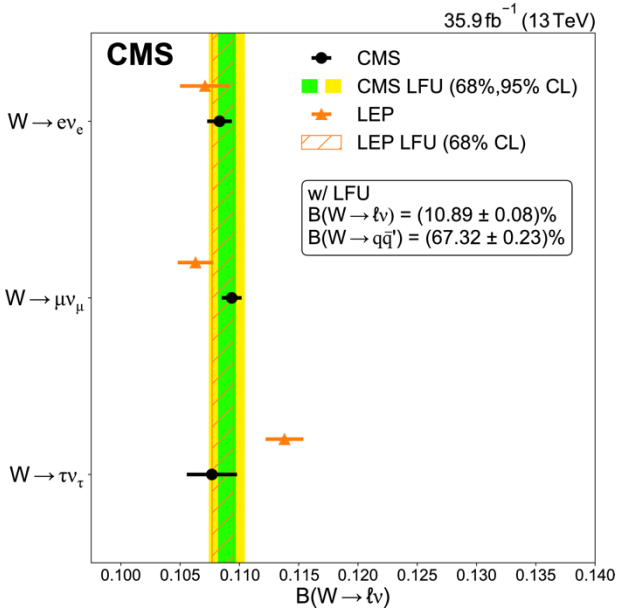
arXiv:2011.12451

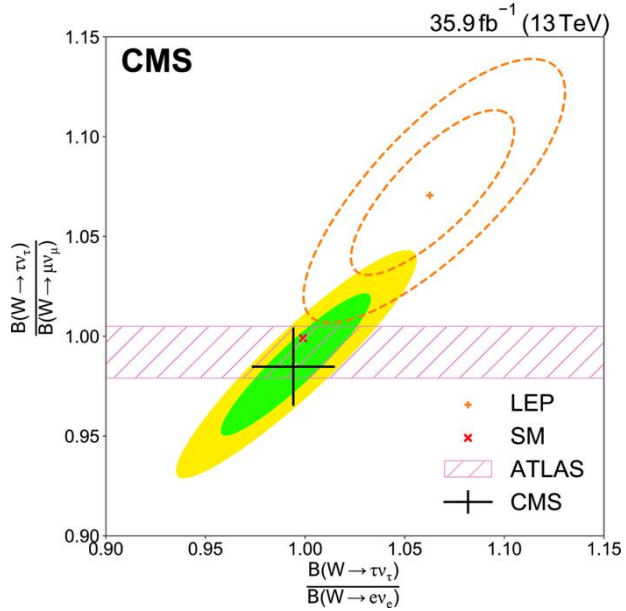
ΔM_W [MeV]	ILC	ILC	ILC	ILC
$\sqrt{s}$ [GeV]	250	350	500	1000
$\mathcal{L}$ [fb^{-1}]	500	350	1000	2000
$P(e^-)$ [%]	80	80	80	80
$P(e^+)$ [%]	30	30	30	30
jet energy scale	3.0	3.0	3.0	3.0
hadronization	1.5	1.5	1.5	1.5
pileup	0.5	0.7	1.0	2.0
total systematics	3.4	3.4	3.5	3.9
statistical	1.5	1.5	1.0	0.5
total	3.7	3.7	3.6	3.9

«.. dominated by the systematic uncertainties from the effective **jet energy scale** which is a challenging demand.. »

W decay branching fractions

LEP2 $W\to\tau\nu$ 2.6σ larger than $e\nu+\mu\nu$ average : not confirmed by more precise recent ATLAS & CMS measurements





	CMS	LEP	ATLAS	LHCb	CDF	D0
$R_{\mu/e}$	1.009 ± 0.009	0.993 ± 0.019	1.003 ± 0.010	0.980 ± 0.012	0.991 ± 0.012	0.886 ± 0.121
$R_{\tau/e}$	0.994 ± 0.021	1.063 ± 0.027	—	—	—	—
$R_{\tau/\mu}$	0.985 ± 0.020	1.070 ± 0.026	0.992 ± 0.013	—	—	—
$R_{\tau/\ell}$	1.002 ± 0.019	1.066 ± 0.025	—	—	—	—

LEP arXiv:1302.3415

	CMS	LEP
$\mathcal{B}(W \to e\bar{\nu}_e)$	$(10.83 \pm 0.01 \pm 0.10)\%$	$(10.71 \pm 0.14 \pm 0.07)\ \%$
$\mathcal{B}(W \to \mu\bar{\nu}_\mu)$	$(10.94 \pm 0.01 \pm 0.08)\%$	$(10.63 \pm 0.13 \pm 0.07)\ \%$
$\mathcal{B}(W \to \tau\bar{\nu}_\tau)$	$(10.77 \pm 0.05 \pm 0.21)\%$	$(11.38 \pm 0.17 \pm 0.11)\ \%$
$\mathcal{B}(W \to q\bar{q}')$	$(67.46 \pm 0.04 \pm 0.28)\%$	—
Assuming LFU		
$\mathcal{B}(W \to \ell\bar{\nu})$	$(10.89 \pm 0.01 \pm 0.08)\%$	$(10.86 \pm 0.06 \pm 0.09)\%$
$\mathcal{B}(W \to q\bar{q}')$	$(67.32 \pm 0.02 \pm 0.23)\%$	$(67.41 \pm 0.18 \pm 0.20)\%$

CMS (all WBRs) arXiv:2201.07861

ATLAS (τ/μ) arXiv:2007.14040 + (τ/e) arXiv:2412.11989

W BR relative precisions

eν(1%) $\mu\nu$ (1%) $\tau\nu$(2%) qq(0.3%)

$\alpha_S(m_W^2)$	$\lvert V_{cs}\rvert$	$\sum_{ij}\lvert V_{ij}\rvert^2$
0.095 ± 0.033	0.967 ± 0.011	1.984 ± 0.021 height

BR(W→qq') derived determinations

W decay branching fractions

FCCee: 12/ab@162 + 5/ab@240 ~ 120M WW decays

ILC : 2/ab @250 GeV, 1/ab with enhanced WW (~40M)

arXiv:2506.15390

Table 3.1: Relative precision on the determination of the W decay branching ratios. Final combined results with LEP2 data are compared to the projected precision obtainable with FCC-ee.

Decay mode relative precision	$B(\mathrm{W} \to e\nu)$	$B(\mathrm{W} \to \mu\nu)$	$B(\mathrm{W} \to \tau\nu)$	$B(\mathrm{W} \to qq)$
LEP2	1.5%	1.4%	1.8%	0.4%
FCC-ee	$3{\cdot}10^{-4}$	$3{\cdot}10^{-4}$	$4{\cdot}10^{-4}$	$1{\cdot}10^{-4}$

Scaling major syst uncertainties with luminosity (as data stat)

Lept universality tests at *3-4 10^{-4} level*

q/ l universality test *10^{-4} level*

$$R_W = \frac{B_q}{1 - B_q} = \left(1 + \frac{\alpha_S(m_W^2)}{\pi}\right) \sum_{i=u,c;j=d,s,b} |V_{ij}|^2.$$

$\Delta R_W^{\ell} = 9\,\Delta B_q \sim 10^{-3}$

stringent test of CKM unitarity in the first two rows

alternatively

➔ $\Delta\alpha_S\,(m_W) \approx (9\,\pi/2)\Delta B_q \approx 10^{-3}$

arXiv:1908.11299

Event selections	B_e	B_μ	B_τ	R_μ	R_τ
All 10	4.2	4.1	5.2	6.1	7.5
9 (not fully-hadronic)	5.9	5.7	6.4	6.1	7.5
9 (not tau-semileptonic)	4.6	4.6	7.8	6.1	10.8
8 (not f-h and not τ-semileptonic)	8.3	8.4	7.8	6.1	12.8
7 (not f-h and not τ-sl and not di-τ)	9.0	9.1	10.6	6.1	16.7

Table 4: Statistical uncertainties, expressed as relative errors in units of 10^{-4} for the leptonic branching fractions of the W boson (B_e, B_μ and B_τ) and the ratios of branching fractions $R_\mu = B_\mu/B_e$, $R_\tau = B_\tau/B_e$. The lines of the table refer to different choices of the included event selections. The values assume ILC measurements at $\sqrt{s} = 250$ GeV using the 45% of the 2 ab^{-1} integrated luminosity with enhanced $e_L^- e_R^+$ collisions, with the same efficiencies and the same background cross sections as in the OPAL measurement [44]. The uncertainties given for R_μ, R_τ are from a separate fit using the (B_e, R_μ and R_τ) parametrization.

W decay branching fractions : systematics

Table 8. W branching ratio systematic error breakdown in units of 10^{-4}

Source	uncertainty			
	$e\nu$	$\mu\nu$	$\tau\nu$	qq
Tracking	2.6	3.8	3.9	3.2
Simulation of calorimeters	1.6	2.4	2.0	5.9
Hadronization models	2.8	2.8	0.7	0.4
Z peak $q\bar{q}$ fragmentation	0.5	0.5	0.5	1.6
Inter-W final state interaction	1.9	1.9	1.9	5.7
Background contamination	1.9	2.4	2.6	6.4
Lepton identification	7.7	5.5	11.2	0.5
Beam-related background	2.5	1.4	15.4	10.2
$\mathcal{O}(\alpha)$ corrections DPA	0.4	1.1	3.1	1.7
Luminosity	0.8	0.5	0.3	1.5
Simulation statistics	1.0	0.7	1.4	1.4
Total	9.6	8.5	20.2	15.2

ALEPH Eur.Phys.J.C 38 (2004) 147

For lept BR will need excellent control of **lepton id** and cross contaminations in signal channels (τ→e,μ and e,μ channels)

Less stringent requirements for syst uncertainty control for hadr BR

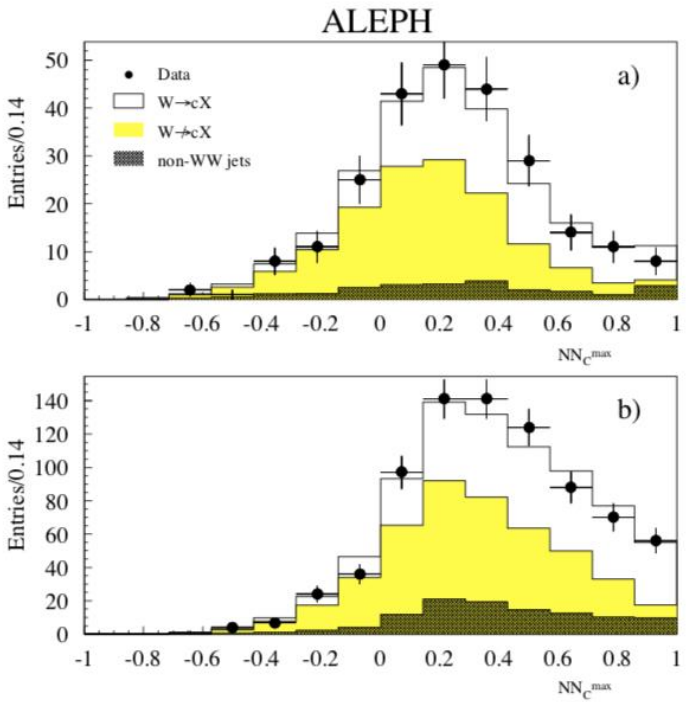
Hadronic W flavor tagging : cX & cs

Charm (+strange) tagging. Done with LEP2 data. Stat extrapolations to (FCCee) 120M WW datasets

ALEPH *Phys. Lett. B* 465 (1999) 349

OPAL *Phys. Lett. B* 490 (2000) 71

ΔR_c^W (stat) FCCee → 2 10^{-4}



$$R_c^W = \frac{|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2}{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 + |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2}$$

Table 2: Systematic errors on R_c^W.

Source	ΔR_c^W (10^{-2})
Background normalization	0.2
Hadronization	2.9
Color reconnection	0.3
Calorimeter calibration	0.9
Tracking error	0.3
Impact parameter resolution	0.4
Mass of the W boson	0.4
Jet algorithm	0.4
Charm production	0.1
Charm fragmentation	0.3
Charm decay properties	0.9
Total Error	3.3

DELPHI *Phys. Lett. B* 439 (1998) 209

Δ$|V_{cs}|$ (stat) FCCee → 3 10^{-4}

Source of Systematic Error	ΔR_c^W 183 GeV	189 GeV
Hadronisation Model	0.011	0.012
Centre–of–mass Energy	0.007	0.005
Mass of the W Boson	0.004	0.003
Charm Fragmentation Function	0.007	0.007
Background Cross-Section	0.006	0.005
Background Composition	0.010	0.009
Charm Hadron Fractions	0.006	0.007
Light Quark Composition	0.007	0.005
Vertex Reconstruction	0.016	0.017
Charm Hadron Lifetimes	0.003	0.002
Charm Decay Multiplicity	0.010	0.010
Lepton Identification	0.012	0.014
Lepton Energy Spectrum	0.003	0.003
Branching ratio Br($c \to \ell$)	0.006	0.006
Total systematic error	0.032	0.032
Statistical error	0.090	0.047
Value of R_c^W	0.493	0.478

challenging syst effects

arXiv:2506.15390 0.05% for |Vcs| including systematics

Hadronic W flavor tagging : bX, bc, bu

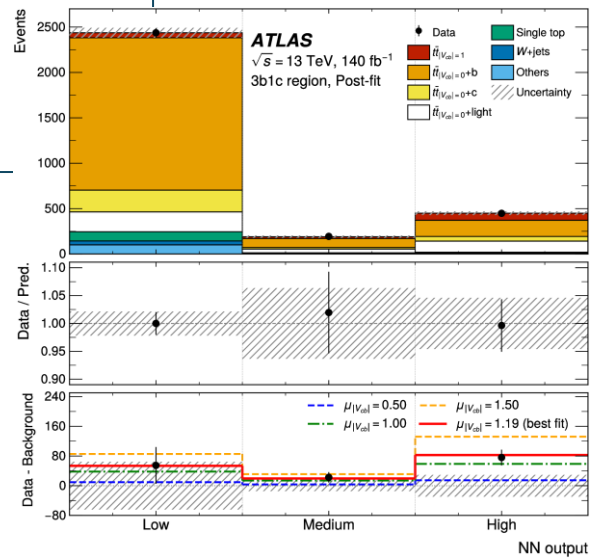
Very small B(W →bX). Not done with LEP data.

$|V_{cb}| = (41.0 \pm 1.4) \times 10^{-3}$ ➔ BR = 5.6 10^{-4} (1.7 10^5 W→cb @FCCee)
$|V_{ub}| = (3.82 \pm 0.24) \times 10^{-3}$ ➔ BR = 4.9 10^{-6} (1.5 10^3 W→ub @FCCee)

With $\boldsymbol{\varepsilon_b}$**=40% and** $\boldsymbol{\varepsilon_c}$**=10^{-3}** @FCCee: N(W→cs)≈100k N(W→bX)≈75k
$\boldsymbol{\Delta}R_b^{\,W}$ (stat)(FCCee) → 0.6% (rel) & $\boldsymbol{\Delta}$ $|V_{cb}|$ (stat) (FCCee) → 0.3% (rel)

First tag $\boldsymbol{\varepsilon_b}$**=40% and** $\boldsymbol{\varepsilon_c}$**=10^{-3}** $\boldsymbol{\varepsilon_{uds}}$**=10^{-5}** Second tag with $\boldsymbol{\varepsilon_c}$**=60% and** $\boldsymbol{\varepsilon_b}$**=0.1** $\boldsymbol{\varepsilon_{uds}}$**=0.2**
@FCCee: N(W→cs,cd)≈20k N(W→cb)≈50k ➔ $\boldsymbol{\Delta}$ $|V_{cb}|$ (stat) (FCCee) → 0.2%(rel)

Inverting second tag could lead to obtain $\boldsymbol{\Delta}$ $|V_{ub}|$ (stat) (FCCee) → ~3-5%(rel)
(less interesting : ~1% from LHCb/Belle2/ FCCee Z)

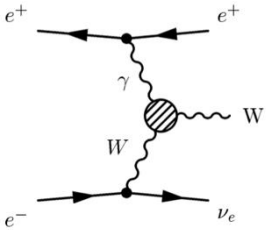


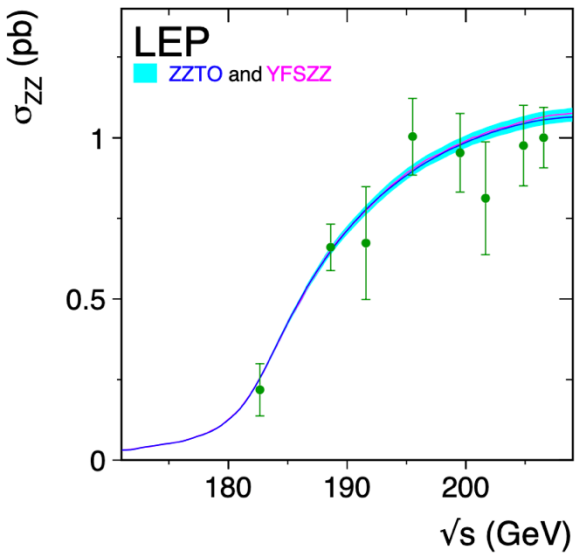
arXiv:2506.15390 0.15% estimate for |Vcb| including systematics

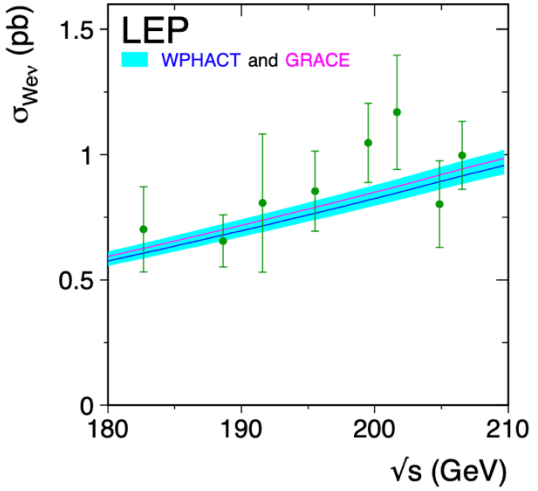
ATLAS arXiv:2603.16414 $|Vcb|=(50+11-14)\times10^{-3}$

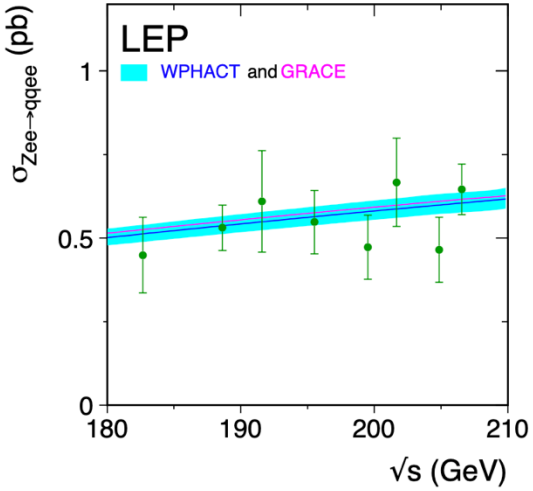
Single W and other 4f processes

Channel relative precision	WW	ZZ	Weν	Zee
LEP2 183-209 GeV	0.8%	5%	8%	6%
FCCee 162 GeV	$2{\cdot}10^{-4}$		$5{\cdot}10^{-4}$	$5{\cdot}10^{-4}$
FCCee 240 GeV	$1{\cdot}10^{-4}$	$5{\cdot}10^{-4}$	$5{\cdot}10^{-4}$	$1{\cdot}10^{-3}$
FCCee 365 GeV	$3{\cdot}10^{-4}$	$1{\cdot}10^{-3}$	$5{\cdot}10^{-4}$	$1{\cdot}10^{-3}$





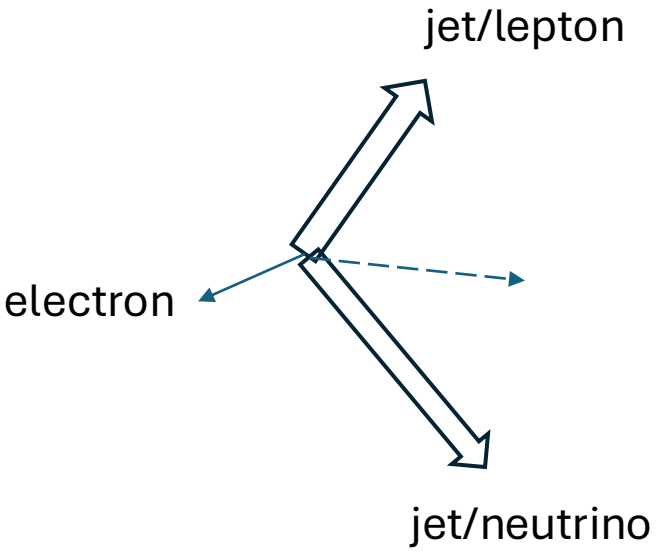




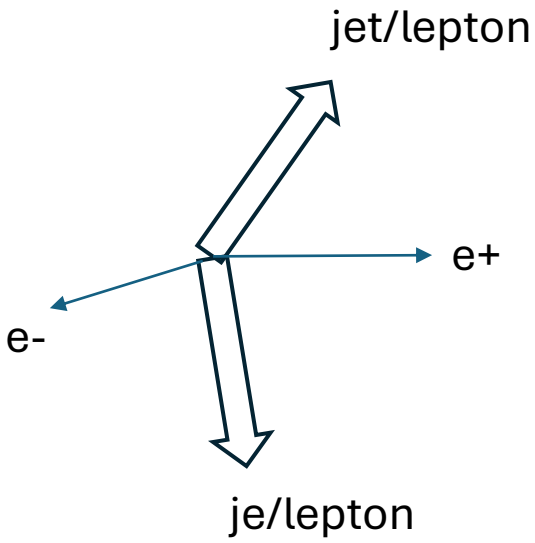
sizeable cross sections down to the Z pole

Single W (and Z) w & w/o eeH

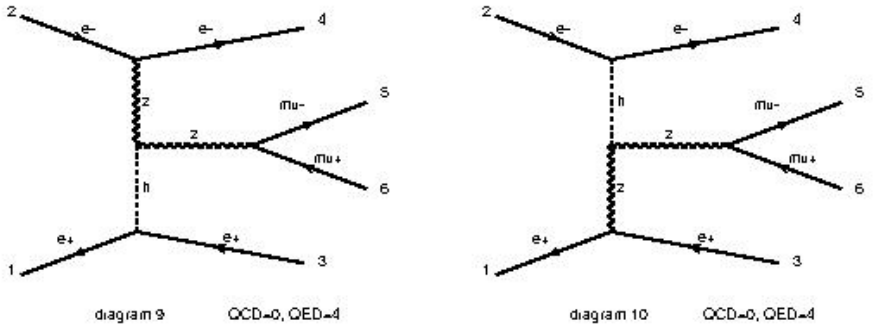
$e^+ e^- \to W e \nu$



$e^+ e^- \to Z e e$



negative interference effects



$$\Delta\sigma\big(Zee(91.2\ GeV)\big) \approx -0.1 - 0.5\ ab$$

$$\Delta\sigma\big(Zee(162\ GeV)\big) \approx -0.1 - 0.5\ fb$$

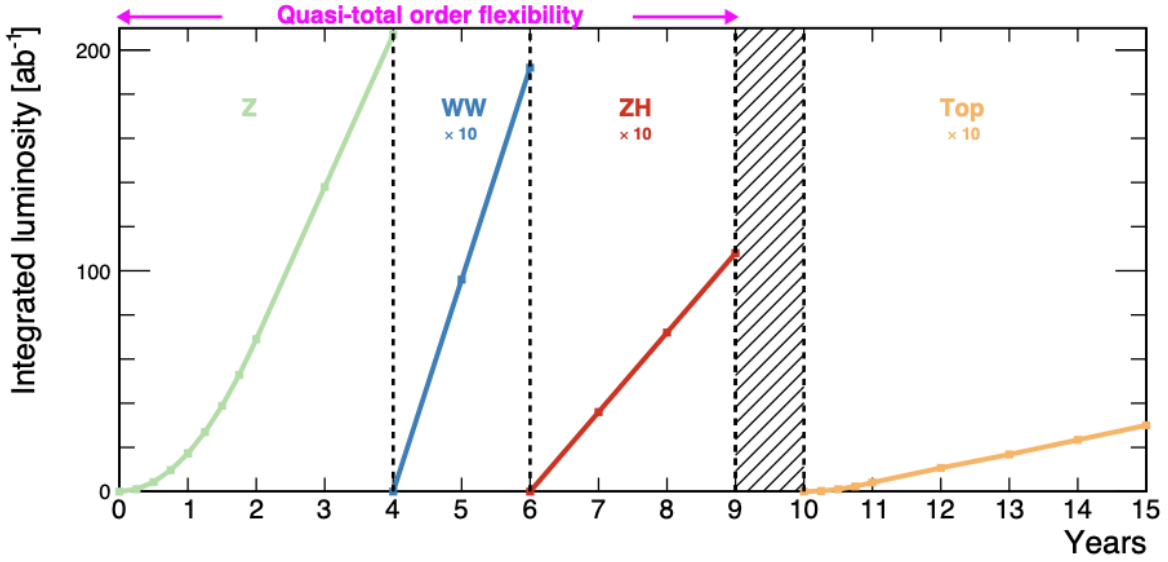
0.1-0.2% effects on total sigmas using rather hard acceptance cuts could be optimized better ?

That's all Folks!

Backup

FCC-ee (4 IPs) arXiv:2505.00272

- 91 GeV 50/ab/y x4y ==200/ab
- 157-162 GeV 9.6/ab/y x2y == 19.2/ab
- 240 GeV 3.6/ab/y x3y == 10.8/ab
- 365 GeV 0.67/ab/y x5y == 3.35/ab



CEPC (1IP) arXiv:2510.05260

- 91 GeV ==13/ab
- 155-170 GeV == 1.2/ab
- 240 GeV == 10/ab

Operation mode	$\sqrt{s}$ (GeV)	SR power (MW)	$\mathcal{L}$ (10^{34} cm^{-2}s^{-1})	$\int \mathcal{L}$/year (ab^{-1})	Years	Total $\int \mathcal{L}$ (ab^{-1})	Event yields
H	240	30	5	0.65	15	10	2.0×10^6
Z	91	12.1	26(*)	3.2	4	13	5.6×10^{11}
W^+W^-	155-170	30	16	1.2	1	1.2	1.0×10^7(†)

LEP3 (2 IPs) arXiv:2504.00541

- 91 GeV 50/ab
- 162 GeV 5.6/ab
- 230 GeV 2.5/ab

	LEP3		
No. of IPs/Xpts	2		
com energy √s (GeV). For final state(X)	230 (ZH)	160 (WW)	91.2 (Z)
Circumference/Length [km]	26.659		
Bending Radius (m)	2958		
Crossing Angle at IP [mrad]	30		
SR Energy Loss/turn [GeV]	5.4	1.3	0.13
Total RF Voltage [GV]	6	1.5	0.5
SR Power /beam (MW)	50		
Beam Current [mA]	9	39	371
Number of bunches/beam	20	220	800
Bunch Intensity (10E11)	2.5	1	2.6
RF frequency. [MHz]	800		
Beam Lifetime (Bhabha+Brem) [min]	17	19	28
Inst. Luminosity/IP [10^34 cm-2s-1]	1.6	6.2	40
Integrated L/IP [ab-1/yr]	0.192	0.7	4.8
Effective Years of Operation	6	4	5.5

LCF arXiv:2503.19983 arXiv:2503.24049

- 91 GeV 0.1/ab
- 250 GeV 3/ab
- 350 GeV 0.4/ab
- 550 GeV 8/ab

	91 GeV	250 GeV	350 GeV	550 GeV	1-3 TeV
$\int \mathscr{L}$ (ab^{-1})	0.1	3	0.4	8	8
beam polarisation (e^-,e^+; %)	80/30	80/30	80/30	80/60	80/20
(− −, −+, +−, ++) (%)	(10,40,40,10)	(5,45,45,5)	(5,68,22,5)	(10,40,40,10)	(10,40,40,10)