ISR Effects and Beamstrahlung

Simulation with GUINEA-PIG

D. Schulte, CERN

Gearing up for Future Colliders — Workshop on Radiative Corrections at Future Lepton Colliders

GUINEA-PIG Introduction

GUINEA-PIG and GUINEA-PIG++ simulate the collision of electron, positron and photon beams
- Private C version (to be published) also allows collision of muons

In 1994, I was simulating the impact of beam-induced background on the detector of a linear collider
- Existing beam-beam simulations at the time (ABEL, MACPAR, RBEAM) had important flaws
- Only ABEL could generate pairs
- So, I wrote GUINEA-PIG (Generator of Unwanted Interactions for Numerical Experiment Analysis – Program Interfaced to GEANT)

The code was developed in C since the floppy with the C++ compiler was defect

In 2005 the code was ported to C++ at LAL in Orsay
- Followed by some corrections of issues from the port and additions later at CERN

Code Fundamentals

GUINEA-PIG is a strong-strong code

- Both beams change their shape significantly during collision
- Very important for linear colliders
  - Focusing of the beams can double the luminosity
- Still important for FCC-ee

It uses a clouds-in-cells algorithm

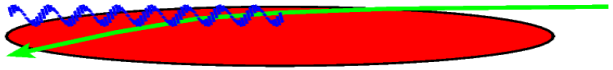
- Beam is cut into slices and a grid put over each slice
- Convolution with FFT then allows to obtain the beam fields

Accelerator and computational parameters are read from file

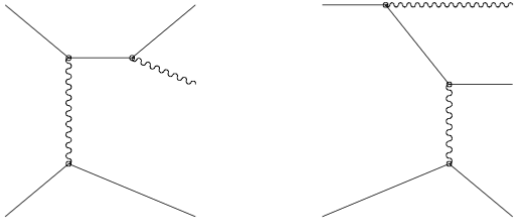
One can generate the initial particles distributions on the fly (e.g. Gaussian distributions)

- But one can also load any mixture of electron, positrons and photons from a file
- Useful to start from tracking results

Lepton and Photon Production

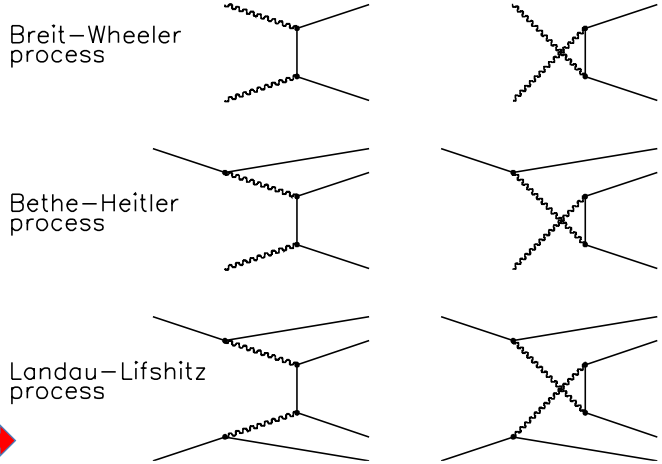


Beamstrahlung

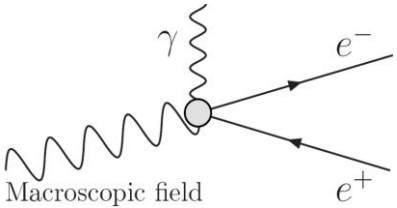


Bremsstrahlung

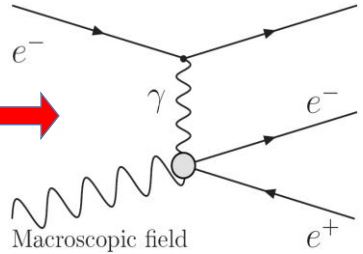
Colliding Photons can produce electron-positron pairs (incoherent pair production) O(10^5) per bunch crossing



Beamstrahlung photons can turn into pair in strong field (coherent pair production) O(1-10^8) per bunch crossing



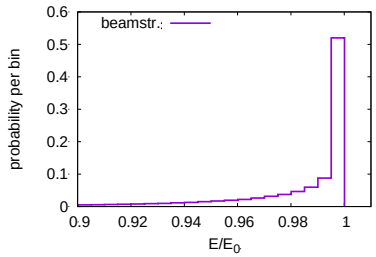
Trident cascade



Other Production

Luminosity spectrum with and without initial state radiation

Stores collisions of particles with given cross section (and spectrum in C version)
More involved forms were/are available



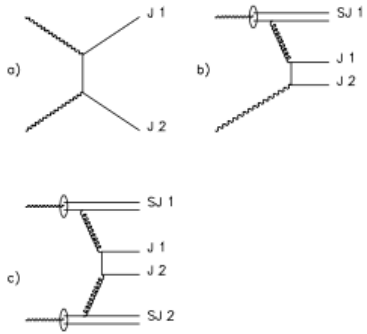
Bhabha scattering

Read Bhabha events from generator and track secondaries to estimate impact on luminosity spectrum reconstruction

$$\frac{d\sigma}{dt} = \frac{2\pi m^2 r_e^2}{s^2}\left[\frac{s^2+u^2}{t^2} + \frac{2u^2}{ts} + \frac{u^2+t^2}{s^2}\right]$$

Minijet background

Using parton (and photon) parametrisations of Glück, Reya and Vogt
Or Drees and Grassi
Tree-level cross sections
Fragmentation outside



[55] M. Drees and K. Grassie. Parametrisations of the photon structure and applications to supersymmetric particle production at HERA. Z. Phys. C, **28** (1985) 451–462.

[56] M. Glück, E. Reya, and A. Vogt. Photonic parton distributions. Physical Review D, **46** (1992) 1973.

Hadronic background

Using parametrisation of total cross section, .e.g from PYTHIA and store initial photons
Events then simulated in PYTHIA and stored in HADES

$$\sigma_{H,3} = 211\,\text{nb}\cdot\left(\frac{s}{\text{GeV}}\right)^{\epsilon} + 297\,\text{nb}\cdot\left(\frac{s}{\text{GeV}}\right)^{-\mu}$$

$$\epsilon = 0.0808 \text{ and } \mu = 0.4525$$

[51] T. Sjoestrand. What is a photon and how should it diffract? Talk given at Photon 95.

Code Fundamentals II

GUINEA-PIG produces beamstrahlung particles, coherent and trident cascade pairs with the same weight as the beam particles

- Charge conservation is strictly observed

Incoherent pairs and bremsstrahlung particles are produced in small number

- Normally they are produced with one macro-particle corresponding to one real particle
- Can tune this
- The weight of the producing photons is not reduced
  - Violates charge conservation typically at a level of 10^{-6}
  - Saved one additional variable per macro-particle

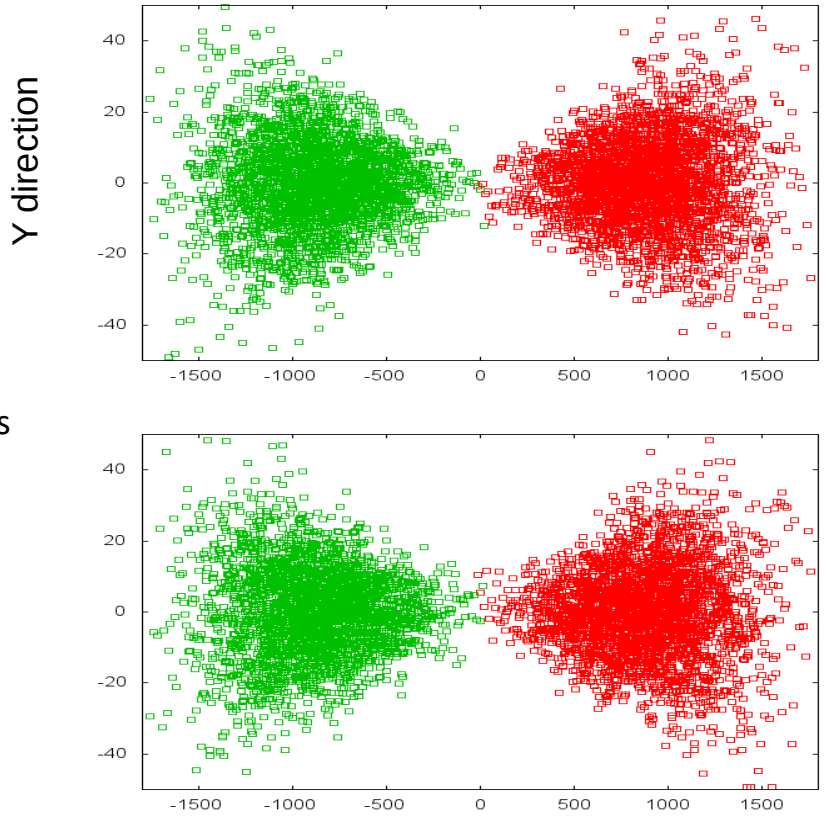
Simulation Codes

Need strong-strong code, e.g.
- CAIN (K. Yokoya et al.)
- GUINEA-PIG (D. Schulte et al.)

- Beams => macro particles
- Beams => slices
- Slices => cells
- The simulation is performed in a number of time steps in each of them
- The macro-particle charges are distributed over cells
  - The forces at the cell locations are calculated
  - The forces are applied to the macro particles
  - The particles are advanced

All simulation performed with GUINEA-PIG

Beam-beam force switched off



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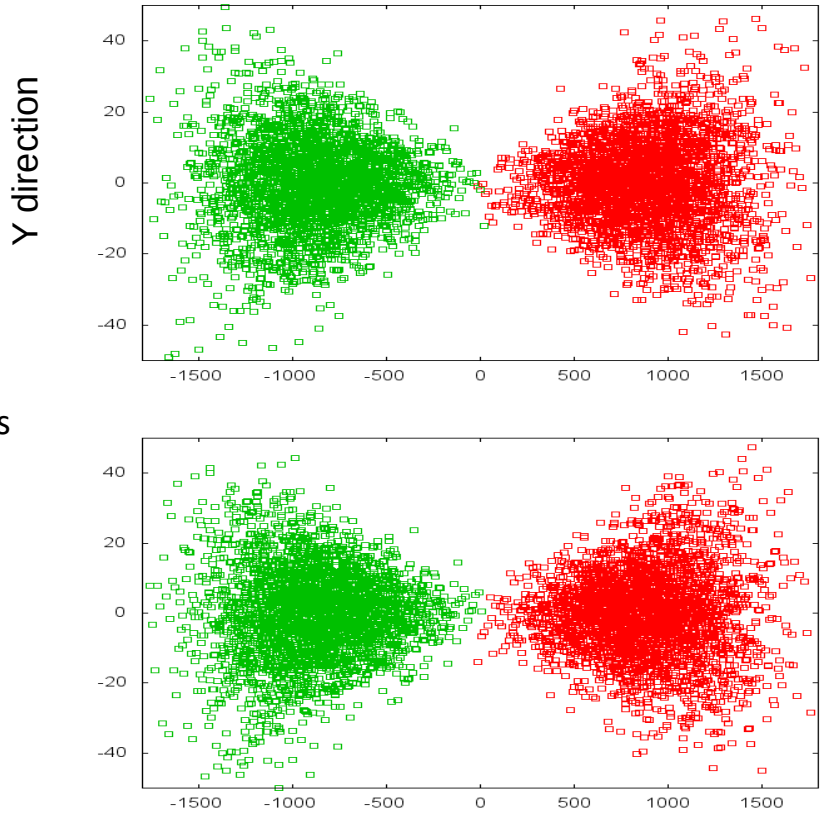


Improvement Potential

Limitations arise from

- Limited modelling of incoming beam
  - In collaboration with accelerators obtain typical beams and establish how important differences are for physics

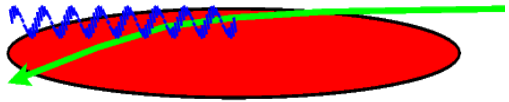
- Limited statistics because of time consumption
  - Speed-up the code (FCC-ee simulations are time consuming because of collision geometry)

- Limited modelling of the physics in the beam-beam collision

Beamstrahlung

Modelled as synchrotron radiation using local bending radius

- This requires among other things that the bending angle is much larger than the synchrotron radiation angle
  - Validity verified for CLIC

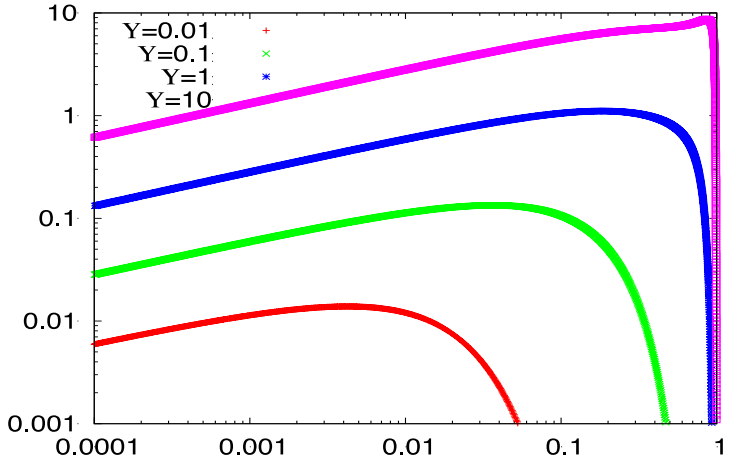


Important is Upsilon parameter, linked to the critical energy

$$\Upsilon = \frac{2}{3}\frac{\hbar\omega_c}{E_0} \qquad \hbar\omega_c = \frac{3}{2}\frac{\gamma^3\hbar c}{\rho}$$

$\Upsilon \ll 1$ is the classical regime
For ILC, FCC-ee and low-energy CLIC

$\Upsilon \gg 1$ the quantum regime
high-energy CLIC



$$x = \frac{\omega}{\omega_c}\frac{E}{E-\hbar\omega} \qquad \mathrm{K}_{\frac{5}{3}} \qquad \mathrm{K}_{\frac{2}{3}} \qquad \frac{\mathrm{d}\,\dot{w}}{\mathrm{d}\,\omega} = \frac{\alpha}{\sqrt{3}\pi\gamma^2}\left[\int_x^\infty \mathrm{K}_{\frac{5}{3}}(x')\mathrm{d}x' + \frac{\hbar\omega}{E}\frac{\hbar\omega}{E-\hbar\omega}\mathrm{K}_{\frac{2}{3}}(x)\right]$$

Examples of ILC and CLIC Main Parameters

Parameter	Symbol [unit]	SLC	ILC	CLIC
Centre of mass energy	E_{cm} [GeV]	92	250	380
Luminosity/IP	L [$10^{34}cm^{-2}s^{-1}$]	0.0003	2.7/2	4.4/2
Luminosity in peak	$L_{0.01}$ [$10^{34}cm^{-2}s^{-1}$]	0.0003	1.0	1.3
Polarization (e-/e+)	[%]	80/0	80/30	80/30
Gradient	G [MV/m]	20	31.5	72
Particles per bunch	N [10^9]	37	20	5.2
Bunch length	σ_z [µm]	1000	300	70
Collision beam size	$\sigma_{x,y}$ [nm/nm]	1700/600	516/7.7	149/2.0
Bunches per pulse	n_b	1	1312	352
Bunch distance	Δt [ns]	-	554	0.5
Repetition rate	f_r [Hz]	120	10	100

FCC-ee Beam Parameters

Parameter	Z	WW	ZH	tt
E_{cm} [GeV]	91.2	160	240	365
L [10^{34} $cm^{-2}s^{-1}$]	141	20	7.5	1.45
Θ_C [mradian]	30			
ε_x [nm]	0.71	2.16	0.66	1.65
$\varepsilon_{y,L}$ / ε_y [pm/pm]	0.87/2.1	1.2/2	0.57/1	0.82/1.32
β_x / β_y [mm]	110/0.7	220/1	240/1	900/1.4
σ_x / σ_y [μm/nm]	8.8/38	22/45	12.6/32	39/43
$\sigma_{E,L}$ / $\sigma_{E,BS}$ [%]	0.04/0.12	0.07/0.1	0.1/0.18	0.15/0.19
$\sigma_{z,L}$ / $\sigma_{z,BS}$ [mm]	5.15/15.2	3.5/5.3	3.3/5.6	1.9/2.3
Φ	25.8	3.6	6.6	0.91
Beam lifetime [60s]	87	75	100	105
L lifetime [60s]	22	16	10	11

Beams collide at large angle

Beam-beam effects increase vertical emittance

Beamstrahlung in collision increases energy spread and bunch length

$$\Phi = \frac{\sigma_{z,\mathrm{BS}}}{\sigma_x^*} \tan\left(\frac{\theta_c}{2}\right)$$

FCC-ee Beamstrahlung

Mean energy loss per passage 0.17*2 MeV=0.34 MeV
Only a few % of the collisions are after emission of a beamstrahlung photon
Energy change particle with beamstrahlung is of $O(10^{-4})$
Average collision energy is impacted by $O(10^{-5})$

Increase of RMS energy spread in FCC-ee at Z0 from beamstrahlung: 52 MeV
Energy spread corresponds to RMS spread over 22500 turns
From injected bunches can converge faster, because the loss is large for initial parameters

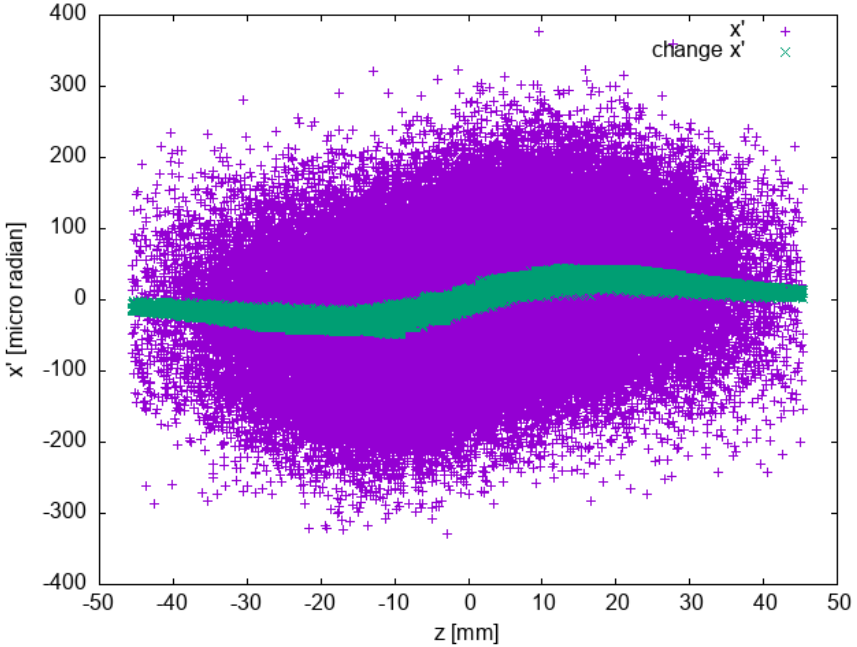
Energy-position correlations may build up
- Larger amplitude particles emit more beamstrahlung

Systematic errors in case non-colliding bunches are used for calibration need exploration

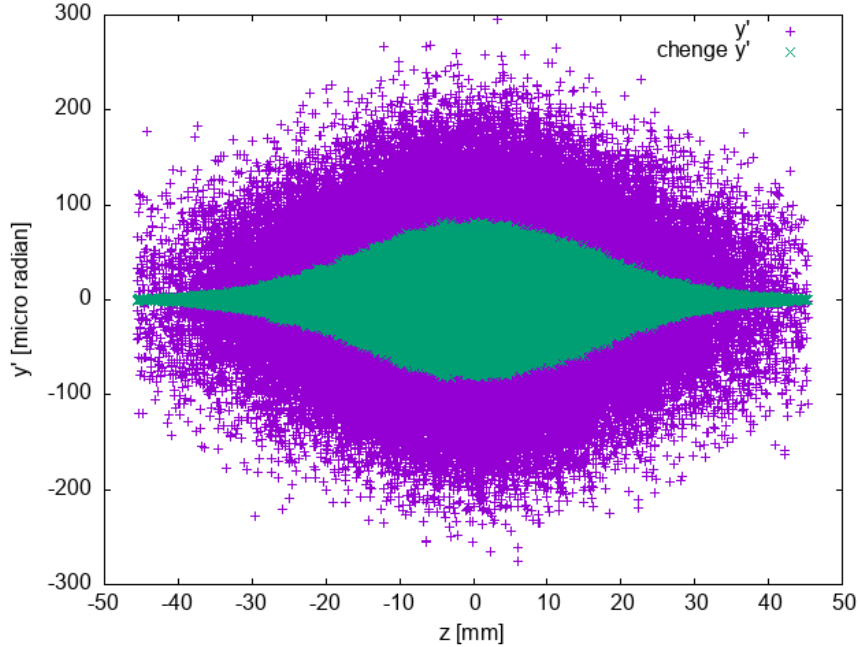
Parameter	Z	tt
L [10^{34} $cm^{-2}s^{-1}$]	141	1.45
β_x / β_y [mm]	110/0.7	900/1.4
σ_x / σ_y [μm/nm]	8.8/38	39/43
Φ [mradian]	25.8	0.9
N_gamma	0.17	0.18
E_gamma [MeV]	2.0	50

Beam-beam Kick

Very small horizontal kick
with systematic component due to the crossing angle

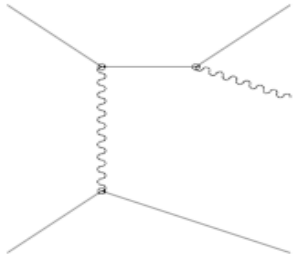


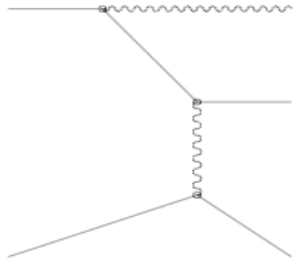
More pronounced vertical kick
Size depends on longitudinal position



Beam Lifetime

Several effects impact beam lifetime
- Drift of particles due to beam-beam effects
- Some particles lose important energy due to beamstrahlung
- Bremsstrahlung
- …





Models:
Compton scattering of virtual photons
Bhabha scattering with photon emission in initial or final state

Parameter	Z	tt
E_{cm} [GeV]	91.2	365
L [10^{34} $cm^{-2}s^{-1}$]	141	1.45
N [10^{10}]	21.8	15.8
N_b	11200	60
Beam lifetime [60s]	87	105
L lifetime [60s]	22	11

Fun fact:

Bremsstrahlung cross section and proton-proton are both around >100 mb
Hence can only get int L = 10 / b per particle
Muons do better, since bremsstrahlung cross section is much smaller

Virtual Photon Approximation

GUINEA-PIG uses Weizsäcker-Williams approximation for pair and bremsstrahlung production

- Cross section for real initial photons is being used
- The upper cut-off depends on the process, the lower on momentum and energy conservation

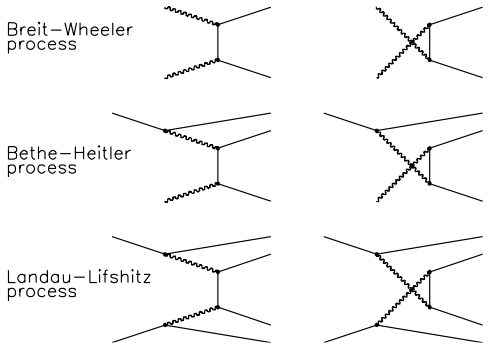
The generator routines have access to the virtuality of the photon and can decide what to do

- For the pairs choice for max q^2 between cut at m^2, $m^2+p_T^2$, s
- For hadrons at $(1\ \mathrm{GeV}+f(s))^2$
- For bremsstrahlung at m^2

$$\frac{\mathrm{d}^2 n_v(x)}{\mathrm{d}\,x\,\mathrm{d}\,Q^2} = \frac{\alpha}{2\pi}\frac{1+(1-x)^2}{x}\frac{1}{Q^2}$$

$$\frac{\mathrm{d}\,n_v(x)}{\mathrm{d}\,x} = \frac{\alpha}{2\pi}\frac{1+(1-x)^2}{x}\ln\frac{\hat{Q}^2}{\check{Q}^2}$$

$$\check{Q}^2 = \frac{x^2 m^2}{1-x}$$



Beam Size Effect

The finite beam size limits the production

- Observed for bremsstrahlung in VEPP-4, HERA and LEP

Different models exist:

- BBBREM assumes that virtual photon is limited by distance of electrons in the beam
  - Density dependent
- Serbo et al. and Piotrzkowski assume that limit arises from beam size
  - Independent of density

- The latter is used in GUINEA-PIG (with a switch and scale factor)
  - Virtual photon is offset with respect to parent particle

In addition, strong fields can further suppress virtual photon spectrum (Baier, Katkov)

[35] A. E. Blinov et al. Phys. Lett. B, **113** (1982) 423.

[36] G. L. Kotkin, V. G. Serbo, and A. Schiller. Processes with large impact parameters at colliding beams. Int. J. Mod Phys. A, **7** (1992) 4707–4745.

[37] J. L. Augneres et al. HERA. A Proposal for a large Electron-Positron Colliding Beam Facility at DESY. DESY HERA 81/10 (1981).

[38] K. Piotrzkowski. Observation of the beam-size effect at HERA. DESY 95-051 (1995).

[39] V. N. Baier and V. M. Katkov. Production of bremsstrahlung during collisions of high-energy particles in a magnetic field. Sov. Phys. Dokl., **17** (1973) 1068.

$$b \approx \hbar/q_\perp \qquad \check{q}_\perp = \hbar/\sigma_y$$

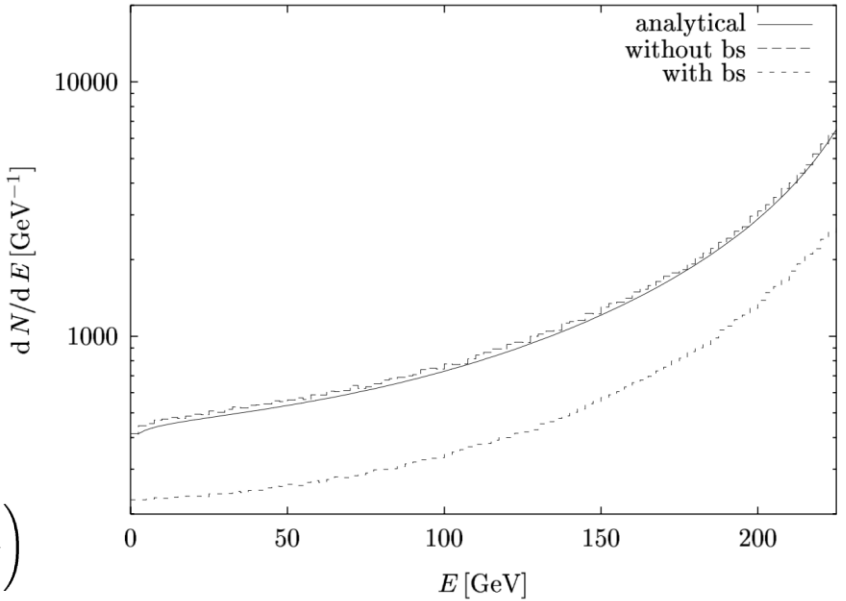
$$\check{q}_\perp \approx xmc \left(\frac{\omega_c}{\omega}\right)^{1/3}$$

Bremsstrahlung

Modelling of Bremsstrahlung as Compton scattering on virtual photon

- Maximum Q at mc

Analytic, simple formula is well reproduced
Beamsize effect has strong impact



$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = 8\pi\frac{r_e^2 m^2}{(s-m^2)^2}\left[\left(\frac{m^2}{s-m^2}+\frac{m^2}{u-m^2}\right)^2+\left(\frac{m^2}{s-m^2}+\frac{m^2}{u-m^2}\right)-\frac{1}{4}\left(\frac{s-m^2}{u-m^2}+\frac{u-m^2}{s-m^2}\right)\right].$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\omega} = 4\alpha r_e^2\frac{1}{\omega}\frac{E-\hbar\omega}{E}\left(\frac{E}{E-\hbar\omega}+\frac{E-\hbar\omega}{E}-\frac{2}{3}\right)\left(\ln\frac{4E^2(E-\hbar\omega)}{m^2c^4\hbar\omega}-\frac{1}{2}\right)$$

Bhabha Scattering

Considered to use Bhabha scattering to determine luminosity spectrum
- Intermediate angle Bhabha can be used to measure the ratio of the colliding particle energies
- Can unfold into two colliding beam energy spectra
- Can refold to obtain centre-of-mass energy spectrum

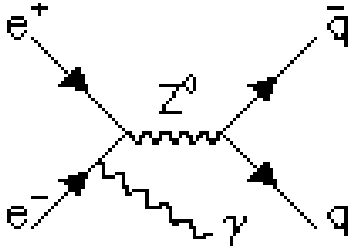
Small deflection of initial particles and of the Bhabha particles can impact the accuracy of this process

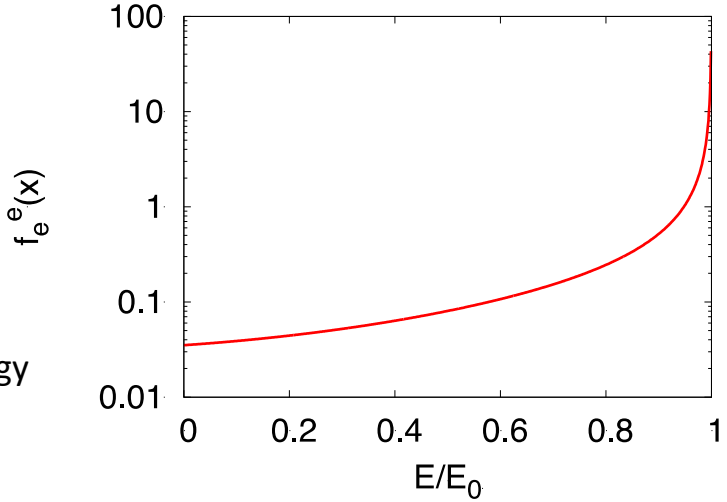
Studies
- Used BHLUMI to generate Bhabha events including first-order radiative corrections
- Loaded them into the colliding beams following the correct probability
- Tracked the particles
- Went through the reconstruction process

C. Rimbault, P. Bambade, K. Mönig and D. Schulte
2007 JINST 2 P09001

Determined that the accuracy was good enough to control the bias to the 10^{-3} –level
- Due to both, beamstrahlung and deflection

Initial State Radiation



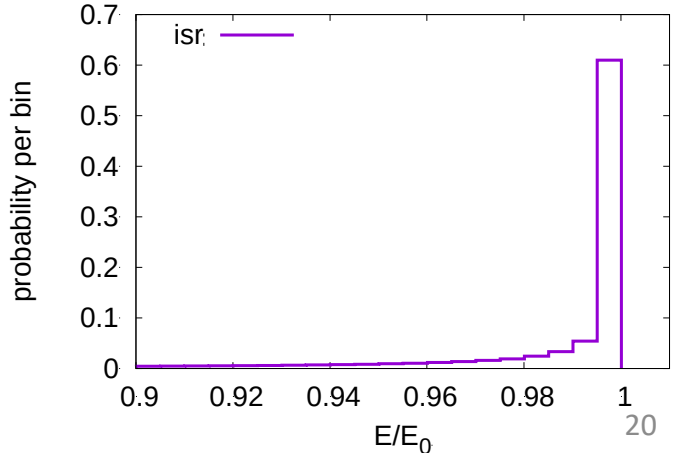


Initial state radiation can be modelled as in PYTHIA
About 65% probability of collision with more than 99% of nominal energy

Can also get collision energies and positions with equal probability and then use a full generator on this
Stub to store collision weighed with given cross section and to reload products later to track through beam fields

$$f_e^e(x,Q^2) = \frac{\beta}{2}(1-x)^{\frac{\beta}{2}-1}\left(1+\frac{3}{8}\beta\right)-\frac{\beta}{4}(1+x)$$

$$\beta = \frac{2\alpha}{\pi}\left(\ln\frac{Q^2}{m^2}-1\right)$$



Luminosity

The initial state radiation can be switched on `(do_isr=1;)` and off `(do_isr=0;)`

Can get collision data with equal weights before or after initial state radiation:

```
do_lumi=1;
num_lumi=1000000;
lumi_p=1e-28;
```

This will store collisions with a cross section of 1b in `lumi.ee.out`. Each time the number of entries exceeds `num_lumi, lumi_p` will be halved and half the events removed
For the CLIC study had large date base and also used a parameterized generator

This allows you to fully control ISR

Sensitivity to Beam Parameters

The initial beam conditions are critical for the collision

The simulations have several uncertainties

- The beam distribution develops over many turns
- Dynamic and static imperfections of the machine
- Beam-beam effect
- Need to establish if this is important for the physics analysis

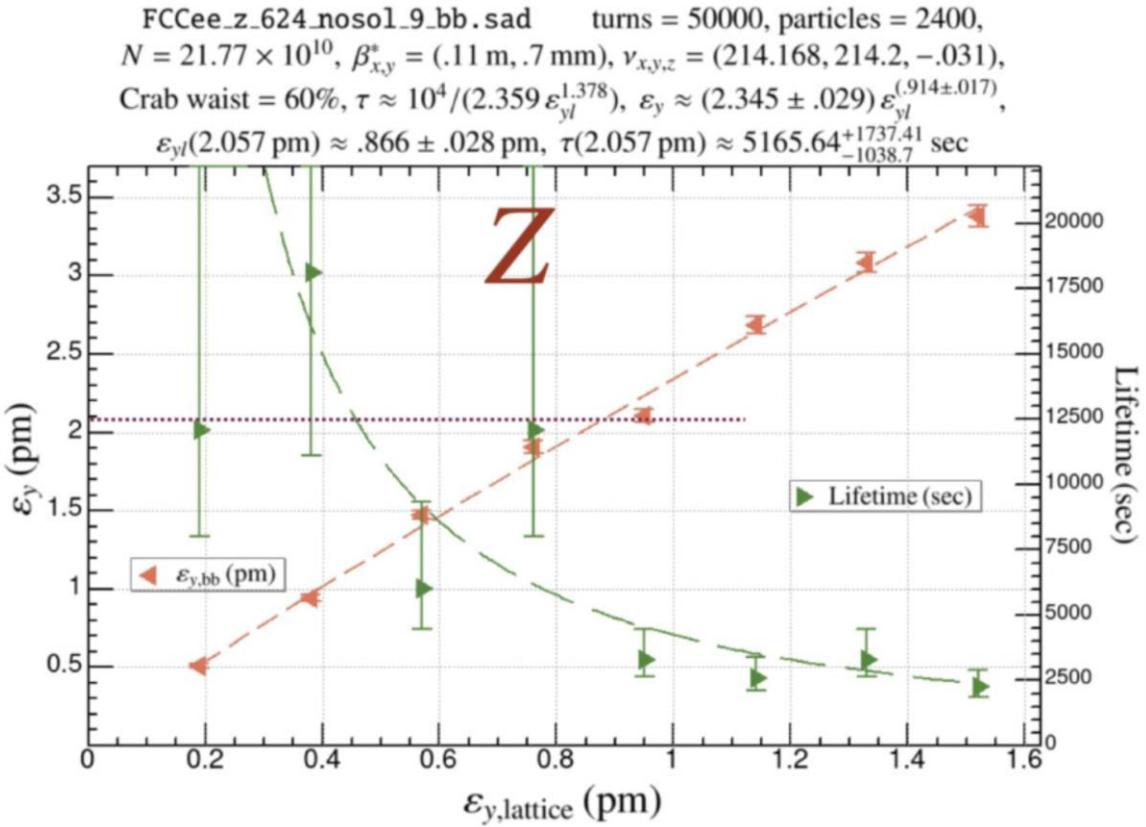
Possible algorithm:

- Obtain beam distributions from multi-turn simulations with different conditions
- Load them into GUINEA-PIG
- Perform the physics study
- To some degree the parameters are often be better known that controlled
    - This may allow to correct some systematic effects

Beam-beam Effect in FCC-ee

Very narrow footprint (much larger vertical tuneshift)

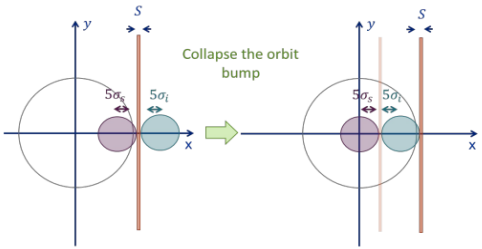
Tracking with beam-beam shows the increase of vertical emittance and beam lifetime

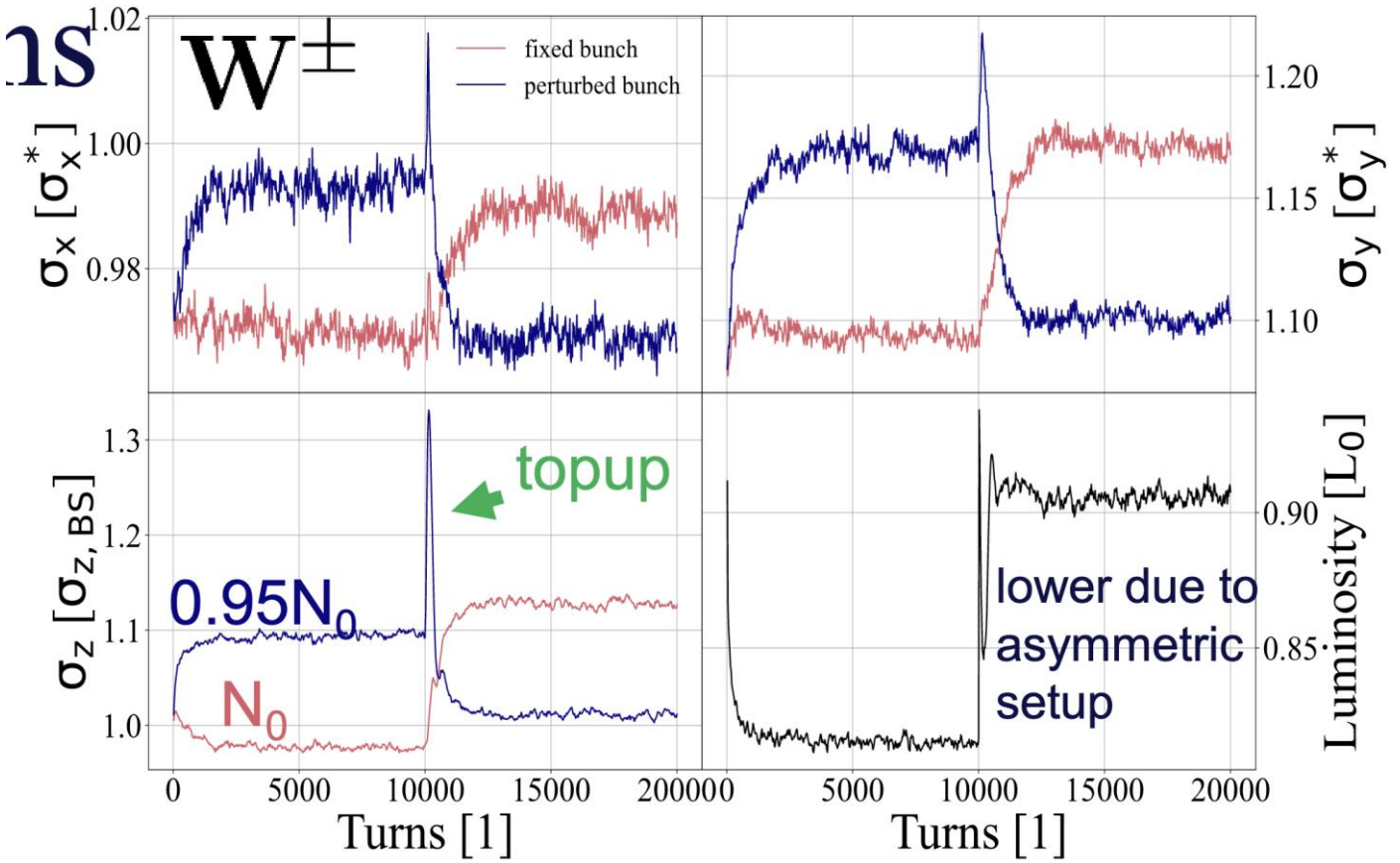


Uneven Bunch Parameters

Inject electrons or positrons

Some dynamic change of bunch parameters





Performance

Code was developed on an 80486 (66MHz) with 4 MB memory (later 8MB)

- E.g. float was used instead of double, where possible
- Flags in code to optimise for each use case
- Counters give time/memory used by different parts of GUINEA-PIG (not visible in ++)

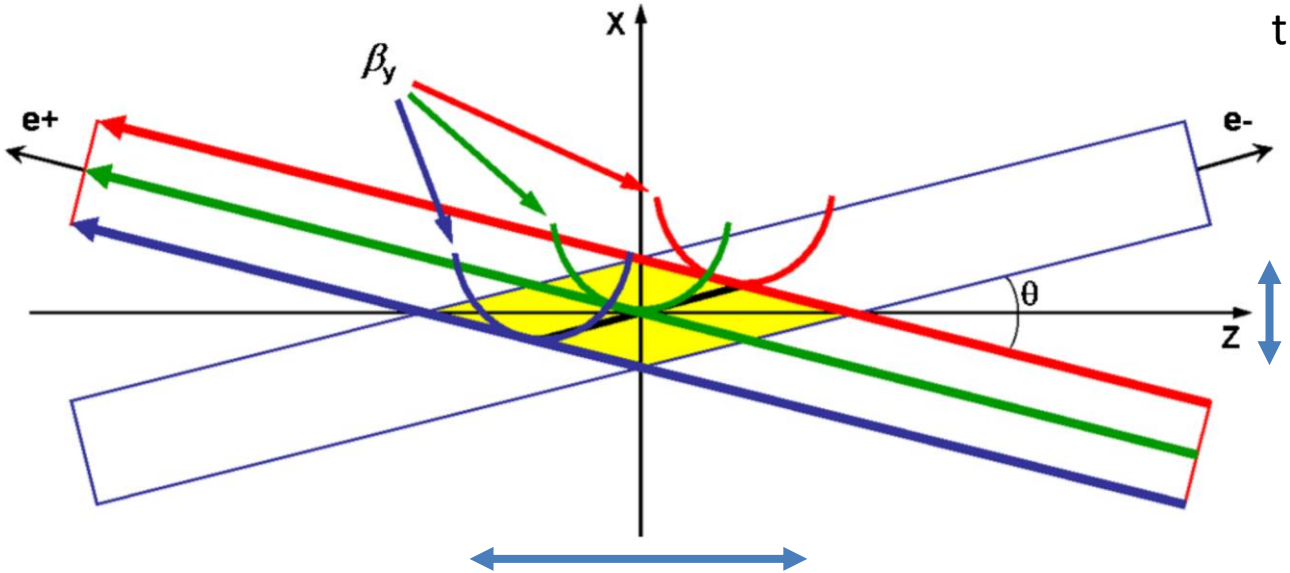
It gives relevant results on a raspberry pi 4 in 8s
On my laptop high resolution collision with no background is 5-15s depending on grid

Note: If you use many particles per beam, use variables electron_ratio and photon_ratio to calibrate the integrated luminosity per collision

Full simulation with incoherent pairs:
LC: O(1-2 minutes), FCC-ee at Z: O(0.2-2h) at tt: O(1-2 minutes)
Difference is dominated by solving the field equations

Calculation Challenge for FCC-ee

GUINEA-PIG uses grids and convolution with FFT to solve potentials (preferred FFTW)



time ~ cells *slices^2

Need to cover both beams

Granularity to resolve transverse beam in the collision

Granularity to resolve longitudinal overlap with enough slices

Need to cover the full collision length

Can this be improved?

Yes

For FCC-ee much of computing time is spent on FFTs and can be further reduced

- CPUs have changed, modification of FFTW runs (already implemented) can give factor of a few speed-up, e.g. on AMD64 CPUs
- Some careful modification of grid system for force calculations can be done

FCC-ee has low level of bremsstrahlung and pair creation per bunch crossing

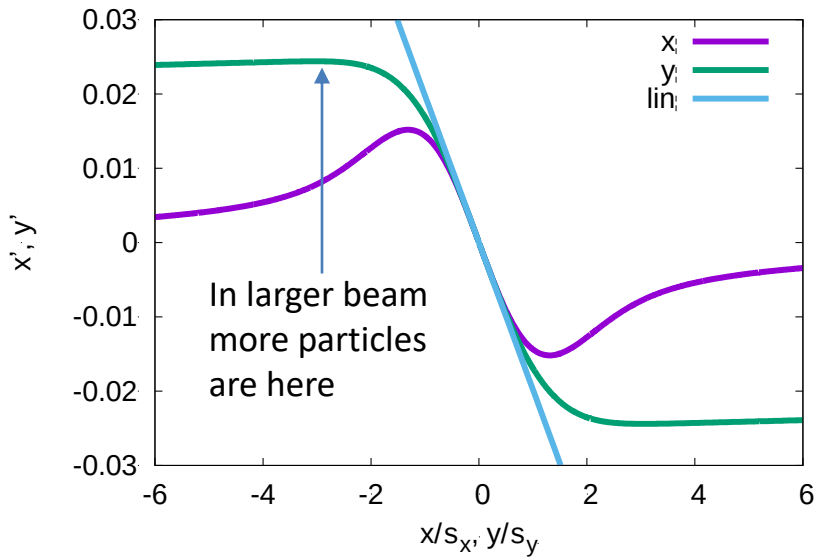
- An existing parameter can reduce secondaries, modify to increase (some logic change)

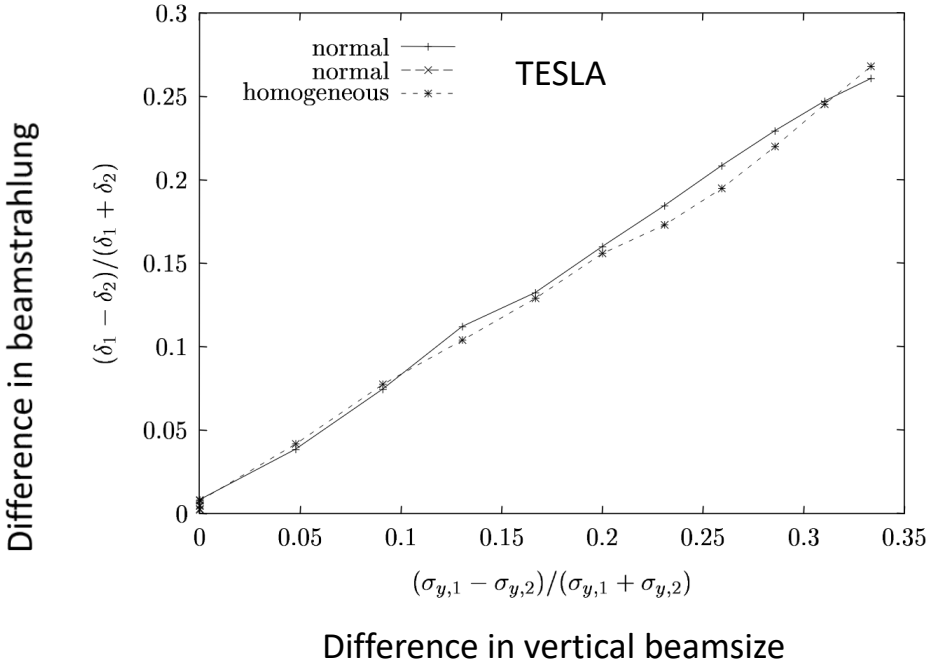
Note: Two variables `electron_ratio=1.0;` and `photon_ratio=1.0;` can be modified in the input file to have a reasonable number of collisions

Beamstrahlung and Luminosity Optimisation

Beamstrahlung can give very useful fast signal for luminosity optimisation

The vertically larger beam emits more beamstrahlung, the smaller less

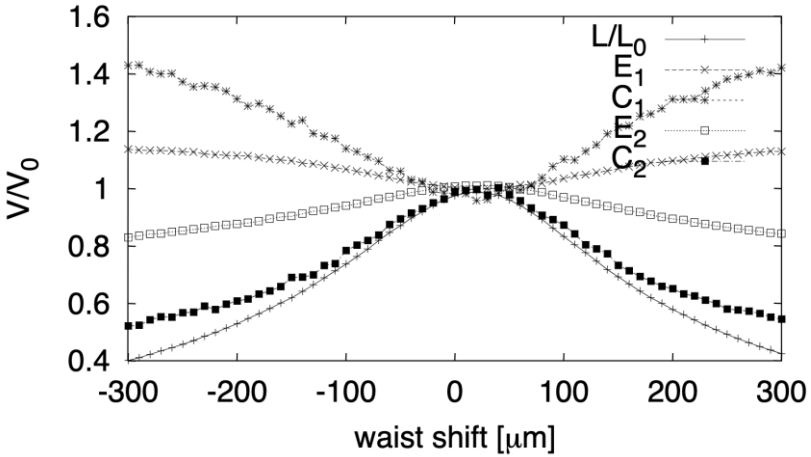




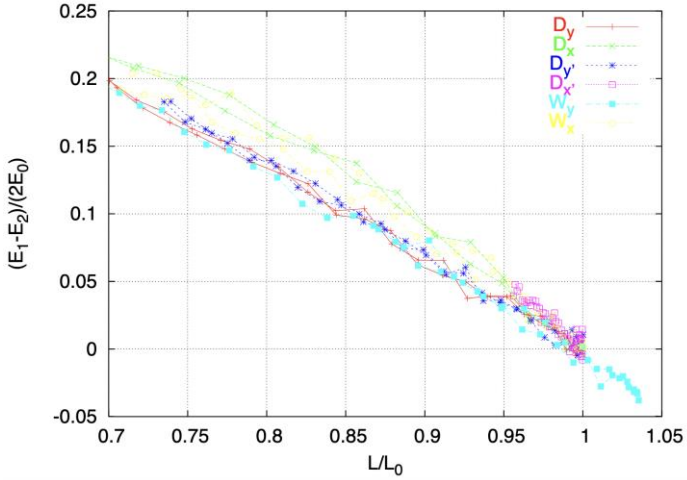
Beamstrahlung and Luminosity Optimisation

Also works for FCC_ee
But have plots for linear colliders only

Optimisation of the waist of one beam (CLIC)



Different causes of beamsize mismatch cause similar signals for given luminosity loss



Used machine learning to optimise CLIC tuning
Could probably use to develop correction of collision conditions for physics

Conclusion

Beam-beam effects are important parameter drivers

The physics models in GUINEA-PIG are essentially valid for future colliders
- Should check if there are some corners that require more precision

Key interfaces allow experts to feed specific generators and feed this back into the tracking
- E.g. Bhabha scattering
- Tracking of secondary particles from other event types is possible but requires some work
- May require a bit of work to bring it up to date

The initial beam conditions are variable and dynamically changing
- Dedicated studies should evaluate the impact on physics
- Need to identify how to monitor the conditions and minimize their impact
- Maybe AI/ML can help

GUINEA-PIG is fast enough for linear colliders and the muon collider
- But slow for FCC-ee because of the specific collision geometry
- This can be improved by an important factor with careful work

Many thanks to people for discussions K. Yokoya, O. Napoli, X. Buffat, P. Kicsiny, …

and my Doktorvater B.H. Wiik† who indirectly made me write GUINEA-PIG

Reserve

Uneven Bunch Parameters

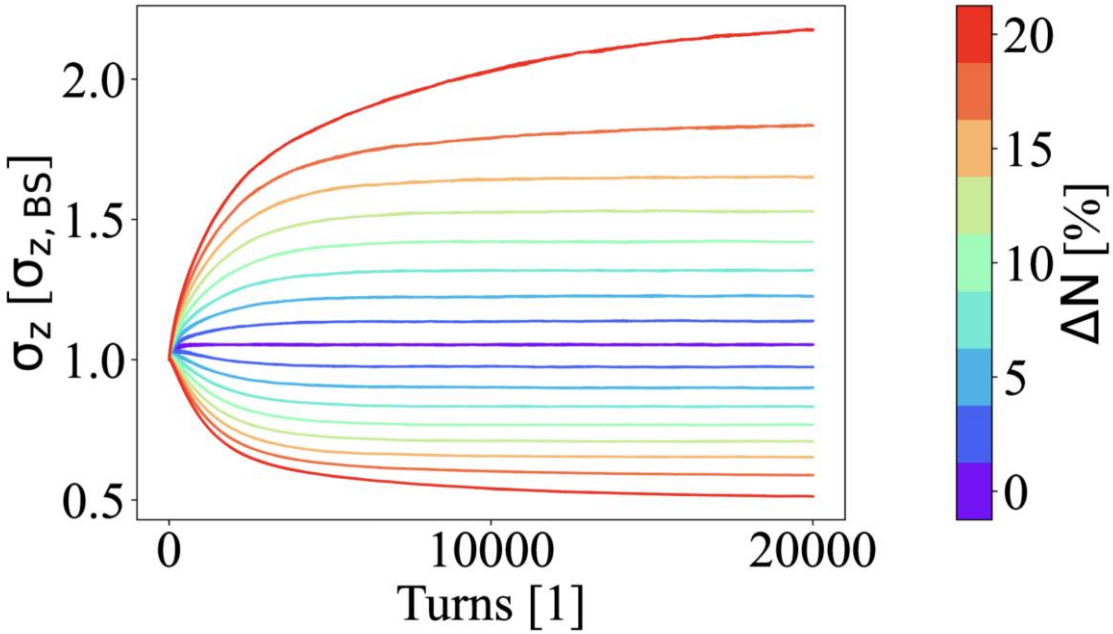
Differences in parameters (charge, emittances, beta-functions, …) between the colliding bunches can lead to an increase of the differences and also to differences in other parameters

One example:

Assume one bunch has more the other less than nominal charge

The bunch with larger charge emits less beamstrahlung because it sees less charge
⇒ It will become shorter
⇒ Its field will increase

The bunch with smaller charge sees more charge and emits more beamstrahlung
⇒ It will become longer
⇒ Its field will become weaker



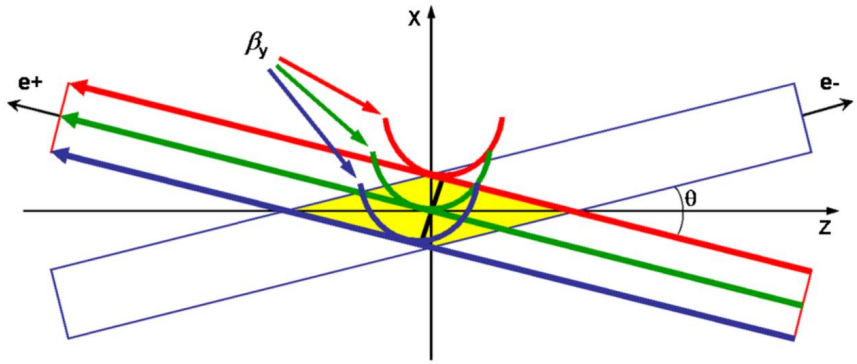
Crossing Angle Consequences

It shortens the region with collisions

- However, the hourglass effect anyway limits the gain further from the collision plane
- The beta-functions need to be comparable to the luminous length

Vertical beam-beam effect is reduced by about factor 26
Luminosity only by a factor of about 5

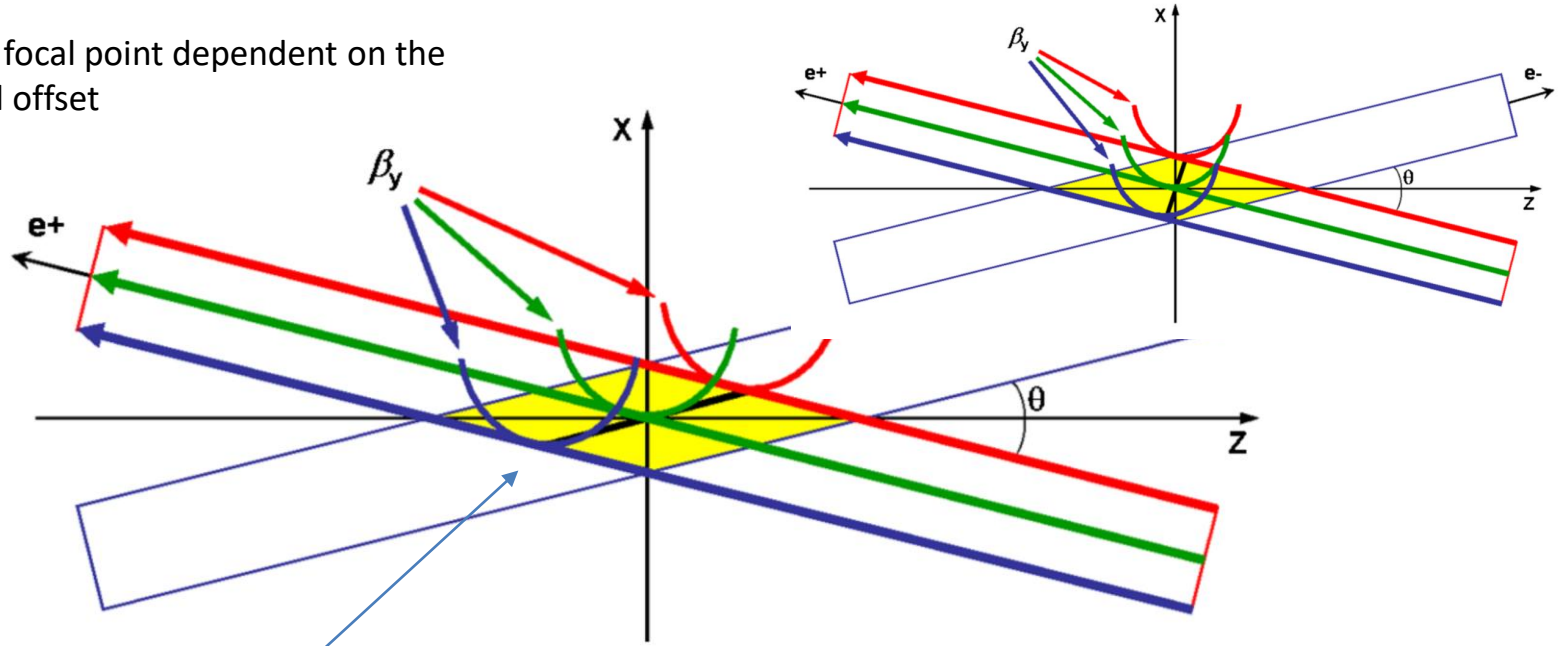
Beamstrahlung is reduced by factor 30, energy spread by about factor 10



Parameter	Z	
L [10^{34} $cm^{-2}s^{-1}$]	141	771
β_x / β_y [mm]	110/0.7	110/0.7
σ_x / σ_y [μm/nm]	8.8/38	8.8/38
Φ [mradian]	25.8	0
N_gamma	0.14	1.4
E_gamma [MeV]	1.7	5.5

Crabbed Waist

Make the focal point dependent on the horizontal offset



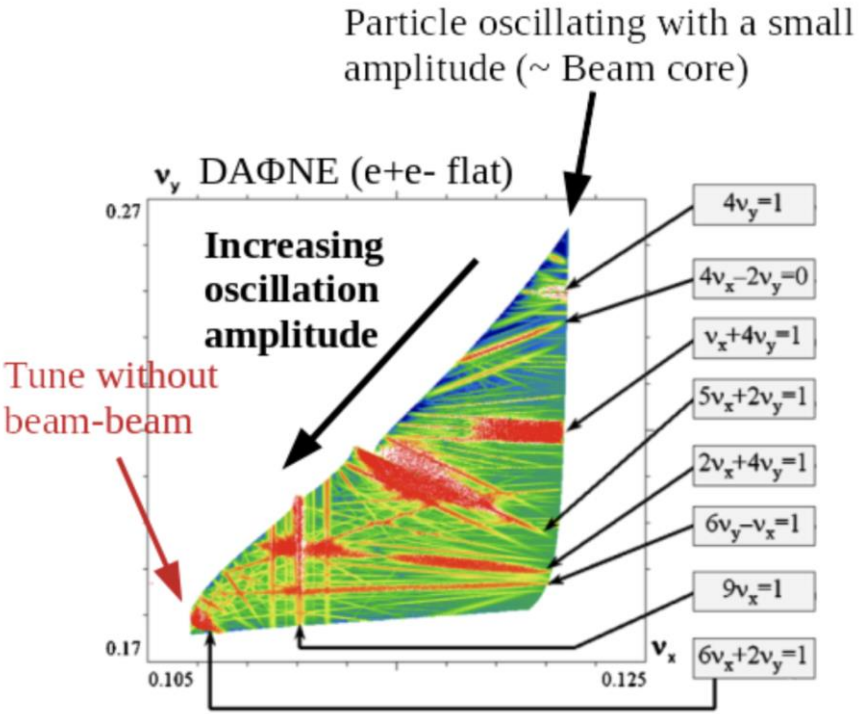
Consider particle with large x but zero y-offset and large y`
Normal waist: kick either up or down
Crabbed waist: to first order no vertical kick

The particles see the largest charge
And has the smallest RMS offset

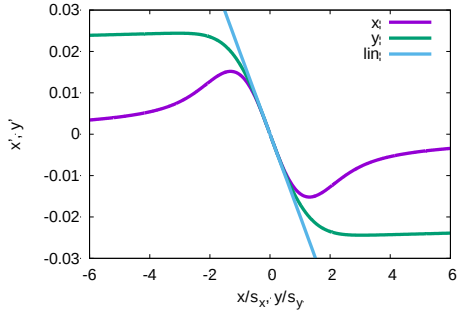
In practice simulations show how much crabbing of the waist is optimal

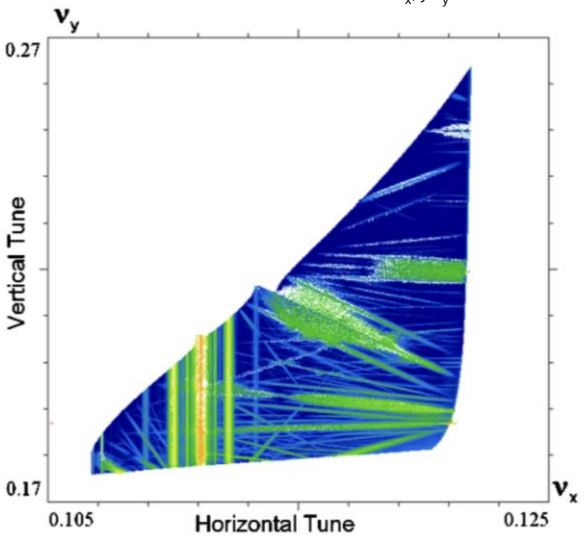
Tune Spread

Crabbed waist makes the particle more stable



(b) DAΦNE [25]



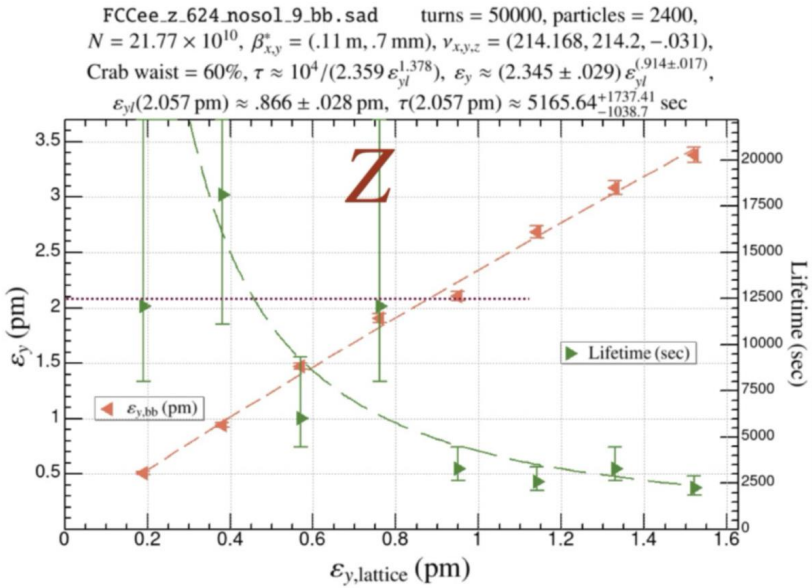


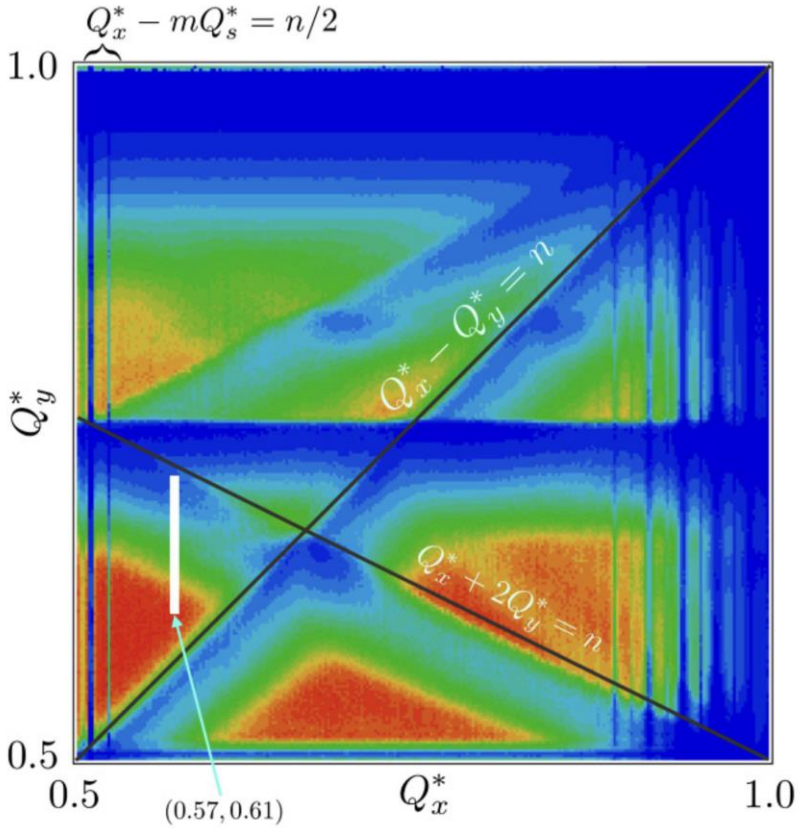
(d) With crab sextupoles

Tune Spread in FCC-ee

Very narrow footprint (much larger vertical tuneshift)

Tracking with beam-beam shows the increase of vertical emittance and beam lifetime

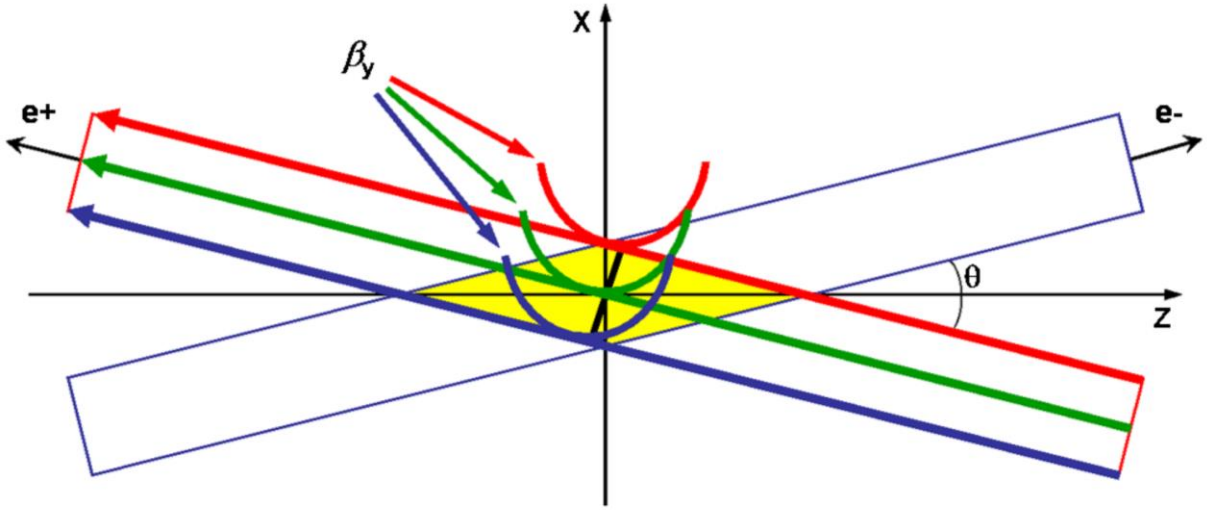




Crossing Angle

Introducing a crossing angle can be understood as increasing σ_x to $\sigma_x\sqrt{1+\emptyset^2}$

(add σ_x z/σ_z to transverse position)



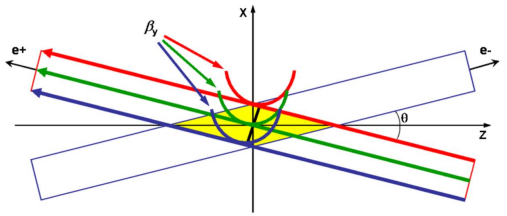
Crossing angle reduces the beam-beam tuneshift to

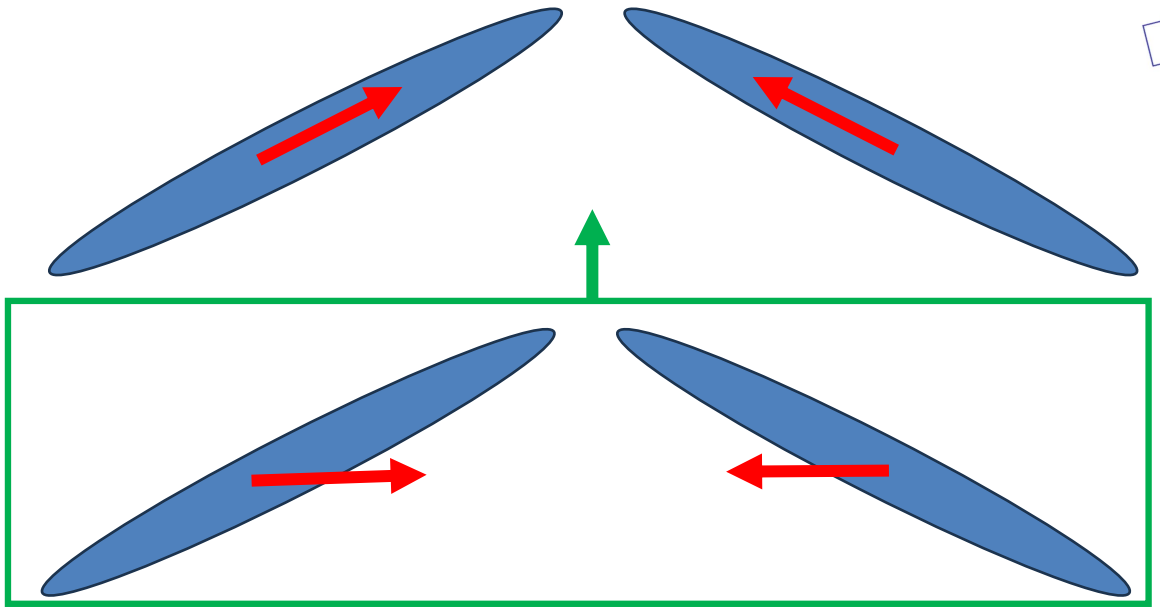
$$\xi_x = \frac{N_b r_e}{2\pi\gamma}\frac{\beta_x^*}{\sigma_x^2(1+\phi^2)} \approx 3\cdot 10^{-3}$$

$$\xi_y = \frac{N_b r_e}{2\pi\gamma}\frac{\beta_y^*}{\sigma_x\sigma_y\sqrt{1+\phi^2}} \approx 0.09$$

Crossing Angle

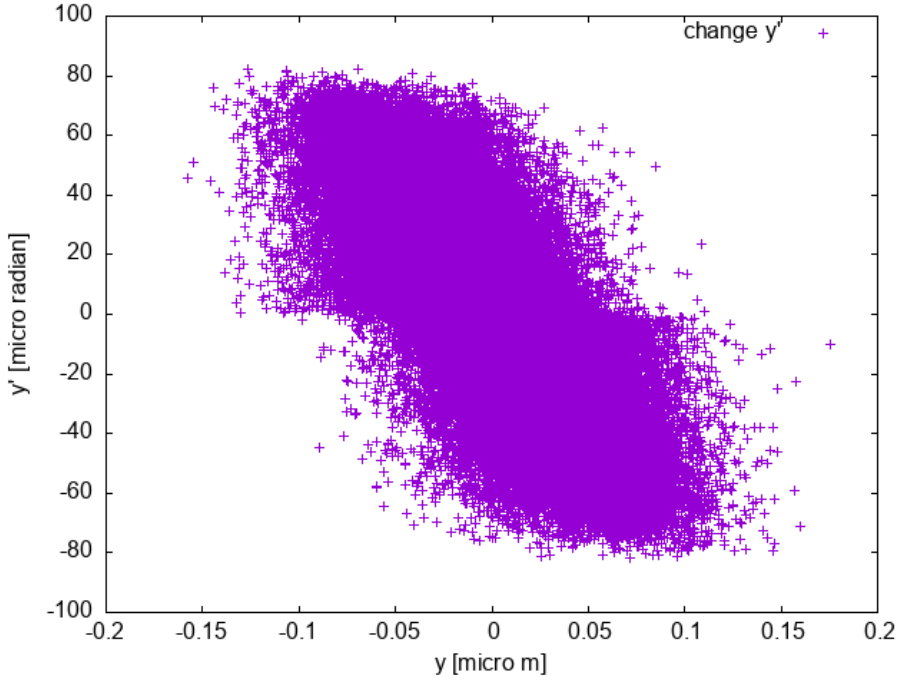
To first order crossing angle can be replaced by co-moving frame and angled bunches





Beam-beam Kick

Focusing vertical kick but strength depends on longitudinal and horizontal position



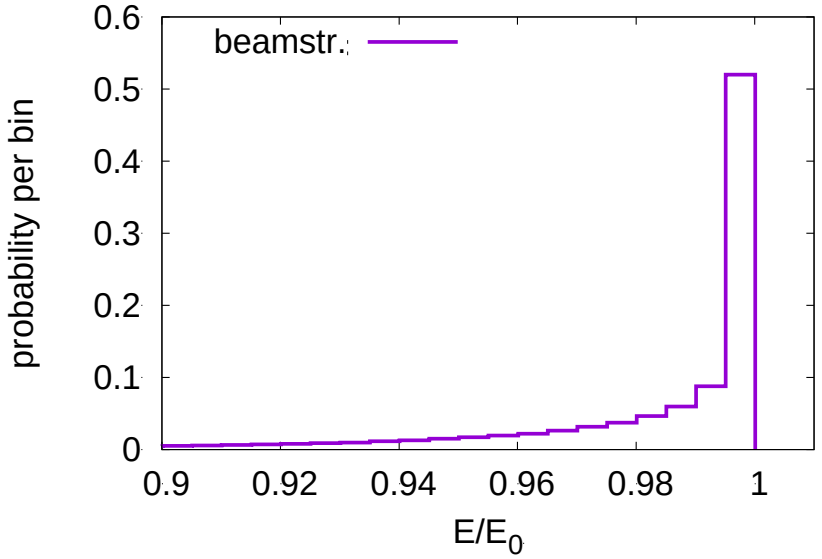
Luminosity Spectrum

Luminosity spectrum for ILC, CLIC at 380GeV looks similar

(beam energy spread is ignored)

Luminosity $L_{0.01}$ above 99% of nominal CMS energy is $L_{0.01}/L = 60\%$

For CLIC380, this has been the design criterion



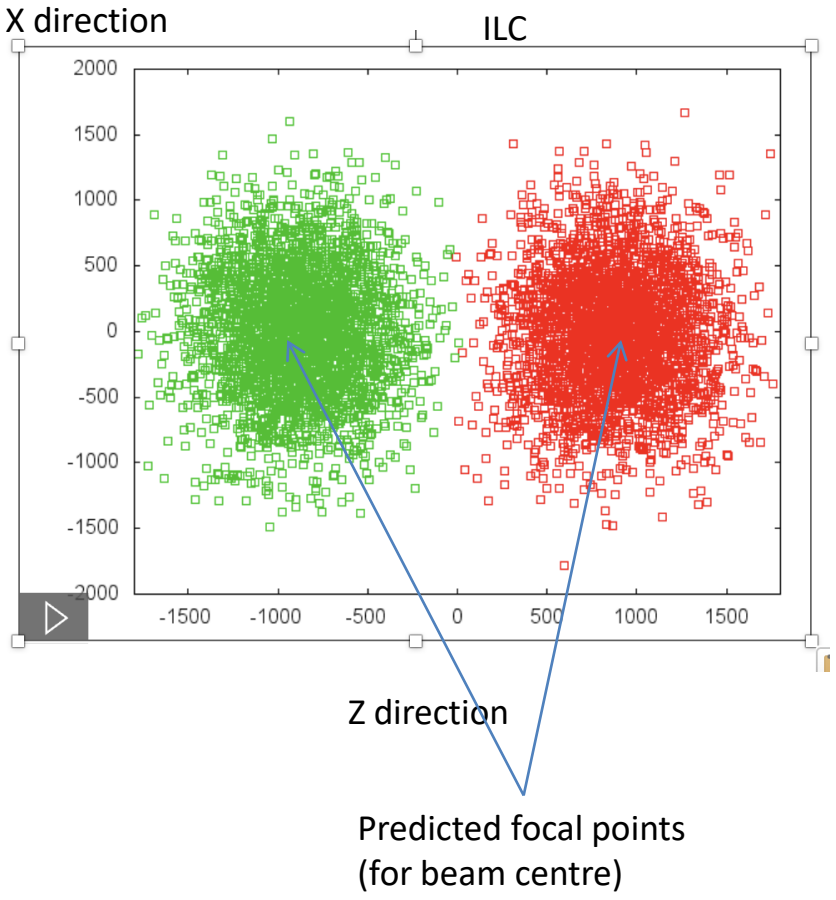
But why did the experiments chose $L_{0.01}/L > 60\%$?

Simulation Codes

Need strong-strong code, e.g.
- CAIN (K. Yokoya et al.)
- GUINEA-PIG (D. Schulte et al.)

- Beams => macro particles
- Beams => slices
- Slices => cells
- The simulation is performed in a number of time steps in each of them
- The macro-particle charges are distributed over cells
  - The forces at the cell locations are calculated
  - The forces are applied to the macro particles
  - The particles are advanced

All simulation performed with GUINEA-PIG



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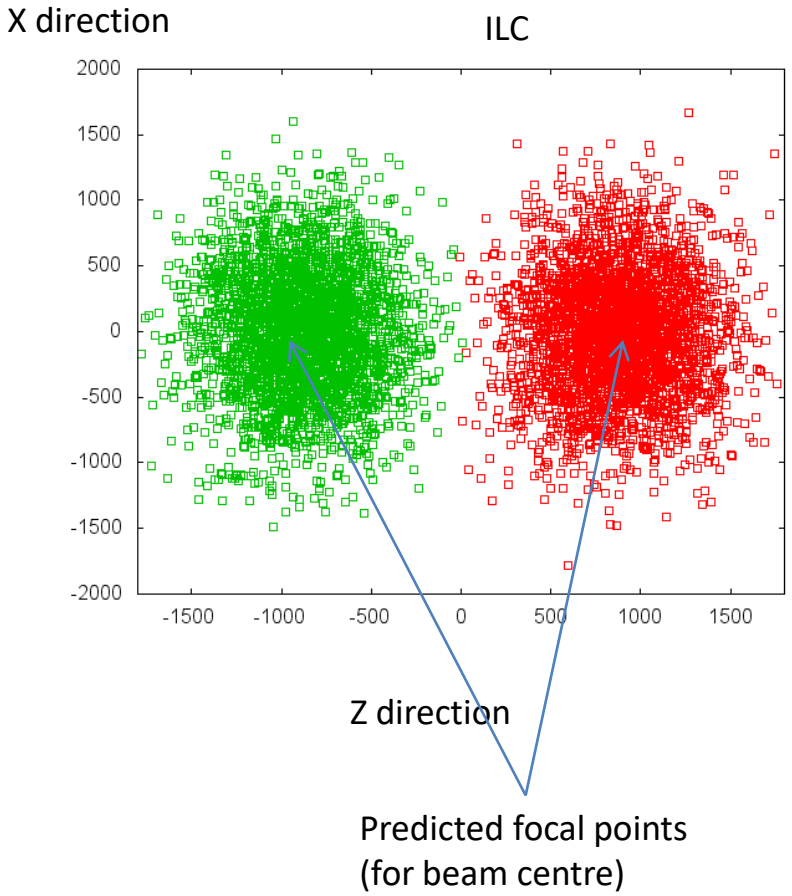

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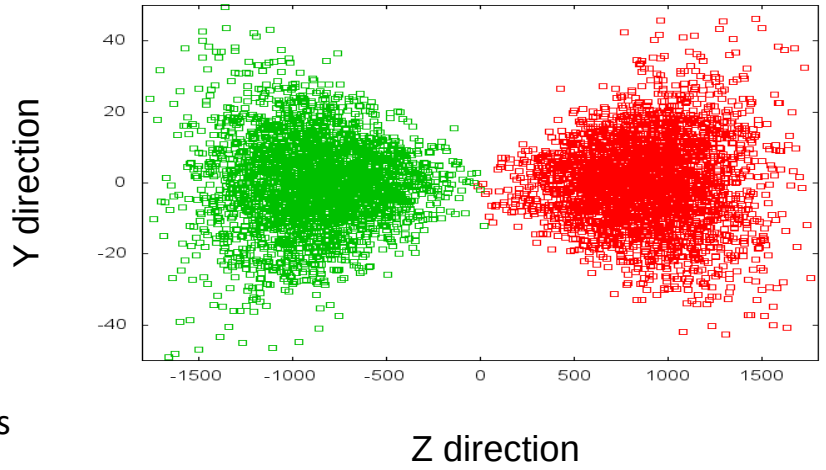
Beam-beam force switched off

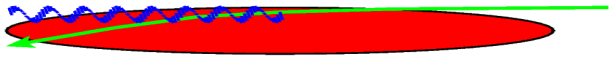


One can see the hourglass effect

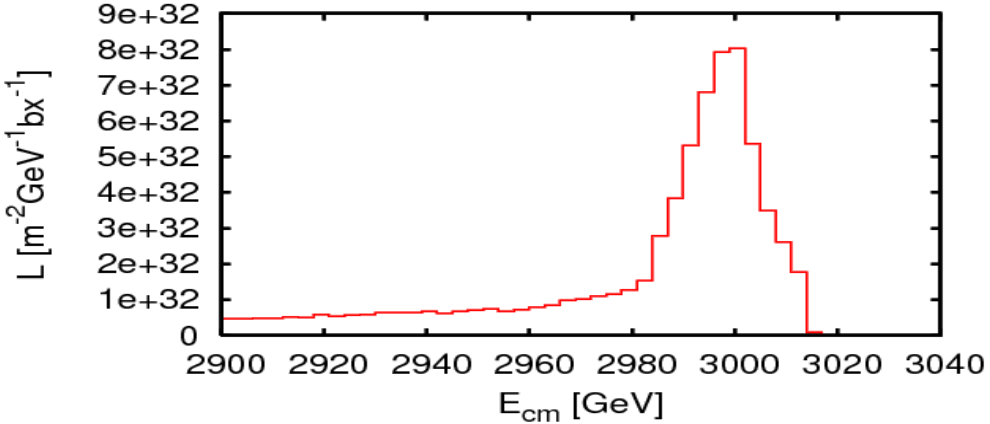
All simulation performed with GUINEA-PIG

Beamstrahlung

$$\mathcal{L} \propto H_D \frac{N}{\sigma_x} N n_b f_r \frac{1}{\sigma_y}$$



Dense beams focus each other
⇒ emitt beamstrahlung



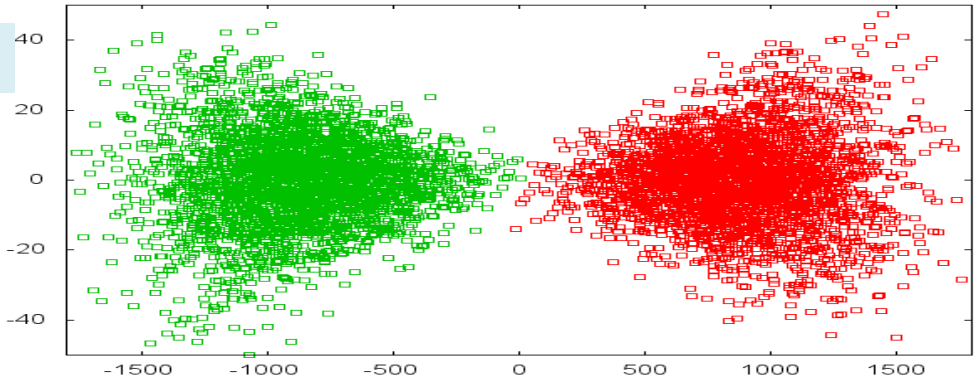
$$\mathcal{L} \propto \frac{N}{\sigma_x \sigma_y}$$

Typically aim for O(1)

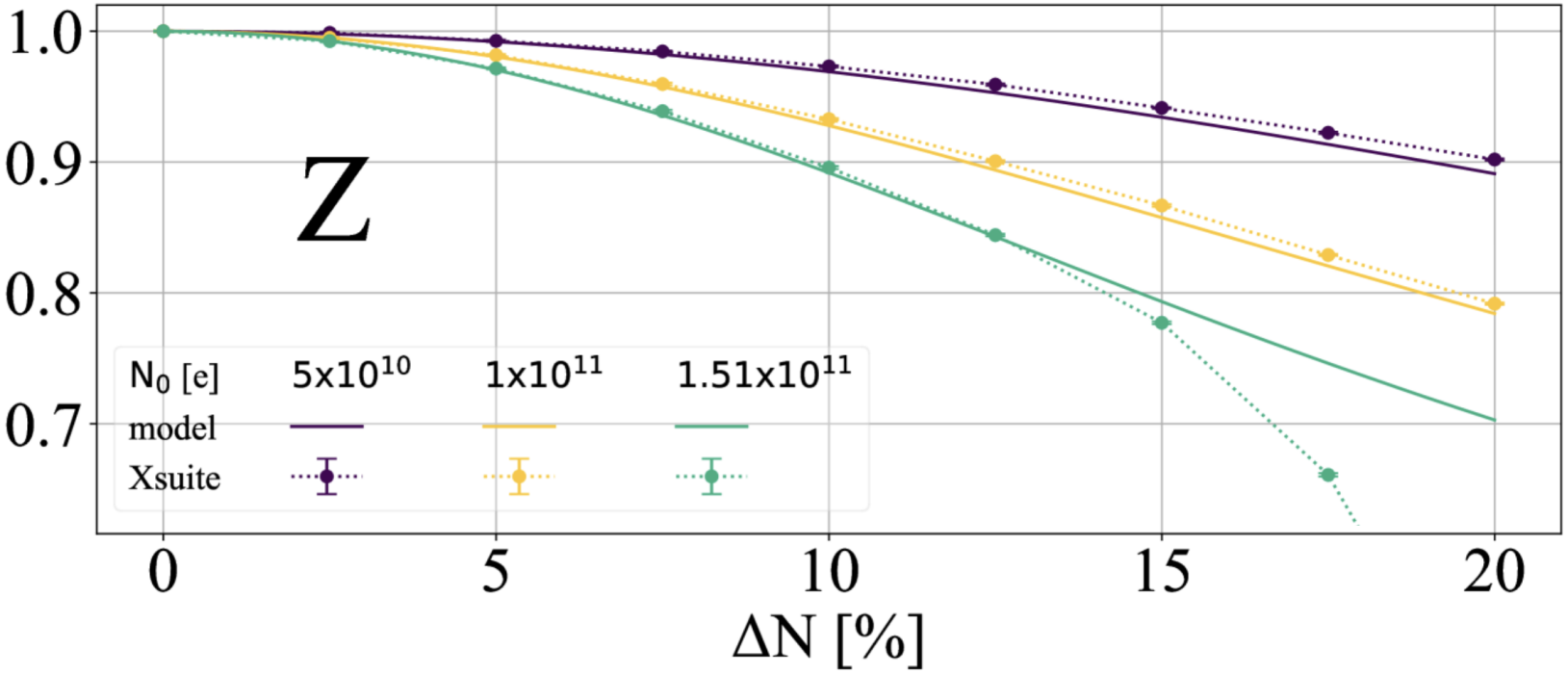
$$n_\gamma \propto E_\gamma \propto \frac{N}{\sigma_x + \sigma_y}$$

$$\sigma_x \gg \sigma_y \qquad \sigma_x + \sigma_y \approx \sigma_x$$

Beam-beam force on



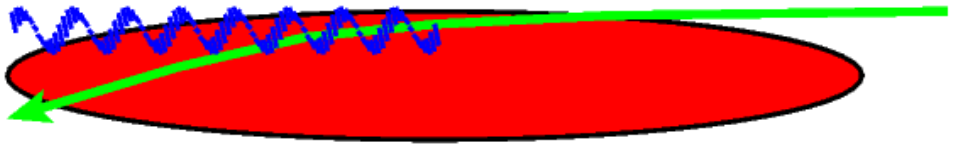
Impact on Luminosity



Also need to consider impact on luminosity measurement

Beamstrahlung

Similar to synchrotron radiation



From local trajectory curvature calculate critical

$$\hbar\omega_c = \frac{3}{2}\frac{\gamma^3\hbar c}{\rho}$$

Define beamstrahlung parameter Upsilon

$$\Upsilon = \frac{2}{3}\frac{\hbar\omega_c}{E_0}$$

Average Upsilon is approximately given by

$$\langle\Upsilon\rangle = \frac{5}{6}\frac{Nr_e^2\gamma}{\alpha(\sigma_x+\sigma_y)\sigma_z}$$

Beamstrahlung

Classical regime $\Upsilon \ll 1$

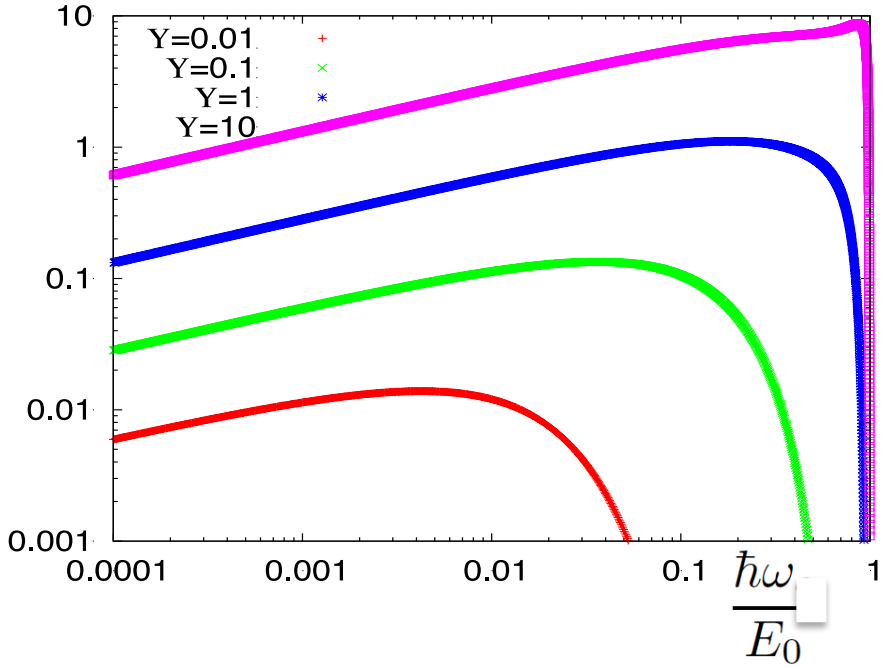
ILC Υ=0.06

CLIC at 380GeV Υ=0.17

Quantum regime $\Upsilon \gg 1$

CLIC at 3TeV Υ=5

Power [a.u.]



With modified Bessel functions $\mathrm{K}_{\frac{5}{3}}$ $\mathrm{K}_{\frac{2}{3}}$

and $x = \frac{\omega}{\omega_c}\frac{E}{E-\hbar\omega}$

The spectrum is given by $\frac{\mathrm{d}\,\dot{w}}{\mathrm{d}\,\omega} = \frac{\alpha}{\sqrt{3}\pi\gamma^2}\left[\int_x^\infty \mathrm{K}_{\frac{5}{3}}(x')\mathrm{d}x' + \frac{\hbar\omega}{E}\frac{\hbar\omega}{E-\hbar\omega}\mathrm{K}_{\frac{2}{3}}(x)\right]$

Model Limits

The modelling of beamstrahlung, bremsstrahlung, incoherent and coherent pair creation, trident cascade process and hadrons simplifies models

Example:
The beamstrahlung is modelled as synchrotron radiation, i.e. as a part of a circular orbit
The total deflection angle divided by the opening cone of the radiation $\eta >> 1$

$$\eta = \gamma\theta_x = D_x\frac{\sigma_x}{\sigma_z}\gamma = \frac{2Nr_e}{\sigma_x + \sigma_y}$$

A shorter bunch would change the spectrum, like the radiation from a short magnet

Virtual Photon Approximation

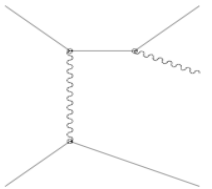
GUINEA-PIG uses Weizsäcker-Williams approximation for pair and bremsstrahlung production

- Cross section for real initial photons is being used
- The cut-off depends on the process

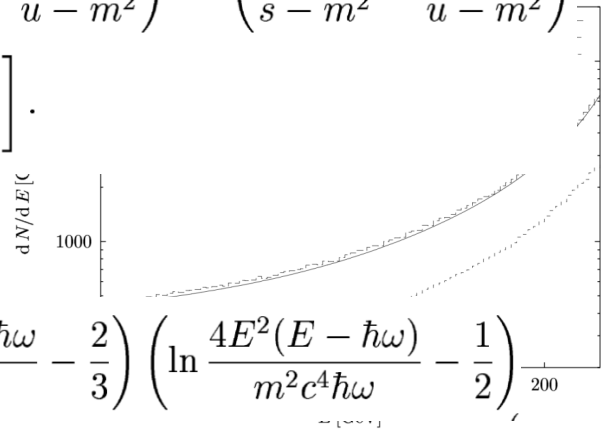
The production routine has access to the virtuality of the photon and can decide what to do

$$\frac{\mathrm{d}^2 n_v(x)}{\mathrm{d}\,x\,\mathrm{d}\,Q^2} = \frac{\alpha}{2\pi}\frac{1+(1-x)^2}{x}\frac{1}{Q^2}$$

$$\frac{\mathrm{d}\,n_v(x)}{\mathrm{d}\,x} = \frac{\alpha}{2\pi}\frac{1+(1-x)^2}{x}\ln\frac{\hat{Q}^2}{\check{Q}^2}$$



$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = 8\pi\frac{r_e^2 m^2}{(s-m^2)^2}\left[\left(\frac{m^2}{s-m^2}+\frac{m^2}{u-m^2}\right)^2+\left(\frac{m^2}{s-m^2}+\frac{m^2}{u-m^2}\right)-\frac{1}{4}\left(\frac{s-m^2}{u-m^2}+\frac{u-m^2}{s-m^2}\right)\right].$$



$$\frac{\mathrm{d}\sigma}{\mathrm{d}\omega} = 4\alpha r_e^2\frac{1}{\omega}\frac{E-\hbar\omega}{E}\left(\frac{E}{E-\hbar\omega}+\frac{E-\hbar\omega}{E}-\frac{2}{3}\right)\left(\ln\frac{4E^2(E-\hbar\omega)}{m^2c^4\hbar\omega}-\frac{1}{2}\right)$$

Model Limits

$$\eta = \gamma\theta_x = D_x \frac{\sigma_x}{\sigma_z}\gamma = \frac{2Nr_e}{\sigma_x + \sigma_y}$$

Virtual Photon Approximation

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- The upper cut-off depends on the process, the lower on momentum and energy conservation

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$$\frac{\mathrm{d}\,n_v(x)}{\mathrm{d}\,x} = \frac{\alpha}{2\pi}\frac{1+(1-x)^2}{x}\ln\frac{Q^2}{\check{Q}^2}$$

$$\check{Q}^2 = \frac{x^2 m^2}{1-x}$$

The finite beam size limits the production

- Observed for breamsstrahlung in VEPP-4, HERA and LEP

Different models exist:

- BBBREM assumes that virtual photon is limited by distance of electrons in the beam
  - Density dependent

$$b \approx \hbar/q_\perp \qquad \check{q}_\perp = \hbar/\sigma_y$$

Serbo et al. and Piotrzkowski assume that limit arises from beam size

- Independent of density
- This is used in GUINEA-PIG (with a switch)
  - Virtual photon is offset with respect to parent particle

Strong fields can further suppress virtual photon spectrum (Baier, Katkov)

$$\check{q}_\perp \approx xmc\left(\frac{\omega_c}{\omega}\right)^{1/3}$$

[35] A. E. Blinov et al. Phys. Lett. B, **113** (1982) 423.

[36] G. L. Kotkin, V. G. Serbo, and A. Schiller. Processes with large impact parameters at colliding beams. Int. J. Mod Phys. A, **7** (1992) 4707–4745.

[37] J. L. Augneres et al. HERA. A Proposal for a large Electron-Positron Colliding Beam Facility at DESY. DESY HERA 81/10 (1981).

[38] K. Piotrzkowski. Observation of the beam-size effect at HERA. DESY 95-051 (1995).

[39] V. N. Baier and V. M. Katkov. Production of bremsstrahlung during collisions of high-energy particles in a magnetic field. Sov. Phys. Dokl., **17** (1973) 1068.

Beamsize Effect

Observed in VEPP-4, HERA and LEP

Different models exist:
Common point: Cut-off in photon in the process

BBBREM assumes that virtual photon is limited by distance of electrons in the beam
- Density dependent

Serbo et al. assume that limit arises from beam size
- Independent of density

This is used in GUINEA-PIG
- Virtual photon is offset with respect to parent particle

$$\check{Q}^2 = \frac{x^2 m^2}{1-x}$$

$$\frac{\mathrm{d}\, n_v(x)}{\mathrm{d}\, x} = \frac{\alpha}{2\pi} \frac{1+(1-x)^2}{x} \ln \frac{\hat{Q}^2}{\check{Q}^2}$$