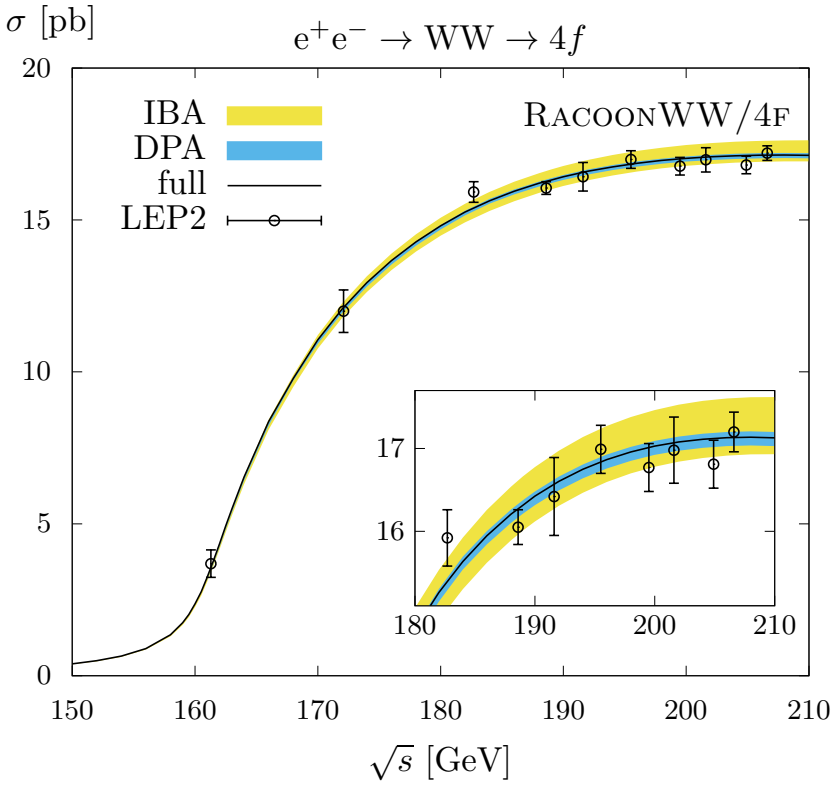

Electroweak Corrections and the Double-Pole-Approximation (+ beyond)

Stefan Dittmaier

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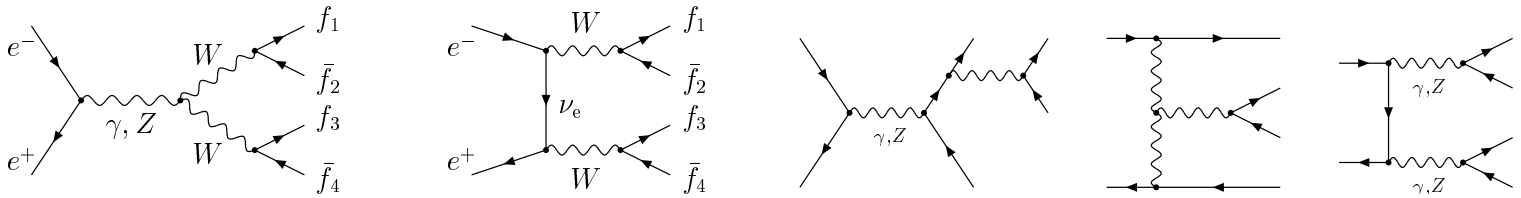
$e^+e^- \to WW \to 4f$ in double-pole approximation (DPA)

Structure of Monte Carlo generators with EW corrections used at LEP2:

RacoonWW (Denner, Dittmaier,Roth,Wackeroth) and

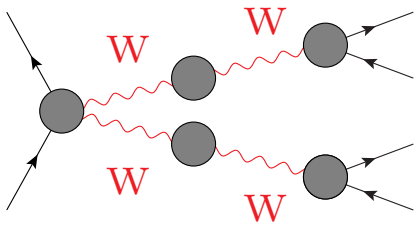
KoralW $\oplus$ *YFSWW* (Jadach,Płaczek,Skrzypek,Ward) include

- full lowest-order matrix elements for $\mathrm{e^+e^-} \to 4f(+\gamma)$



signal diagrams background diagrams etc.

- non-universal electroweak corrections DPA



leading term in expansion about W resonances

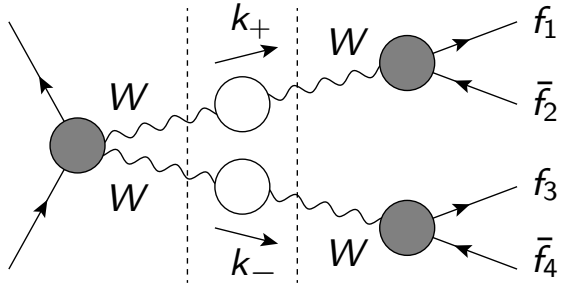
$\hookrightarrow$ contributions:

- corrections to $\mathrm{ee} \to \mathrm{WW}$ Böhm et al. '88; Fleischer et al. '89
- corrections to $\mathrm{W} \to f\bar{f}'$ Bardin et al. '86; Jegerlehner '86; Denner, Sack '90
- non-factorizable photonic corrections Melnikov, Yakovlev '96; Beenakker et al. '97; Denner et al. '97

- improvements by leading higher-order corrections

Virtual corrections in DPA:

- Factorizable corrections:



on-shell production on-shell decays

$$\mathcal{M}^{\mathrm{e^+e^-\to WW\to 4}f}_{\mathrm{virt,fact,DPA}} = \frac{R(M_\mathrm{W}^2, M_\mathrm{W}^2)}{(k_+^2 - M_\mathrm{W}^2 + \mathrm{i}M_\mathrm{W}\Gamma_\mathrm{W})(k_-^2 - M_\mathrm{W}^2 + \mathrm{i}M_\mathrm{W}\Gamma_\mathrm{W})}$$

with the gauge-independent residue

$$\begin{aligned} R(M_\mathrm{W}^2, M_\mathrm{W}^2) = \sum_{\text{W-pols}} \Big(& \delta\mathcal{M}^{\mathrm{e^+e^-\to W^+W^-}}\, \mathcal{M}^{\mathrm{W^+\to}f_1\bar{f}_2}_{\mathrm{Born}}\, \mathcal{M}^{\mathrm{W^-\to}f_3\bar{f}_4}_{\mathrm{Born}} \\ & + \mathcal{M}^{\mathrm{e^+e^-\to W^+W^-}}_{\mathrm{Born}}\, \delta\mathcal{M}^{\mathrm{W^+\to}f_1\bar{f}_2}\, \mathcal{M}^{\mathrm{W^-\to}f_3\bar{f}_4}_{\mathrm{Born}} \\ & + \mathcal{M}^{\mathrm{e^+e^-\to W^+W^-}}_{\mathrm{Born}}\, \mathcal{M}^{\mathrm{W^+\to}f_1\bar{f}_2}_{\mathrm{Born}}\, \delta\mathcal{M}^{\mathrm{W^-\to}f_3\bar{f}_4} \Big) \end{aligned}$$

containing the corrections to on-shell production and decay

- Non-factorizable corrections:

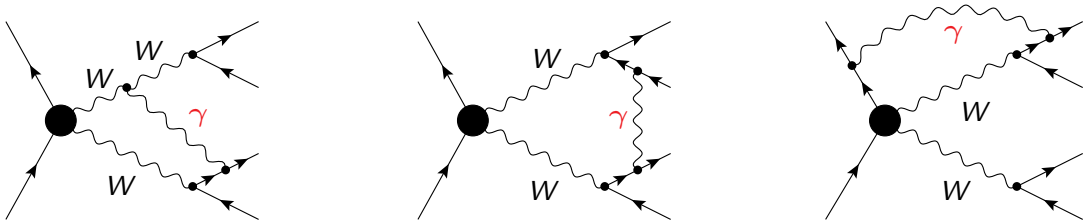
$$\mathcal{M}^{e^+e^-\to WW\to 4f}_{\text{virt,nonfact,DPA}} = \delta\mathcal{M}^{e^+e^-\to 4f}\big|_{\text{doubly-resonant part}} - \mathcal{M}^{e^+e^-\to WW\to 4f}_{\text{virt,fact,DPA}}$$

$$= \mathcal{M}^{e^+e^-\to WW\to 4f}_{\text{Born,DPA}}\ \delta_{\text{virt,nonfact,DPA}}$$

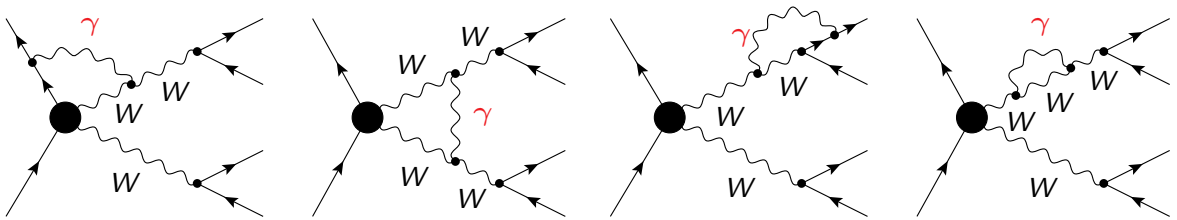
Features of $\delta_{\text{virt,nonfact,DPA}}$ analogous to single-resonance case:

- gauge independent, no mass singularities
- compensates IR singularities of W bosons in $\mathcal{M}^{e^+e^-\to WW\to 4f}_{\text{virt,fact,DPA}}$
- no factorization of Breit–Wigner-type resonances (complicated dependence on off-shellness $k^2_\pm$ of W bosons)

Manifestly non-factorizable diagrams:

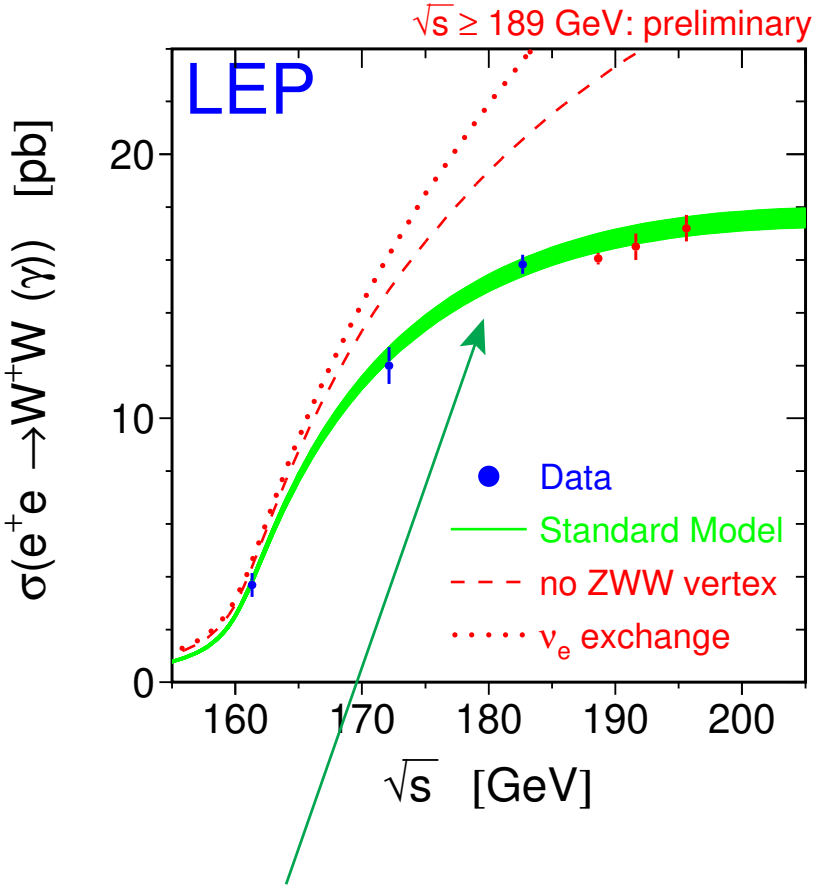


Diagrams contributing to factorizable and non-factorizable RCs:



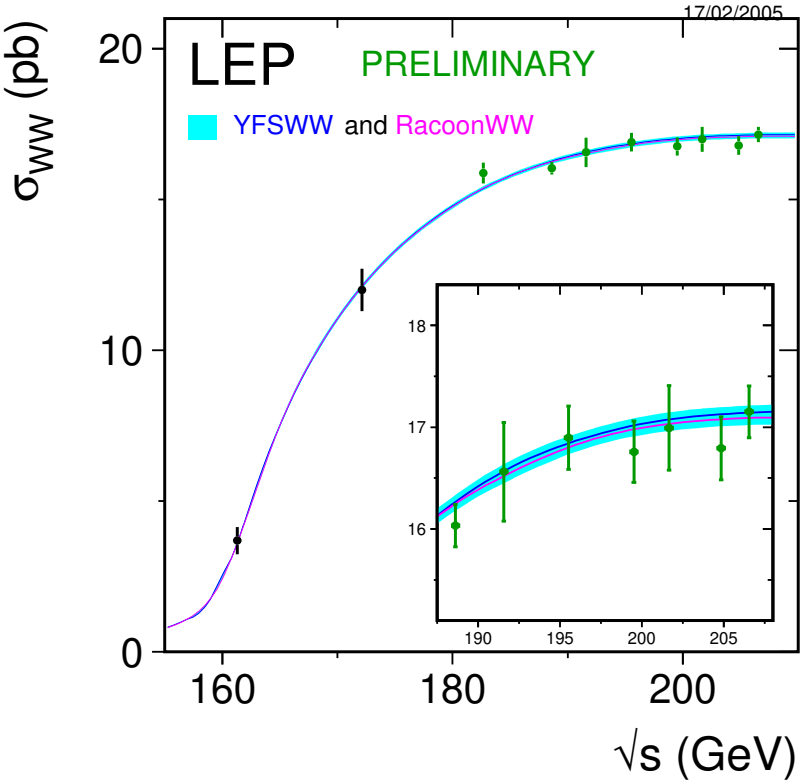
Total WW cross section at LEP2

Status of 1999: (LEPEWWG '99)



GENTLE (Bardin et al.)
only universal EW corrections
↪ theoretical uncertainty $\sim \pm 2\%$

Final result: (LEPEWWG '05)



YFSWW (Jadach et al.) / RacoonWW (Denner et al.)
non-universal corrections included
↪ th. uncertainty $\sim \pm 0.5\%$ for $\sqrt{s} > 170\,\mathrm{GeV}$

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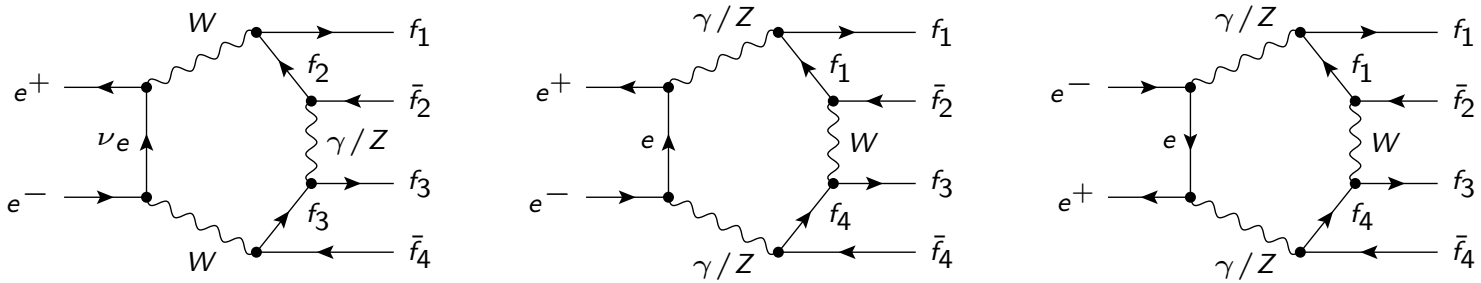
Full NLO corrections to (charged-current) $e^+e^- \to 4f$

Denner et al. hep-ph/0502063; hep-ph/0505042

Features of the calculation:

- # 1-loop diagrams $\sim$ 1200, loops up to 6-point integrals
- complex-mass scheme introduced for W resonances
- loop integrals with complex W/Z masses, new tensor reduction methods
  $\hookrightarrow$ triggered development of library COLLIER (Denner et al. '16)
- nowadays $2 \to 4$ NLO calculations doable with one-loop generators
  MADGRAPH (Frederix et al. '18), OPENLOOPS (Buccioni et al. '19),
  RECOLA (Actis et al. '12,'16), ...
- most complicated diagrams:



Complex-mass scheme at NLO

Denner et al. hep-ph/0505042; Denner, S.D. 1912.06823
(see also Frederix et al., 1804.10017)

mass^2 = location of propagator pole in complex p^2 plane

↪ complex mass renormalization:

$$\underbrace{M^2_{\mathrm{W},0}}_{\text{bare mass}} = \mu^2_{\mathrm{W}} + \underbrace{\delta\mu^2_{\mathrm{W}}}_{\text{ren. constant}}, \quad \text{etc.}$$

Gauge invariance by complex weak mixing angle:

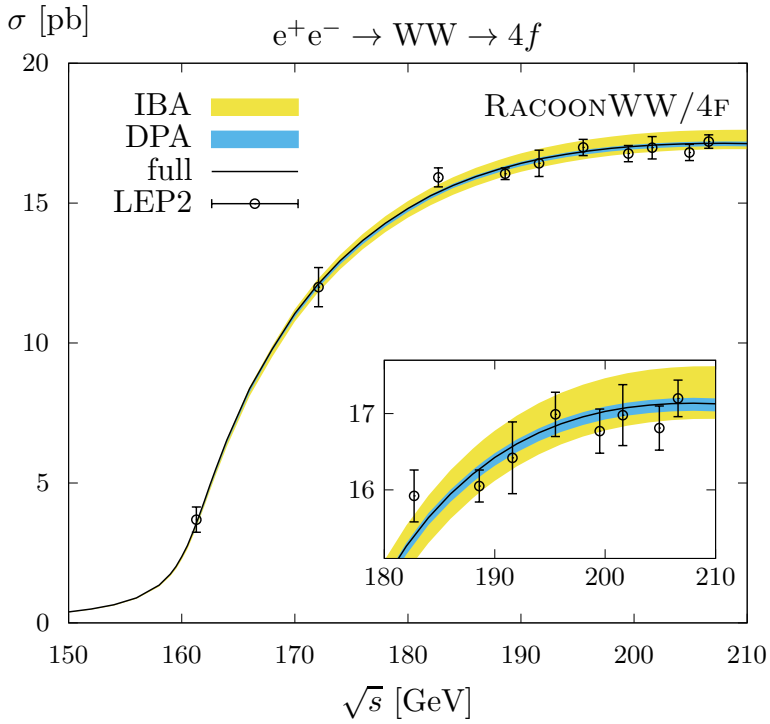
$$c_{\mathrm{W}} = \frac{\mu_{\mathrm{W}}}{\mu_{\mathrm{Z}}}, \qquad \frac{\delta c^2_{\mathrm{W}}}{c^2_{\mathrm{W}}} = \frac{\delta\mu^2_{\mathrm{W}}}{\mu^2_{\mathrm{W}}} - \frac{\delta\mu^2_{\mathrm{Z}}}{\mu^2_{\mathrm{Z}}}$$

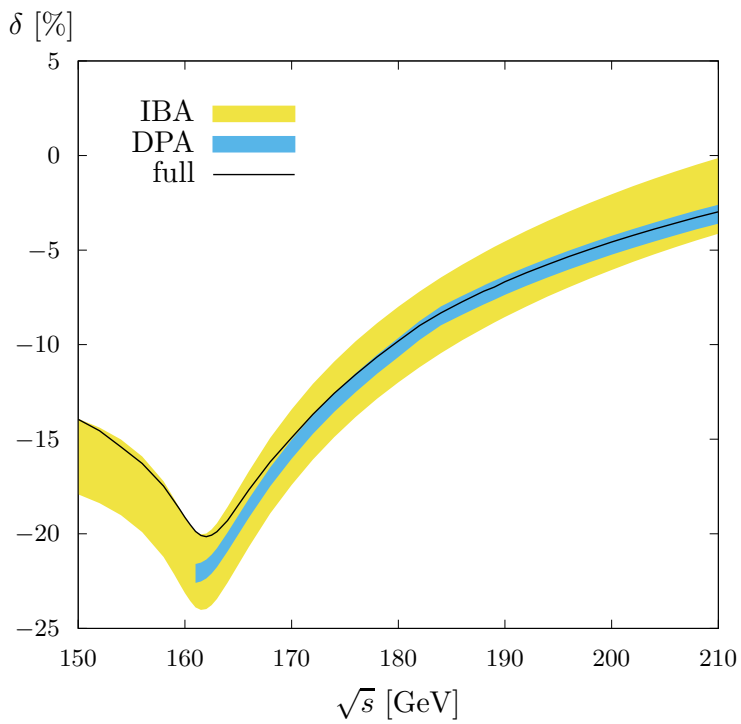
Features of the complex-mass scheme:

- ⊕ perturbative calculations as usual (with complex masses and couplings)
- ⊕ no double counting of contributions (bare Lagrangian unchanged!)
- ⊕ gauge invariance (ST identities, gauge-parameter independence)
- ⊕ NLO accuracy everywhere in phase space
- ▶ spurious terms are beyond NLO, but spoil unitarity
- ▶ complex gauge-boson masses also in loop integrals (all known)
- ⊖ unstable particles only allowed as resonances (not as external states)
- ⊖ generalization to NNLO not yet known (but expected to work)

State-of-the-art prediction of σ_{WW} in LEP2 energy range

Denner, S.D., 1912.06823





- IBA = based on leading-log ISR and universal EW corrections ($\Delta \sim 2\%$)
  $\hookrightarrow$ shows large ISR impact near threshold (also by GENTLE)
- DPA = "Double-Pole Approximation" (leading term of resonance expansion)
  $\hookrightarrow$ $\Delta \sim 0.5\%$ above threshold, not applicable at threshold RacoonWW, YFSWW
- "full" = full NLO prediction for $e^+e^- \to 4f$ via charged current Denner et al. '05
  + leading-log improvements for ISR beyond NLO
  $\hookrightarrow$ $\Delta \sim 0.5\%$ everywhere

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W-boson mass measurements vs. prediction from μ decay

LC: LC Vision Coll., 2503.19983 FCC-ee: Freitas et al., 1906.05379

	experimental accuracy				theory uncertainty				
		σ_{WW} @ threshold			intrinsic			parametric	
	current	LEP2	LC	FCC-ee	current	source	prospect	prospect	source
ΔM_W[MeV]	10	200	2	0.5−1	3−4	$\alpha^3, \alpha^2\alpha_s$	1	1(0.6)	$\Delta\alpha_{had}$

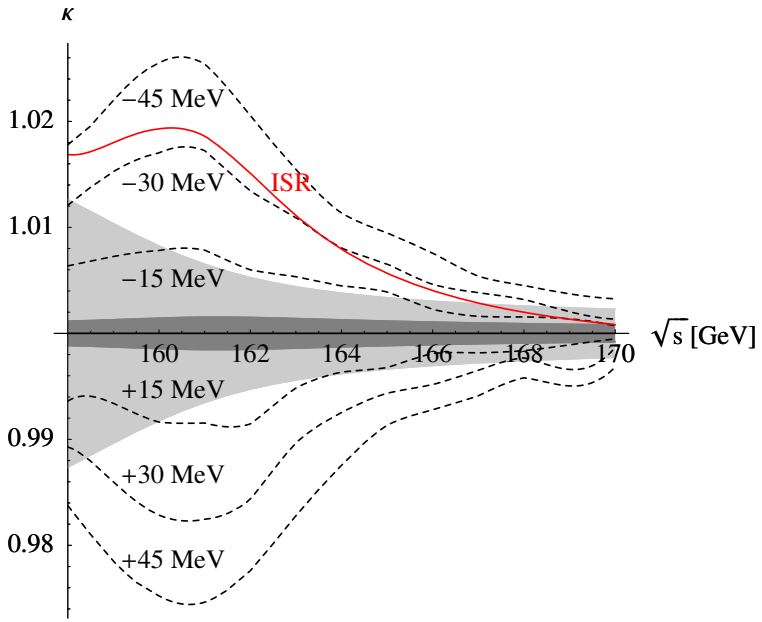
complicated reconstructions

basically counting experiments

M_W calculated from μ decay

Sensitivity of σ_{WW} to M_W:

Beneke et al. '07



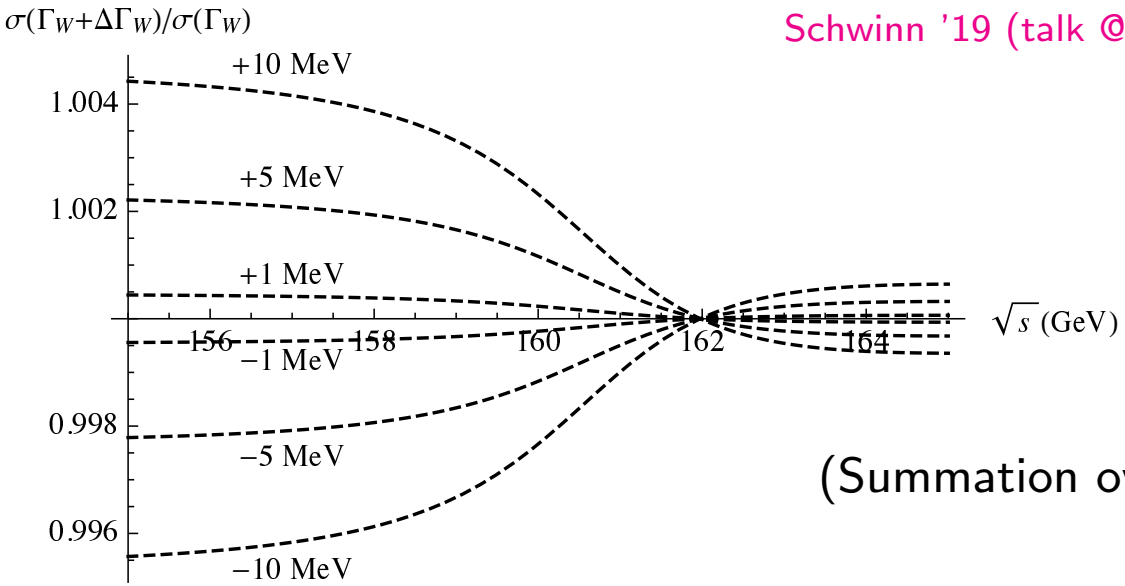
$$\kappa = \frac{\sigma_{WW}(s, M_W + \delta M_W)}{\sigma_{WW}(s, M_W)}$$

$\Delta\kappa = 0.1\%\,(0.02\%) \leftrightarrow \delta M_W = 1.5\,(0.3)\,\mathrm{MeV}$
for $\sqrt{s} = 161\,\mathrm{GeV}$

$\Rightarrow$ FCC-ee requires $\Delta_{TH} \sim 0.02\%$ in σ_{WW}

Shaded areas / ISR curve:
some uncertainties of NLO(EFT) calculation,
improveable via full NLO(ee→4f) and NNLO(EFT)

Γ_W determination from energy scan @ WW threshold:



Schwinn '19 (talk @ FCC-ee Workshop, CERN)

(Summation over all $4f$ states)

Simultaneous fit of M_W and Γ_W by scan of σ_{WW}:

- FCC-ee study: 1703.01626
  2-point fit ($15\,\mathrm{ab}^{-1}$): $\Delta M_W = 0.41\,\mathrm{MeV},\ \Delta\Gamma_W = 1.1\,\mathrm{MeV}$
- CEPC study: 1812.09855
  3-point fit ($2.6\,\mathrm{ab}^{-1}$): $\Delta M_W = 1\,\mathrm{MeV},\ \Delta\Gamma_W = 2.8\,\mathrm{MeV}$

Improvements for $\sigma_{\rm WW}$ @ threshold via EFT

Beneke et al. '07; Actis et al. '08

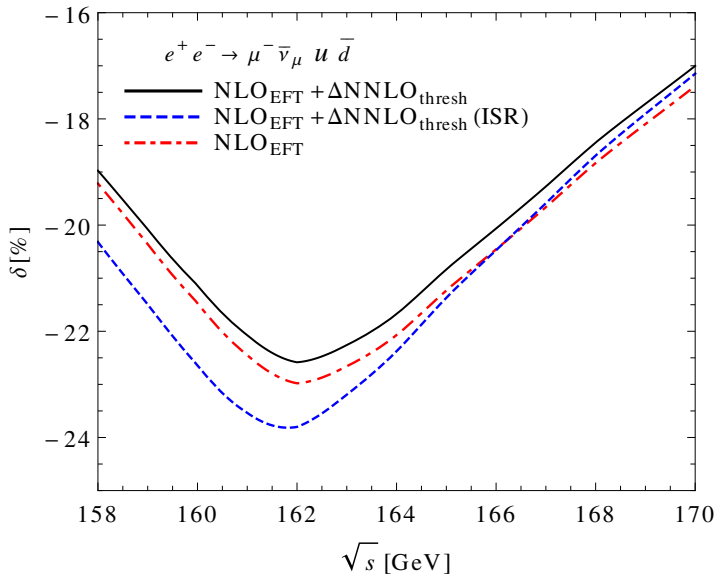
EFT provides expansion of $\sigma_{\rm WW}$ for $\beta = \sqrt{1-4M_{\rm W}^2/s} \sim \sqrt{\Gamma_{\rm W}/M_{\rm W}} \sim \sqrt{\alpha}$:

$$\sigma_{\rm WW} = C\alpha^2\beta\Big[\, 1 + c^{(0)}\beta \qquad \text{LO}$$

$$+ \alpha\left(\frac{c_1^{(1)}}{\beta} + c_2^{(1)}\ln\beta L_{\rm e} + c_3^{(1)} L_{\rm e} + c_4^{(1)} + c_5^{(1)}\beta\right) \qquad \text{NLO}$$

$$+ \alpha^2\underbrace{\left(\frac{c_1^{(2)}}{\beta^2} + \frac{c_2^{(2)}}{\beta} + c_3^{(2)}\ln^2\beta L_{\rm e}^2 + c_4^{(2)}\ln\beta L_{\rm e}^2 + \ldots\right)}_{\text{leading NNLO parts known}} + \ldots\Big] \qquad \text{NNLO}$$

↓ required for FCC-ee



ISR enhancement factor $L_{\rm e} = \ln(m_{\rm e}/M_{\rm W})$

Resummation of leading $(\alpha L_{\rm e})^n$ and subleading $\alpha(\alpha L_{\rm e})^{n-1}$ ISR necessary!

→ talk by M. Beneke

Theory issues in scan of $\sigma_{\mathrm{WW}}(s)$ over WW threshold

- Definition of σ_{WW} via $4f$ final states
  - $\mathrm{e}^{\pm}$ final states: separation or inclusion of single-W channels?
    $\hookrightarrow$ σ_{tot} versus fiducial cross sections
  - Hadronic final states: separation of multi-jet events (2j,3j,4j,...)
    $\hookrightarrow$ issues with Bose–Einstein correlations, colour reconnection, etc.
  - Cleanest final states: $\mu\nu_\mu + 2\mathrm{jets}$
    $\hookrightarrow$ σ_{tot} possible, no BE/colour reconnection issues
  - Abandon σ_{tot} at all due to extrapolation uncertainties?
- Required for the best achievable theory prediction for σ_{WW}:
  - Full NLO $\mathrm{e}^+\mathrm{e}^- \to 4f$ prediction for each $4f$ type
    (interferences with ZZ and forward-$\mathrm{e}^{\pm}$ channels)
    $\hookrightarrow$ nowadays straightforward
  - NNLO EW corrections at least in threshold region (e.g. via EFT)
  - leading 3-loop Coulomb-enhanced EFT corrections
  - matching of all NLO, NNLO, EFT, ... ingredients for $\mathrm{e}^+\mathrm{e}^- \to 4f$
  - convolution of matched and corrected XS with higher-order ISR

  $\hookrightarrow$ Estimate of theory uncertainty:
  $\Delta \sim 0.01\text{–}0.04\%$ for σ_{WW} @ threshold Freitas et al., 1906.05379

Further considerations about EW corrections to $e^+e^- \to 4f$:

- further investigations at NLO of full and threshold EFT results desirable
  $\hookrightarrow$ necessary for steps towards NNLO
- basic requirement at NNLO:
  control implicit NNLO effects in NLO contributions!
  - DPA NLO: hard to control (impossible?)
    due to "on-shell projection" of kinematics
  - full NLO: spurious width terms, but controllable
  - NLO EFT: systematic expansion order by order
- methods for NNLO corrections:
  - DPA NNLO: extremely hard, maybe feasible
  - full NNLO: out of reach in foreseeable future
  - NNLO EFT: should be feasible
- QED ISR corrections: PDFs known to NNLO
  Blümlein et al., 1107.4638, 2004.04287;
  Stahlhofen 2508.16964; Schnubel, Szafron, 2509.09618

  $\hookrightarrow$ NNNLO (+more?) required

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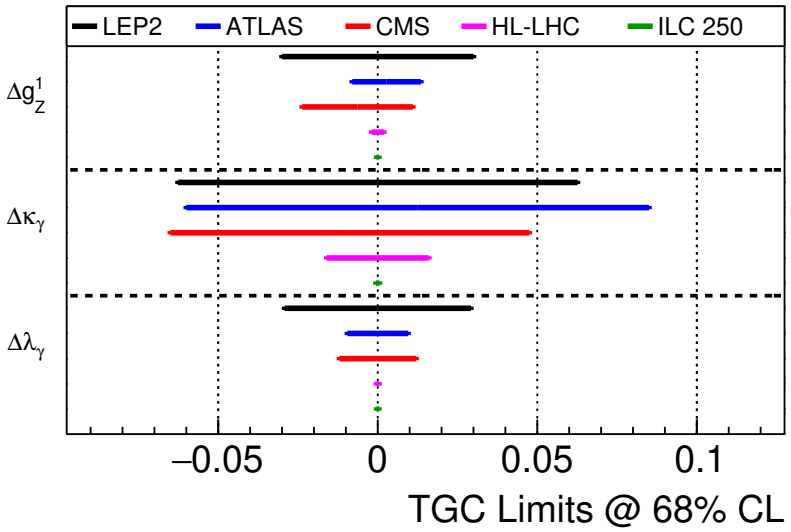
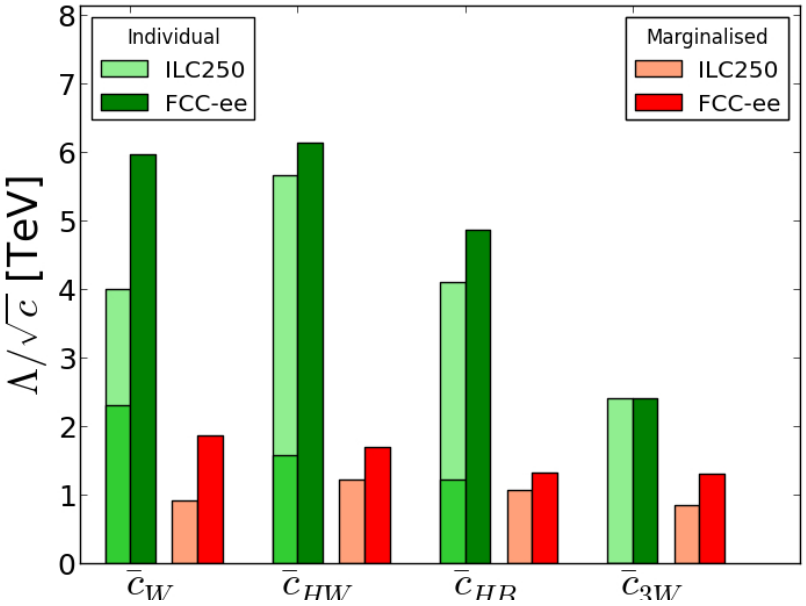
WW production beyond LEP2 energy range

- Ideal for precision study of anomalous TGCs
  (no formfactors for damping required)
- SMEFT framework:
  sensitivity to dim-6 operators complementary to Higgs analyses Ellis, You '15

Bambade et al. '19





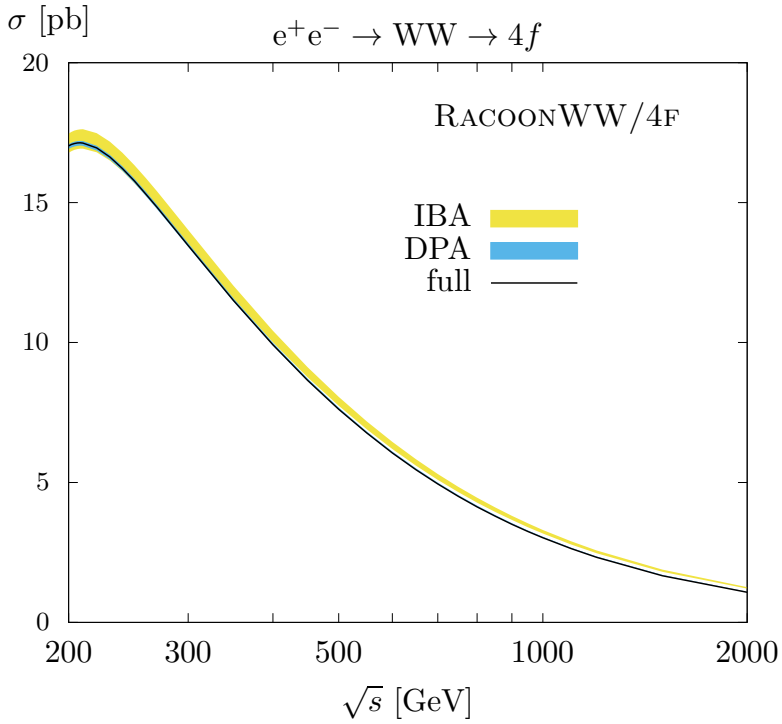
- Impact of $\Delta\kappa_\gamma$ on $\mathrm{d}\sigma_{\mathrm{WW}}$:

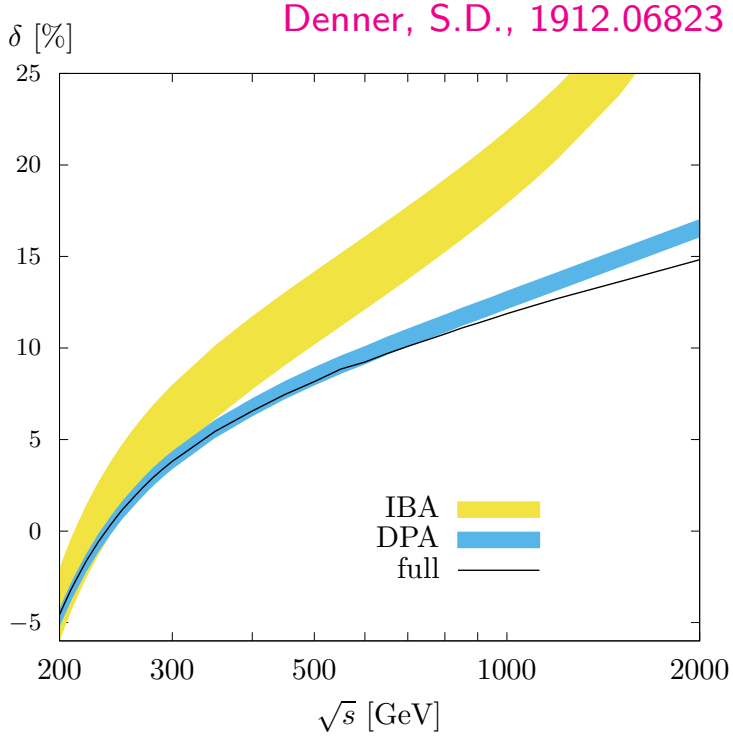
$\sqrt{s}/\mathrm{GeV}$	200	250	500
$\Delta\kappa_\gamma$	0.05	0.004	0.001
$\mathrm{d}\sigma_{\mathrm{WW}}(\kappa_\gamma)/\mathrm{d}\sigma^{\mathrm{SM}}_{\mathrm{WW}} - 1$	3%	$\sim 0.5\%$	$\sim 0.5\%$

$\hookrightarrow$ SM precision limits reach in TGCs for moderate $\sqrt{s}$!

Fixed-order NLO + leading-log ISR prediction:

Denner, S.D., 1912.06823





Note: large non-universal weak corrections + sizeable off-shell effects

Achieved precision:

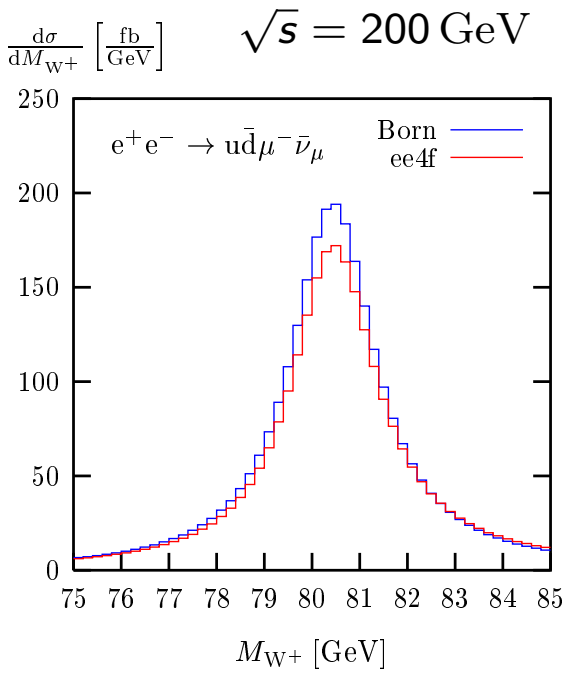
- by full NLO for $e^+e^- \to 4f$ + leading NNLO corrections + ISR resummation
- estimate: $\Delta \sim 0.5\%$ in distributions ($\sim 1\%$ in tails) up to $\sqrt{s} \sim 1\,\mathrm{TeV}$

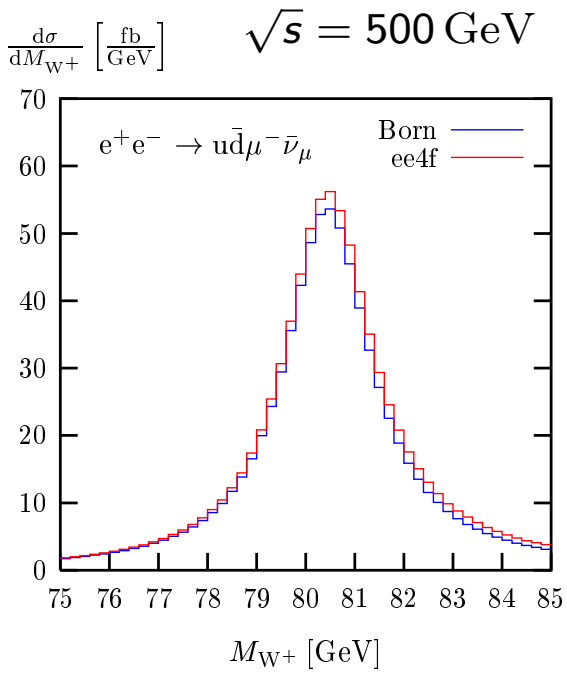
Higher precision for distributions needed at future colliders (some 0.1%?)!

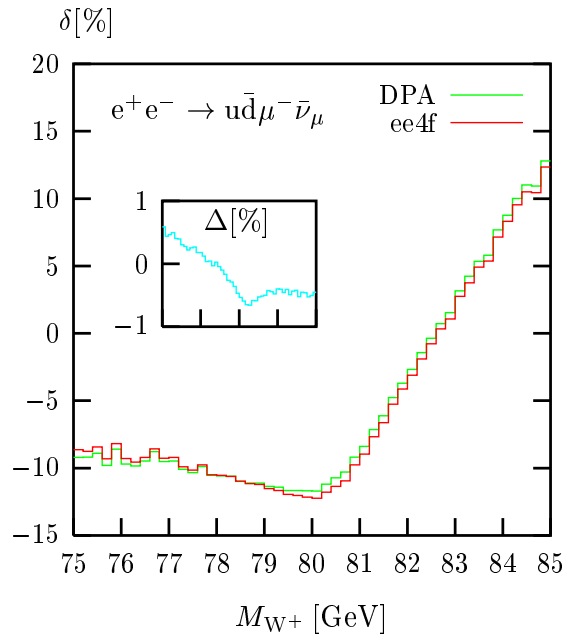
$\hookrightarrow$ Full NLO + NNLO EW in DPA + higher orders for ISR?!

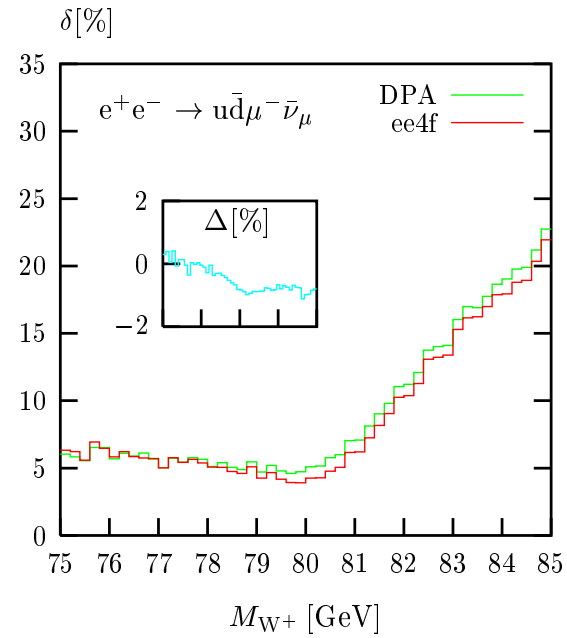
Distribution in the invariant mass of the W^+

Denner et al., hep-ph/0505042



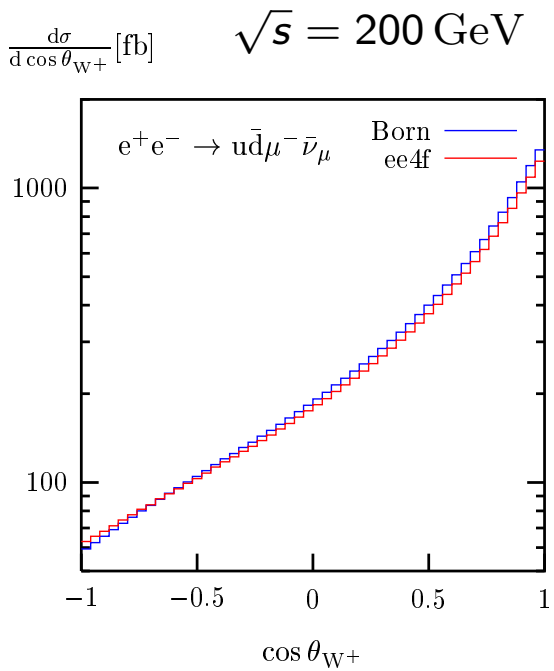


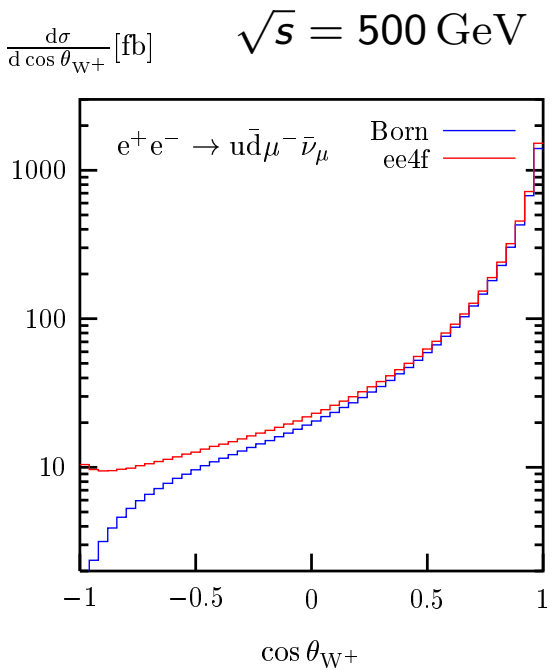


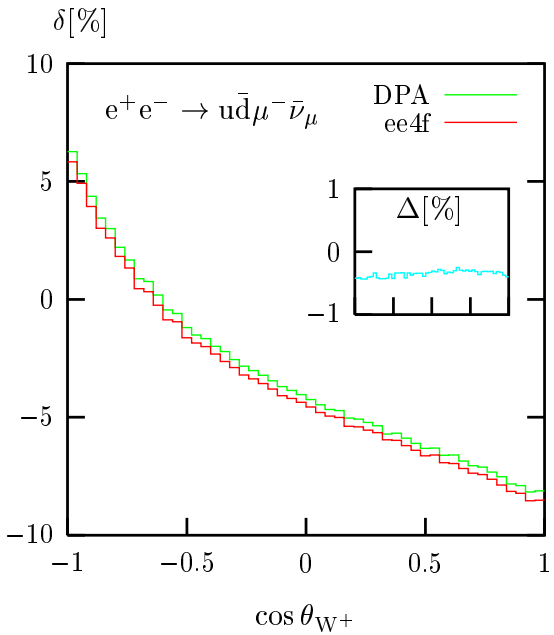


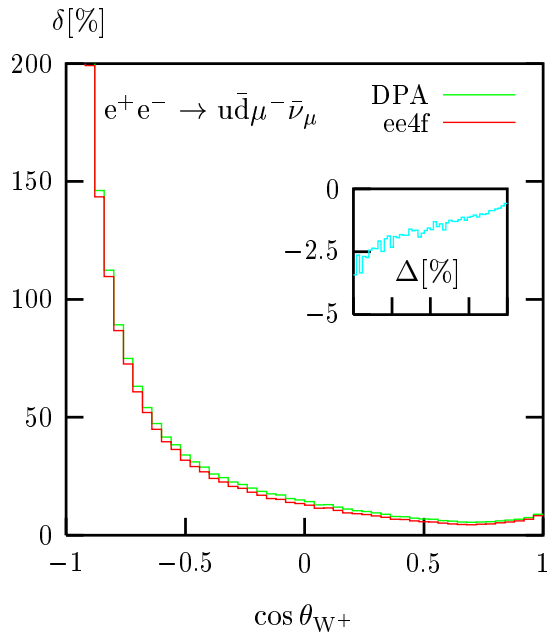
Distribution in the cosine of the W^+ production angle

Denner et al., hep-ph/0505042



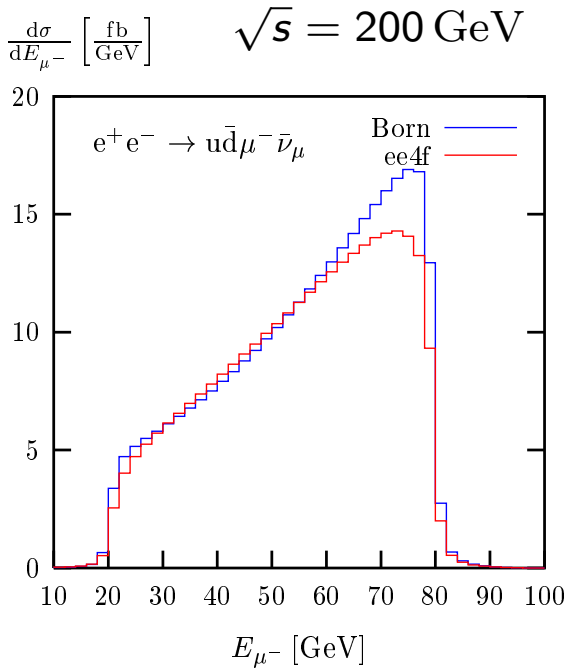


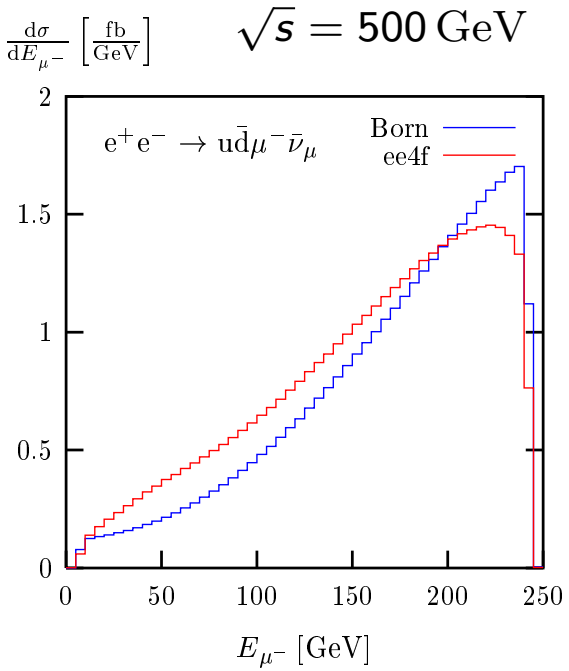


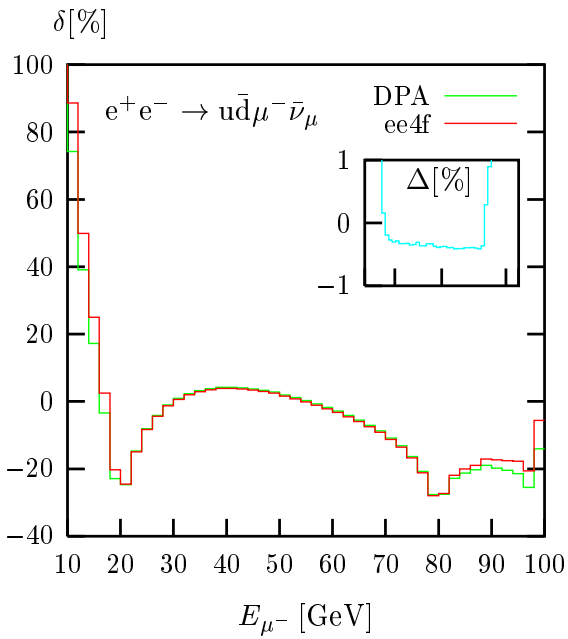


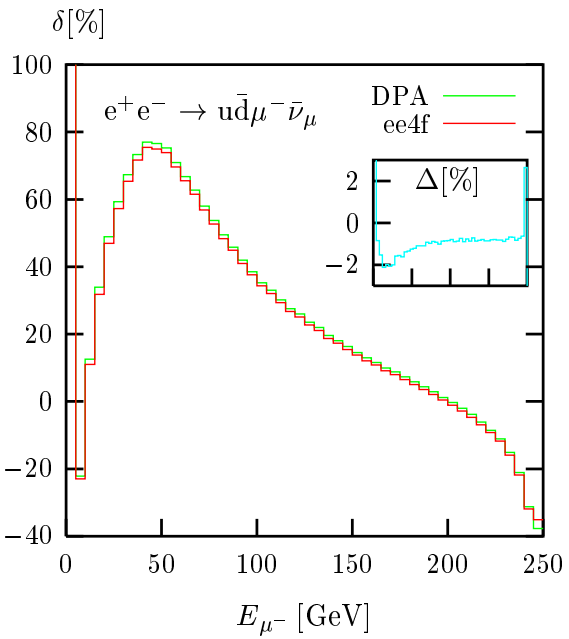
Distribution in the energy of the μ^-

Denner et al., hep-ph/0505042









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Conclusions

σ_{WW} @ WW threshold:

- ΔM_{W} promised to $\sim 0.5{-}1\,\mathrm{MeV}$ @ FCC-ee $\quad(\sim 2\,\mathrm{MeV}$ @ LC)
- $\frac{\Delta\sigma_{\mathrm{WW}}}{\sigma_{\mathrm{WW}}} \sim 0.02{-}0.05\%$ for FCC-ee
- road to the required precision:
  - full LO + NLO predictions for $\mathrm{ee} \to 4f$
  - + NNLO EW corrections @ threshold, e.g. via dedicated EFT
  - + higher-order Coulomb + ISR corrections

$\mathrm{d}\sigma_{\mathrm{WW}}$ @ higher energies:

- $\sim$ few 0.1% precision
- NNLO EW corrections via double-pole approximation most promising

Predictions for W-pair production at any FC:

$\hookrightarrow$ huge technical + conceptual challenges, but next steps very clear!

Prospects for WW @ threshold: (my optimistic assessment)

$\frac{\Delta\sigma_{\mathrm{WW}}}{\sigma_{\mathrm{WW}}} \lesssim 0.1\%$: 100%

$\sim 0.02{-}0.05\%$: 90% realism + 10% optimism

(anticipating progress, many years of hard work)