Event generators for Bhabha scattering

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Gearing up for Future Colliders
Workshop on Radiative Corrections at Future Lepton Colliders

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Outline

- Bhabha & luminosity
- The LEP era: Monte Carlo generators (and integrators) for Bhabha
- Bhabha at flavour factories
- Present (& future)
- Alternative to Bhabha: $e^+e^- \to \gamma\gamma$

Reference processes for luminosity

- Instead of getting the luminosity from machine parameters, it's more effective to exploit the relation

$$\sigma = \frac{N}{L} \quad \rightarrow \quad L = \frac{N_{\text{ref}}}{\sigma_{\text{theory}}} \qquad \frac{\delta L}{L} = \frac{\delta N_{\text{ref}}}{N_{\text{ref}}} \oplus \frac{\delta \sigma_{\text{theory}}}{\sigma_{\text{theory}}}$$

- Reference (*normalization*) processes are required to have a clean topology, high statistics and be calculable with high theoretical accuracy

⋆ At LEP and future high-energy e^+e^- machines **small-angle** Bhabha scattering is the golden process

⋆ **Large-angle** processes as Bhabha, $e^+e^- \to \gamma\gamma$, $e^+e^- \to \mu^+\mu^-$ are golden processes at flavour factories

- At future machines, the theoretical accuracy must go a step further: $\mathcal{O}(10^{-3}) \to \mathcal{O}(10^{-4})$

↪ **Inclusion of QED radiative corrections is mandatory**

Why SA or LA?

- at LEP and future high-energy colliders, the choice is small-angle Bhabha scattering (SABS)
  - ⇝ Z contribution is tiny ($\simeq 0.3\%$), t-channel γ-exchange overwhelmingly dominates
  - ⇝ dedicated detectors for luminosity installed
- at flavour factories, the central region is preferred (LABS)
  - ⇝ Z tiny due to $\sqrt{s}$, t-channel is larger than s-channel
  - ⇝ no dedicated luminometers
- in any case, it's essentially a QED process

A recent example: $\mathbf{N}_\nu$ from $\mathbf{\Gamma}_Z^{\rm inv}$ at LEP $\mathbf{Z}$ peak measurements

- assuming lepton universality

$$N_\nu \left(\frac{\Gamma_{\nu\bar{\nu}}}{\Gamma_{ll}}\right)_{\rm SM} = \sqrt{\frac{12\pi R_l^0}{\sigma_{\rm had}^0 m_Z^2}} - R_l^0 - (3+\delta_\tau)$$

$$\mathbf{N_\nu} = \mathbf{2.9840 \pm 0.0082}$$

$$\delta N_\nu \simeq 10.5\frac{\delta n_{\rm had}}{n_{\rm had}} \oplus 3.0\frac{\delta n_{\rm lept}}{n_{\rm lept}} \oplus 7.5\frac{\delta\mathcal{L}}{\mathcal{L}}$$

$$\frac{\delta\mathcal{L}}{\mathcal{L}} = 0.061\% \Longrightarrow \delta N_\nu = 0.0046$$

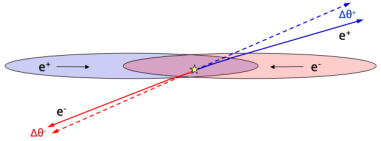
ADLO, SLD and LEPEWWG, Phys. Rept. 427 (2006) 257, hep-ex/0509008

2σ away from SM: hint for BSM? Right handed neutrinos?

Beam-beam effects studied in detail recently

G. Voutsinas, E. Perez, M. Dam, P. Janot, arXiv:1908.01704

- systematics bias on the acceptance due to e.m. beam-beam interactions $\Longrightarrow$ underestimate of luminosity by $\sim 0.1\%$



- together with an update on Bhabha cross sections (see later) $\Longrightarrow$ Luminosity

P. Janot, S. Jadach, arXiv:1912.02067

$$\mathbf{N_\nu = 2.9963 \pm 0.0074}$$

Luminosity is a key quantity for a precision $\mathbf{e^+e^-}$ collider

Classification of QED corrections

$$\sigma(L,\ell) \sim \alpha^k \sum_{m=0}^{\infty} \alpha^m \sum_{i=0}^{m} \sum_{j=0}^{m} c_m^{\{i,j\}} L^i \ell^j$$

$$L = \ln \frac{Q^2}{m_e^2} \qquad \text{collinear log}$$

$$\ell = \ln \frac{Q^2}{(\Delta E_\gamma)^2} \qquad \text{IR log}$$

- $L \simeq 24$ for $\sqrt{s} = 100$ GeV, $L \simeq 17$ for $\sqrt{-t} = 3$ GeV
- the size of the $L^i \ell^j$ coefficients strongly depends on event selection criteria and to some extent on the underlying process
- a framework for logs resummation is mandatory
  - ↪ YFS resummation
  - ↪ QED Parton Shower

QED resummation

- Instead of focussing on calculating order-by-order exact corrections in α for a given process, **QED Parton Shower** and **Yennie-Fraustchi-Suura resummation** take a different point of view as starting point:

  they aim at calculating **approximate** and **"universal"** corrections **up to all orders**, by including (the important, leading) contributions arising from soft and/or collinear regions

- They rely on the general property of **factorization of soft/collinear divergencies** (enhancements) in QED, which leads to **exponentiation**

⇝ Sometimes, in some phase-space regions, for some observables, for certain experimental cuts, you better have an approximate resummed result than a fixed-order one

$$\alpha < \alpha^2 L^2 \quad \text{somewhere}$$

Summary of QED (photonic) radiative corrections

LO	α^0		
NLO	αL	α	
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$
h.o.	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^n$	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^{n-1}$	$\dots$

Blue: some resummation framework

Summary of QED (photonic) radiative corrections

LO	α^0		
NLO	αL	α	
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$
h.o.	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^n$	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^{n-1}$	$\dots$

Red: some resummation framework matched to NLO

Summary of QED (photonic) radiative corrections

LO	90%		
NLO	10%	0.5%	
NNLO	0.5%	0.05%	0.01%
h.o.	0.01%	$\cdots$	$\cdots$

possible additional enhancements from infra-red logs induced by events selection

Yennie-Frautschi-Suura soft exponentiation

✓ The core concepts are unravelled in this work

D. R. Yennie, S. C. Frautschi and H. Suura

"The infrared divergence phenomena and high-energy processes", Ann. Phys. 13, 379 (1961)

⇝ Many Monte Carlos for LEP (and LHC) developed by Jadach, Płaczek, Ward, Was and colleagues on this framework

(`Koral[W/Z]`, `BH[LUMI/WIDE]`, `YFS[WW3/ZZ]`, `WINAC`, `KKMC`)

see talk by Zbigniew

Notably `BH[LUMI/WIDE]` for Bhabha

Jadach, Placzek, Ward, PLB **390** (1997), 298

Jadach, Placzek, Richter-Was, Ward, Was, CPC **102** (1997), 229

S. Jadach, Richter-Was, Ward, Was, CPC **70** (1992), 305

⇝ Nowadays YFS is the basis for QED radiation resummation in `Sherpa`.

Applied also to (future) e^+e^- machines

Price, Krauss, PRD 113 (2026) 7, 073008

Krauss, Price, Schönherr, SciPost Phys. 13, 026 (2022)

and Alan's talk

YFS: master formulae

- After factorizing out all soft virtual and soft real corrections, you end up with something like

$$\mathrm{d}\sigma = \sum_{n_\gamma=0}^{\infty} \frac{e^{Y(\Omega)}}{n_\gamma!}\,\mathrm{d}\Phi_Q \left[\prod_{i=1}^{n_\gamma} \mathrm{d}\Phi_i^\gamma\, \tilde{S}\,(k_i)\,\Theta(k_i,\Omega)\right]\left(\tilde{\beta}_0 + \sum_{j=1}^{n_\gamma} \frac{\tilde{\beta}_1(k_j)}{\tilde{S}\,(k_j)} + \sum_{\substack{j,k=1\\ j<k}}^{n_\gamma} \frac{\tilde{\beta}_2(k_j,k_k)}{\tilde{S}\,(k_j)\,\tilde{S}\,(k_k)} + \cdots\right)$$

  where

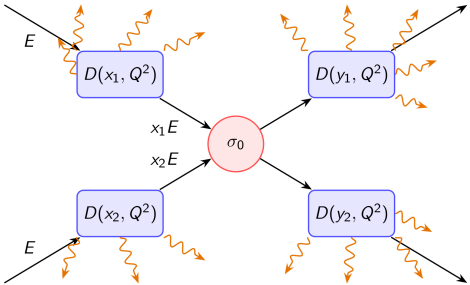
- $e^{Y(\Omega)}$ resums all soft virtual and soft real emissions, $\tilde{S}(k_i)$ are eikonal factors, $\tilde{\beta}_n$ are IR-subtracted matrix elements remnants (with n photons)
- fully exclusive event generation
- inclusion of exact higher-order matrix elements is "natural": $\tilde{\beta}_1 \neq 0$ matches to NLO, $\tilde{\beta}_2 \neq 0$ matches to NNLO, ...

towards NNLO EW matching → Price, Krauss, PRD 113 (2026) 7, 073008

- two flavours:
  - **EEX**: exclusive exponentiation: works at $|\mathcal{M}|^2$ level
  - **CEEX**: coherent exclusive exponentiation: works at $\mathcal{M}$ level: implemented in `KKMC`, more "difficult" to implement

QED collinear Structure Functions

- PS algorithms rely on QCD-inspired Structure Function approach to radiative corrections



- If we are interested only in photon radiation, $D(x, Q^2)$ are the **Leading-Log non-singlet QED SF**

  Kuraev Fadin, Nicrosini Trentadue, Cacciari Deandrea Montagna Nicrosini, Arbuzov, ...

- Recent renewed interest, going to **NLL** and **NNLL**

  Frixione, Bertone/Cacciari/Frixione/Stagnitto

  see talks by Giovanni & Stefano & Kay

Bhabha generators and integrators for LEP

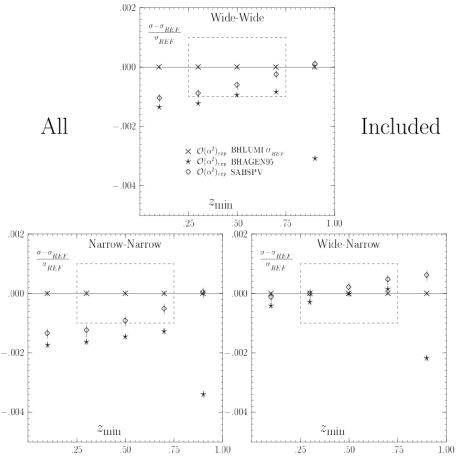
Event generators for Bhabha scattering, conveners Jadach, Nicrosini, CERN Workshop on LEP2 Physics, feb 1996, hep-ph/9602393

- BHAGEN95 (integrator): Caffo, Czyż, Remiddi
  Structure functions + NLO
- BHAGENE3 (event generator): Field, Riemann
  Large angles, NLO + h.o. in some sort of soft approximation
- BHLUMI (event generator): Jadach, Richter-Was, Ward, Was
  YFS resummation on e^+ and e^- legs, improved with collinear logs, exact NLO. Taylored for SABS
- BHWIDE (event generator): Jadach, Płaczek, Ward
  "BHLUMI" for large angles. Includes up-down interferences
- NLLBA (semi-analytical): Arbuzov, Kuraev, Trentadue
  Structure functions with information from NLO matrix elements
- SABSPV (integrator): Cacciari, Montagna, Nicrosini, Piccinini
  Structure functions supplemented with NLO (additive or factorized form)
- UNIBAB (event generator): Anlauf, Ohl
  Large angles, ISR and FSR with a sort of QED shower

Comparison of Bhabha generators and integrators for LEP

form Arbuzov et al., PLB 383 (1996) 238

All



Included

Figure 2: Monte Carlo results for various symmetric/asymmetric versions of the CALO2 ES, for matrix elements beyond first order. Z exchange, up-down interference and vacuum polarization are switched ON. The center of mass energy is $\sqrt{s} = 92.3$ GeV. z_{min} is the cut condition on the final-state energies, defined as $E_+E_-/E^2 \geq z_{min}$, $E_{-,+}$ being the final-state energy of the electron and positron clusters, respectively. The *fiducial* angular range is 0.024-0.058 rad (for more details on the ES, the reader is referred to [1]).

Theoretical accuracy: LEP & future machines

from Jadach, Płaczek, Skrzypek, Ward, Yost, PLB 790 (2019) 314

Table 1
Summary of the total (physical+technical) theoretical uncertainty for a typical calorimetric detector. For LEP1, the above estimate is valid for a generic angular range within 1°–3° (18–52 mrads), and for LEP2 energies up to 176 GeV and an angular range within 3°–6°. Total uncertainty is taken in quadrature. Technical precision included in (a).

Type of correction/error	LEP1		LEP2	
	1996	1999	1996	1999
(a) Missing photonic $\mathcal{O}(\alpha^2)$ [4,5]	0.10%	0.027%	0.20%	0.04%
(b) Missing photonic $\mathcal{O}(\alpha^3 L_e^3)$ [6]	0.015%	0.015%	0.03%	0.03%
(c) Vacuum polarization [7,8]	0.04%	0.04%	0.10%	0.10%
(d) Light pairs [9,10]	0.03%	0.03%	0.05%	0.05%
(e) Z and s-channel γ [11,12]	0.015%	0.015%	0.0%	0.0%
Total	0.11% [12]	0.061% [13]	0.25% [12]	0.12% [13]

Table 2
Summary of the total (physical+technical) theoretical uncertainty for a typical calorimetric LEP luminosity detector within the generic angular range of 18–52 mrad. Total error is summed in quadrature.

Type of correction/error	1999	Update 2018
(a) Photonic $\mathcal{O}(L_e\alpha^2)$	0.027% [5]	0.027%
(b) Photonic $\mathcal{O}(L_e^3\alpha^3)$	0.015% [6]	0.015%
(c) Vacuum polariz.	0.040% [7,8]	0.013% [26]
(d) Light pairs	0.030% [10]	0.010% [18,19]
(e) Z and s-channel γ exchange	0.015% [11,12]	0.015%
(f) Up-down interference	0.0014% [28]	0.0014%
(f) Technical Precision	–	(0.027)%
Total	0.061% [13]	0.038%

Table 3
Anticipated total (physical+technical) theoretical uncertainty for a FCC-ee luminosity calorimetric detector with the angular range being 64–86 mrad (narrow), near the Z peak. Description of photonic corrections in square brackets is related to the 2nd column. The total error is summed in quadrature.

Type of correction/error	Update 2018	FCC-ee forecast
(a) Photonic $[\mathcal{O}(L_e\alpha^2)]$ $\mathcal{O}(L_e^2\alpha^3)$	0.027%	0.1×10^{-4}
(b) Photonic $[\mathcal{O}(L_e^3\alpha^3)]$ $\mathcal{O}(L_e^4\alpha^4)$	0.015%	0.6×10^{-5}
(c) Vacuum polariz.	0.014% [26]	0.6×10^{-4}
(d) Light pairs	0.010% [18,19]	0.5×10^{-4}
(e) Z and s-channel γ exchange	0.090% [11]	0.1×10^{-4}
(f) Up-down interference	0.009% [28]	0.1×10^{-4}
(f) Technical Precision	(0.027)%	0.1×10^{-4}
Total	0.097%	1.0×10^{-4}

Theoretical accuracy: LEP & future machines

from Jadach, Płaczek, Skrzypek, Ward, Yost, PoS ICHEP2024 (2025) 848

Table 1
Summary of the total (physical+technical) theoretical uncertainty for a typical calorimetric detector. For LEP1, the above estimate is valid for a generic angular range within 1°–3° (18–52 mrads), and for LEP2 energies up to 176 GeV and an angular range within 3°–6°. Total uncertainty is taken in quadrature. Technical precision included in (a).

Type of correction/error	LEP1		LEP2	
	1996	1999	1996	1999
(a) Missing photonic $\mathcal{O}(\alpha^2)$ [4,5]	0.10%	0.027%	0.20%	0.04%
(b) Missing photonic $\mathcal{O}(\alpha^3 L_e^3)$ [6]	0.015%	0.015%	0.03%	0.03%
(c) Vacuum polarization [7,8]	0.04%	0.04%	0.10%	0.10%
(d) Light pairs [9,10]	0.03%	0.03%	0.05%	0.05%
(e) Z and s-channel γ [11,12]	0.015%	0.015%	0.0%	0.0%
Total	0.11% [12]	0.061% [13]	0.25% [12]	0.12% [13]

Forecast study for FCCee$_{M_Z}$		
Type of correction / Error	Published [2]	Redone
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.10×10^{-4}	0.10×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.06×10^{-4}	0.06×10^{-4}
(b') Photonic $\mathcal{O}(\alpha^2 L_e^0)$		0.17×10^{-4}
(c) Vacuum polariz.	0.6×10^{-4}	0.6×10^{-4}
(d) Light pairs	0.5×10^{-4}	0.27×10^{-4}
(e) Z and s-channel γ exch.	0.1×10^{-4}	0.1×10^{-4}
(f) Up-down interference	0.1×10^{-4}	0.08×10^{-4}
Total	1.0×10^{-4}	0.70×10^{-4}

Forecast			
Type of correction / Error	ILC$_{500}$	ILC$_{1000}$	CLIC$_{3000}$
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.13×10^{-4}	0.15×10^{-4}	0.20×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.27×10^{-4}	0.37×10^{-4}	0.63×10^{-4}
(c) Vacuum polariz.	1.1×10^{-4}	1.1×10^{-4}	1.2×10^{-4}
(d) Light pairs	0.4×10^{-4}	0.5×10^{-4}	0.7×10^{-4}
(e) Z and s-channel γ exch.	$1.0 \times 10^{-4(*)}$	2.4×10^{-4}	16×10^{-4}
(f) Up-down interference	$< 0.1 \times 10^{-4}$	$< 0.1 \times 10^{-4}$	0.1×10^{-4}
Total	1.6×10^{-4}	2.7×10^{-4}	16×10^{-4}

Figure 3: On the left, lumi theory outlook when redone as in Refs. [4, 5] – lattice [7–9] treatment of (c) could reduce it by ∼ 6. On the right, expectations for higher energies.

Luminosity at flavour factories (2000 – present)

- The requirements at flavour factories [DAΦNE (Frascati), KEK (Tsukuba), PEP-II (SLAC), BEPC (Beijing), VEPP-2M (Novosibirsk)] are to control the luminosity at the $\mathcal{O}(0.1\%)$
- The golden process is large-angle Bhabha, cross-checked with $e^+e^- \to \gamma\gamma$
- `BabaYaga@NLO` was (is) developed as dedicated tool for luminometry (and more)

  CC, Montagna, Nicrosini, Piccinini et al.

  $\rightsquigarrow$ it is based on a QED Parton Shower matched to NLO

  $\rightsquigarrow$ it's a fully exclusive event generator

$D(x,Q^2)$ is the solution of the QED DGLAP equation

$$Q^2\frac{\partial}{\partial Q^2}D(x,Q^2)=\frac{\alpha}{2\pi}\int_x^1\frac{dy}{y}P_+(y)D(\frac{x}{y},Q^2)$$

$$P_+(x)=\frac{1+x^2}{1-x}-\delta(1-x)\int_0^1\frac{1+t^2}{1-t}dt$$

which can be solved analytically (but some "exclusive" information is lost because integrated out) or by a **Monte Carlo iterative solution** (the **Parton Shower**, which is "exclusive")

$$\begin{aligned}D(x,Q^2) &= \Pi(Q^2,m^2)\delta(1-x)\\ &+\int_{m^2}^{Q^2}\Pi(Q^2,s')\frac{ds'}{s'}\Pi(s',m^2)\frac{\alpha}{2\pi}\int_0^{x_+}dyP(y)\delta(x-y)\\ &+\int_{m^2}^{Q^2}\Pi(Q^2,s')\frac{ds'}{s'}\int_{m^2}^{s'}\Pi(s',s'')\frac{ds''}{s''}\Pi(s'',m^2)\times\\ &\left(\frac{\alpha}{2\pi}\right)^2\int_0^{x_+}dx_1\int_0^{x_+}dx_2P(x_1)P(x_2)\delta(x-x_1x_2)+\cdots\end{aligned}$$

PS: pros and cons

$$\Pi(Q^2, m^2)_\epsilon = e^{-\frac{\alpha}{2\pi}\log\frac{Q^2}{m^2}\int_0^{1-\epsilon} dx P(x)} = e^{-\frac{\alpha}{2\pi}\log\frac{Q^2}{m^2} I_+}$$

is the Sudakov Form Factor, which exponentiates approximate virtual and soft emission up to all orders

✓ Advantages:

- ⇝ the number of emitted photons is not limited (shower)
- ⇝ at each branching, kinematical variables are generated and photons' momenta can be reconstructed $\rightarrow$ **fully exclusive event generation**
- ⇝ it can be truncated at $\mathcal{O}(\alpha^n)$ and consistently compared to fixed-order N^{n}LO calculations.

✗ Disadvantages:

- ⇝ initial-final state radiation interference effects are not naturally included, but they can be recovered by choosing an appropriate photons' angular distribution (eikonal, YFS-inspired)

CC, PLB 520 (2001) 16

$$I(k) = \sum_{i,j} \eta_i \eta_j \frac{p_i \cdot p_j}{(p_i \cdot k)(p_j \cdot k)} E_\gamma^2$$

- ⇝ the $I(k)$ choice pretty much fixes the Q^2 scale: $\int I(k)\, d\cos\theta_\gamma \rightarrow \log\frac{st}{um_e^2} - 1$ (for Bhabha)

PS: matching (BabaYaga solution)

- At its LL level, it misses already corrections at $\mathcal{O}(\alpha)$: a matching to NLO is needed
- We unfold the iterative $D(x, Q^2)$ MC solution, so that the LL corrected cross section can be cast in the form

$$d\sigma_{LL}^{\infty} = \Pi(Q^2, \varepsilon) \sum_{n=0}^{\infty} \frac{1}{n!} |\mathcal{M}_{n,LL}|^2 \, d\Phi_n,$$

$$|\mathcal{M}_{1,LL}|^2 = \frac{\alpha}{2\pi} \frac{1+z^2}{1-z} I(k) \, |\mathcal{M}_0|^2 \, \frac{8\pi^2}{E^2 z(1-z)}$$

- The multi-differential phase-space is kept **exact**
- Any approximation is shifted on matrix elements
- A mapping of momenta is needed (as always in resummed MCs)

PS: matching (BabaYaga solution)

$$d\sigma^{\infty}_{matched} = F_{SV}\ \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty}\frac{1}{n!}\ \left(\prod_{i=0}^{n} F_{H,i}\right)\ |\mathcal{M}_{n,LL}|^2\ d\Phi_n,$$

- F_{SV} and F_H are built with NLO and their LL counterparts. They are collinear and infrared safe, no double counting of LL terms
- the cross-section is still fully differential
- its $\mathcal{O}(\alpha)$ expansion coincides by construction with $d\sigma^{\text{NLO}}$
- resummation of LL higher-orders, beyond NLO, is preserved
- it can be expanded at $\mathcal{O}(\alpha^2)$ and compared to exact NNLO corrections

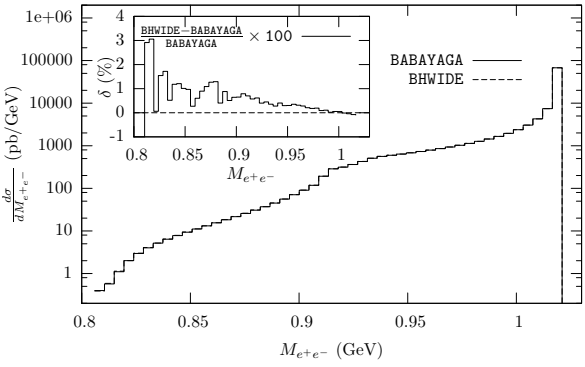
✓ Successfully applied to match QED NLO to PS in `BabaYaga@NLO`, EWK NLO to PS in `Horace` (neutral and charged Drell-Yan) and `Hto4l` ($H \to 4\ell$)

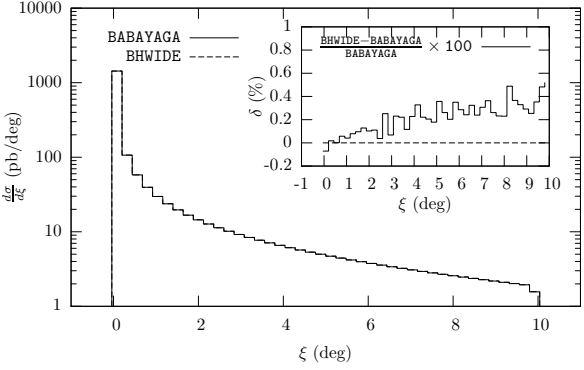
↪ The theoretical error is shifted to NNLO $\mathcal{O}(\alpha^2 L)$ terms (typically of $\mathcal{O}(0.1\%)$)

NLO matched PS vs NLO YFS

- distributions: BabaYaga@NLO vs. BHWIDE (Bhabha $e^+e^- \to e^+e^-(+n\gamma)$, at KLOE)

from Balossini et al., NPB 758 (2006) 227





ξ = acollinearity

$\Delta\sigma/\sigma \simeq 10^{-4}$ (only photonic RC)

Theoretical error in Bhabha at flavour factories

main conclusion of the luminosity section of EPJC 66 (2010) 585

Putting the various sources of uncertainties (for large–angle Bhabha) all together...

Source of error (%)	$\Phi-$factories	$\sqrt{s} = 3.5$ GeV	$B-$factories
$\lvert\delta^{\rm err}_{\rm VP}\rvert$ [Jegerlehner]	0.00	0.01	0.03
$\lvert\delta^{\rm err}_{\rm VP}\rvert$ [HMNT]	0.02	0.01	0.02
$\lvert\delta^{\rm err}_{{\rm SV},\alpha^2}\rvert$	0.02	0.02	0.02
$\lvert\delta^{\rm err}_{{\rm HH},\alpha^2}\rvert$	0.00	0.00	0.00
$\lvert\delta^{\rm err}_{{\rm SV,H},\alpha^2}\rvert$ [in progress]	0.05	0.05	0.05
$\lvert\delta^{\rm err}_{\rm pairs}\rvert$	0.03	0.016	0.03
$\lvert\delta^{\rm err}_{\rm total}\rvert$ linearly	0.12	0.1	0.13
$\lvert\delta^{\rm err}_{\rm total}\rvert$ in quadrature	0.07	0.06	0.06

- For the experiments on top of and closely around the J/ψ resonance, the accuracy slightly deteriorates, because of the differences between the predictions of independent $\Delta\alpha^{(5)}_{\rm had}(q^2)$ routines

⋆ The error estimate is comparable with that achieved for small–angle Bhabha luminosity monitoring at LEP

✓ A new community looking at flavour factories gathered for MC comparisons and development. Bhabha is not the main focus, but anyway part of the effort

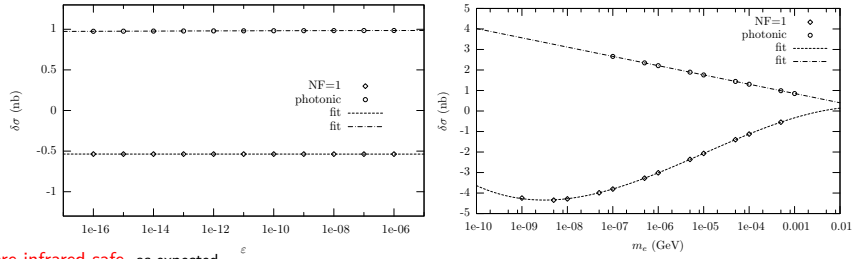
radiomontecarlow2.gitlab.io

Comparison with NNLO calculation for double [virtual + soft]

Comparison of $\sigma_{\rm SV}^{\alpha^2}$ calculation of `BabaYaga@NLO` with

Using realistic cuts for luminosity @ KLOE

- A. Penin (PRL **95** (2005) 010408 & Nucl. Phys. **B734** (2006) 185, i.e. NNLO photonic corrections): as a function of the log of the soft photon cut-off and of a fictitious electron mass



★ differences are infrared safe, as expected

★ $\delta\sigma(\text{photonic})/\sigma_0 \propto \alpha^2 L$, as expected

- Numerically, for various selection criteria at the Φ and B factories

$$\sigma_{\rm SV}^{\alpha^2}(\text{Penin}) - \sigma_{\rm SV}^{\alpha^2}(\texttt{BabaYaga@NLO}) \;<\; 0.02\% \times \sigma_0$$

Bhabha at NNLO: McMule

Banerjee, Engel, Schalch, Signer, Ulrich, PLB 820 (2021), 136547

- McMule implements *fixed-order* NNLO Bhabha scattering for low energies:
  - ↪ it includes the full set of corrections
  - ↪ numerically stable evaluation of *real-virtual* diagrams (e.g. pentagons) with the next-to-soft stabilization

see talk by Sara

  - ↪ not yet an event generator, but work is in progress

- Comparison to NNLO-truncated BabaYaga@NLO at KLOE (only photonic corrections)

	$\sigma/\mu\text{b}$	
	McMule	BABAYAGA
$\sigma^{(0)}$	6.8557	6.8557
$\sigma^{(1)}$	-0.7957	-0.7957
$\sigma^{(2)}$	0.0312	0.0267
σ_2	6.0912	6.0868

$$\frac{\Delta\sigma}{\sigma_{\text{NNLO}}} \simeq 0.07\%$$

Multi-purpose automatic MCs

- `MadGraph5_aMC@NLO` (uses beamstrahlung & ISR with QED structure fuctions, linked to `eMELA`)
- `Sherpa` (implements YFS EEX, beam energy spread)

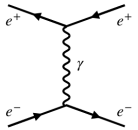
talk by Alan

- `WHIZARD` (includes ISR with QED structure fuctions & beamstrahlung with `CIRCE2`)

from F.P. Ucci, LoopFest XXIV

New Physics in Luminosity?

Standard Model

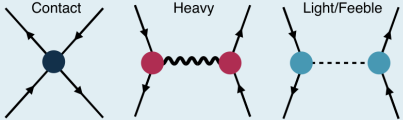


$$\sigma^{(n)} = \left(\frac{\alpha}{\pi}\right)^n \log^n \frac{Q^2}{m_e^2}$$

From the SM side you need **Monte Carlo Generators** able to simulate collisions at this level of precision

$$\left.\frac{\delta\sigma_{\rm SM}}{\sigma_{\rm SM}}\right|_{\rm th}^{\rm FCC} \sim 10^{-5}$$

New Physics



$$\frac{\delta\sigma_{\rm NP}}{\sigma_{\rm SM}} \propto \frac{2{\rm Re}\,\mathscr{M}_{\rm SM}\mathscr{M}^{\dagger}_{\rm NP}}{|\mathscr{M}_{\rm SM}|^2}$$

New Physics d.o.f. could contaminate the SABS
At what level?

$$\frac{\delta\sigma_{\rm NP}}{\sigma_{\rm SM}} \simeq ?$$

New Physics contamination in SA Bhabha?

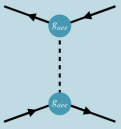
from F.P. Ucci, LoopFest XXIV

$e^+e^- \to e^+e^-$: Light New Physics

New Physics contamination to precision luminosity measurements at future e+e- colliders
M. Chiesa, C. L. Del Pio, G. Montagna, O. Nicrosini, F. Piccinini, F.P.U.
Phys. Rev. D. 112 (2025) 1

(Pseudo)scalar ALPs

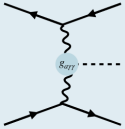
$g_{aee}(\bar{e}\, i\gamma_5 e)a$



(g_{aee}, m_a)
$\simeq (3\times10^{-3}, 1\,\text{GeV})$

ALP mixing with photons

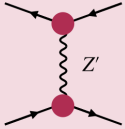
$\frac{1}{4}g_{a\gamma\gamma}(F_{\mu\nu}\tilde{F}^{\mu\nu})a$



$g_{a\gamma\gamma} \simeq 2\times10^{-4}\,\text{GeV}^{-1}$

Dark Vectors

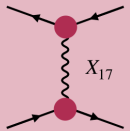
$g'_A\bar{e}\left(\gamma^\mu\gamma_5\right)e\,V_\mu$



(g'_V, M_V)
$\simeq (3\times10^{-4}, 1\,\text{GeV})$

X17

$g'_V\left(\bar{e}\,\gamma^\mu\, e\right)V_\mu$



$(g'_V, M_{X_{17}})$
$(6\times10^{-4}, 17\,\text{MeV})$

The LNP contribution is **negligible** at 10^{-5} level

from F.P. Ucci, LoopFest XXIV

$e^+e^- \to e^+e^-$: Heavy New Physics

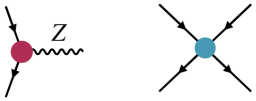
Standard Model Effective Field Theory

$\hat{O}_i^{(6)}$ Operators with same fields and symmetries of the SM

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \sum_i \frac{C_i}{\Lambda_{\text{NP}}^2}\hat{O}_i^{(6)} + \mathcal{O}(\Lambda_{\text{NP}}^{-4})$$

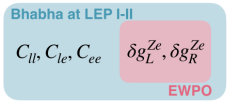
The deviation is computed taking into account **correlations** between WCs

$$(\delta \pm \Delta\delta)_{\text{SMEFT}} = \frac{1}{\sigma_{\text{SM}}}\left(\sigma^{(6)} \pm \sqrt{\sum_{ij}\sigma_i^{(6)}V_{ij}\sigma_j^{(6)}}\right)$$



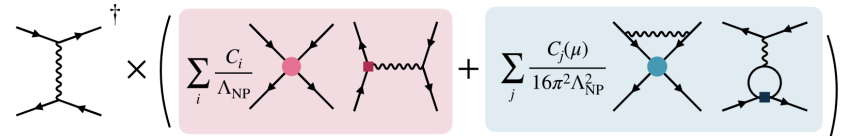
$\delta g_L^{Ze}, \delta g_R^{Ze}$ $\quad$ C_{ll}, C_{le}, C_{ee}

WCs are obtained by **global fits** from LEP data



from F.P. Ucci, LoopFest XXIV

$e^+e^- \to e^+e^-$: Heavy New Physics



Exp.	$[\theta_{\min}, \theta_{\max}]$	$\sqrt{s}$ [GeV]	$(\delta \pm \Delta\delta)_{\rm SMEFT}$	$\Delta L/L$
FCC	$[3.7°, 4.9°]$	91	$(-4.2 \pm 1.7) \times 10^{-5}$	$< 10^{-4}$
		160	$(-1.3 \pm 0.5) \times 10^{-4}$	
		240	$(-2.9 \pm 1.2) \times 10^{-4}$	10^{-4}
		365	$(-6.7 \pm 2.7) \times 10^{-4}$	
ILC	$[1.7°, 4.4°]$	250	$(-1.2 \pm 0.5) \times 10^{-4}$	$< 10^{-3}$
		500	$(-4.9 \pm 1.9) \times 10^{-4}$	
CLIC	$[2.2°, 7.7°]$	1500	$(-9.7 \pm 3.9) \times 10^{-3}$	$< 10^{-2}$
		3000	$(-4.2 \pm 1.7) \times 10^{-2}$	

SM NLO corrections to dim-6 operators appearing at LO

$$\frac{\sigma^{(6)}_{{\rm NLO},i}}{\sigma^{(6)}_{{\rm LO},i}} \simeq \frac{g^2_{\rm SM}}{16\pi^2} \log \frac{M_Z^2}{|t|} \simeq \mathcal{O}(1\%)$$

Dim-6 operators loop insertions

$$\frac{\sigma^{(6)}_{{\rm NLO},j}}{\sigma_{\rm SM}} \simeq \frac{|t|}{\Lambda^2_{\rm NP}} \frac{C_j}{16\pi^2} \log \frac{\Lambda^2_{\rm NP}}{|t|} \simeq 2 \times 10^{-4} C_j$$

from F.P. Ucci, LoopFest XXIV

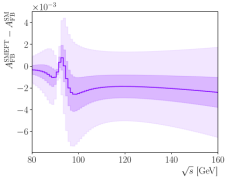
$e^+e^- \to e^+e^-$: Heavy New Physics

Z peak runs — FCC-ee

We use the FB asymmetry as a function of $\sqrt{s_\alpha}$

$$\sum_{i\in 4f} \frac{C_i}{\Lambda^2_{\rm NP}} \left[\frac{(\sigma_{\rm F}-\sigma_{\rm B})^{(6)}_i}{(\sigma_{\rm F}-\sigma_{\rm B})_{\rm SM}} - \frac{(\sigma_{\rm F}+\sigma_{\rm B})^{(6)}_i}{(\sigma_{\rm F}+\sigma_{\rm B})_{\rm SM}} \right]_\alpha = \frac{\Delta A^0_{{\rm FB},\alpha}}{A^0_{{\rm FB},\alpha}},$$

To fit the three WCs we can use three points



$\sqrt{s_1} = 89\,\text{GeV}$
$\sqrt{s_2} = 93\,\text{GeV}$
$\sqrt{s_3} = 98\,\text{GeV}$

In 6 months of run on every point

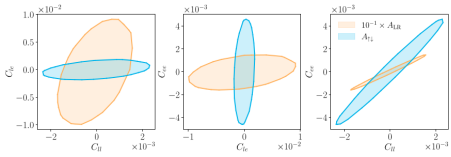
$\Delta C_{4f} < 10^{-2}$ ⟶ $\delta_{\rm SMEFT} < 10^{-5}$

250 GeV run — ILC

For polarised beams A_{LR} is not sensitive to all WCs.
We propose another polarisation asymmetry

$$A^-_{\uparrow\downarrow}(P_{e^+}, P_{e^-}, \cos\theta) = \frac{d\sigma(P_{e^+}, P_{e^-}) - d\sigma(P_{e^+}, -P_{e^-})}{d\sigma(P_{e^+}, P_{e^-}) + d\sigma(P_{e^+}, -P_{e^-})}$$

Up-down asymmetry



We calculate the 68% CLs using

$$\chi^2 = \sum_{\alpha=1}^{n} \frac{\left(A^0_{\rm pol} - A^{\rm th}_{\rm pol}(\vec{C}_{4f})\right)^2_\alpha}{(\Delta A^0_{\rm pol})^2_\alpha}$$

⟶ $\delta_{\rm SMEFT} < 10^{-7}$

$e^+e^- \to \gamma\gamma$ as luminosity reference process

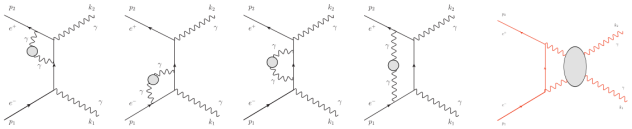
CC et al., PLB 798 (2019) 134976

- at LO, it's a pure QED process, also at high energies
- `BabaYaga@NLO` implements it, also including NLO EWK corrections
- at NLO, no fermionic loops enter yet, nor ***hadronic loops***
- hadronic vacuum polarization and its uncertainty enter at NNLO
- HVP uncertainty can become a limiting factor for luminometry with SABS

$e^+e^- \to \gamma\gamma$ as luminosity reference process

from F. Piccinini, 7th FCC Physics Workshop

Rough estimate of (NNLO) VP hadronic corrections (and uncertainties)



$$\sigma^{\text{NNLO}}_{\Delta\alpha_{had}} \pm \delta\sigma \overset{\text{very naive!}}{\approx} \left(\sigma^{\text{NLO}}_{\text{QED}} - \sigma^{\text{LO}}\right) \times \left[\Delta\alpha_{had}(s) \pm \delta\Delta\alpha_{had}\right]$$

$\sqrt{s}$ (GeV)	$\Delta\alpha_{had}(s)^*$	$\delta\sigma/\sigma_{LO}$ [1]	$\delta\sigma/\sigma_{LO}$ [2]
91	$(276.7 \pm 1.2) \cdot 10^{-4}$	$2.8 \cdot 10^{-5}$	$3.7 \cdot 10^{-6}$
160	$(309.1 \pm 1.2) \cdot 10^{-4}$	$3.0 \cdot 10^{-5}$	$3.8 \cdot 10^{-6}$
240	$(333.2 \pm 1.2) \cdot 10^{-4}$	$3.1 \cdot 10^{-5}$	$3.9 \cdot 10^{-6}$
365	$(358.5 \pm 1.2) \cdot 10^{-4}$	$3.4 \cdot 10^{-5}$	$4.0 \cdot 10^{-6}$

- LbL contribution, with its uncertainty, should be quantified

*from F. Jegerlehner's recent `hadr5n16.f`

$e^+e^- \to \gamma\gamma$ as luminosity reference process

from F. Piccinini, 7th FCC Physics Workshop

$e^+e^- \to \gamma\gamma$: Pros and cons

- ✓ at LO, purely QED process, ***at any energy***
- ✓ at NLO, weak corrections (loops with Z & $W^\pm$), but not fermionic loops yet (in particular, ***no hadronic loops***)
- ✓ hadronic vacuum polarization (**and its uncertainty**) enters only at NNLO (2-loops, order α^2)
- ✓ $d\sigma/d\cos\theta \sim 1/\sin^2\theta$) $\Longrightarrow$ lowest angle acceptance less critical than for Bhabha

- ✗ Large Bhabha background, in particular at Z pole
- ✗ At NNLO also Ligh-by-Light contribution present, (**with its uncertainty**)
- ✗ Statistics lower than Bhabha for respective typical event selections
- ✗ Lack of independent MC codes for cross-checks/validation

✓ the last item is out-of-date: `McMule` now includes $e^+e^- \to \gamma\gamma$ at NNLO

Engel, Rocco, Signer, Ulrich, PLB 874 (2026) 140236

Conclusions & Outlook

- From the `BabaYaga` side (in the near future):
  - ⋆ we must fully port it to FCC energies (for Bhabha and $e^+e^- \to \mu^+\mu^-$): full set of NLO EW corrections, validation in SA and LA scenarios
  - ⋆ it's natural to try to implement a NLL QED PS for improved accuracy

  exploratory work in progress with Stefano & Giovanni

- On general grounds:
  - ⋆ LEP & flavour factories experience: development and comparison of independent MC is vital to firmly assess the theoretical accuracy, even more so given the unprecedented precision needed
  - ⋆ also internal building blocks must be double/triple-checked independently (e.g. NNLO amplitudes)
  - ⋆ (the confusing) beamstrahlung must be included in our MCs: we'd better do it from the beginning
  - ⋆ at such precision level, NP contamination needs to be addressed