Overview of Z+H production at lepton colliders

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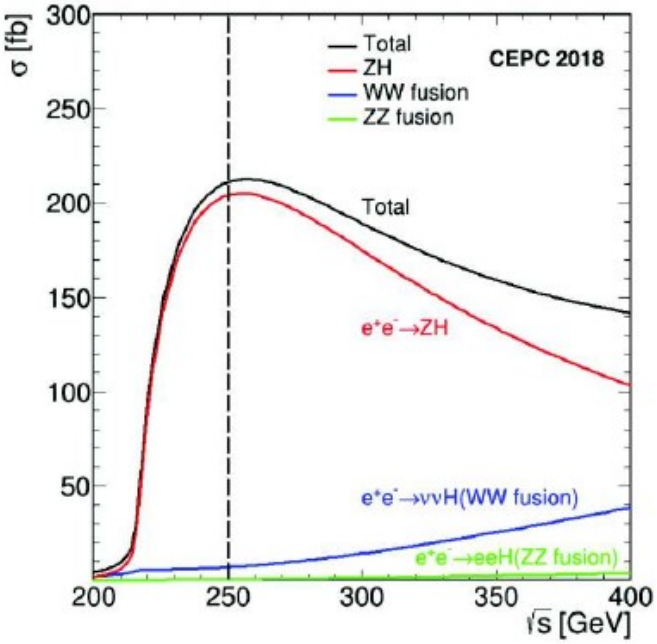
designed by Wannapik, https://www.wannapik.com/vectors/25151

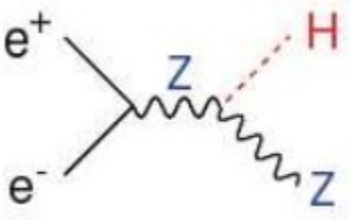
Higgs production experimental goals

- **ee→HZ:** dominant Higgs prod. process at e+e- colliders below 500 GeV
- Expected precision (240~250 GeV):

ILC/LCF	0.6%	[ESPPU ‘26]
CEPC	0.3-0.5%	[1811.10545]
FCC-ee	0.2-0.3%	[ESPPU ‘26]

(exclusive H→bb – inclusive H→any)

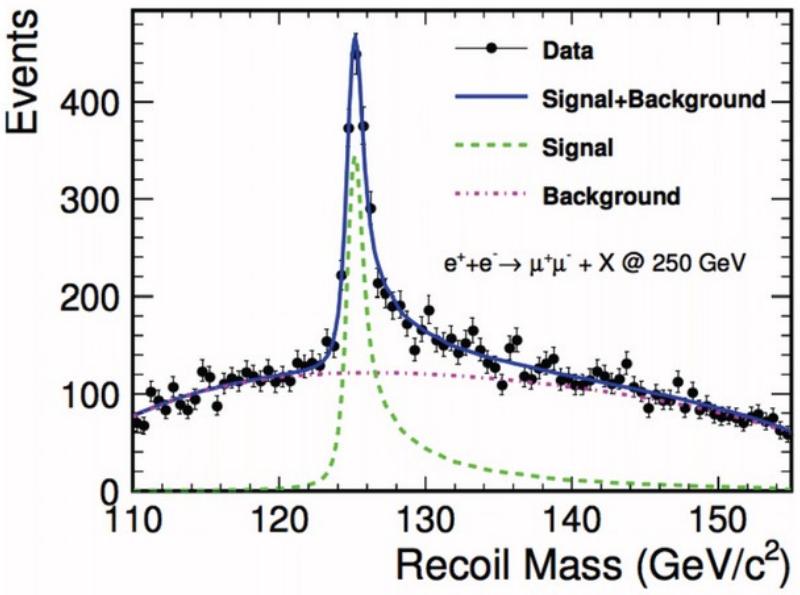




Higgs decay experimental goals

- Use Z→ recoil: model-independent tagging of any H decays
- Improved precision without exotic decay modes

	FCC-ee	ILC/LCF
H → bb	0.2%	0.4%
H → cc	1.6%	2.5%
H → ττ	0.6%	1.0%
H → WW	0.8%	1.4%
H → ZZ	2.5%	5.5%
H → gg	0.8%	2.1%
H → γγ	3.6%	10%

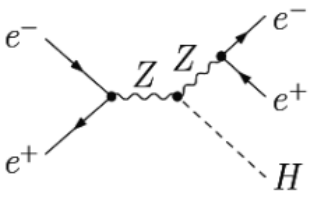


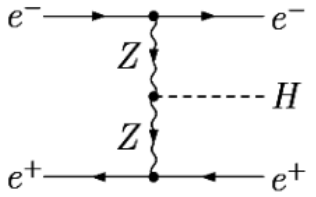
[Peskin, Fujii, Grojean, '17]

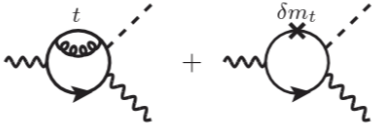
(numbers from contrib. to [ESPPU '26])

Theory calculations for ee→ZH

- NLO corrections to ZH production [Kniehl, '92] [Denner, Küblbeck, Mertig, Böhm '92]
- NLO corrections to H*ff* production
  [Denner, Dittmaier, Roth, Weber, '03] [Boudjema et al., 04]
- NLO corrections now automated, e.g.
  - MadGraph5_aMC@NLO [Frederix et al., 1804.10017]
  - OpenLoops [Buccioni et al., 1907.13071]
  - …
- Higher-order ISR effects [Greco et al., 1711.00826]
- Mixed EW-QCD NNLO $\mathcal{O}(\alpha\alpha_s)$ corrections
  [Gong, Li, Xu, Yang, Zhao, 1609.03955] [Sun, Feng, Jia, Sang, 1609.03995]
- $\mathcal{O}(N_f\alpha^2)$ corrections (N_f = # of closed fermion loops) [Freitas, Song, 2209.07612]
- Full NNLO electroweak matrix elements [Chen et al., 2209.14953]







Higher-order ISR

- Factorization of soft and collinear photon emission [Greco et al., 1711.00826]

$$\mathrm{d}\sigma(s) = \int \mathrm{d}x_1 \mathrm{d}x_2 D(x_1,s) D(x_2,s) \mathrm{d}\sigma_0(x_1 x_2 s) \Theta(\text{cuts}).$$

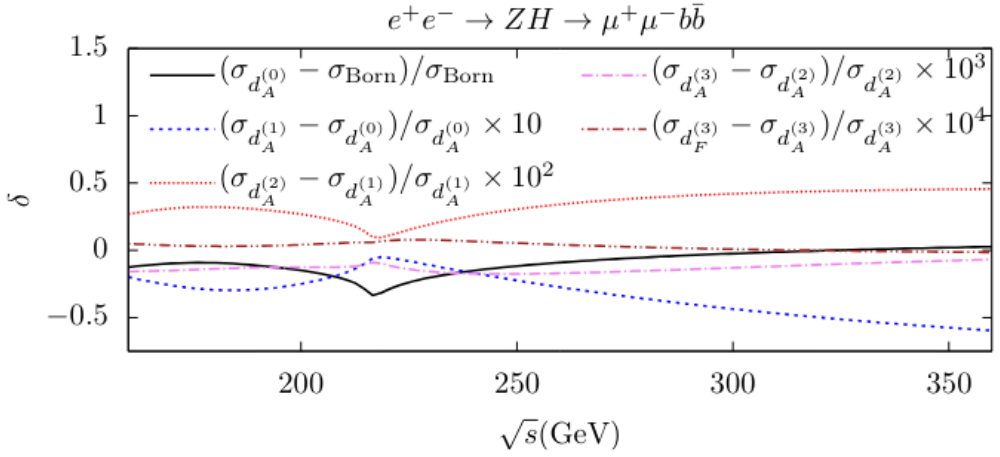
- Soft-photon exponentiation (Gribov-Lipatov):

$$D_{GL}(x,s) = \frac{\exp\left[\frac{1}{2}\beta\left(\frac{3}{4}-\gamma_E\right)\right]}{\Gamma\left(1+\frac{1}{2}\beta\right)} \frac{1}{2}\beta\,(1-x)^{\frac{1}{2}\beta-1}, \qquad \beta = \frac{2\alpha}{\pi}(L-1), \qquad L = \ln(s/m_e^2).$$

- Adding explicit coll. logs:

$$\begin{aligned} D_F(x,s) &= D_{GL}(x,s) \sum_{i=1}^{3} d_F^{(i)} \\ d_F^{(1)}(x,s) &= \frac{1}{2}(1+x^2), \\ d_F^{(2)}(x,s) &= \frac{1}{4}\frac{\beta}{2}\left[-\frac{1}{2}(1+3x^2)\ln x - (1-x)^2\right], \end{aligned}$$

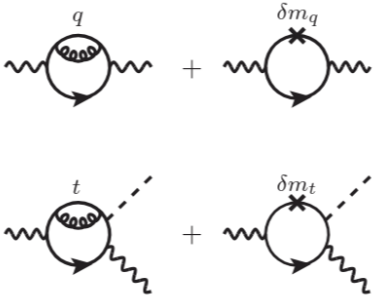
...



NNLO $\mathcal{O}(\alpha\alpha_s)$ corrections

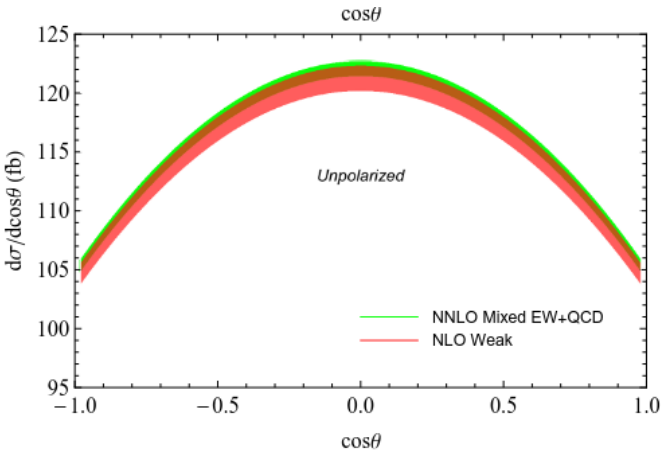
[Gong, Li, Xu, Yang, Zhao, 1609.03955] [Sun, Feng, Jia, Sang, 1609.03995]

- Most difficult contributions are 2-loop vertex diagrams with top quarks
- Reduction to MIs with Reduze 2 and FIRE
  [v.Manteuffel and Studerus, 1201.4330] [Smirnov, 1408.2372]
- Evaluation of MIs with large-m_t expansion (valid for $\sqrt{s} < 2m_t$) and sector decomposition
- Corrections 0.5–1%



	$\alpha(0)$ scheme	G_μ scheme
σ^{LO} [fb]	223.14	239.64
σ^{NLO} [fb]	229.78	232.46
$\sigma^{\mathrm{NNLO,EW\times QCD}}$ [fb]	232.21	233.29

- Reduced scheme dependence



Input/renormalization schemes

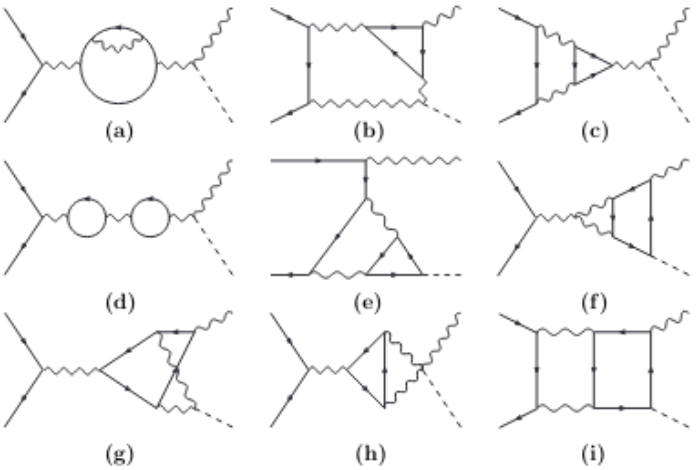
- α(0)–m_W–m_Z scheme:
  - OS mass renormalization (complex propagator pole)

$$m_Z = m_Z^{\exp}\,[1 + (\Gamma_Z^{\exp}/m_Z^{\exp})^2]^{-1/2},$$
$$\Gamma_Z = \Gamma_Z^{\exp}\,[1 + (\Gamma_Z^{\exp}/m_Z^{\exp})^2]^{-1/2}.$$

  - Electroweak couplings in terms of α(0) = $e^2/(4\pi)$, $g = \dfrac{e}{\sin\theta_W} = \dfrac{e}{\sqrt{1 - m_W^2/m_Z^2}}$
    → dependence on $\Delta\alpha = 1 - \alpha(m_Z)/\alpha(0)$
- G_μ–m_W–m_Z scheme:
  - Electroweak couplings defined through G_μ: $\dfrac{G_\mu}{\sqrt{2}} = \dfrac{g^2}{8m_W^2}(1 + \Delta r)$
    → absorbs dependence on Δα

NNLO electroweak corrections

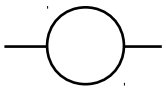
- $\mathcal{O}(\alpha^2)$ corrections with closed fermion loop
  [Freitas, Song, 2209.07612]
  - Vertex+box diagrams with method based on dispersion rels. (no red. to MIs)
  - OS renormalization
  - ISR QED corrections factorized



- Full NNLO electroweak matrix elements
  [Chen et al., 2209.14953]
  - Reduction to MIs using reconstruction from finite fields with LiteRed, FiniteFlow [Lee, 1310.1145] [Peraro, 1905.08019]
    (with block-triangular relations [Liu, Ma, 1801.10523] [Guan, Liu, Ma, 1912.09294])
  - MIs evaluated with series solution of diff. eqs.
  - Bare matrix element with UV/IR divergences

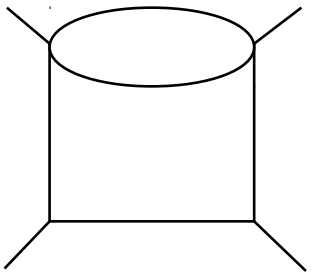
Fermionic NNLO corrections

- Basic idea: use dispersion relation for sub-loop



$$= B_0(p^2, m_1^2, m_2^2) = \int_{(m_1+m_2)^2}^{\infty} \mathrm{d}\sigma \, \frac{\Delta B_0(\sigma, m_1^2, m_2^2)}{\sigma - p^2 - i\epsilon},$$

$$\Delta B_0(\sigma, m_1^2, m_2^2) \equiv \frac{1}{\pi} \mathrm{Im}\, B_0(\sigma, m_1^2, m_2^2)$$



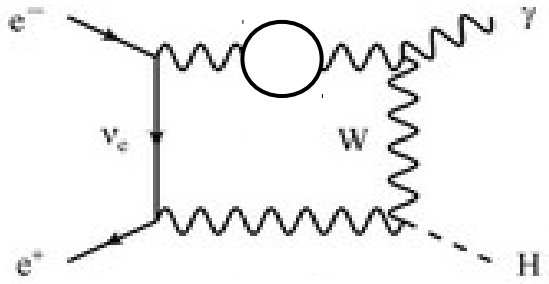
$$= - \int d\sigma \, \Delta B_0(\sigma, ...) \, D_0(..., \sigma)$$

[Bauberger, Berends, Bohm, Buza, hep-ph/9409388]

Fermionic NNLO corrections

- Basic idea: use dispersion relation for sub-loop
- including numerator terms:

$$\left[\quad\right]_{\mathrm{T}} = \int d\sigma \, \frac{c_0 \Delta B_0(\sigma, ...) + c_1 \Delta B_1(\sigma, ...) + c_{00} \Delta B_{00}(\sigma, ...) + ...}{\sigma - p^2 - i\epsilon}$$



$$= - \int d\sigma \, [c_0 \Delta B_0 + c_1 \Delta B_1 + c_{00} \Delta B_{00} + ...] \times [a_1 D_0 + a_2 D_1 + ... + a_n C_0 + a_{n+1} C_1 + ...]$$

(coefficients depend on masses, external momenta and σ)

Fermionic NNLO corrections

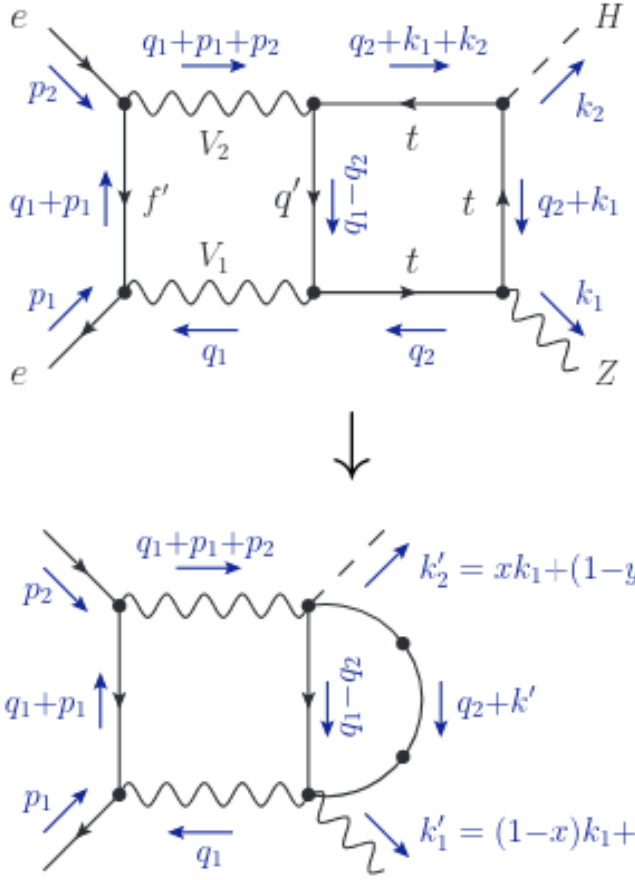
- Introduce Feynman parameters

- q_2 loop $= \int dx\,dy\,\frac{\partial^2}{\partial(m'^2)^2}\int_{\sigma_0}^{\infty} d\sigma\,\frac{\Delta B_0(\sigma, m'^2, m_{q'}^2)}{\sigma - \tilde{q}_1^2}$

$$\tilde{q}_1 = q_1 + k' + i\epsilon,$$

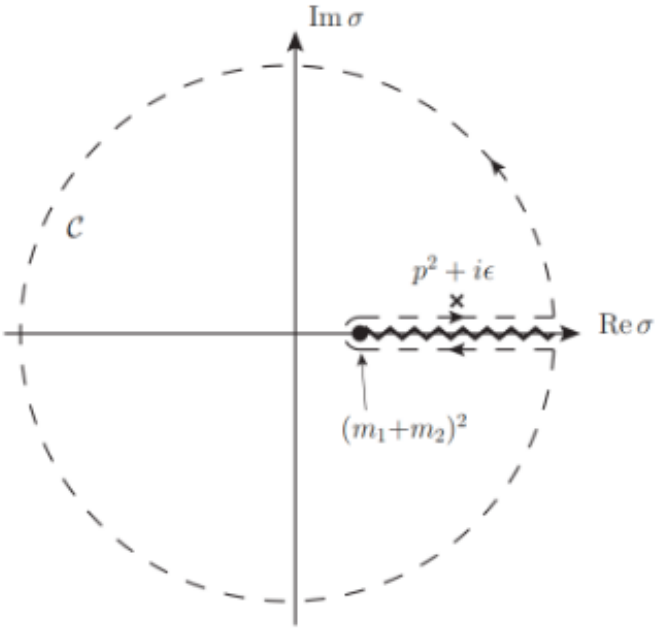
$$m'^2 = m_t^2 - xy(k_1+k_2)^2 - (1-x-y)(xk_1^2 + yk_2^2).$$

- Similarly use Feynman pars. for other box and vertex diagrams



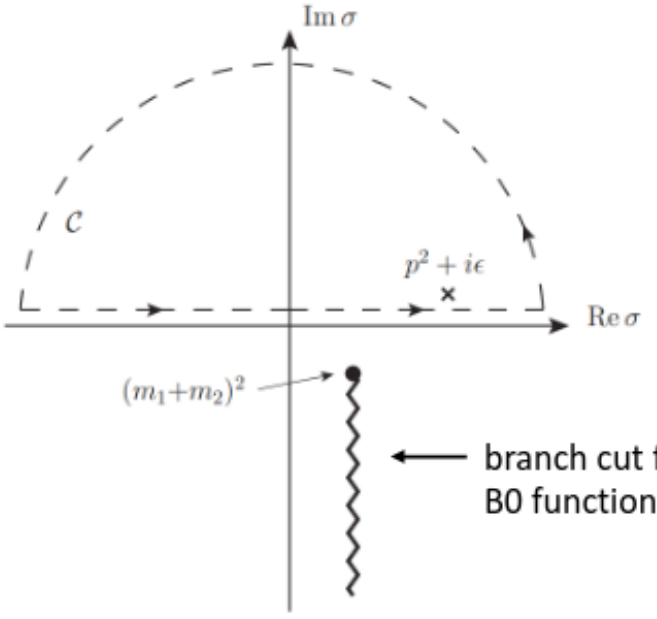
Fermionic NNLO corrections

$m_1^2 \geq 0\ ,\ m_2^2 \geq 0$



integration contours

$m_1^2 > 0\ ,\ m_2^2 < 0$



formulas

$$B_0(p^2, m_1^2, m_2^2) = \frac{1}{2\pi i} \oint d\sigma \frac{B_0(\sigma, m_1^2, m_2^2)}{\sigma - p^2 - i\epsilon}$$
$$= \int_{(m_1+m_2)^2}^{\infty} d\sigma \frac{\Delta B_0(\sigma, m_1^2, m_2^2)}{\sigma - p^2 - i\epsilon}$$

$$B_0(p^2, m_1^2, m_2^2) = \frac{1}{2\pi i} \oint_C d\sigma \frac{B_0(\sigma, m_1^2, m_2^2)}{\sigma - p^2 - i\epsilon}$$
$$= \frac{1}{2\pi i} \int_{-\infty}^{+\infty} d\sigma \frac{B_0(\sigma, m_1^2, m_2^2)}{\sigma - p^2 - i\epsilon}$$

Fermionic NNLO corrections

- UV divergences will cause the num. integral to diverge
- Need to subtract terms so make integral finite
- Subtraction terms simple enough to integrate analytically and add back

$$|M_0 M_2^*| \sim \int dx \int dy \int d\sigma \times \underbrace{[\text{integrand}]}_{\text{UV div}}$$

$$= \int dx \int dy \int d\sigma \times \underbrace{[\text{integrand} - I_{\text{subtra}}]}_{\text{UV finite, integrate numerically}}$$

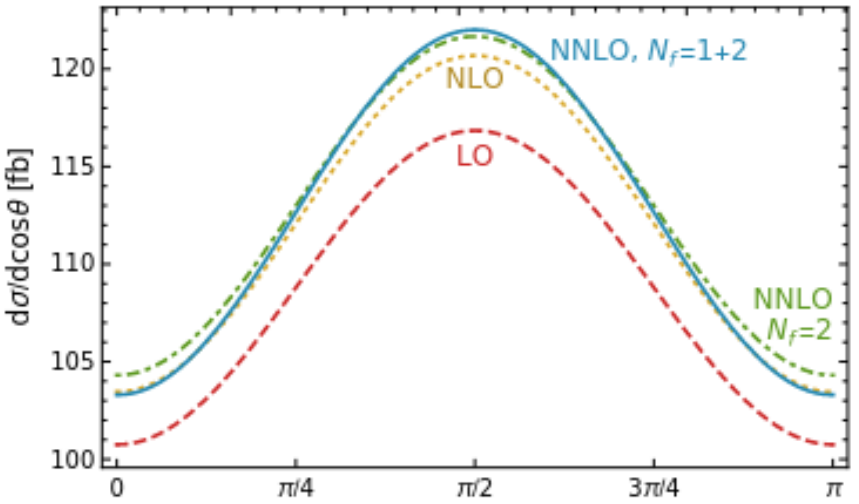
$$+ \int dx \int dy \int d\sigma \times \underbrace{[I_{\text{subtra}}]}_{\text{UV div, integrate analytically}}$$

NNLO electroweak corrections

- Numerical impact of $\mathcal{O}(N_f\alpha^2)$: 0.5–1%

[Freitas, Song, 2209.07612]
[Freitas, Song, Xie, 2305.16547]

	$\alpha(0)$ scheme	G_μ scheme
$\sigma^{\rm LO}$ [fb]	223.14	239.64
$\sigma^{\rm NLO}$ [fb]	229.78	232.46
$\sigma^{\rm NNLO,EW\times QCD}$ [fb]	232.21	233.29
$\sigma^{\rm NNLO,EW}$ [fb]	233.86	233.98



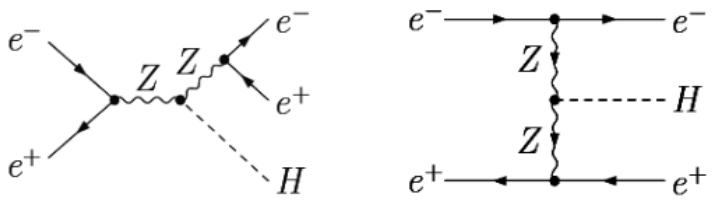
- Error estimate from missing bosonic $\mathcal{O}(\alpha^2)$:
  - scheme dependence: 0.1%
  - $|\mathcal{M}_{(1,\rm bos)}|^2$: 0.3%
- Rough error estimate from N3LO, counting prefactors: < 0.1% [ESPPU, 2511.03883 + in prep.]

exp. precision:

ILC/LCF	0.6%
CEPC	0.3-0.5%
FCC-ee	0.2-0.3%

Combination of ZH production and Z decay

- Z boson has large decay width
  → more complete description $e^+e^- \to Hf\bar{f}$



Possible schemes for accounting Z resonance:

- Complex-mass scheme: [Denner, Dittmaier, Roth, Wieders, hep-ph/0505042]

  $m_Z^2 \to m_Z^2 - i m_Z \Gamma_Z$ everywhere, including $s_W^2 = 1 - m_W^2/m_Z^2$

  - preserves gauge-invariance, simple book-keeping
  - would require full $e^+e^- \to Hf\bar{f}$ @ NNLO

- Pole scheme: Expand MI about complex pole [Willenbrock+Valencia, Sirlin, Stuart, …]

$$\mathcal{M}_{ee\to\mu\mu H} = \frac{[A]_{p_z^2=s_0}}{p_z^2 - s_0} + [B]_{p_z^2=s_0} + (p_z^2 - s_0)[C]_{p_z^2=s_0} + \dots ,$$

Combination of ZH production and Z decay

- Leading-pole term factorizes into (prod)×(decay)

$$\mathcal{M}_{ee\to\mu\mu H} = \frac{[A]_{p_z^2=s_0}}{p_z^2 - s_0} + [B]_{p_z^2=s_0} + (p_z^2 - s_0)[C]_{p_z^2=s_0} + \dots ,$$

$$A = \frac{\Gamma_{\rm prod}\Gamma_{\rm dec}}{1+\Sigma'_Z} ,$$

$$B = \frac{\Gamma_{\rm prod}\Gamma'_{\rm dec} + \Gamma'_{\rm prod}\Gamma_{\rm dec}}{1+\Sigma'_Z} - \frac{\Gamma_{\rm prod}\Gamma_{\rm dec}\Sigma''_Z}{2(1+\Sigma'_Z)^2} + \mathcal{M}_{\rm bkgd} ,$$

$$C = \dots ,$$

"bkgd": diagrams without res. Z

Focusing on leading *A* term:

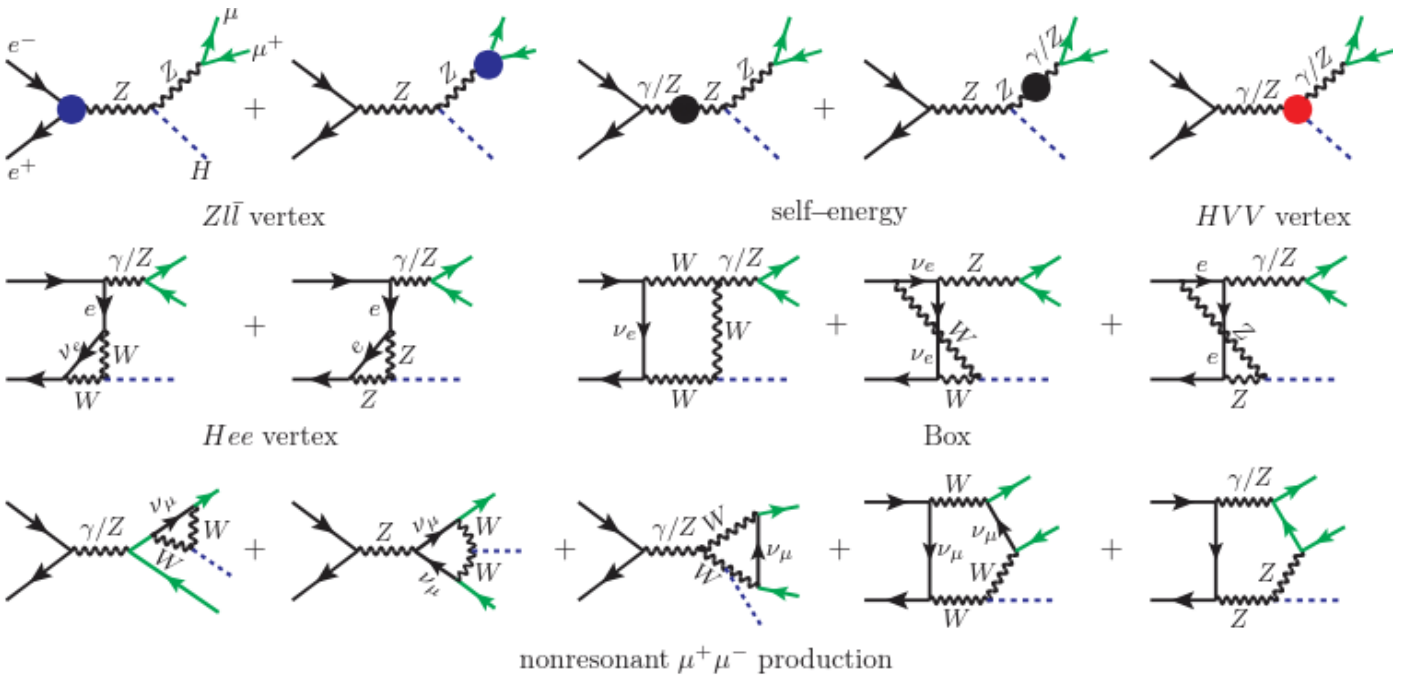
$$\frac{d^2\sigma}{d\cos\theta\, dp_z^2} = \frac{\beta}{32\pi s}\,|\mathcal{M}^{\rm eff}_{\rm prod}|^2 \frac{\pi^{-1} m_Z \Gamma_{Z\to\mu\mu}}{(p_z^2 - m_Z^2)^2 + m_Z^2\Gamma_Z^2} ,$$

$$\mathcal{M}^{\rm eff}_{\rm prod} \equiv \frac{\Gamma_{\rm prod}}{\sqrt{1+\Sigma'_Z}} ,$$

$$\Gamma_{Z\to\mu\mu} \equiv \frac{|\Gamma_{\rm dec}|^2}{16\pi\, m_Z(1+\Sigma'_Z)} ,$$

For sub-% precision: A @ NNLO
B @ NLO

Combination of ZH production and Z decay



[Chen, Feng, Jia, Sang, 1811.05453]

NLO calc. of B possible with autom. tools

Avoid double-counting with leading A term

$C = \ldots,$

B @ NLO

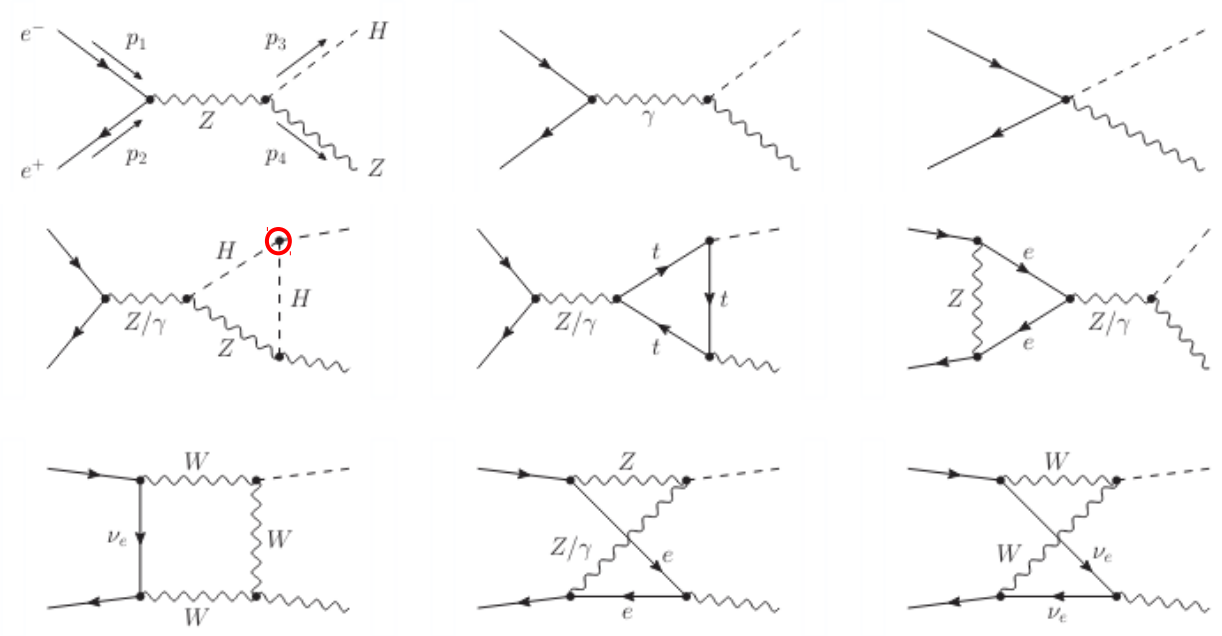
“bkgd”: diagrams without res. Z

Other considerations

- Matching with MC generators for exclusive simulation of photon / fermion-pair emission (only ISR) → talk by M. Schoenherr
- Non-interfering backgrounds: $e^+e^- \to b\bar{b}f\bar{f}$, etc.
  → need NLO corrections?

BSM effects

- Can use precision studies of ee→ZH to probe BSM physics (specific models or SMEFT/HEFT)
- NLO effects can be relevant, in particular for Higgs self-coupling



[Asteriadis et al., 2409.11466]

- = SMEFT op. insertion

BSM effects

- Can use precision studies of ee→ZH to probe BSM physics (specific models or SMEFT/HEFT)
- SMEFT dim-6 insertions:

$$\lambda = \sqrt{2}G_\mu M_H^2\left\{1 + X_H + \frac{1}{2\sqrt{2}G_\mu\Lambda^2}(C_{HD} - 4C_{H\Box})\right\} + \frac{3C_H}{\sqrt{2}G_\mu\Lambda^2}\,,$$

$$y_t = \sqrt{2}m_t(\sqrt{2}G_\mu)^{1/2}\left\{1 + \frac{1}{2}X_H + (\sqrt{2}G_\mu)^{-3/2}\frac{C_{u\phi}[3,3]}{2\sqrt{2}m_t\Lambda^2}\right\},$$

$$\overline{g} = 2M_W(\sqrt{2}G_\mu)^{1/2}\left\{1 + \frac{1}{2}X_H\right\},$$

$$\overline{g}' = 2(\sqrt{2}G_\mu)^{1/2}\sqrt{M_Z^2 - M_W^2}\left\{1 + \frac{1}{2}X_H\right\} - \frac{1}{2(\sqrt{2}G_\mu)^{1/2}\Lambda^2}\left\{4M_W C_{\phi WB} + \frac{M_Z^2}{\sqrt{M_Z^2 - M_W^2}}C_{\phi D}\right\},$$

$$X_H \equiv \frac{1}{2\sqrt{2}G_\mu\Lambda^2}\left\{C_{ll}[1,2,2,1] + C_{ll}[2,1,1,2] - 2(C_{\phi l}^{(3)}[1,1] + C_{\phi l}^{(3)}[2,2])\right\},$$

[Asteriadis et al., 2409.11466]

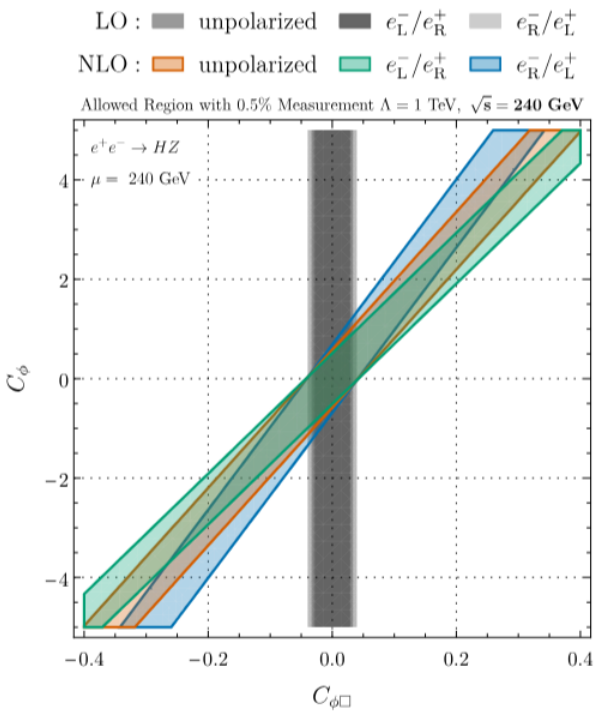
Need to renormalize SMEFT Wilson coeffs:

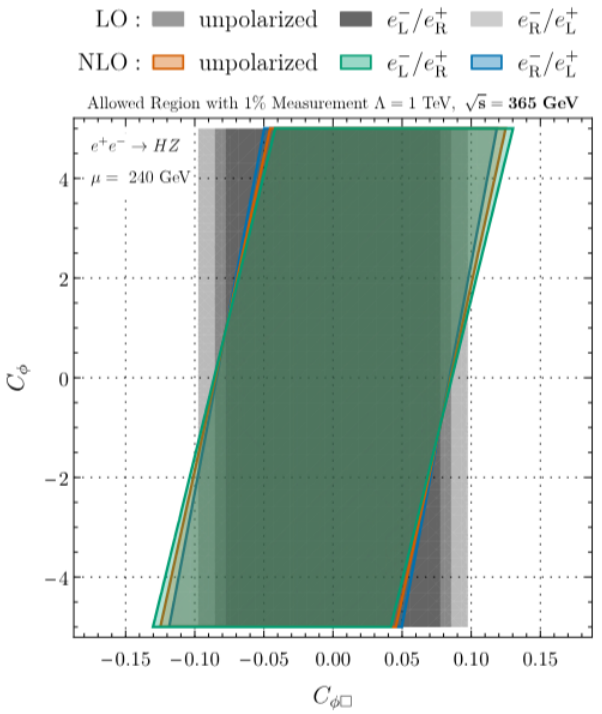
$$C_i(\mu) = C_{0,i} - \frac{1}{2}\left[\frac{1}{\epsilon}\bar{\mu}^{2\epsilon}\right]\frac{1}{16\pi^2}\gamma_{ij}C_j(\mu)\,,$$

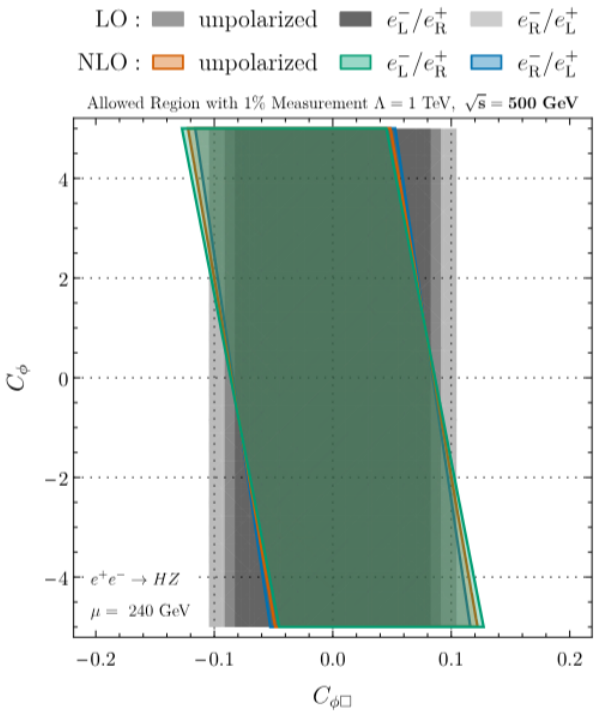
BSM effects

- Can use precision studies of ee→ZH to probe BSM physics (specific models or SMEFT/HEFT)
- SMEFT dim-6 insertions:

[Asteriadis et al., 2409.11466]







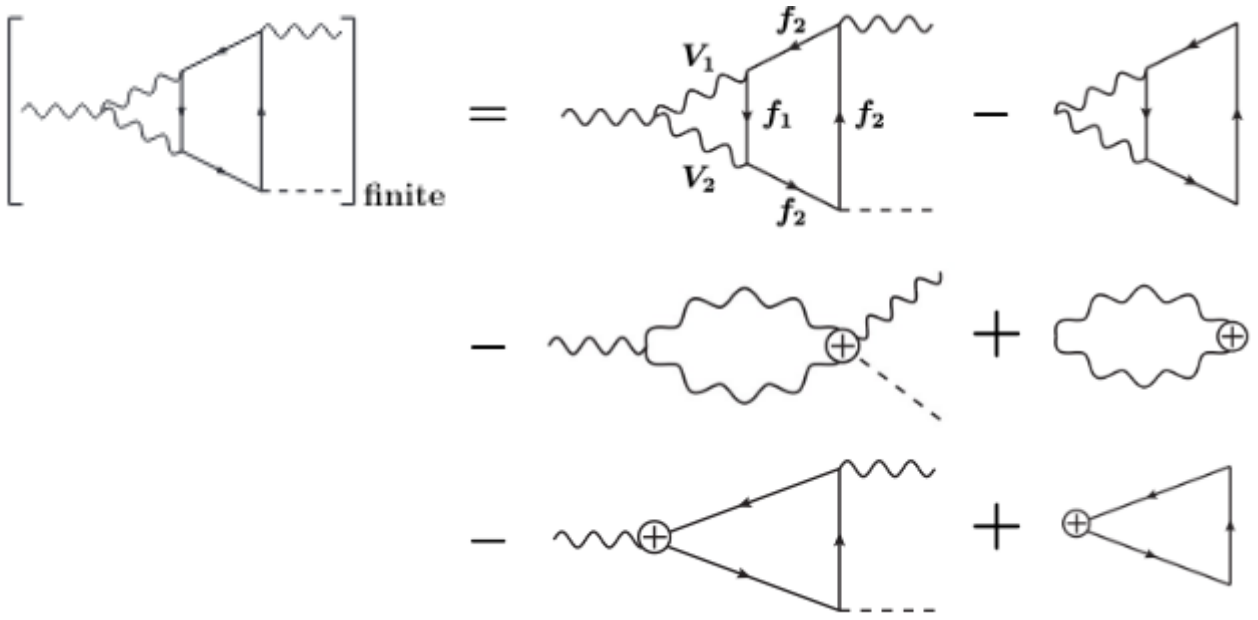
Conclusions

- Exp. sub-% target precision for $ee \to ZH$
- Need full NNLO corrections, maybe partial N3LO?
  - Techniques exist, but further improvements needed
- Combination with $ee \to Z^* H \to ff H$
  - No obstacle, but implementation work
- Matching with QED shower/evolution
- BSM effects in loops can be important

Backup

Fermionic NNLO corrections

- Separate treatment for global divergence and two sub-loop divergences



- Each subtraction term can be written as dispersion integral
  → total disp. integral becomes finite

Fermionic NNLO corrections

- Diagram generation with **`FeynArts`** [Hahn, 2001]
- Algebraic manipulations in **`Mathematica`**
- No IBP reduction
- Numerical integration in C++
  - Passarino-Veltman functions (D_0, D_1, etc.) from **`LoopTools`** [Hahn, Perez-Victoria, 1999]
  - Adaptive Gauss integration
- Overall precision limited by floating point numbers
  - ~3 digits with double precision (minutes running time)
  - >6 digits with quadruple precision (up to few hours running time)
- Approach applicable to EW corrections for other 2→2 processes