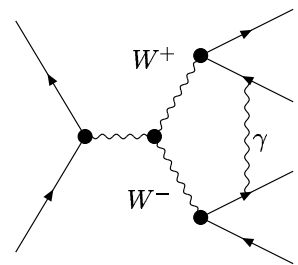
W-Mass from W^+W^- threshold and Unstable Particle EFT

M. Beneke (TU München)

Gearing up for Future Colliders – Workshop on Radiative Corrections at Future Lepton Colliders, CERN, June 29 – July 3, 2026

- Introduction
- Unstable particle EFT (UPEFT)
- Four-fermion production near $\sqrt{s} = 2m_W$ with UPEFT
- Present status
- The step to NNLO ($\mathrm{N}^{5/2}\mathrm{LO}$)
- Discussion points



TECHNISCHE UNIVERSITÄT MÜNCHEN

MB, Chapovsky, Signer, Zanderighi, PRL93:011602,2004; NPB686:205-247,2004 (EFT formalism)
MB, Falgari, Schwinn, Signer, Zanderighi, NPB792:89, 2008 (0707.0773 [hep-ph]) (NLO WW threshold)
Actis, MB, Falgari, Schwinn, Nucl.Phys.B807:1, 2009 (0807.0102 [hep-ph]) (WW threshold beyond NLO)
Schwinn, C., Prospects for higher-order corrections to W-pair production near threshold in the EFT approach, CERN Yellow Reports: Monographs 3 (2020), FCC-ee 77-90

Introduction

- Four-fermion production near $\sqrt{s} = 2m_W$

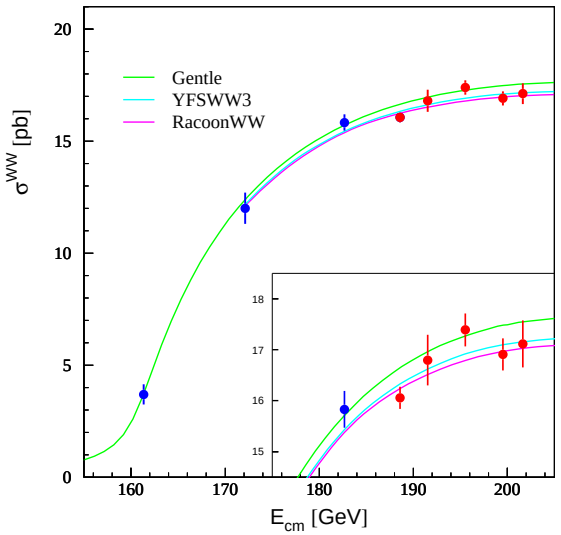
$$e^+e^- \to W^+W^-X \to \mu^-\bar{\nu}_\mu u\bar{d} + X$$

- Threshold region

$$\sqrt{s} = 2m_W \pm \text{few} \times \Gamma_W \approx (155 - 170)\,\text{GeV}$$

Focus on semi-leptonic final state ($\approx 1/27\times$ total)

$$\sigma(161\,\text{GeV}) \approx 120\,\text{fb}$$



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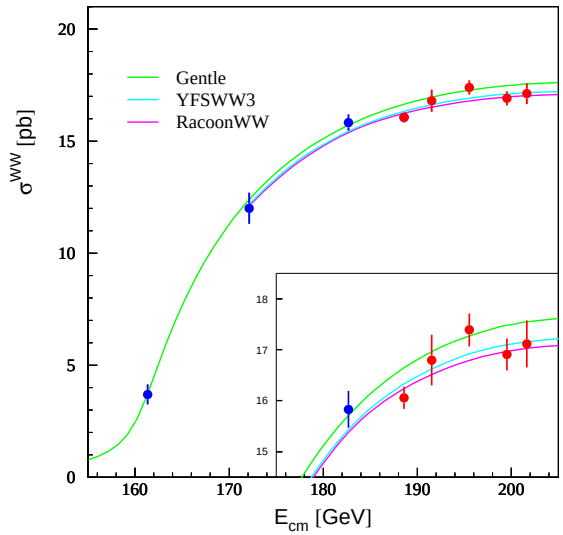


	$\delta\sigma$	$(\delta\sigma)/\sigma$	δm_W
LEP2 (fig)		$\gtrsim 5\%$	$\gtrsim 100$ MeV
theory status*	0.2 fb	0.2%	3 MeV
FCCee target	0.02 fb	$2 \cdot 10^{-4}$	0.3 MeV

*NLO complex mass + $N^{3/2}LO$ UPEFT excluding ISR uncertainty

Theory improvement by a factor of 10 needed for FCCee. $(\alpha_2/\pi)^2 \sim 10^{-4}$. NNLO might just be enough, but full NNLO EW $2 \to 4$ is current beyond reach

Unstable particle production

Always a two-scale problem:

$$\left\{M, \sqrt{s}\right\}, \qquad \left\{\Gamma, \sqrt{s} - M\right\}$$

- Perturbation expansions do not work for the production of resonances ("unstable particles") even for weak coupling, because

$$M^2/(s - M^2) \sim M^2/(M\Gamma) \sim 1/g^2$$

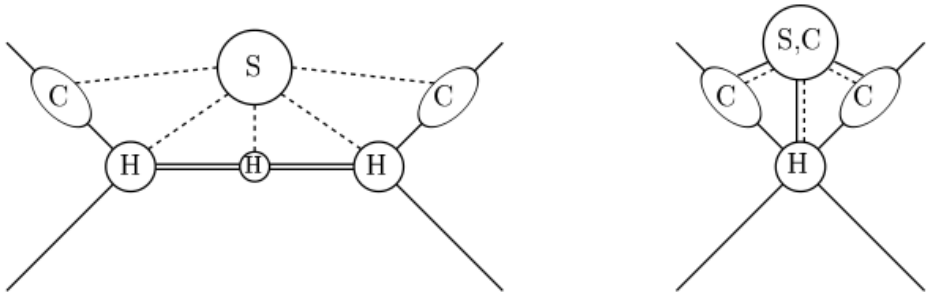
  Systematic expansion / resummation?
  Calculates too little and too much (since positive powers of $(s - M^2)/M^2 \ll 1$)

- Complex mass scheme solves the first problem, but still calculates too much.

- Unstable particle EFT resums only what is needed and expands whenever allowed in $(s - M^2)/M^2, \Gamma/M \sim g^2$ to achieve a prescribed accuracy in $(g^2)^n$.
  Clarifies that Dyson resummation is enough, manifestly gauge-invariant.

  Useful if complex mass scheme calculation is not feasible or too costly or if logs have to be summed.

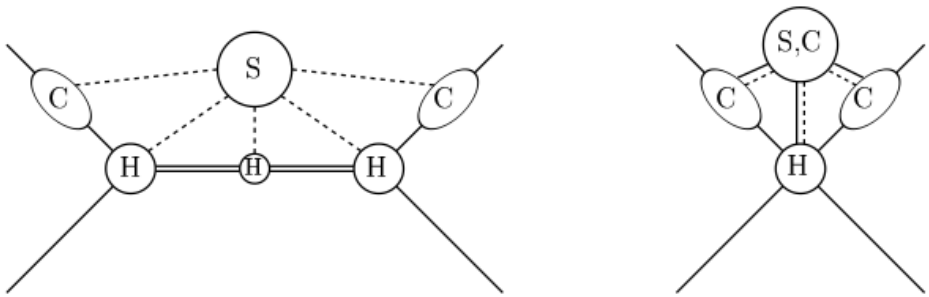
Key formula



Forward-scattering amplitude including non-resonant production in UPEFT

$$i\mathcal{A} = \sum_{k,l} \int d^4x \, \langle e^+e^- | T(i\mathcal{O}_p^{(k)}(0) i\mathcal{O}_p^{(l)}(x)) | e^+e^- \rangle_{|\mathcal{L}_{\text{eff}}} + \sum_k \langle e^+e^- | i\mathcal{O}_{4e}^{(k)}(0) | e^+e^- \rangle_{|\mathcal{L}_{\text{eff}}}$$

Key formula



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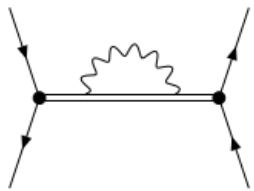
- left: resonant production
  includes Dyson resummation, complicated dependence on $E \equiv \sqrt{s} - M$
  Effective Lagrangian $\mathcal{L}_{\text{eff}}$ for nearly on-shell particles – HQET-like for single resonance / NREFT-like for pair production
- right: nonresonant production
  The local four-electron operator includes off-shell WW or single / no W intermediate states.
  Simple polynomial dependence on E.

Separation is gauge-invariant, because matching to gauge-invariant operators $\mathcal{O}$.

Example diagram



Separate hard and soft contributions to the 1-loop self-energy $\Pi(s) = \Pi_h(s) + \Pi_s(s)$, then expand in $\delta \equiv (s - M^2)/M^2$:

$$\Pi_h(s) = \hat{M}^2 \sum_l \delta^l \, \Pi^{(1,l)}$$

The expansion coefficients are distributed in the EFT as follows:

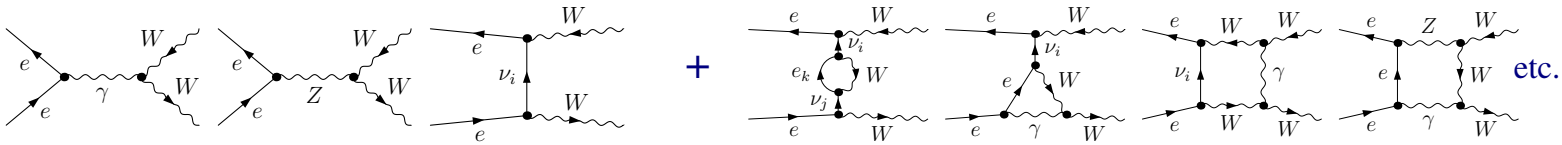
- $\Pi^{(1,0)}$ (gauge-invariant) $\rightarrow$ residual complex mass $\Delta^{(1)}$ resonance propagator (LO effect!)
- $\Pi^{(1,1)}$ (gauge-dependent) $\rightarrow$ production or decay operator $\mathcal{O}_p^{(k)}(0)$ (at NLO)
- $\Pi^{(1,2)}$ (gauge-dependent) $\rightarrow$ non-resonant production-decay operator $\mathcal{O}_{4e}^{(k)}(0)$ (NNLO)
- Π_s is reproduced by the effective theory self-energy

And so on in higher order in δ and α.

$$i\,\mathcal{A} = \sum_{k,l} \int d^4x \, \langle e^+e^- | T(i\mathcal{O}_p^{(k)}(0) i\mathcal{O}_p^{(l)}(x)) | e^+e^- \rangle_{|\mathcal{L}_{\text{eff}}} + \sum_k \langle e^+e^- | i\mathcal{O}_{4e}^{(k)}(0) | e^+e^- \rangle_{|\mathcal{L}_{\text{eff}}}$$

Resonant term

Matching of the leading production operator

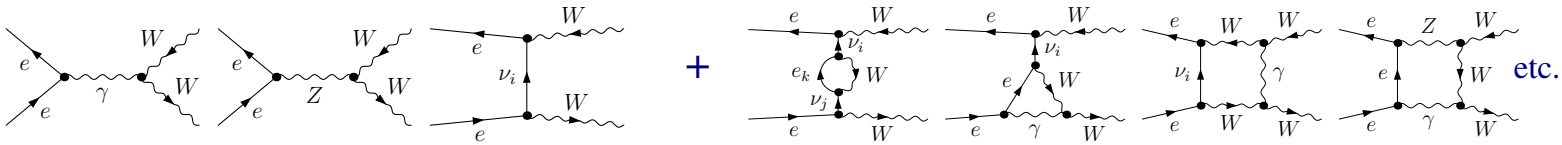


$$\mathcal{O}_p^{(0)} = \frac{\pi\alpha_{ew}C_{p,LR}}{M_W^2}\left(\bar{e}_{c2,L}\gamma^{[i}n^{j]}e_{c1,L}\right)\left(\Omega_-^{\dagger i}\Omega_+^{\dagger j}\right), \qquad C_{p,LR} \approx 1 - \frac{5\alpha}{\pi} + \text{2-loop}$$

At LO in the expansion around threshold, only the t-channel diagram contributes.

Resonant term

Matching of the leading production operator



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At LO in the expansion around threshold, only the t-channel diagram contributes.

Non-relativistic effective Lagrangian for a unstable massive vector field with $U(1)_{em}$ gauge interaction

$$\mathcal{L}_{\text{eff}} = \sum_{\mp}\left[\Omega_{\mp}^{\dagger i}\left(iD_s^0 + \frac{\vec{\partial}^2}{2M_W} - \frac{\Delta}{2}\right)\Omega_{\mp}^i + \Omega_{\mp}^{\dagger i}\frac{(\vec{\partial}^2 - M_W\Delta)^2}{8M_W^3}\Omega_{\mp}^i\right]$$
$$+ \int d^3r\left[\Omega_-^{\dagger i}\Omega_-^i(x+\vec{r})\right]\left(-\frac{\alpha_{\text{QED}}}{r}\right)\left[\Omega_+^{\dagger j}\Omega_+^j\right](x) + \ldots$$

- contains Coulomb force between W bosons, non-factorizable photon exchange etc.

Non-resonant term

Non-resonant $\mathrm{N}^{1/2}\mathrm{LO}$ and $\mathrm{N}^{3/2}\mathrm{LO}$ corrections at $\mathcal{O}(\alpha^3)$
Part of Born cross section in standard PT

- Matching coefficient of

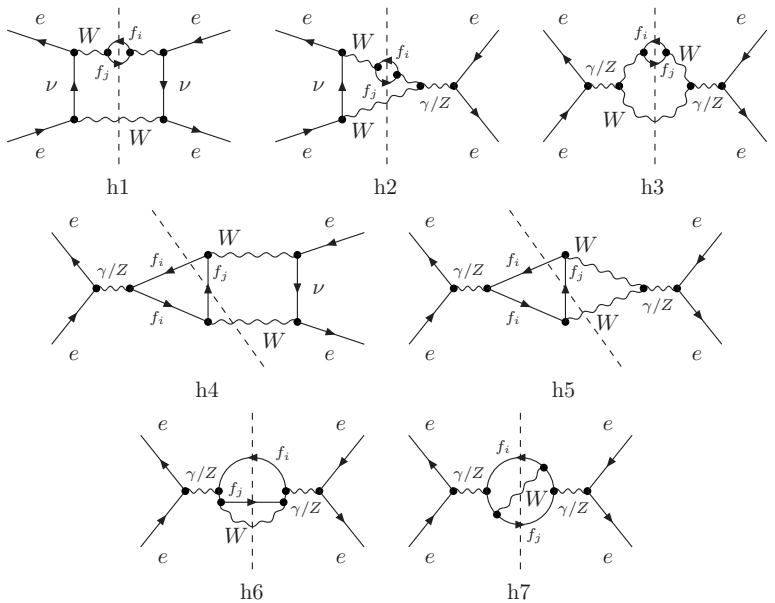
$$\mathcal{O}_{4e}^{(k)} = \frac{C_{4e}^{(k)}}{M_W^2}(\bar{e}_{c1}\Gamma_1 e_{c2})(\bar{e}_{c2}\Gamma_2 e_{c1})$$

  Computed in standard PT with propagator $-ig_{\mu\nu}/(p^2 - M_W^2)$.

- Two-loop cut diagrams result in

$$\sigma_{LR}^{(1/2+3/2)} = \frac{4\alpha^3}{27 s_w^6 s}\left[K_{h1} + K_{h2}\frac{E}{M_W}\right]$$

- Three-loop cut diagrams give another $\mathrm{N}^{3/2}\mathrm{LO}$ term $\propto \alpha^4$ but energy-independent (up to flux factor).



Scales, parameters, power counting

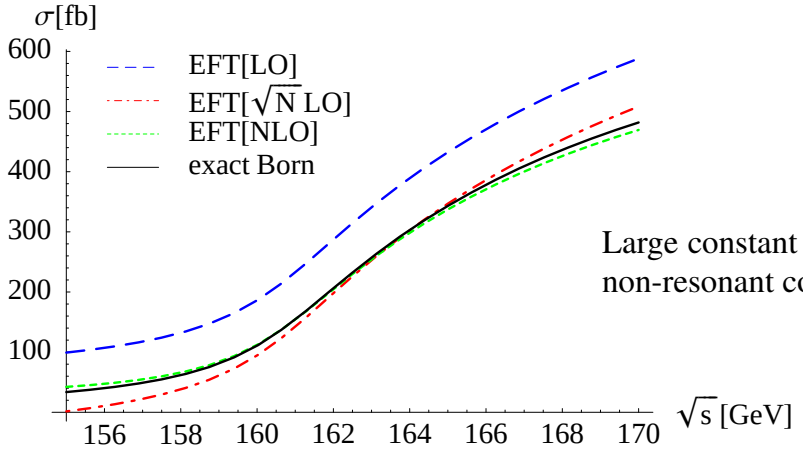
Comparison of W^+W^- and $t\bar{t}$ threshold

- WW pair production near threshold is dominated by electroweak interactions (in leading orders), top pair production by the strong interaction.

	WW	$t\bar{t}$
α_{ew}	δ (def.)	δ^2
α_{em}	δ	δ^2
α_s	$\sqrt{\delta}$	δ (def.)
Γ/M	δ	δ^2
$v^2 \equiv (\sqrt{s} - [2M + i\Gamma])/M$	δ	δ^2
g^2/v (Coulomb)	$\sqrt{\delta}$	1

- Both require non-relativistic + unstable particle EFT.
  Top threshold is dominated by strong Coulomb force, width effects are sub-dominant
  W threshold is dominated by instability and larger non-resonant effects, Coulomb force enhanced but not $\mathcal{O}(1)$.
- For WW expansion in $\sqrt{\delta}$: LO, N$^{1/2}$LO, NLO, ... due to Coulomb enhancement and α_s counting.

Standard Born (LO) cross section (without ISR) from UPEFT



Large constant negative shift by $N^{1/2}$LO leading non-resonant contribution

	$\sigma(e^- e^+ \to \mu^- \bar{\nu}_\mu u\bar{d})$(fb)				
$\sqrt{s}$ [GeV]	EFT(LO)	EFT($\sqrt{N}$LO)	EFT(NLO)	EFT($N^{\frac{3}{2}}$LO)	exact Born
155	101.61	1.62	43.28	31.30	34.43(1)
158	135.43	39.23	67.78	62.50	63.39(2)
161	240.85	148.44	160.45	160.89	160.62(6)
164	406.8	318.1	313.5	318.8	318.3(1)
167	527.8	442.7	420.4	429.7	428.6(2)
170	615.5	533.9	492.9	505.4	505.1(2)

Born cross section should always be treated exactly

Initial-state radiation

- Perform a "QCD-style" calculation of the "partonic" short-distance cross section with massless electrons in the $\overline{\text{MS}}$ scheme for collinear divergences.
- Physical cross section is

$$\sigma(s) = \int_0^1 dx_1 dx_2 f_{e/e}(x_1) f_{e/e}(x_2)\, \hat{\sigma}_h^{\overline{\text{MS}}}(x_1 x_2 s).$$

  $\overline{\text{MS}}$ electron distribution function depends on m_e, but not on $\sqrt{s}$, M, Γ.

- Sums large collinear logs $\alpha \ln(2m_W/m_e)$.
  Should also sum logs of Γ/M (similar to threshold summation of $\ln(1-x)$ in DY production in QCD.)
- In practice, by what is now called "massification", converted $\hat{\sigma}^{\overline{\text{MS}}} \to \hat{\sigma}_h^{\text{conv}}$,

$$\sigma(s) = \int_0^1 dx_1 \int_0^1 dx_2\, \Gamma_{ee}^{\text{LL}}(x_1)\Gamma_{ee}^{\text{LL}}(x_2)\hat{\sigma}_h^{\text{conv}}(x_1 x_2 s)\,,$$

  with electron structure functions $\Gamma_{ee}^{\text{LL}}(x)$ as in the LEP2 Yellow Book.

C. Schwinn, 1905.05078

Table B.7.1: Comparison of the strict electroweak NLO results (without QCD corrections, second Coulomb correction and ISR resummation) in the EFT approach to the full NLO[ee4f] calculation and the DPA implementation of Ref. [11].

	$\sigma(e^-e^+ \to \mu^-\bar{\nu}_\mu u\bar{d}\,X)$(fb)			
$\sqrt{s}$ (GeV)	Born	NLO(EFT) [14]	ee4f [12]	DPA [12]
161	150.05(6)	107.34(6)	106.33(7)	103.15(7)
170	481.2(2)	379.03(2)	379.5(2)	376.9(2)

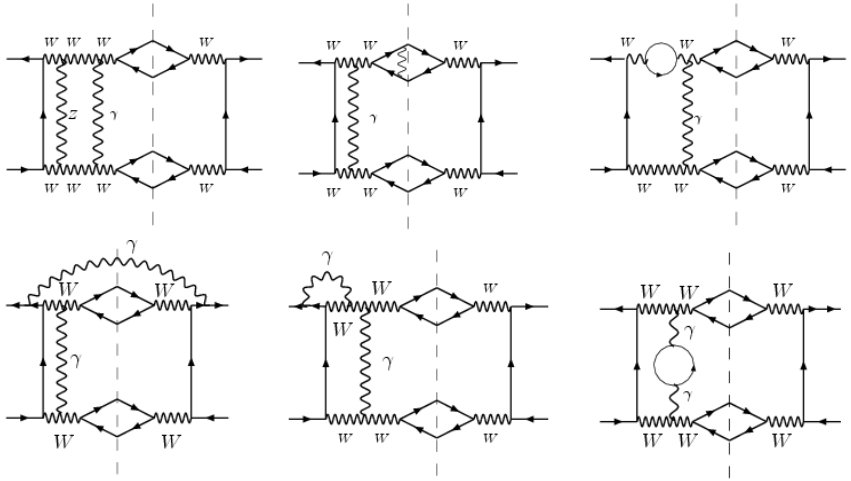
[14] update of MB, Falgari, Schwinn, Signer, Zanderighi 0707.0773

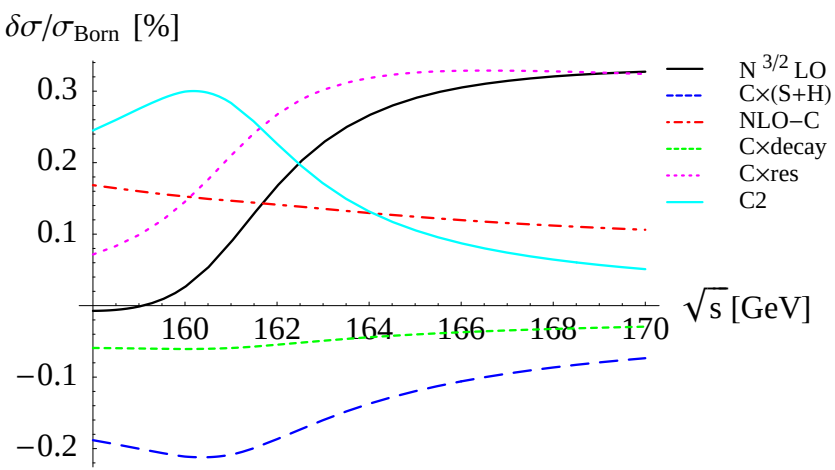
[12] update of Denner, Dittmaier, Roth, Wieders, hep-ph/05052063

The 1 fb difference between NLO(EFT) and ee4f is or the expected size of the missing $\mathcal{O}(\alpha^4)$ non-resonant term at $N^{3/2}LO$, which counts as NLO in the standard expansion.

Beyond NLO

Some NNLO terms in ee4f counting are $N^{3/2}LO$ in EFT counting and already known, all involving at least one Coulomb photon exchange.





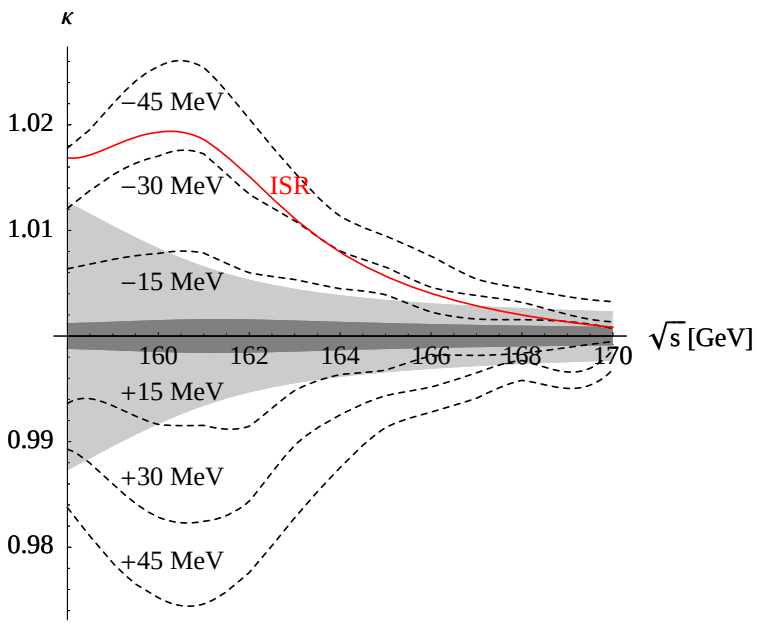
In total a small correction:

$[\delta m_W]_{\rm BeyondNLO} \approx 3\,{\rm MeV}$

Uncertainty on m_W from ISR

Sensitivity to m_W and theoretical uncertainty
Variation of cross section normalized to standard input

- ISR resummation has only LL accuracy. Different LL-accurate implementations give sizeable difference.
- At the point of maximal sensitivity large uncertainty from current implementation of ISR ($\delta m_W \approx 30\,\mathrm{MeV}$)
- General issue for FCCee - more accurate ISR treatment required.
- Probably also need a less inclusive treatment.



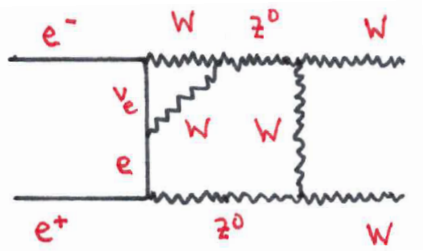
Achieving NNLO ($N^{5/2}LO$) precision with UPEFT

Many terms. Here highlight those that may pose computational challenges. See also Schwinn, section B.7 in CERN Yellow Report 1905.05078.

① Two-loop EW on-shell $e^+e^- \to W^+W^-$ amplitude
(and 2-loop EW W decay)

One-loop was 2.4% correction. Simple estimate based on geometric progression gives for two-loop

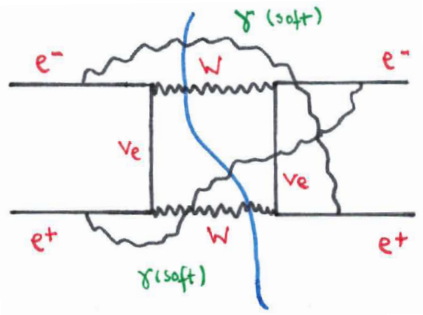
$$\frac{\delta\sigma}{\sigma} \approx 6 \cdot 10^{-4}$$



↪ relevant, seems doable (?)

② Two-loop non-factorizable corrections

Required for consistent cancellation of hard-soft factorization scale dependence at two-loop of the resonant term.
QED version of the two-loop Drell-Yan soft function which is known.



↪ easy

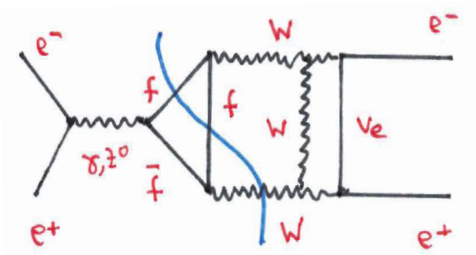
Achieving NNLO ($N^{5/2}LO$) precision with UPEFT

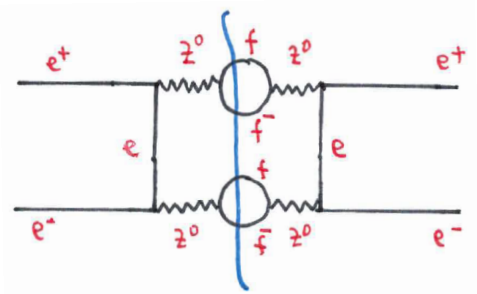
③ Non-resonant matching

For $N^{5/2}LO$ need two-loop correction to $e^+e^- \to W + 2f + X$ (one-shell W) and one-loop correction $e^+e^- \to 4f + X$.
Computations done with $\Gamma_W = 0$ and dim reg.
Non-resonant two-loop $e^+e^- \to 4f$ contribution is higher order.

$$\sigma_{\text{nonres}} = \alpha^3 \left[\# + \# \frac{E}{m_W} + \# \frac{E^2}{m_W^2} \right] + \alpha^4 \left[\# + \# \frac{E}{m_W} \right] + \alpha^5 \#$$

$N^{1/2}$ ✓ $N^{3/2}$ ✓ $N^{5/2}$ $N^{3/2}$ $N^{5/2}$ $N^{5/2}$





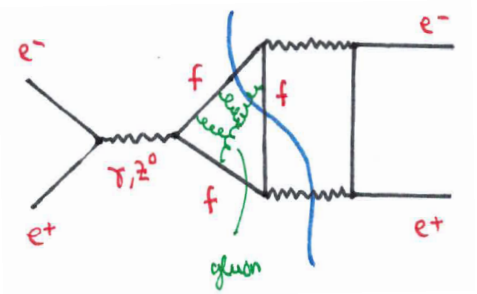
↪ $N^{5/2}LO$ appears very hard. The hard pieces are energy-independent might be absorbed in a floating normalization (???).
Fermion triangle graphs seem to be less important than bubbles.

Achieving NNLO ($N^{5/2}LO$) precision with UPEFT

④ QCD effects up to $\mathcal{O}(\alpha_s^4)$ for NNLO [recall $\alpha_s^2 \sim \alpha_{\rm EW}$]

Once quarks are present in a diagram, can replace every EW loop by two QCD loops.
Same for number of final-state particles.

- Γ_W (known) and $Z_W^{\rm OS}$ to four loops in QCD, and 3-loop mixed $\alpha\alpha_s^2$.
- On-shell $e^+e^- \to WW$ amplitude, 3-loop mixed $\alpha\alpha_s^2$
- $e^+e^- \to W + 2\text{f} + \text{X}$ (one-shell W): 3-loop QCD corrections and 2-loop mixed QCD-EW contribute at NNLO to the non-resonant term, expanded in E/m_W.
- 1-loop QCD correction to $e^+e^- \to 4\text{f} + X$, expanded in E/m_W (non-resonant term, NNLO)



↪ 2nd and 3rd items are hard. The highest order is always energy-independent.
Some topologies might be numerically suppressed

Cuts and EFT

So far totally inclusive final state, except for fermion flavour quantum numbers.
Extract cross section from the forward-scattering amplitude

$$\hat{\sigma} = \frac{1}{s} \operatorname{Im} \mathcal{A}(e^- e^+ \to e^- e^+)_{|\mu^- \bar{\nu}_\mu u \bar{d}}$$

Cuts are not straightforward in the EFT approach: may introduce new scales regions.
Example: Invariant mass cuts $|M^2_{u\bar{d}} - M^2_W|, |M^2_{\mu\bar{\nu}_\mu} - M^2_W| < \Lambda^2$

- Loose cut: $\Lambda \sim M_W$

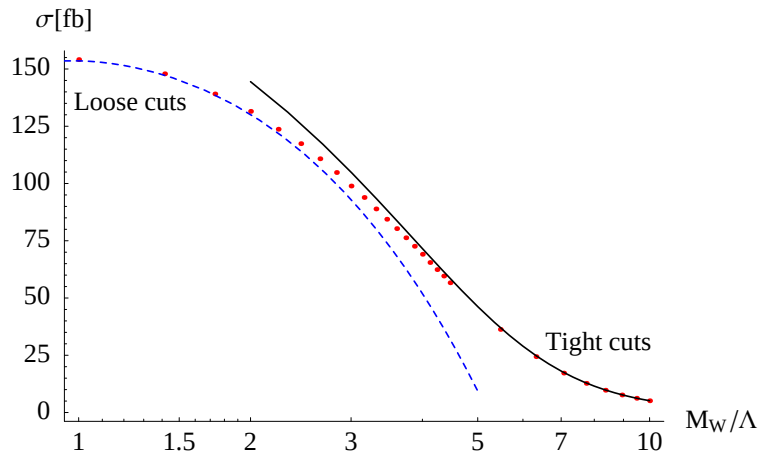
  No modification of potential loops (momenta always within the cut by power counting).

  Cut affects the matching coefficient of the four-electron operator (non-resonant terms).

- Tight cut: $\Lambda \sim \sqrt{M_W \Gamma_W}$

  Four-electron operator (non-resonant terms) does not contribute at all.

  Cut affects loop calculations in the effective theory.



Red dots: Born cross section ($\sqrt{s} = 161$ GeV, WHIZARD)

ee4f / WW LEP 2 cuts

W mass measurement uses almost the inclusive cross section.
Cuts for the cross section measurement at $\sqrt{s} = 161\,\text{GeV}$ used at LEP2:

Cut	$\sigma_{\text{Born}}(e^-e^+ \to \mu^-\overline{\nu}_\mu u\bar{d})$(fb)	$\sigma_{\text{cut}}/\sigma_{\text{tot}}$
–	154.18(5)	
$\lvert\vec{p}_\mu\rvert > 20$ GeV	153.71(5)	99.69(5) %
$M_{\mu\nu} > 55$ GeV, 40 GeV $< M_{jj} <$ 120 GeV	150.61(5)	97.68(5) %
$\theta_{\mu j} > 15$ degrees	149.35(5)	96.87(5) %
$\lvert\cos\theta_\nu\rvert < 0.95$	148.28(5)	96.17(5) %
all	140.03(5)	90.82(5) %

2nd and 3rd column: Effect of LEP phase-space cuts on the Born cross section at $\sqrt{s} = 161$ GeV

Strategy: Use fully differential NLO ee4f computation (complex mass scheme) including all cuts + EFT calculation of the NNLO terms without cuts ($\approx 7\%$ error on a small correction).

Discussion points

1. How does the sensitivity to m_W degrade if one lets a strictly constant normalization float? Compare to accuracy of luminosity measurement.
2. Are there some N3LO terms in standard ee4f counting that are needed to achieve FCCee precision?
3. UPEFT simplifies analytic computations but breaks it into many terms. Instead of UPEFT matching simply do a region identification and expansion to sufficient depth?

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3. UPEFT simplifies analytic computations but breaks it into many terms. Instead of UPEFT matching simply do a region identification and expansion to sufficient depth?
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Discussion points

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6. Are the LEP2 cuts still representative for FCCee? Can measurements be done more inclusively? How inclusive with respect to hadrons?