QED parton showers for lepton colliders and matching the NLO EW

Marek Schönherr*

IPPP, Durham University

*in collaboration with Lois Flower (Liverpool) and Joanne Roper (Durham)

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2. QED dipole shower
3. MC@NLO EW
4. MC@NLO QCD+EW with resonances
5. Conclusions

Introduction

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2 QED dipole shower
3 MC@NLO EW
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Introduction

We need **highest-precision event generators** to be able to exploit the projected recorded data of current and future collider experiments to the fullest.

→ see Paolo's and Roberto's talk

Translation of parton shower experience from hadron colliders to lepton collider initial state radiation far from trivial. Different solutions exist.

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QED dipole showers

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Parton showers

Objective

Construct a universal approximation for higher-order corrections

$$\mathrm{d}\sigma_{n+1} = \mathrm{d}\sigma_n \sum \mathrm{d}\Phi_1 \, \mathcal{K}(\Phi_1) \qquad\qquad \mathrm{d}\Phi_1 = \mathrm{d}t \, \mathrm{d}z \, \mathrm{d}\phi \; J(t, z, \phi)$$

with the splitting functions $\mathcal{K}(t, z, \phi)$ reproducing the soft and collinear limits of the $(n+1)$-particle x-section wrt. the n-particle x-section.
t, z, and ϕ parametrise the one-parton phase space.

Construction

Generate hard scattering $\mathrm{d}\sigma_0$ first, then evolve outwards

FS: forward evolution, $t \searrow$

IS: backward evolution, $t \searrow$, $x \nearrow$

$\Rightarrow$ PDF/SF is needed as input, not dynamically generated
IS shower unfolds the existing PDF/SF in $\mathrm{d}\sigma_0$

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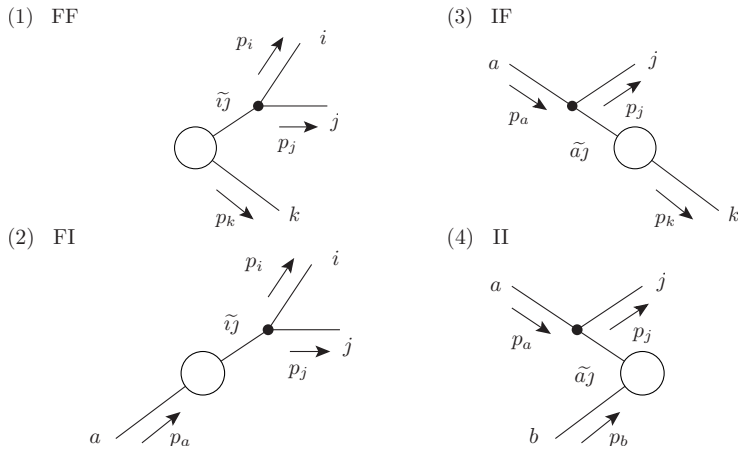
Dipole showers

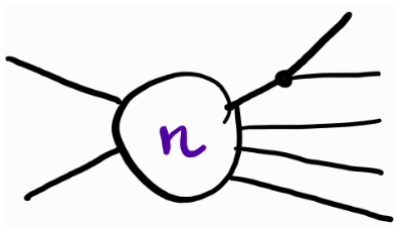


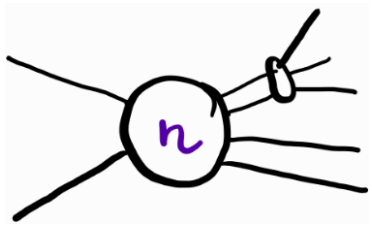
Construction

$$\mathrm{d}\sigma_{n+1} = \mathrm{d}\sigma_n \sum_{\widetilde{ij},\widetilde{k}} \mathrm{d}\Phi_1 \, \mathcal{K}_{\widetilde{ij},\widetilde{k}}(\Phi_1^{\widetilde{ij},\widetilde{k}})$$

Dipole splitting functions $\mathcal{K}_{\widetilde{ij},\widetilde{k}}$ for splitting $\widetilde{ij}\,\widetilde{k} \to ij\,k$

Collinear limit: $\mathcal{K}_{\widetilde{ij}} = \sum_{\widetilde{k}} \mathcal{K}_{\widetilde{ij},\widetilde{k}}$ **Soft limit:** $\mathcal{K}_{\widetilde{ij},\widetilde{k}}$





Collinear limit spread over many dipoles,
coherence and consistence are fundamental

QCD parton showers and the large-N_c limit

Full soft-collinear splitting function:

$$\mathcal{K}_{\widetilde{ij},\widetilde{k}} = -\mathbf{T}_{\widetilde{k}}\mathbf{T}_{\widetilde{ij}} \otimes \mathbf{V}_{\widetilde{ij},\widetilde{k}}$$

where the matrix-valued colour correlators have entries of indeterm. sign, and matrix-valued splitting operator $\mathbf{V}_{\widetilde{ij},\widetilde{k}}$

The large-N_c limit.

$$-\mathbf{T}_{\widetilde{k}}\mathbf{T}_{\widetilde{ij}} \longrightarrow \begin{cases} C_F + \mathcal{O}(1/N_c^2) & \text{if } \widetilde{ij} = q \\ C_A/2 + \mathcal{O}(1/N_c^2) & \text{if } \widetilde{ij} = g \\ \mathcal{O}(1/N_c^2) & \text{otherwise} \end{cases} \quad \text{and } \widetilde{k} \text{ colour neighbour,}$$

This reduces the matrix-valued colour correlators to Casimirs ($\in \mathbb{R}$).

Spin-averaging. Replace $\mathbf{V}_{\widetilde{ij},kt} \to \langle \mathrm{V}_{\widetilde{ij},\widetilde{k}} \rangle$.

The splitting functions $\mathcal{K}$ now are real-valued positive definite functions.

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QED parton showers

Well-understood abelian limit of QCD parton showers which suffer from non-existing large-N_c limit. QED is U(1), and $\frac{1}{1}$ is not so small.
$\Rightarrow$ all charge partners of equal importance, but contrib. with diff. signs.

Collinear limit:

coll. emission pattern trivial as only determined by charge of emitter

Soft limit:

sum of eikonals of alternating sign,
neglecting neg. dipoles gets soft limit wrong

alt: multipole rad. Yennie, Frautschi, Suura '61

MS, Krauss '08; Kleiss, Verheyen '17

Additional complication: QED result of a broken gauge group
$SU(2)_L \times U(1)_Y \to U(1)_{\text{QED}}$
in addition to massive fermions
$\Rightarrow$ resonances everywhere

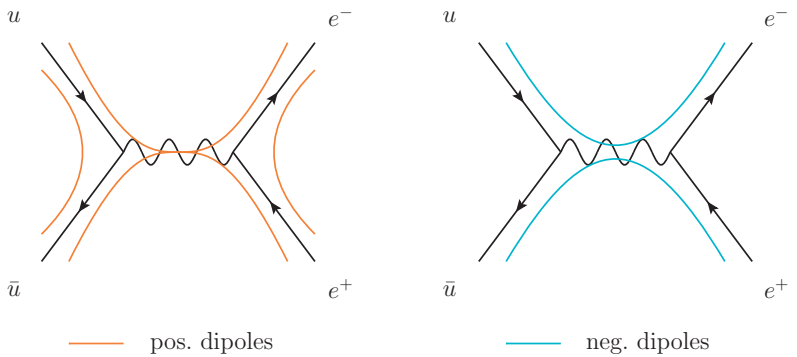
QED dipoles and soft eikonal

Main issue: QED is U(1)

equivalent of large-N_c expansion not very meaningful

need full multipole picture for soft-photon coherence

Example: $u\bar{u} \to e^+e^-$, 6 dipoles (4 opposite sign, 2 same sign)



need all dipoles for correct soft-collinear structure,
all dipoles contribute on equal footing
can be handled by weighted shower

QED dipoles

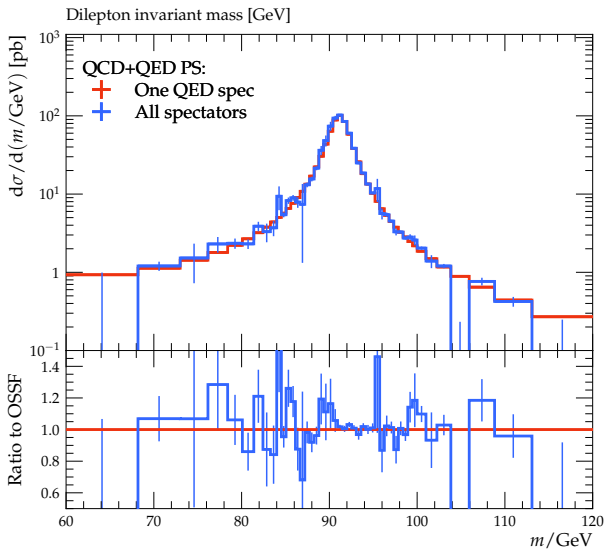
Photon splittings $\gamma \to f\bar{f}$

Pure collinear splitting, free to choose recoil partner to ensure momentum conservation

Flower, MS '22

Problem: every $\gamma \to f\bar{f}$ and $g \to q\bar{q}$ introduces $2n+2$ new opposite-sign and $2n$ new like-sign QED dipoles.

However, in strongly ordered parton shower each such $f\bar{f}$-pair is produced at a well-separated scales and radiation separates into local-dipoles, ie. $2n$ opposite-sign/like-sign dipole pairs that cross the intermediate γ/g cancel pairwise.



QED dipoles – initial state evolution

$$\mathrm{d}\sigma_{n+1} = \mathrm{d}\sigma_n \sum_{\widetilde{ij},\widetilde{k}} \mathrm{d}\Phi_1 \, \mathcal{K}_{\widetilde{ij},\widetilde{k}}(\Phi_1^{\widetilde{ij},\widetilde{k}}) \qquad \mathrm{d}\Phi_1^{\widetilde{ij}\widetilde{k}} = \mathrm{d}t \, \mathrm{d}z \, \mathrm{d}\phi \; J(t,z,\phi)$$

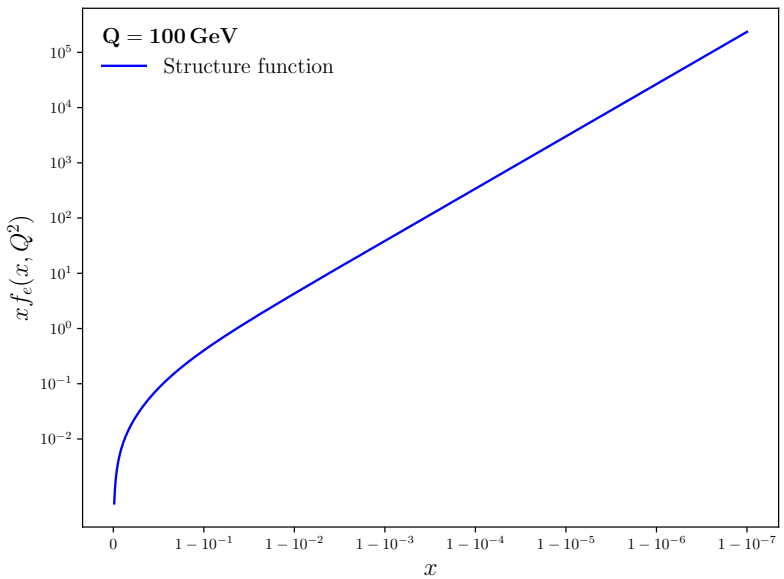
When initial state partons are involved, either as emitter or spectator, they change their momentum fraction x, inducing a change of the structure function associated with it.

$$J_{\mathrm{SF}} = \frac{f_a\left(\frac{x_{\widetilde{ai}}}{z}, t_i\right)}{f_{\widetilde{ai}}(x_{\widetilde{ai}}, t_i)}$$

While in a proton PDF, all f_a are regular as $x \to 1$, this is not the case for the electron structure function.

The electron structure function

$$f_e(x, Q^2) = \beta \frac{\exp\left(-\gamma_e \beta + \frac{3}{4}\beta_S\right)}{\Gamma(1+\beta)} (1-x)^{\beta-1} + \sum_{n=1}^{\infty} \beta_H^n \mathcal{H}_n(x)$$



$$\beta = \frac{\alpha(Q^2)}{\pi}\left[\log\frac{Q^2}{m_e^2} - 1\right]$$

Lepton structure function
LL solution to DGLAP eq. with initial conditions $f_e(x, m_e^2) = \delta(1-x)$

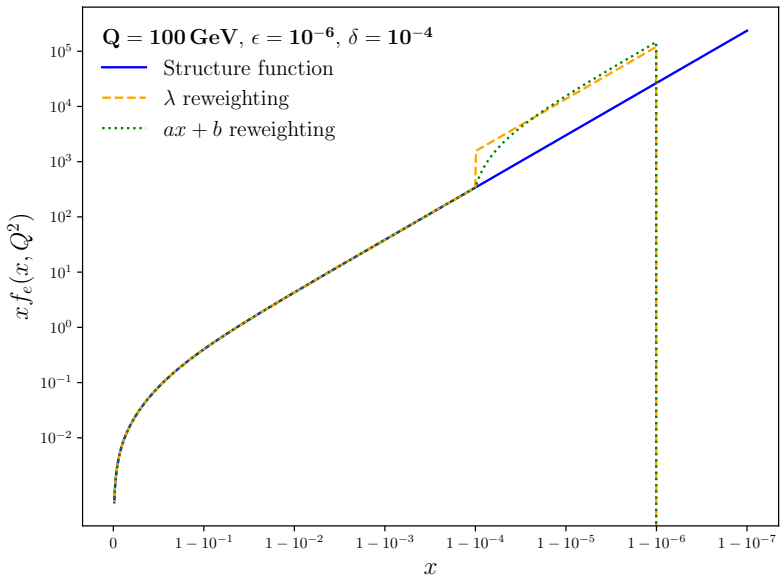
towards higher-orders Frixione '19
Stahlhofen '25; Schnubel, Szafron '25

full PDFs Frixione et.al '19,'23

Integrable singularity at $x = 1$ at any order.

.. and numerical integration

$$f_e(x, Q^2) = \beta \frac{\exp\left(-\gamma_e \beta + \frac{3}{4}\beta_S\right)}{\Gamma(1+\beta)} (1-x)^{\beta-1} + \sum_{n=1}^{\infty} \beta_H^n \mathcal{H}_n(x)$$



Integrable singularity at $x = 1$ is problematic for num. integration

Solution:

$$W_e = \begin{cases} f_e(x) & x \in [0, 1-\delta] \\ w(x) f_e(x) & x \in [1-\delta, 1-\epsilon] \\ 0 & x \in [1-\epsilon, 1] \end{cases}$$

Determine $w(x)$ such that

$$\int_{1-\delta}^{1} \mathrm{d}x\, W_e = \int_{1-\delta}^{1} \mathrm{d}x\, f_e$$

Problems:

- W_e for $w(x) = \text{const.}$ not continuous
- W_e does no longer solves DGLAP eq.

Numerical stability

Flower, MS '26

Process	Rescaling	$\delta = 10^{-4}$	Cross section (pb)		
	ϵ	10^{-9}	10^{-8}	10^{-7}	10^{-6}
$e^+e^- \to \nu_\mu\bar{\nu}_\mu$	λ	2869.0 ± 1.6	2869.2 ± 1.3	2870.5 ± 1.1	2869.7 ± 1.0
91.2 GeV	$ax+b$	2868.3 ± 1.6	2870.4 ± 1.5	2870.0 ± 1.1	2869.9 ± 1.0
$e^+e^- \to \nu_\mu\bar{\nu}_\mu$	λ	0.8370 ± 0.0005	0.8368 ± 0.0003	0.8368 ± 0.0003	0.8371 ± 0.0003
500 GeV	$ax+b$	0.8370 ± 0.0004	0.8368 ± 0.0003	0.8366 ± 0.0003	0.8373 ± 0.0003
$e^+e^- \to ZH$	λ	0.1209 ± 0.0006	0.1212 ± 0.0006	0.1220 ± 0.0006	0.1223 ± 0.0006
365 GeV	$ax+b$	0.1210 ± 0.0007	0.1211 ± 0.0006	0.1218 ± 0.0007	0.1219 ± 0.0007

Process	Rescaling	$\epsilon = 10^{-8}$	Cross section (pb)		
	δ	10^{-7}	10^{-6}	10^{-5}	10^{-4}
$e^+e^- \to \nu_\mu\bar{\nu}_\mu$	λ	2876.0 ± 7.0	2870.7 ± 3.3	2872.0 ± 1.8	2872.1 ± 1.2
91.2 GeV	$ax+b$	2871.1 ± 8.0	2873.0 ± 3.8	2871.6 ± 2.1	2871.6 ± 1.2
$e^+e^- \to \nu_\mu\bar{\nu}_\mu$	λ	0.8393 ± 0.0012	0.8381 ± 0.0007	0.8376 ± 0.0005	0.8375 ± 0.0003
500 GeV	$ax+b$	0.8400 ± 0.0014	0.8384 ± 0.0007	0.8378 ± 0.0005	0.8376 ± 0.0004
$e^+e^- \to ZH$	λ	0.1236 ± 0.0010	0.1235 ± 0.0005	0.1230 ± 0.0003	0.1231 ± 0.0002
365 GeV	$ax+b$	0.1248 ± 0.0012	0.1236 ± 0.0006	0.1232 ± 0.0003	0.1230 ± 0.0002

Fine for now. Still good enough for 10^{-5} accuracy?

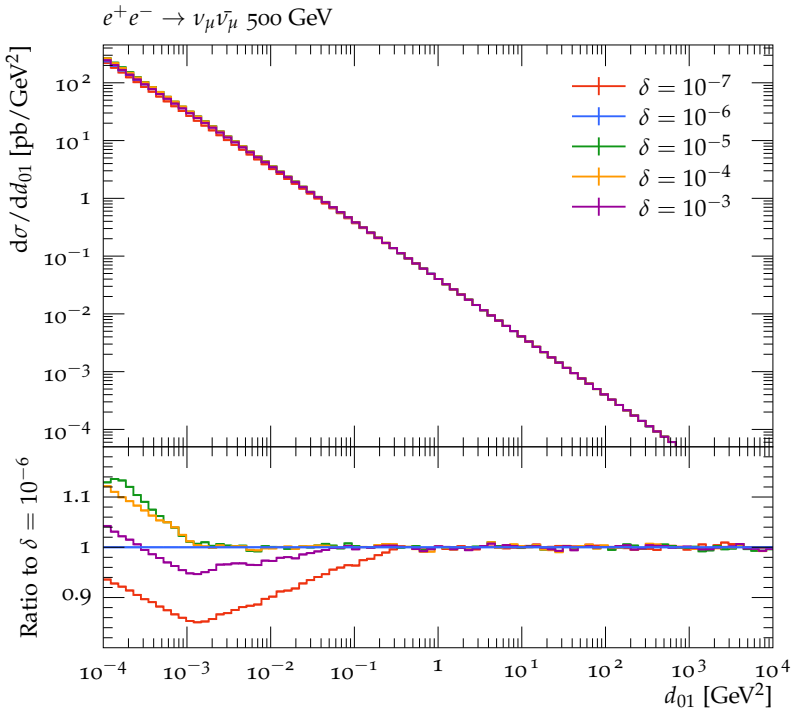
Interplay with initial state QED shower

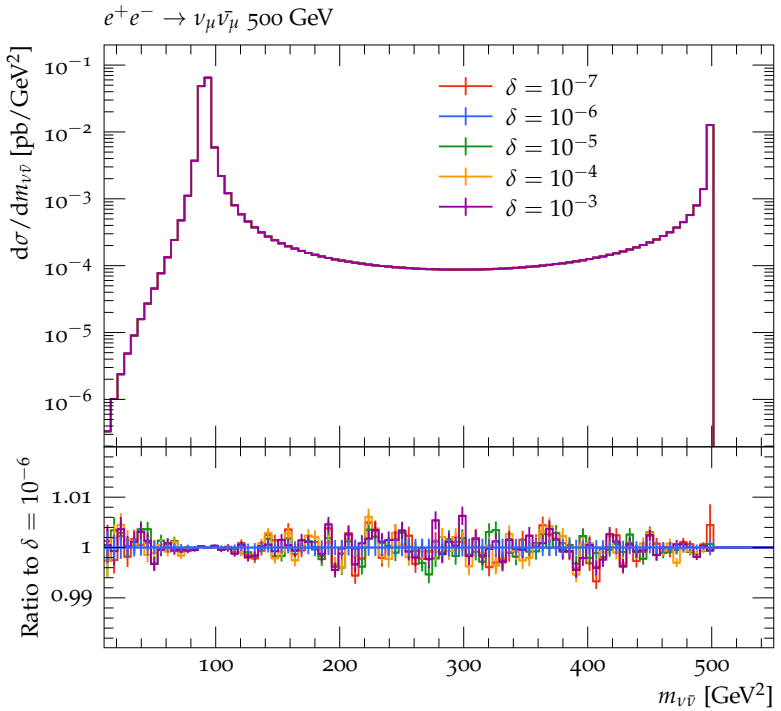
Initial state shower

- backwards evolution from hard scattering to incident beam particles
  $t_{n+1} < t_n$, but $x_{n+1} > x_n$
- contrary to hadron colliders, electron structure function enhances evolution to larger x,
  in particular high probability to evolve from moderate x, eg. $x \approx 0.5$, to large $x \lesssim 1$
- ϵ and δ imply $x_{\text{max}} < 1$
  choice: $x_{\text{max}} = 1 - \delta$ (largest x of ME config),
  allow one emission into $[1 - \delta, 1 - \epsilon]$, do not evolve further,
  important that W_e continuous at $x = 1 - \delta$
- infrared regularisation through cut-off t_c,
  QED IR free, hence $t_c \propto m_e$

In practice – LO+PS QED $e^+e^- \to \nu_\mu \bar{\nu}_\mu$ @ 500 GeV

Dependence on δ





- dependence below accuracy if δ small enough, default $\delta = 10^{-4}$

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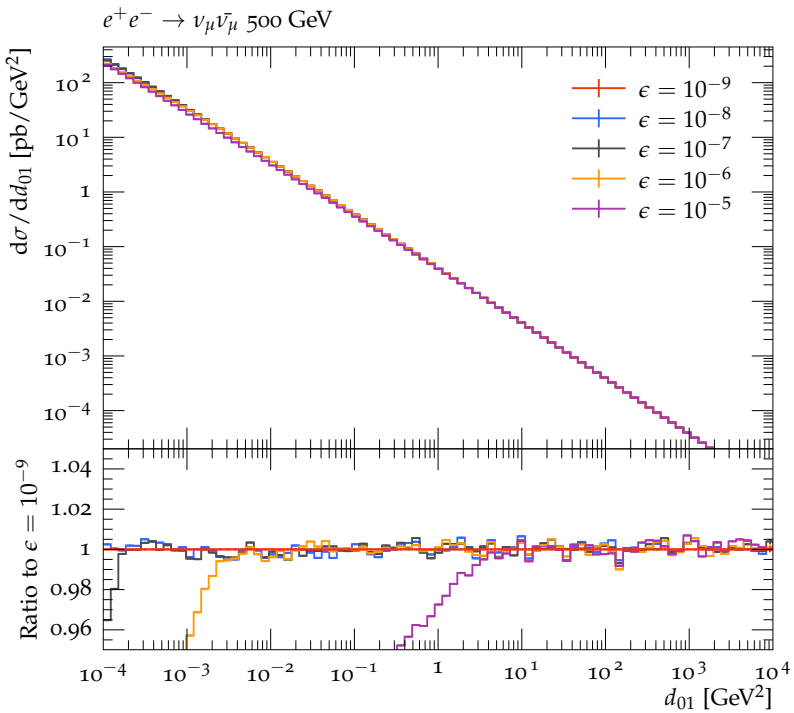
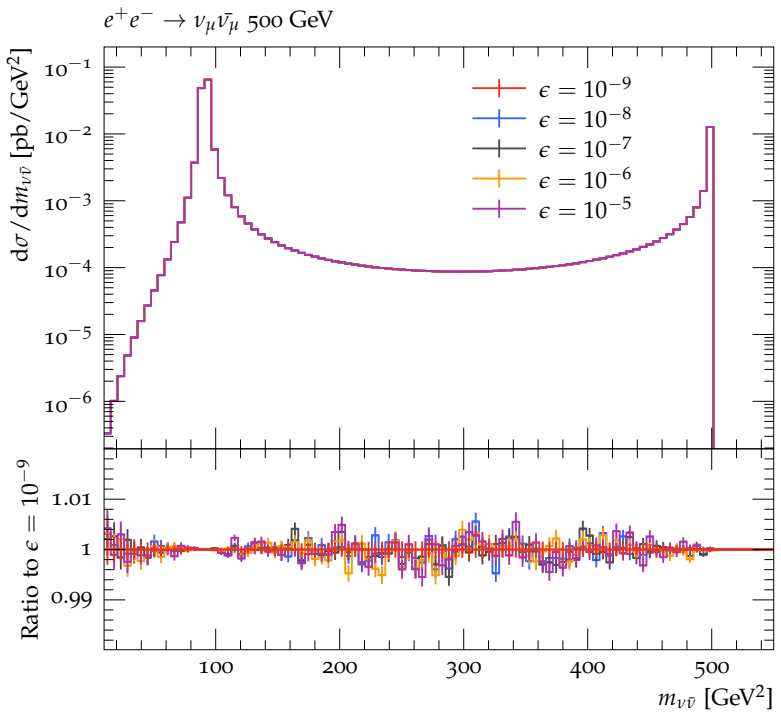
Dependence on ϵ

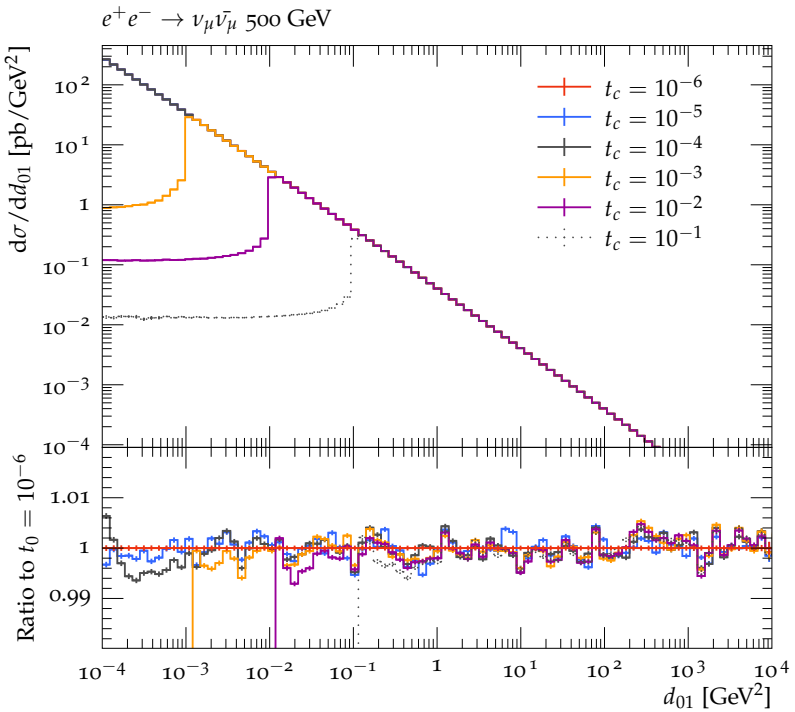


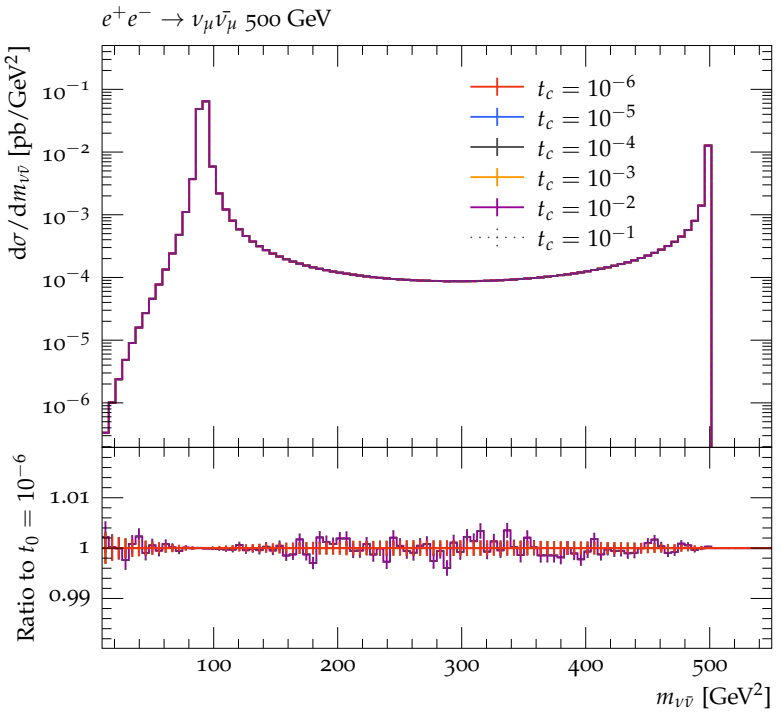


- dependence below accuracy if ϵ small enough, default $\epsilon = 10^{-8}$

In practice – LO+PS QED $e^+e^- \to \nu_\mu \bar{\nu}_\mu$ @ 500 GeV

Dependence on t_c





- dependence below accuracy if t_c small enough, default $t_c = m_e^2$

MC@NLO EW for lepton colliders

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MC@NLO EW for lepton colliders

MC@NLO matching in SHERPA

$$\langle \mathcal{O} \rangle = \int \mathrm{d}\Phi_B \, \overline{\mathrm{B}}(\Phi_B) \, \mathcal{F}^A(t_0, O) + \int \mathrm{d}\Phi_R \, \mathrm{H}(\Phi_R) \, \mathcal{F}(t_1, O)$$

MC@NLO EW follows the established NLO QCD matching methods

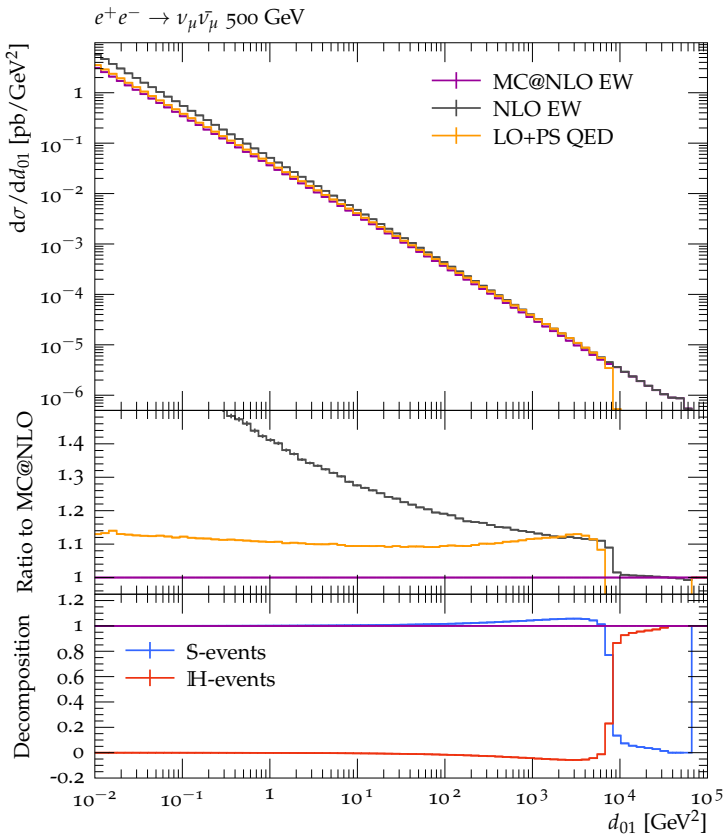
Höche, Krauss, MS, Siegert '11

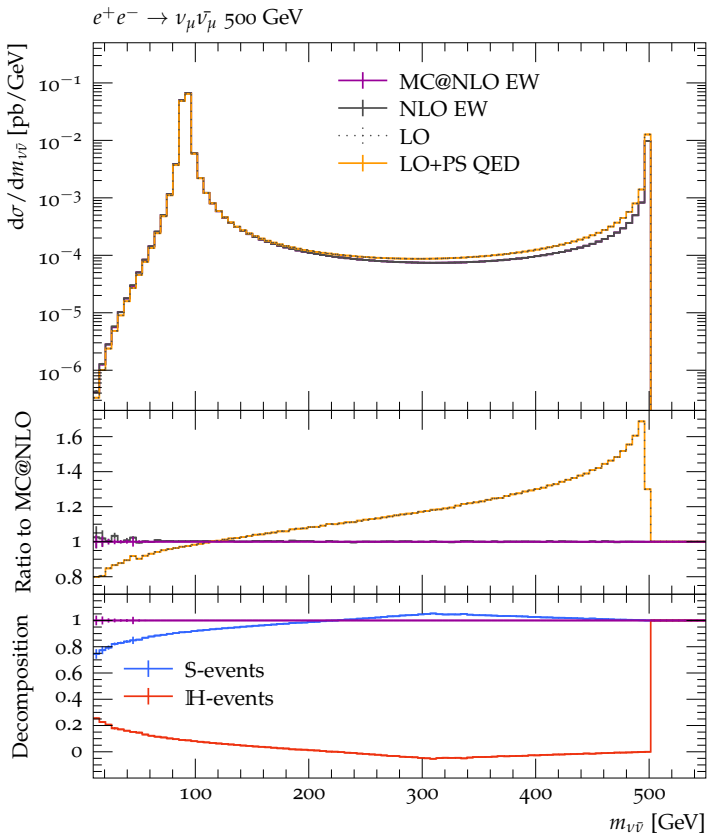
$$\begin{aligned}
\overline{\mathrm{B}}(\Phi_B) &= \mathrm{B}(\Phi_B) + \tilde{V}(\Phi_B) + \mathrm{I}^S(\Phi_B) \\
&\quad + \int \mathrm{d}\Phi_1 \sum_i \left[\mathrm{D}_i^A(\Phi_B, \Phi_1) - \mathrm{D}_i^S(\Phi_B, \Phi_1) \right] \\
\mathrm{H}(\Phi_R) &= \mathrm{R}(\Phi_R) - \sum_i \mathrm{D}_i^A(\Phi_B^i, \Phi_1^i)
\end{aligned}$$

with the spin-correlated splitting functions D_i^A
and IR subtraction $\mathrm{I}^S = \sum_i \int \mathrm{d}\Phi_1 \, \mathrm{D}_i^S$.

MC@NLO QED $e^+e^- \to \nu_\mu \bar{\nu}_\mu$ @ 500 GeV

Flower, MS '26

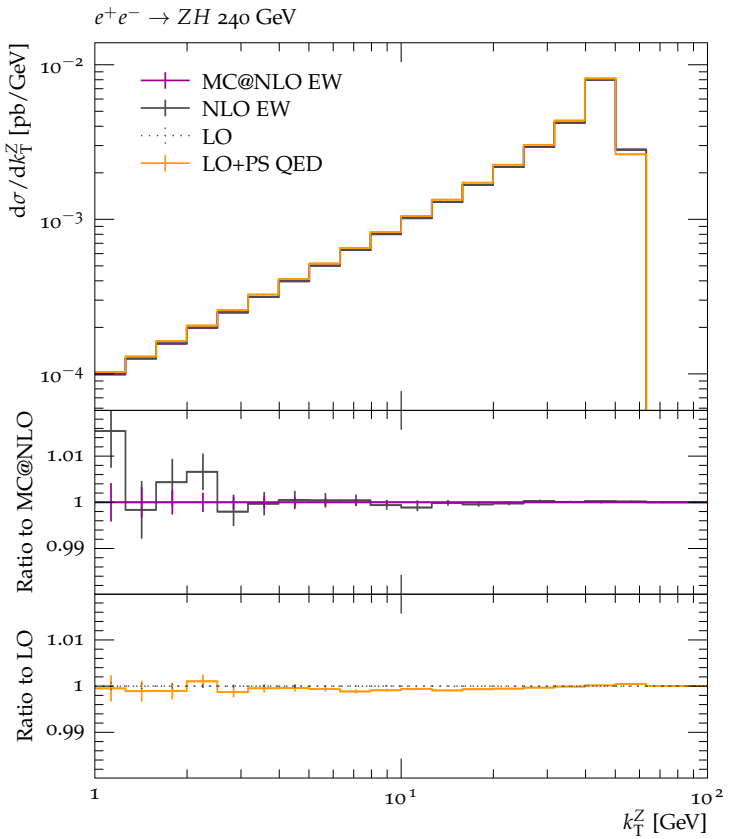


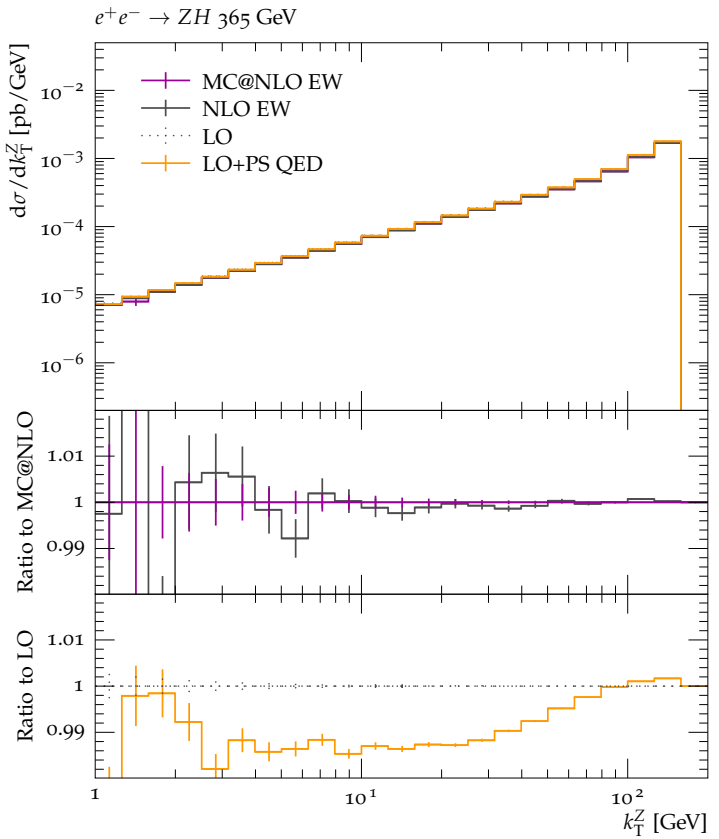


- LO and LO+PS identical → PS reproduces DGLAP
- $m_{\nu\nu}$ identical between NLO EW and MC@NLO EW (should be)

MC@NLO QED $e^+e^- \to ZH$ @ 240 GeV & 365 GeV

Flower, MS '26





- small inclusive NLO correction
- parton shower effects small on inclusive distributions

MC@NLO and resonances

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QED showers and resonances

QCD parton shower can largely avoid resonances as the top quark is the only colour-charged resonance in the SM.

Resonances are ubiquitous in the the EW SM. All processes at the LHC and future colliders contain resonances, if not at LO, then at higher orders.

Soft-collinear limit of splitting functions has no information of transverse momentum balance (recoil), in principle arbitrary.

Recoil assignment (momentum conservation) must not alter virtuality of resonance.

Infrared subtraction and parton shower splitting kernels follow the same physics principles.

for FKS Jezo, Nason '15, Frederix et.al '16
for ALARIC Höche, Reichelt '26

QCD+QED dipole shower evolution

The **emission probability**

$$\mathrm{d}\sigma_{n+1} = \mathrm{d}\sigma_n \sum_{\widetilde{ij},\widetilde{k}} \mathrm{d}\Phi_1 \, \mathcal{K}_{\widetilde{ij},\widetilde{k}}(\Phi_1^{\widetilde{ij},\widetilde{k}}) \qquad \mathrm{d}\Phi_1^{\widetilde{ij}\widetilde{k}} = \mathrm{d}t \, \mathrm{d}z \, \mathrm{d}\phi \; J(t,z,\phi)$$

with combined kernel $\mathcal{K}_{\widetilde{ij},\widetilde{k}} = \mathcal{K}^{\mathrm{QED}}_{\widetilde{ij},\widetilde{k}} + \mathcal{K}^{\mathrm{QED}}_{\widetilde{ij},\widetilde{k}}$ is the **sum** of QCD and QED emissions.

The **no-emission probability** (Sudakov form factor), however,

$$\Delta_n = \exp\left[-\int \mathrm{d}\Phi_1 \, \mathcal{K}_{\widetilde{ij},\widetilde{k}}(\Phi_1)\right] = \Delta_n^{\mathrm{QCD}} \cdot \Delta_n^{\mathrm{QED}}$$

is the **product** of QCD and QED no-emission probabilties.

Interleaved shower not simply the sum of QCD and QED evolution.

Renance awareness

Flower, Roper, MS to appear

Resonant vs. non-resonant

Not a clearly defined concept, define
$\Delta = |\sqrt{s} - m|/\Gamma$ as measure of offshellness Kallweit, Lindert, Pozzorini, MS '17

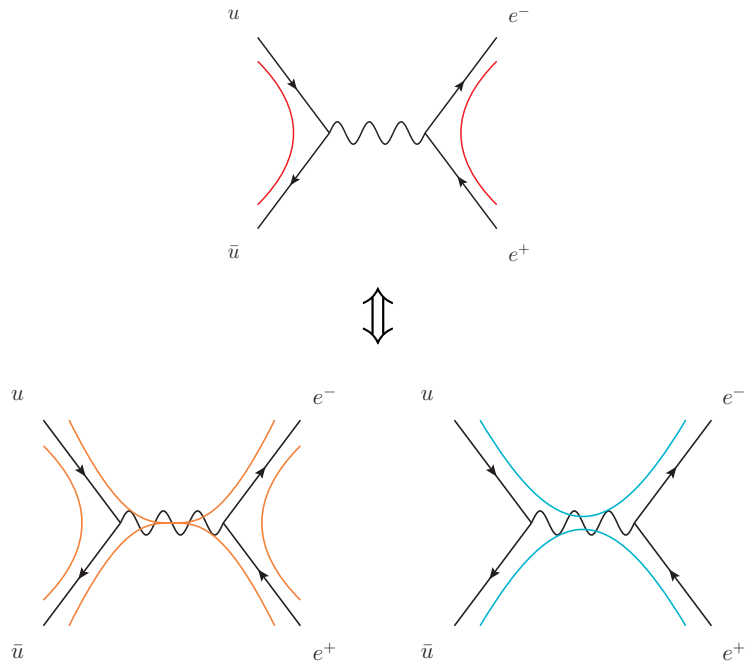
$\Rightarrow$ **introduce** $\Delta_{\text{res}} \sim 2..10$, with $\Delta < \Delta_{\text{res}}$ resonant

Physical picture on resonance

$t \gtrsim \Gamma^2$ photon emission **can** resolve the resonance, production and decay emit separately.

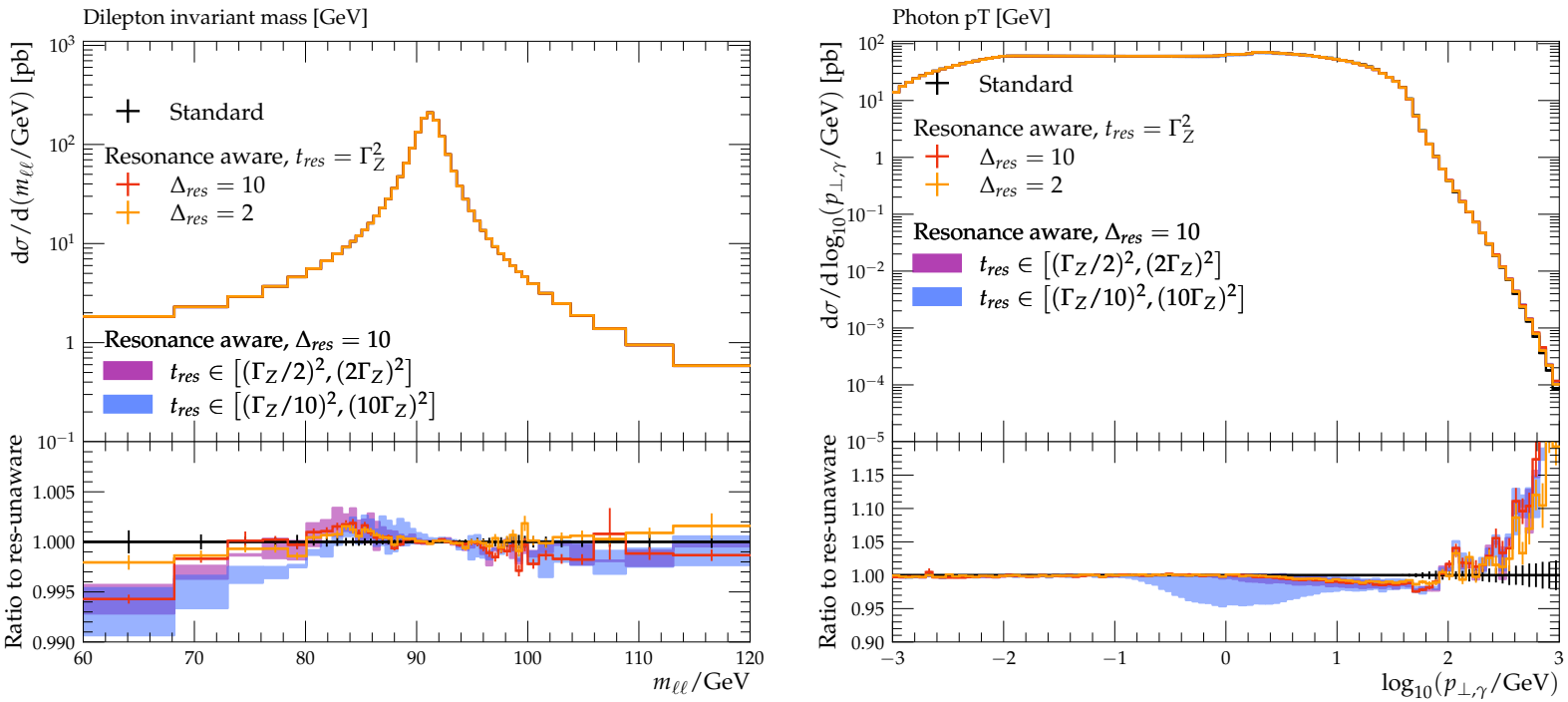
$t \lesssim \Gamma^2$ photon emission **cannot** resolve the resonance, production and decay emit coherently.

$\Rightarrow$ **introduce** $t_{\text{res}} \sim \Gamma^2$



Preliminary: MC@NLO QCD+EW $pp \to \ell\ell$ @ LHC

Flower, Roper, MS to appear



- resonance awareness relevant at subpercent accuracy
- but full infrared structure relevant for emission properties

Conclusions

e^+e^- colliders offer great physics opportunities, but we need the tools to seize them!

- QED dipole showers implement natively the soft and collinear limits of QED radiation, and can be straight forwardly interleaved with QCD dipole showers
- QED initial state showers for lepton initial states at lepton colliders need some care regarding integrable singularity at $x = 1$ in lepton structure function
- can produce negative weights when $n_{\text{ch}} > 2$, but physics rescues us
- MC@NLO EW proceeds along the same lines as MC@NLO QCD
- resonance awareness necessary for many processes

http://sherpa.hepforge.org

Thank you!

Backup