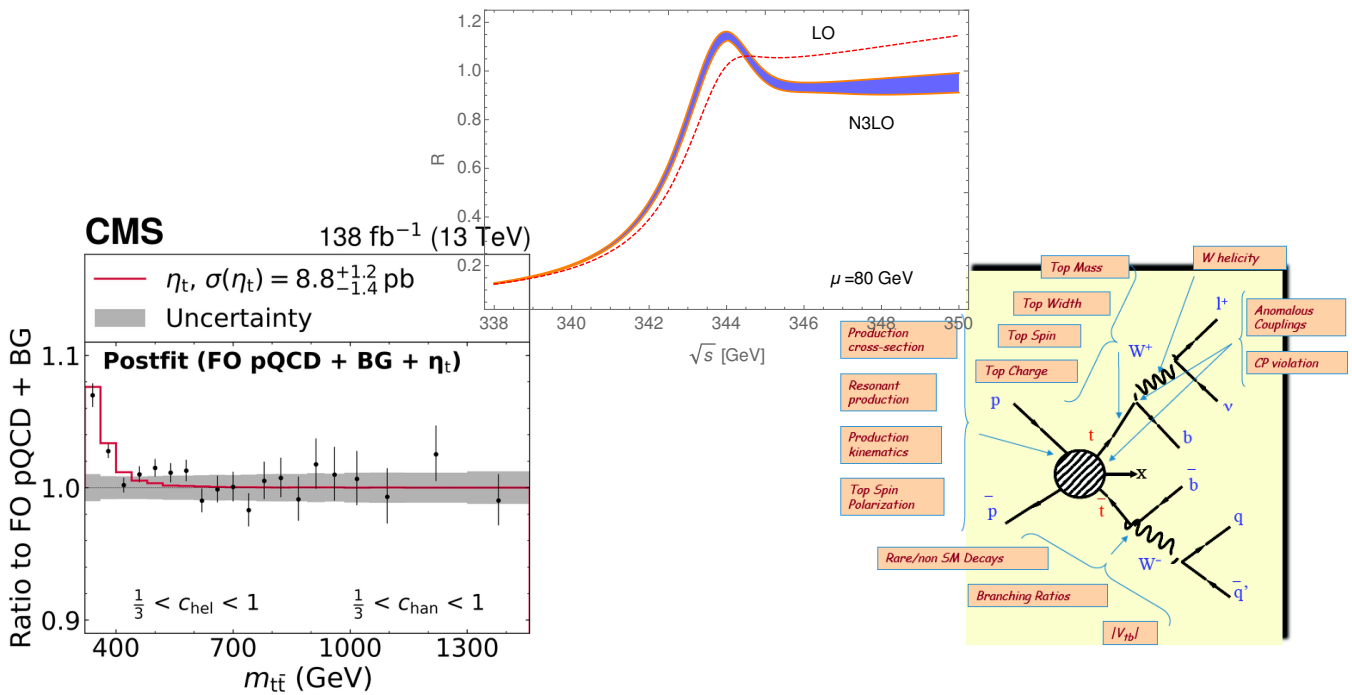
ttbar production at threshold: from ee to pp

M. Beneke (TU München)

LHC TOP WG meeting : Toponium Day
January 14, 2026



Pair production near threshold

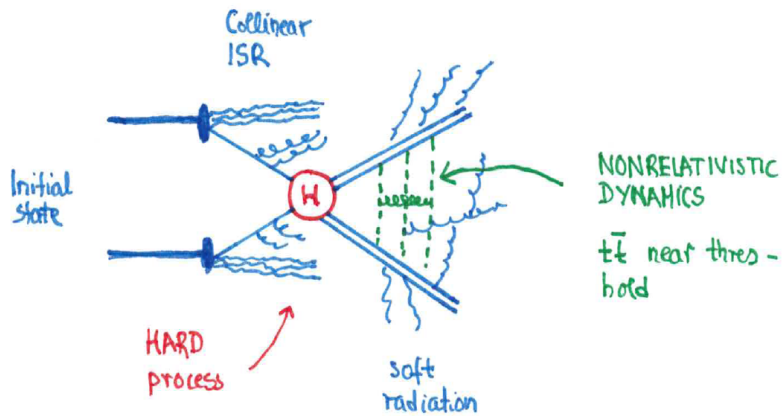
$$ij\,(q\bar{q}, qg, gg, e^+e^-) \to t\bar{t} + X$$

$$d\sigma = \sum_{i,j} \int_{4\bar{m}_t^2}^{s} d\hat{s}\, \mathcal{L}_{ij}(\hat{s}/s, \mu_f)\, d\hat{\sigma}_{ij}(\hat{s}, m_t; \mu_f)$$

$$\beta = \sqrt{1 - 4m_t^2/\hat{s}}$$

"Non-perturbative" despite small couplings:

- Soft gluon (photon) resummation, Sudakov logarithms: $g^2 \ln^2 \beta \sim 1$.
- Strong Coulomb force: $g^2/\beta \sim 1$



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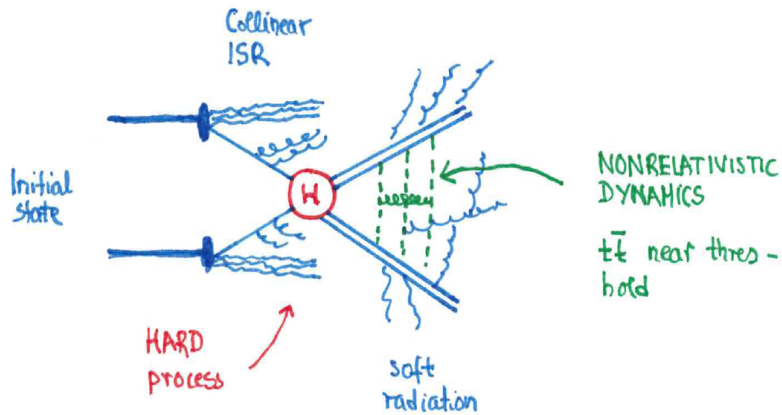


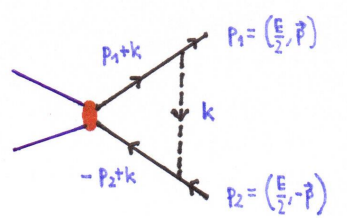
All order Soft-Coulomb factorization [MB, Falgari, Schwinn, 2009, 2010], valid to $O(\beta^2)$

$$\hat{\sigma}(\beta, \mu) = \sum_i H_i(M, \mu) \int d\omega \sum_{R_\alpha} J_{R_\alpha}(E - \frac{\omega}{2})\, W_i^{R_\alpha}(\omega, \mu)$$

Toponium from Feynman diagrams

- Coulomb force: dominant contribution to loops from $k^0 \ll \vec{k} \sim m_t v$.

$$\frac{g_s^2}{\vec{k}^2} \xrightarrow{\text{FT}} -\frac{\alpha_s}{r}$$



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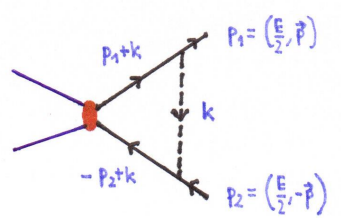

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- Sum ladder diagrams, square amplitude $J(E) \propto \text{Im}\,[G_C(E)]$ $\quad (E = \sqrt{s} - 2m_t = m_t v^2)$

$$G_C^{\overline{\text{MS}}}(E) = \frac{m^2}{4\pi}\left[-\sqrt{-\frac{E}{m}} - \alpha_s C_F\left\{\frac{1}{2}\ln\left(\frac{-4mE}{\mu^2}\right) - \frac{1}{2} + \gamma_E + \Psi(1-\lambda(E))\right\}\right]$$

$$\text{Im}\,G_C(E) = \sum_{n=1}^{\infty} \underbrace{\frac{1}{8}\left(\frac{\alpha_s C_F m}{n}\right)^3 \delta(E-E_n)}_{\text{bound states}} + \underbrace{\theta(E)\,\frac{m^2}{4\pi}\,\frac{\pi\alpha_s C_F}{1-e^{-\frac{\pi\alpha_s C_F}{v}}}}_{\text{continuum}}$$

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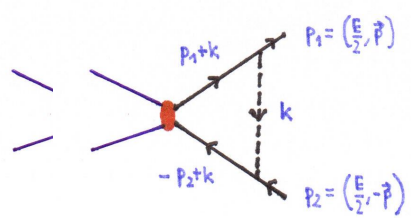

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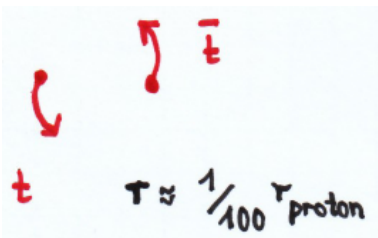


- For E near E_n, the chromostatic force bends $t\bar{t}$ into a bound orbit with radius $r \approx 10^{-17}$ m.

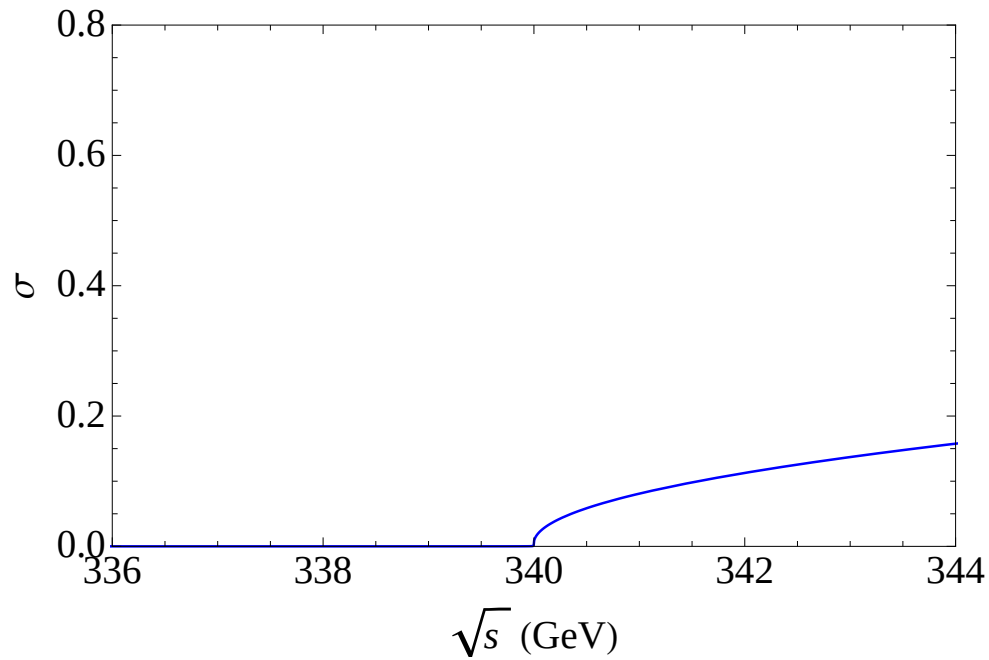
  Smallest non-elementary structure in the Standard Model of particle physics.

- Revolution time $t \approx 2\pi r/v \approx 10^{-24}$ s $\approx 2 \times \tau_{\text{top}}$

  Very short-lived resonance due to top decay.

$$\mathrm{Im}\, G_C(E) \qquad (m_t = 170\,\mathrm{GeV})$$

Stable top quark, no strong interaction



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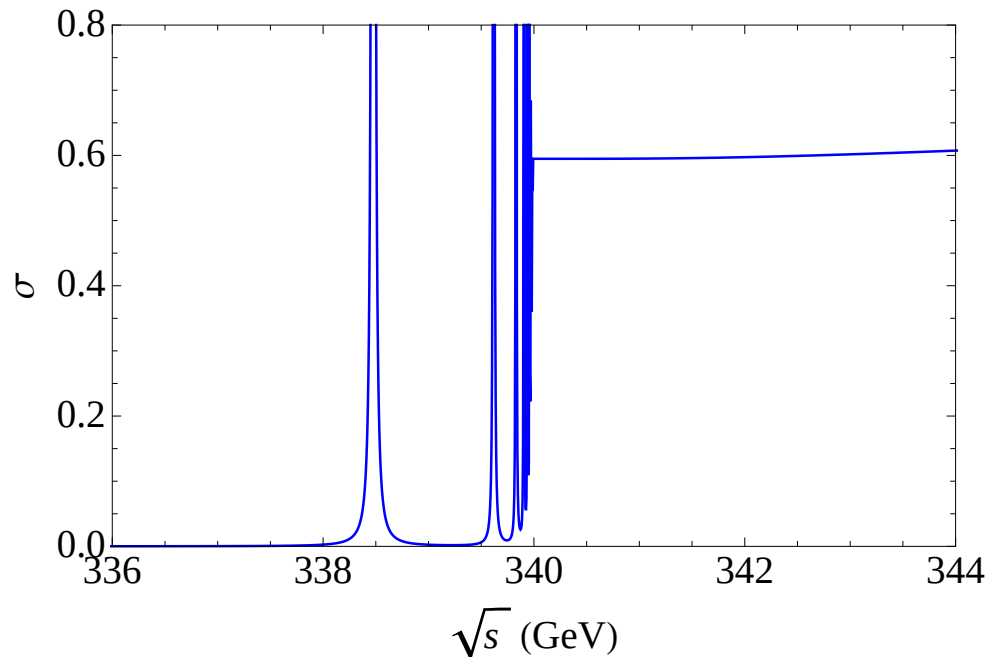
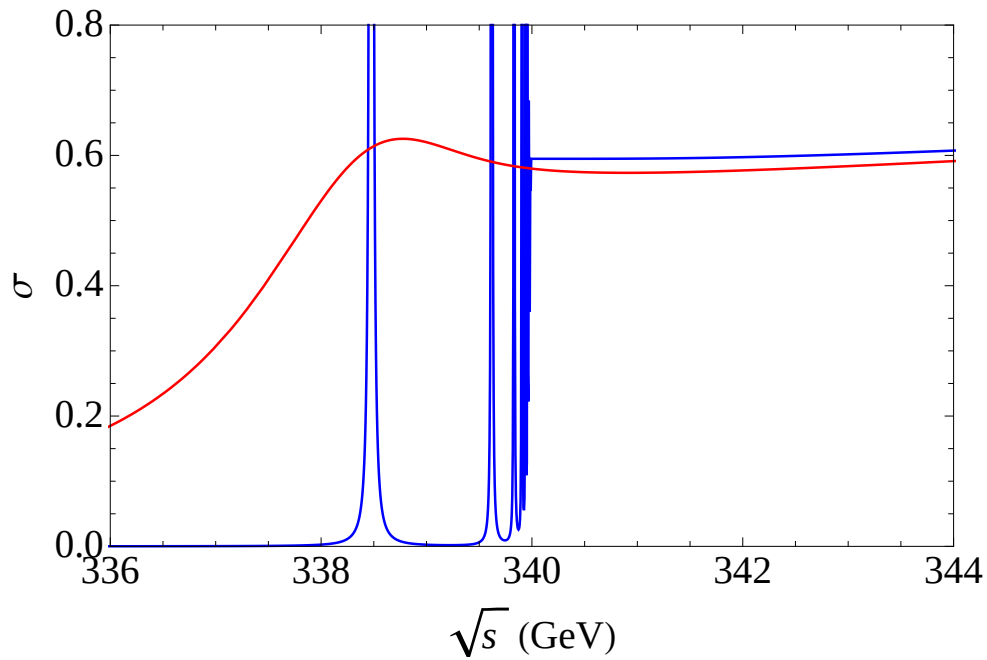


$$\mathrm{Im}\, G_C(E+i\Gamma)$$

Unstable top quark, with strong Coulomb force



How to calculate systematically

$$d\hat{\sigma} = \Sigma(\alpha_s, \sqrt{\hat{s}})$$

$$= \alpha_s^2 \left[f_{\mathrm{LO}}(\beta) + \alpha_s f_{\mathrm{NLO}}(\beta) + \ldots \right] \qquad \alpha_s \text{ small, } \beta \sim O(1)$$

$$= \alpha_s^2 \beta \left[g_{\mathrm{LO}}(\alpha_s/\beta) + \alpha_s\, g_{\mathrm{NLO}}(\alpha_s/\beta) + \ldots \right]$$

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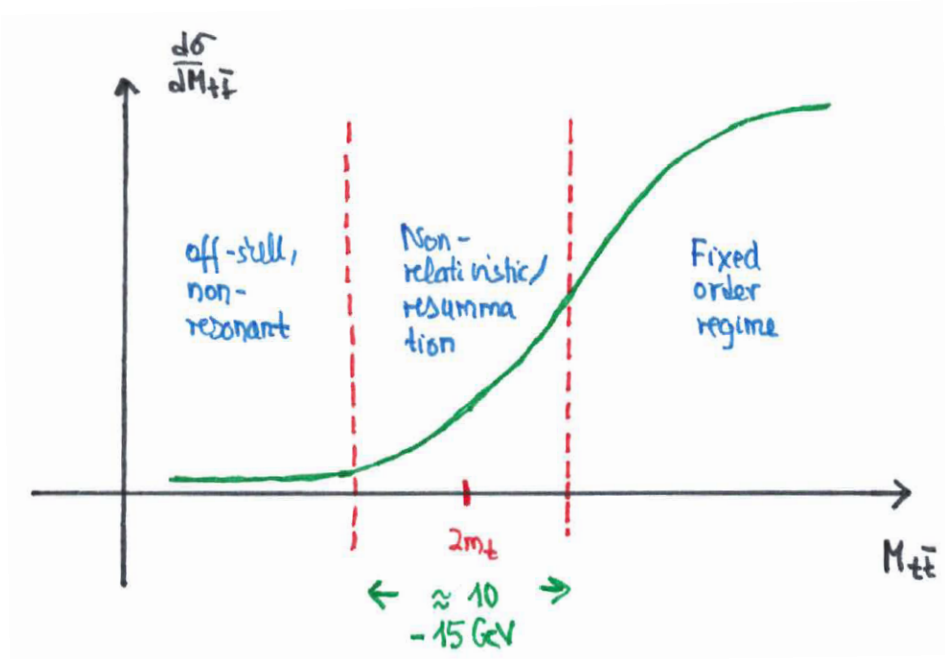

How to calculate systematically in the threshold region: (P)NRQCD

Match short-distance quantum loops to local (production + interaction) vertices ($r \sim 1/m_t$) and instantaneous potentials ($r \sim 1/(m_t\beta)$) order by order, than compute with (P)NRQCD effective Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{PNRQCD}} = {} & \psi^\dagger \Big(iD_0 + i\frac{\Gamma_t}{2} + \frac{\boldsymbol{\partial}^2}{2m} + \frac{\boldsymbol{\partial}^4}{8m^3} \Big)\psi + \chi^\dagger \Big(iD_0 - i\frac{\Gamma_t}{2} - \frac{\boldsymbol{\partial}^2}{2m} - \frac{\boldsymbol{\partial}^4}{8m^3} \Big)\chi \\ & + \int d^{d-1}\mathbf{r} \Big[\psi^\dagger\psi\Big](x+\mathbf{r}) \Big(-\frac{\alpha_s C_F}{r} + \delta V(r,\boldsymbol{\partial}) \Big) \Big[\chi^\dagger\chi\Big](x) \\ & - g_s \psi^\dagger(x)\boldsymbol{x}\boldsymbol{E}(t,\mathbf{0})\psi(x) - g_s \chi^\dagger(x)\boldsymbol{x}\boldsymbol{E}(t,\mathbf{0})\chi(x)\end{aligned}$$

- The leading-order Coulomb potential is part of the unperturbed Lagrangian. The propagator is the Coulomb Green function $G_c(\boldsymbol{r},\boldsymbol{r}';E)$
- Perturbations consist of kinetic energy corrections, perturbation potentials, and ultrasoft gluon interactions.

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Message 1: Non-relativistic theory is not an uncontrolled approximation. The $t\bar{t}$ threshold and toponium resonances can be sytematically computed in the same expansion parameter α_s as in the relativistic fixed-order regime.

$e^+e^- \to t\bar{t}$ NNNLO QCD result

- e^+e^- — N3LO [MB, Kiyo, Marquard, Penin, Piclum, Steinhauser, 1506.06864]
  Overview of theory status: 2024 v2 of MB, Kiyo, Schuller, 1312.4791, experimental study for FCCee Defranchis et al. 2503.18713
- pp — NLO [Kiyo et al. 0812.0919], $\text{NNLO}_{\text{approx}}$ [MB, Falgari, Klein, Schwinn, 1109.1536]

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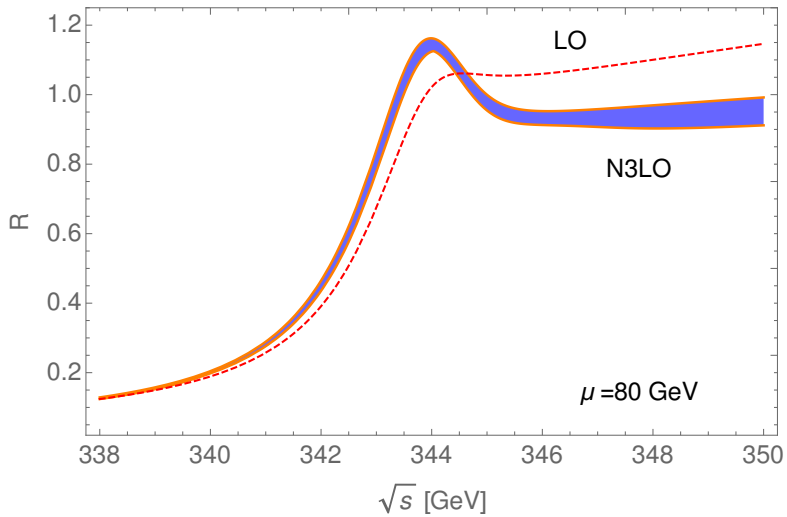

[MB, Kiyo, Marquard, Penin, Piclum, Steinhauser, 1506.06864, figure from MB, Kiyo 2409.05960]

Photon exchange and Z-vector coupling only.

$m_{t,\text{PS}}(20\,\text{GeV}) = 171.5\,\text{GeV}$, $\Gamma_t = 1.33\,\text{GeV}$,

$\alpha_s(m_Z) = 0.1185 \pm 0.006$,

- Quantum corrections are important. At least NNLO required (strengthens the force)

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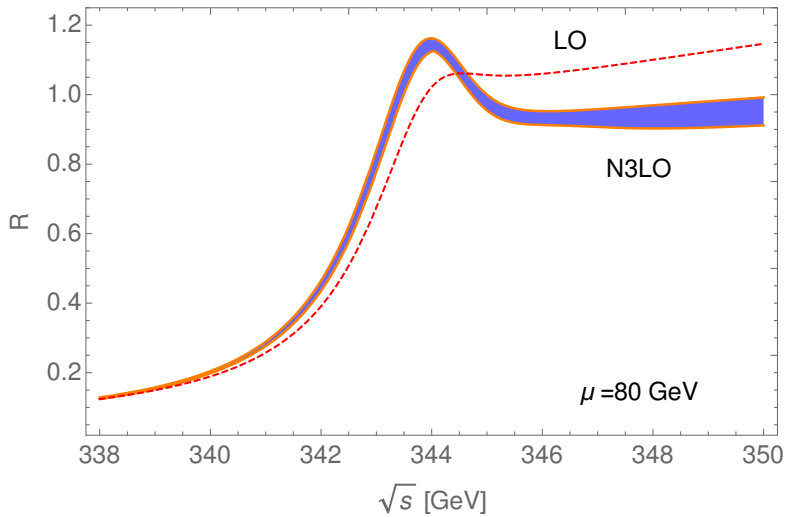

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Message 2: To discover toponium one needs to measure locally the resonance peak in a window of approx 2 GeV to distinguish it from continuum corrections.

From QCD $t\bar{t}$ threshold in e^+e^- to a realistic prediction for $W^+W^-b\bar{b}$

- Top pole mass must not be used.
- Include P-wave production
- QED effects
- Electroweak matching coefficients absorptive parts and electroweak corrections in general
- Higgs contributions
- Non-resonant contributions: $\sigma_{e^+e^-\to W^+W^-b\bar{b}_{\text{nonres}}}(\mu_w)$
  Mostly inclusive, possibly invariant mass cuts.
- QED initial state radiation
  Formally NNLO, but large logs of m_e. Effectively LO.

See back-up slides for references and presently achieved accuracies. Effects are not small relative to N3LO QCD.

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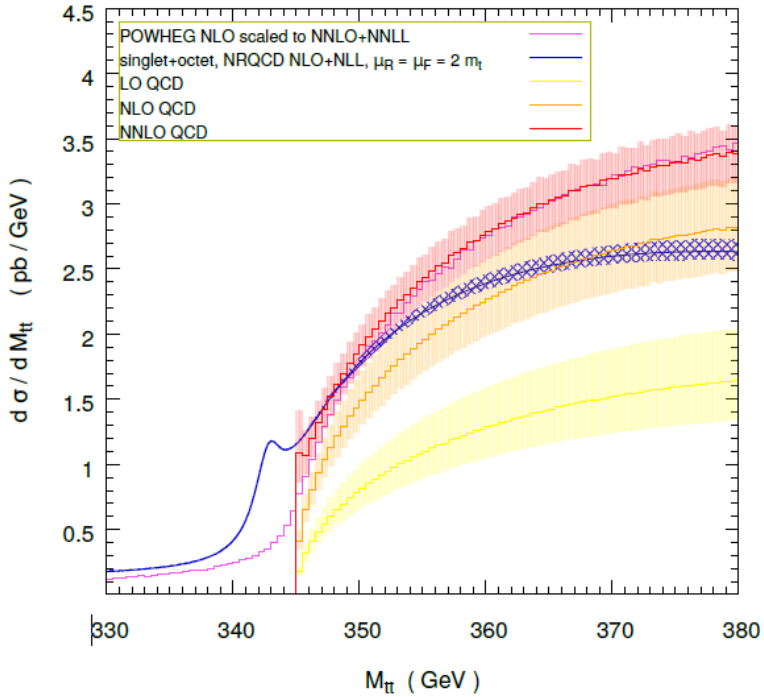
For ttbar in pp, can presently only dream of these accuracies, both experimentally and theoretically.

$pp \to t\bar{t}$ invariant mass distribution near threshold



[Kiyo, Kühn, Moch, Steinhauser, Uwer, 0812.0919, figure from update Garzelli et al. 2412.16685, see also Sumino, Yokoya, 1007.0075]

$$\sum_{pp'=q\bar{q},gg} \mathcal{L}_{pp'}(4m_t^2/s)\hat{\sigma}^{\text{thr,LO}}_{pp'\to t\bar{t}X}(E) = \frac{4\pi^2\alpha_s^2}{m_t^4} \sum_{pp'=q\bar{q},gg} \sum_{R_\alpha=1,8} \mathcal{L}_{pp'}(4m_t^2/s)\,\sigma^{R_\alpha}_{pp'}\,\text{Im}\left[G_C^{R_\alpha}(E)\right]$$

$$\sigma^1_{gg} = \frac{1}{96}\,, \qquad \sigma^8_{gg} = \frac{5}{192}\,, \qquad \sigma^1_{q\bar{q}} = 0\,, \qquad \sigma^8_{q\bar{q}} = \frac{1}{9}\,.$$

At LHC gg octet dominates from a few GeV above threshold. Near threshold gg singlet, which in turn is in the $^1S_0$ spin state.

$pp \to t\bar{t}\,X$ total cross section

- Top++ [Czakon, Mitov, 1112.5675]

  NNLO + soft-gluon NNLL

- TOPIXS [MB, ... Piclum ... et al., 1206.2454]

  NNLO + soft-gluon NNLL + non-relativistic resummation incl. bound states
  + $\text{N3LO}_{\text{approx}}$ (from v3, [Piclum, Schwinn, 1801.5788])

https://users.ph.nat.tum.de/t31software/topixs/

Welcome to the TOPIXS webpage

TOPIXS computes the inclusive top-quark pair production cross section in hadron collisions at NNLL accuracy in soft and Coulomb resummation, including the contribution from bound states, as described in

1. M. Beneke, P. Falgari, S. Klein, and C. Schwinn
   Hadronic top-quark pair production with NNLL threshold resummation
   Nucl. Phys. B **855** (2012) 695, arXiv:1109.1536 [hep-ph]
2. M. Beneke, P. Falgari, S. Klein, J. Piclum, C. Schwinn, M. Ubiali, and F. Yan
   Inclusive top-pair production phenomenology with TOPIXS
   JHEP **1207** (2012) 194, arXiv:1206.2454 [hep-ph]

Download

The current version is topixs-3.0 (Aug 24, 2018).

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	Tevatron	LHC ($\sqrt{s} = 7$ TeV)	LHC ($\sqrt{s} = 14$ TeV)
BS	$0.014\,^{+0.011+0.005}_{-0.009-0.004}$	$0.67\,^{+0.33+0.12}_{-0.31-0.10}$	$3.1\,^{+1.7+0.5}_{-1.6-0.4}$
δCb_1	-0.140	2.81	7.8
δCb_2	-0.052	0.13	-0.3

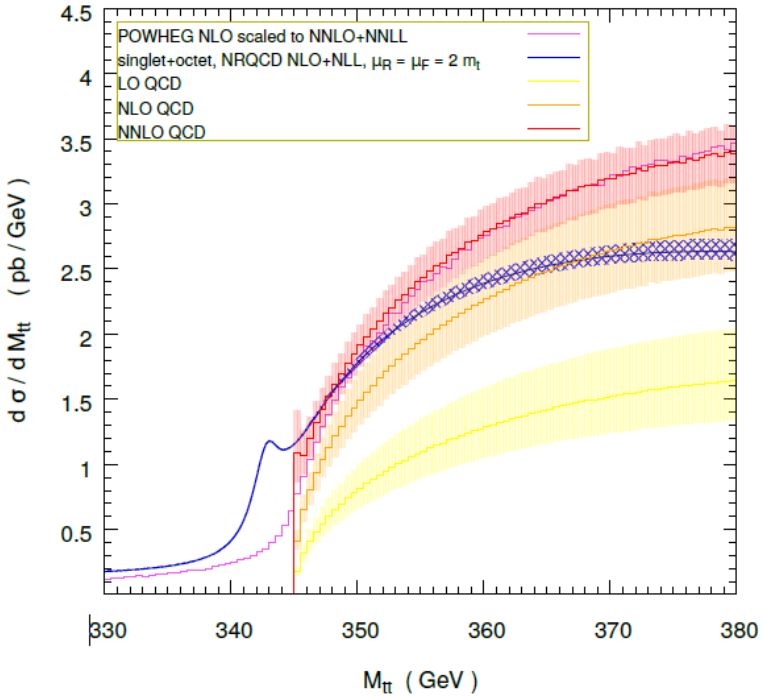
Table 6: Bound-state and Coulomb contributions for $m_t = 173.3$ GeV, Method 2. All numbers in pb. [Table from 1109.1536]

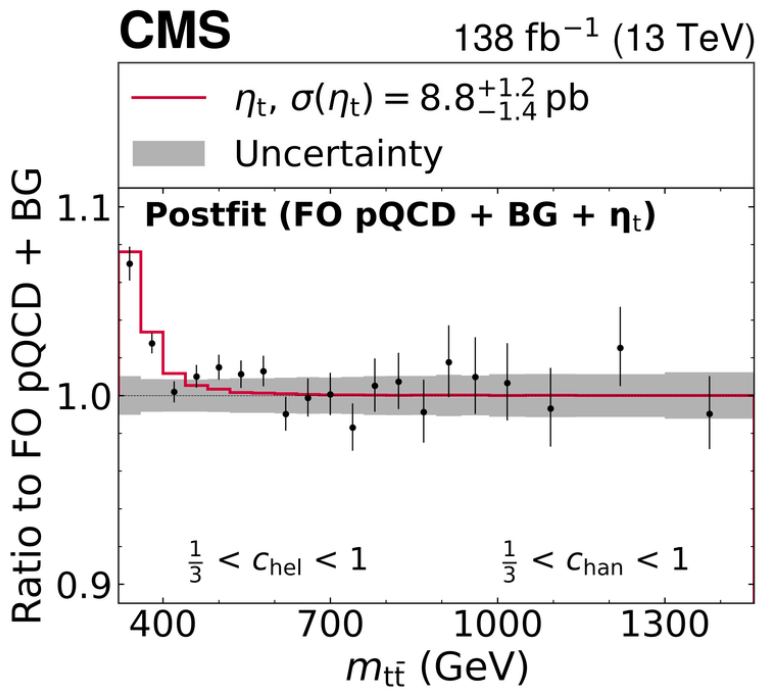
Bound states only

Continuum *beyond* NNLO

Includes soft-gluon resummation for bound-state formation

$pp \to t\bar{t}$ invariant mass distribution near threshold





CMS [2503.22382] and ATLAS [CONF-2025-008] observe an excess consistent with ${}^1S_0$ in their lowest two $m_{t\bar{t}}$ bins (approx $m_{t\bar{t}} < 400\,\text{GeV}$) of width $\approx 50\,\text{GeV}$

CMS: $8.8^{+1.2}_{-1.4}$ pb ATLAS: 9.0 ± 1.3 pb

relative to NNLO+NNLL total cross section from top++2.0 [Czakon et al.]

Divergence in fixed-order calculations?

$$\hat{\sigma}/\alpha_{\text{eff}}^2 \propto \text{Im}\, G_C(E) = \sum_{n=1}^{\infty} \frac{1}{8} \left(\frac{\alpha_{\text{eff}} m}{n}\right)^3 \delta(E - E_n) + \underbrace{\theta(E)\, \frac{m^2}{4\pi}\, \frac{\pi \alpha_{\text{eff}}}{1 - e^{-\frac{\pi \alpha_{\text{eff}}}{v}}}}_{v + c\alpha_{\text{eff}} + \sum_{n=1} c_n \alpha_{\text{eff}}^{2n+1}/v^{2n}}$$

- The usual fixed-order calculation of the total cross section or any cross section in a bin including the threshold is ill-defined starting from N^{4}LO due to $1/\beta^4$ term:

$$\int_0^{\beta_{\max}} d\beta\, \frac{8\beta m_t^2}{s(1-\beta^2)^2}\, \mathcal{L}_{ij}(\beta) \times \alpha_{\text{eff}}^2 \beta \times \frac{\alpha_{\text{eff}}^4}{\beta^4} = \infty$$

  Would already appear at N^{3}LO, but there happens to be no α_s^3/β^3 term at this order.

- Resummation into the Sommerfeld factor cures the divergence, but is a small effect on the total XS and should not be required to compute the total XS or a wide enough bin.

  Note: $m_{t\bar{t}}^{\max} = 360\,\text{GeV} \longrightarrow \beta_{\max} \approx 0.27 > \alpha_{\text{eff}} \approx 0.18$

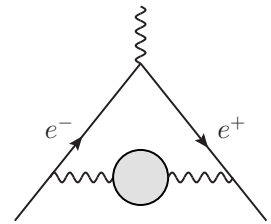
  Fixed-order should work.

Non-relativistic "parton-hadron duality" [MB, Ruiz-Femenia, 1606.02434]

Simpler QED example: Photon vacuum polarization contribution to the anomalous magnetic moment of the electron

$$a_e^{(\mathrm{vp})} = -\frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Pi\left(\frac{-x^2}{1-x}\,m^2\right) = \frac{\alpha}{\pi^2}\int_0^\infty \frac{ds}{s}\,\mathrm{Im}\,\Pi(s)\,K(s)$$

$$K(s) = \int_0^1 dx\,\frac{x^2(1-x)}{x^2+(1-x)s/m^2} \qquad \Pi = \frac{2\pi\alpha}{m^2}\,G_C$$

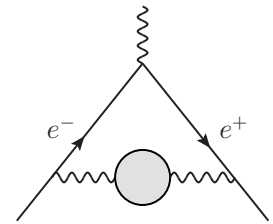


Non-relativistic "parton-hadron duality" [MB, Ruiz-Femenia, 1606.02434]

Simpler QED example: Photon vacuum polarization contribution to the anomalous magnetic moment of the electron

$$a_e^{(\mathrm{vp})} = -\frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Pi\Big(\frac{-x^2}{1-x}\,m^2\Big) = \frac{\alpha}{\pi^2}\int_0^\infty \frac{ds}{s}\,\mathrm{Im}\,\Pi(s)\,K(s)$$

$$K(s) = \int_0^1 dx\,\frac{x^2(1-x)}{x^2+(1-x)s/m^2} \qquad \Pi = \frac{2\pi\alpha}{m^2}\,G_C$$



Correct result at order $\mathcal{O}(\alpha^5)$ (=N3LO) can be obtained in two ways:

- "non-perturbative/resummed" ("hadronic")

$$a_e^{(\mathrm{vp})} \supset \underbrace{\frac{\alpha^5}{4\pi}\,\zeta_3\,K(4m_e^2)}_{\text{positronium}} - \underbrace{\frac{\alpha^5}{8\pi}\,\zeta_3\,K(4m_e^2)}_{\text{resummed continuum}}$$

  Note: expanding the continuum in α before integrating misses the 2nd term (no $1/\beta^3$ term)

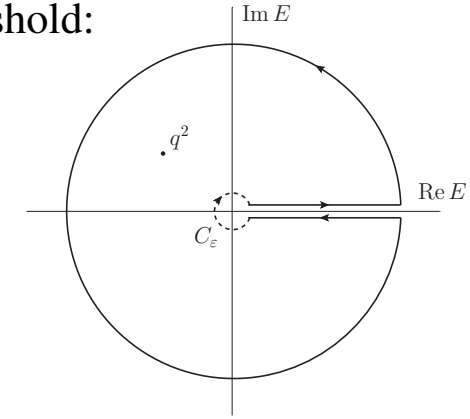
- "perturbative/fixed order" ("partonic")

$$\Pi^{(4)}(E) \supset -\frac{\alpha^4}{8}\,\frac{\zeta_3}{E/m} \Rightarrow \mathrm{Im}\,\Pi^{(4)}(E) \supset \mathrm{const}\times\delta(E) \quad\Rightarrow\quad a_e^{(\mathrm{vp})} \supset \frac{\alpha^5}{8\pi}\,\zeta_3\,K(4m_e^2)$$

Threshold-subtracted dispersion relations

Dispersion relation needs to be subtracted not only at infinity but also at threshold:

$$\Pi^{(n)}(q^2) = \frac{q^2}{2\pi i}\,\Pi^{(n)}_{C_\varepsilon}(q^2) + \frac{q^2}{\pi}\int_{(2m+\varepsilon)^2}^{\infty} ds\,\frac{\mathrm{Im}\,\Pi^{(n)}(s+i\eta)}{s\,(s-q^2)}$$



For $n > 4$ both terms depend on ε and can be rearranged into a subtraction of the dispersion integrand [see MB, Ruiz-Femenia, 1606.02434]

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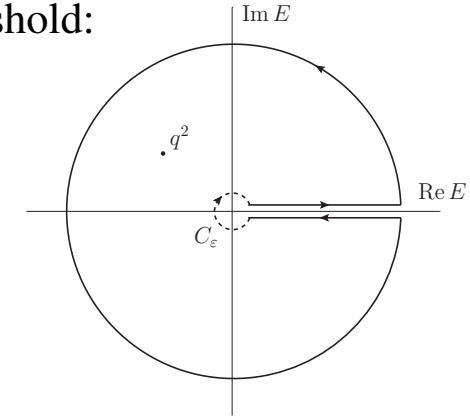

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Back to $t\bar{t}$

- $\Pi \to A(t\bar{t} \to t\bar{t})_{\text{forward}}$, $\mathrm{Im}\,\Pi \to \hat{\sigma}_{t\bar{t}}$, kernel $\to$ parton luminosity.

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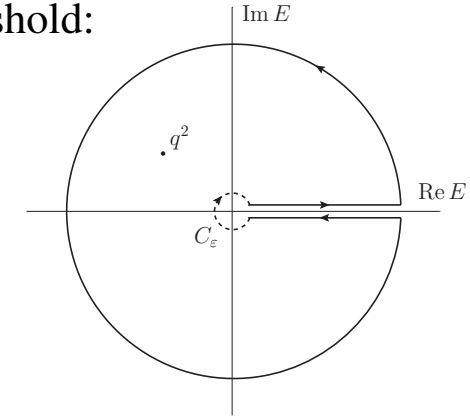

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- $\Pi \to A(t\bar{t} \to t\bar{t})_{\text{forward}}$, $\mathrm{Im}\,\Pi \to \hat{\sigma}_{t\bar{t}}$, kernel $\to$ parton luminosity.
- No need for Coulomb resummation and bound states for wide-enough bins: Fixed-order computations can be done with subtracted on convolutions with parton luminosity.
- But: if calculation is performed as phase-space integral of virtual + real at N3LO, the delta-function term will be missed. Additional N3LO contribution

$$\Delta\sigma_{t\bar{t}} \approx 0.6\,\text{pb} \qquad \text{at } \sqrt{s}_{\text{LHC}} = 13\,\text{TeV}$$

- It is automatically included in the "non-perturbative/resummed" calculation (as in TOPIXS).

Message 3: For $t\bar{t}$ invariant mass bins as wide as presently used by CMS/ATLAS, the enhancement can in principle be obtained from fixed-order perturbative calculations. Beyond the current NNLO, this is subtle and requires to properly take into account the analytic properties of the forward scattering amplitude near $4m_t^2$.

Summary

- Non-relativistic EFT is a systematic approach to calculate the top pair production cross section near threshold and toponium properties in an expansion in α_s. NNLO is desirable for good accuracy but not yet available.

- To discover toponium, one needs invariant mass resolution of a few GeV.

- For the total cross section and wide enough invariant mass bins, the threshold enhancement can be computed in standard perturbation theory with some subtleties starting at N3LO.

- When NRQCD and nonperturbative resummation is employed, the effect of bound-states and Coulomb enhancement is smaller than what is observed. However, it is not possible at this moment to disentangle the effect from other perturbative N3LO corrections.

- Nonetheless, CMS/ATLAS have probably seen the first indications of the strong Coulomb force predicted by QCD that can bind top-anti-top into a fleeting resonance.

Backup slides

$e^+e^- \to t\bar{t}X$, QCD N3LO ingredients

- Bound state quantities (S-wave)
  - E_n – Kniehl, Penin, Smirnov, Steinhauser (2002); MB, Kiyo, Schuller (2005); Penin, Sminrov, Steinhauser (2005)
  - $|\psi_n(0)|^2$ – MB, Kiyo, Schuller (2007); MB, Kiyo, Penin (2007)
- Matching coefficients
  - a_3 – Anzai, Kiyo, Sumino (2009); Smirnov, Sminrov, Steinhauser (2009)
  - c_3 – Marquard, Piclum, Seidel, Steinhauser (2014) [2009]
- Continuum (PNRQCD correlation function)
  - ultrasoft – MB, Kiyo (2008)
  - potential – MB, Kiyo, Schuller [1312.4791], MB, Kiyo [2409.05960] in preparation [2007]
  - P-wave – MB, Piclum, Rauh [1312,4792

Note: logarithmically enhanced 3rd order terms known before or resummed [Hoang et al. 2001-2013; Pineda et al. 2002-2007]. But non-log terms are as large in individual terms.
2nd order available since end of 1990s.

From QCD $t\bar{t}$ threshold in e^+e^- to a realistic prediction for $W + W^- b\bar{b}$

- Axial-vector Z-coupling (not a non-QCD effect), P-wave production
  N3LO [MB, Piclum, Rauh, 2013] [starts at NNLO]
- QED effects
  N3LO [MB, Maier, Rauh, Ruiz-Femenia, 2017] [starts at NLO, Pineda, Signer, 2006; MB, Jantzen, Ruiz-Femenia, 2010]
- Electroweak matching coefficients absorptive parts [Hoang, Reisser, 2004] and electroweak corrections in general [Guth, Kühn, 1992]
  NNLO [MB, Maier, Rauh, Ruiz-Femenia, 2017] [starts at NNLO] $[\alpha_{\text{EW}} \sim \alpha_s^2]$
- Higgs contributions [Eiras, Steinhauser, 2006; MB, Maier, Piclum, Rauh, 2015]
  N3LO [starts at NNLO]
- Non-resonant contributions: $\sigma_{e^+e^- \to W^+W^- b\bar{b}_{\text{nonres}}}(\mu_w)$
  Mostly inclusive, possibly invariant mass cuts.
  NNLO [MB, Maier, Rauh, Ruiz-Femenia, 2017] [starts at NLO, MB, Jantzen, Ruiz-Femenia, 2010; Penin, Piclum, 2011, Jantzen, Ruiz-Femenia, 2013; Ruiz-Femenia, 2014]]
- QED initial state radiation
  Formally NNLO, but large logs of m_e. Effectively LO. [MB, Maier, Rauh, Ruiz-Femenia, 2017]

e^+e^-, beyond N3LO QCD: Higgs + EW+QED+non-resonant+ISR

[MB, Maier, Piclum, Rauh, 2015; MB, Maier, Rauh, Ruiz-Femenia, 1710.10429]

Inclusive $e^+e^- \to W^+W^- b\bar{b}$ cross section with

left: N3LO Higgs and NNLO (EW+QED+non-resonant)

right: initial state radiation

$m_{t,\mathrm{PS}}(20\,\mathrm{GeV}) = 171.5\,\mathrm{GeV}$, $\Gamma_t = 1.33\,\mathrm{GeV}$, $\alpha_s(m_Z) = 0.1185 \pm 0.006$, $\sin^2\theta_W = 0.2229$,
$\mu = (50 \ldots 80 \ldots 350)\,\mathrm{GeV}$, $\mu_w = 350\,\mathrm{GeV}$.