Femtoscopy

Intermediate energy heavy-ion collisions

G. Verde, December 2, 2025

Workshop on Particle Correlations and Femtoscopy

Budapest May 18-22, 2026

18th Edition!

Occasion to discuss femtoscopy and its interdisciplinary aspects from low to high energies

Stay tuned!

- WPCF 2024, Toulouse, France
- WPCF 2023, Catania, Italy
- WPCF 2022, East Lansing, MI, USA
- WPCF 2020, Montego Bay, Jamaica (cancelled)
- WPCF 2019, Dubna, Russia
- WPCF 2018, Kraków, Poland
- WPCF 2017, Amsterdam, Netherlands
- WPCF 2015 , Warsaw, Poland
- WPCF 2014 , Gyöngyös, Hungary
- WPCF 2013 , Acireale, Italy
- WPCF 2012, Frankfurt, Germany
- WPCF 2011 , Tokyo, Japan
- WPCF 2010, Kiev, Ukraine
- WPCF 2009, Geneva, Switzerland
- WPCF 2008 , Cracow, Poland
- WPCF 2007 , Santa Rosa, USA
- WPCF 2006 , Sao Paulo, Brazil
- WPCF 2005 , Kroměříž, Czech Republic

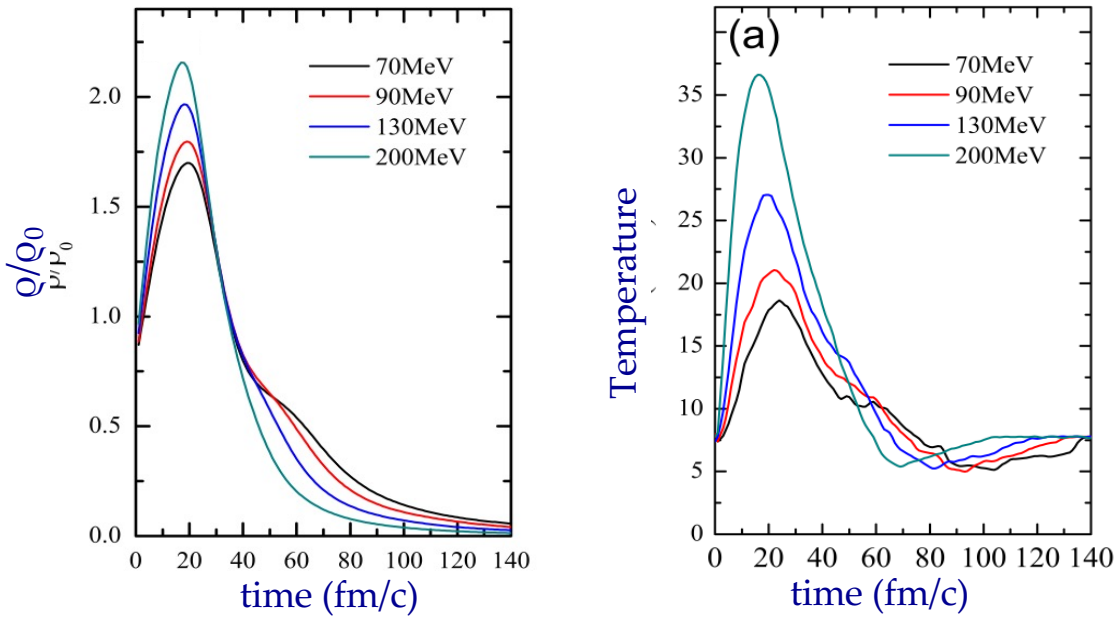
Ouline

- Why femtoscopy at lower energies
  - "low" doesn’t mean bad… :D
  - Complexity of emitting sources
- Long-lived emitting sources and source profiles
- Transport modeling Femtoscopy – pBUU vs HIC data at FRIB (E/A<100 MeV)
- Conclusions and perspectives

Space-time scales at intermediate energies

- HIC at intermediate energies: low energies but complicated (and quantum) systems

Quantum Molecular Dynamics simulations – Au + Au central collisions

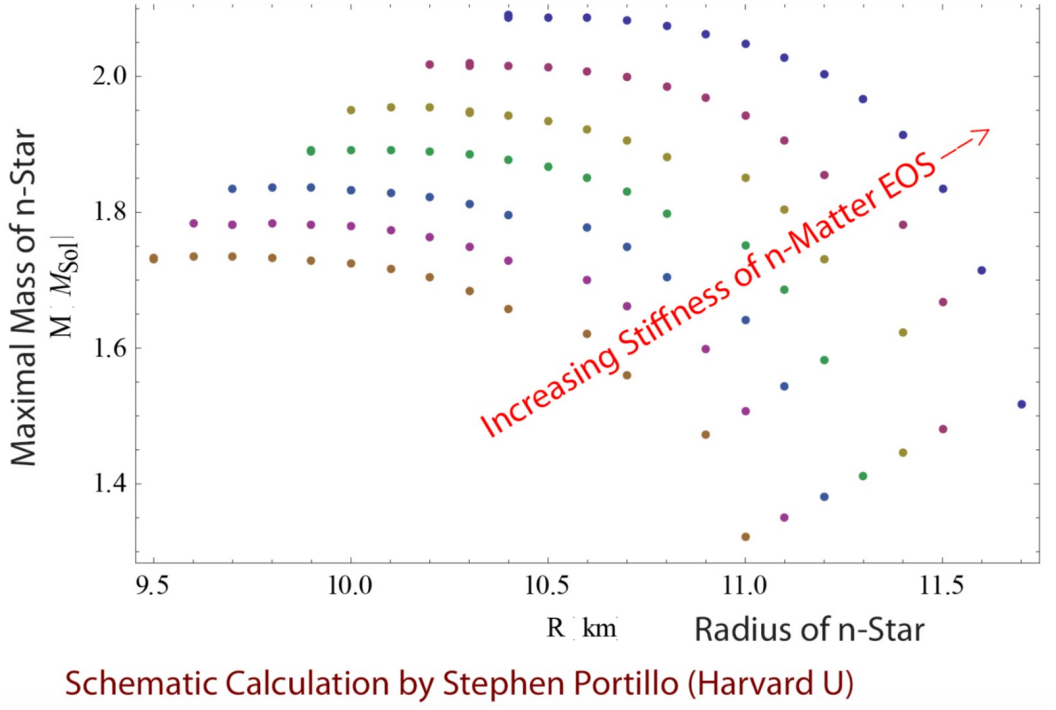


Is it worth the effort?

- Study of the EoS: how does a nuclear many-body problem react to compression and heating?

  → direct link to nucleon-nucleon interaction as in ordinary matter

- Astrophysics: neutron stars and links to Gravitational Waves



Schematic Calculation by Stephen Portillo (Harvard U)

$$E(\rho,\alpha) = E(\rho,0) + S(\rho)\alpha^2$$

$\alpha = (\rho_n - \rho_p)/\rho$

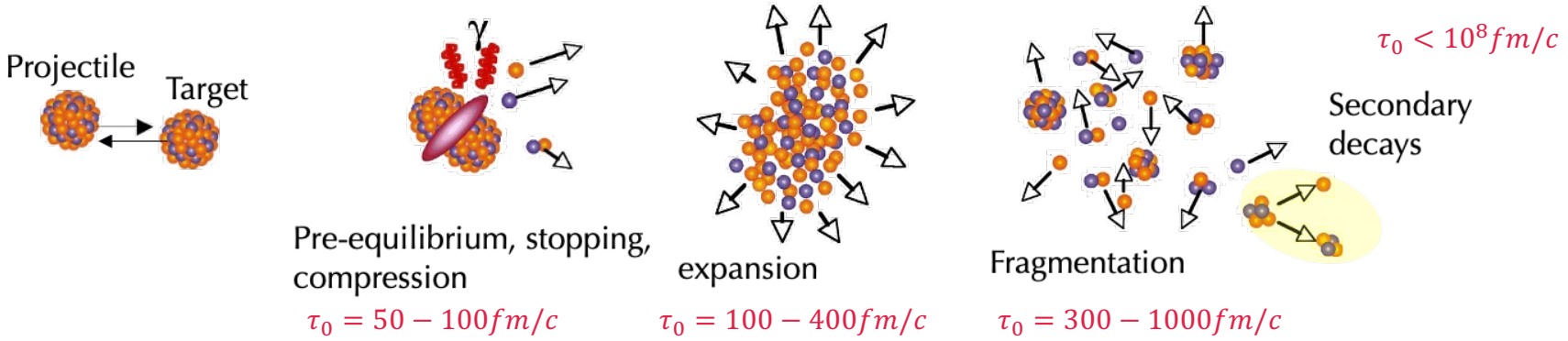
$S(\rho)$ = symmetry energy

Stay tuned on talks by

M. Colonna, M. Kurami-Nishimura

Important but too fast...

- HIC at intermediate energies: low energies but complicated (and quantum) systems



$^8Be_{g.s.} \rightarrow \alpha + \alpha$	$\tau_0 \approx 1.2 \cdot 10^{-16} s \approx 3.5 \cdot 10^7 fm/c$	$\rightarrow L \approx 10nm(\beta = 0.3) \approx 73nm(\beta = 0.9)$
$^{12}C_{Hoyle} \rightarrow (^8Be_{g.s.} + \alpha \rightarrow)\alpha + \alpha + \alpha$	$\tau_0 \approx 8 \cdot 10^{-17} s \approx 2.4 \cdot 10^7 fm/c$	$\rightarrow L \approx 7.5nm(\beta = 0.3) \approx 50nm(\beta = 0.9)$

Tracking detection techniques cannot be used - need to backtrace the history of the reaction

Femtoscopy

Koonin-Pratt Equation

$$R(\vec{q},\vec{P}) = \int d\vec{r} \cdot S_{\vec{P}}(\vec{r}) \cdot K(\vec{r},\vec{q})$$

Kernel

$|\Psi(r,q)|^2$

Correlation function

(Experimental data)

Source function: probability density of emitting a pair of particles separated by r (at at the time the second one is emitted) - space-time ambiguity can be resolved with 3D studies

Solving the equation by numerical inversion

you know the interaction (K) and use the data (R) to deduce the source function S(r)

or

you know (by other ways) the source function S(r) and use the data (R) to deduce the interaction (K)

Femtoscopy

Koonin-Pratt Equation

$$R(\vec{q},\vec{P}) = \int d\vec{r} \cdot S_{\vec{P}}(\vec{r}) \cdot K(\vec{r},\vec{q})$$

Kernel

$|\Psi(r,q)|^2$

Intermediate energies: emitted particles ar nucleons and clusters of nucleons

Example: d-α, p-d, p-α, etc. -- we have (old) data on elastic and inelastic scattering

- study of q-dependence of scattering phase shifts
- solve the Shrödinger Equation in partial wave decomposition up to l=3, 4, …

Solving the equation by numerical inversion

you know the interaction (K) and use the data (R) to deduce the source function S(r)

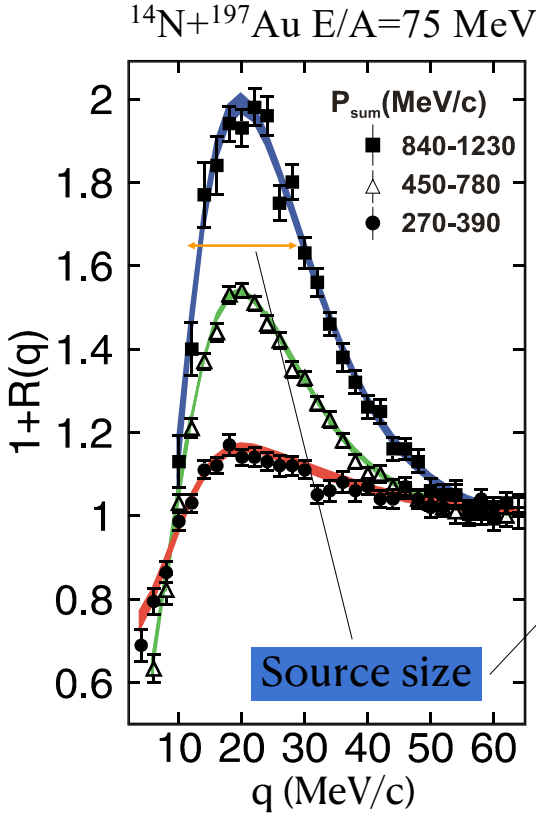
or

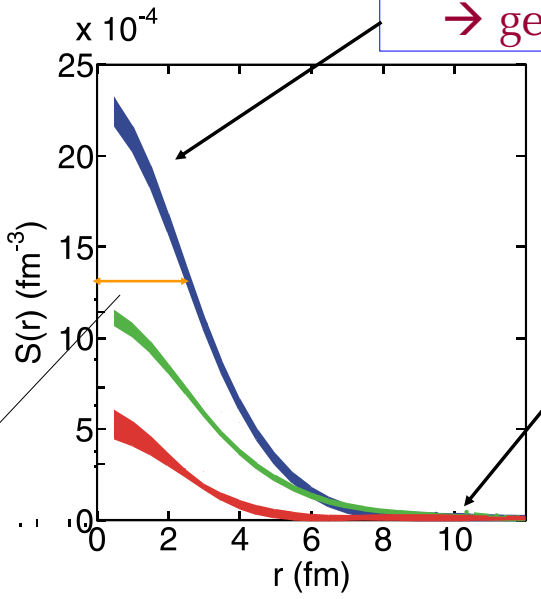
you know (by other ways) the source function S(r) and use the data (R) to deduce the interaction (K)

Inverting the KP eqn: Imaging

$$R(q) = 4\pi \int dr \cdot r^2 \cdot S(r) \cdot K(r,q)$$

G. Verde, P. Danielewicz et al.,
PRC65, 069604 (2002)





dynamical source (pre-eq.)
→ geometry-dominated

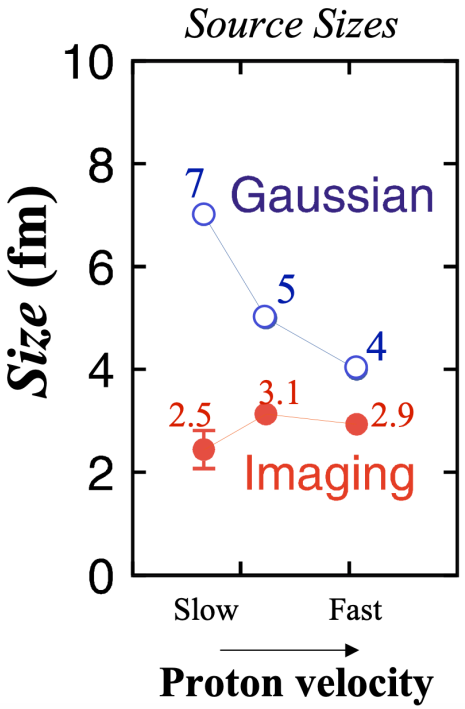
evap. Sources:
secondary decays
→ space-time

Y_{total}=Dynamical + Sec. Decays

Y_{fast} + Y_{slow}

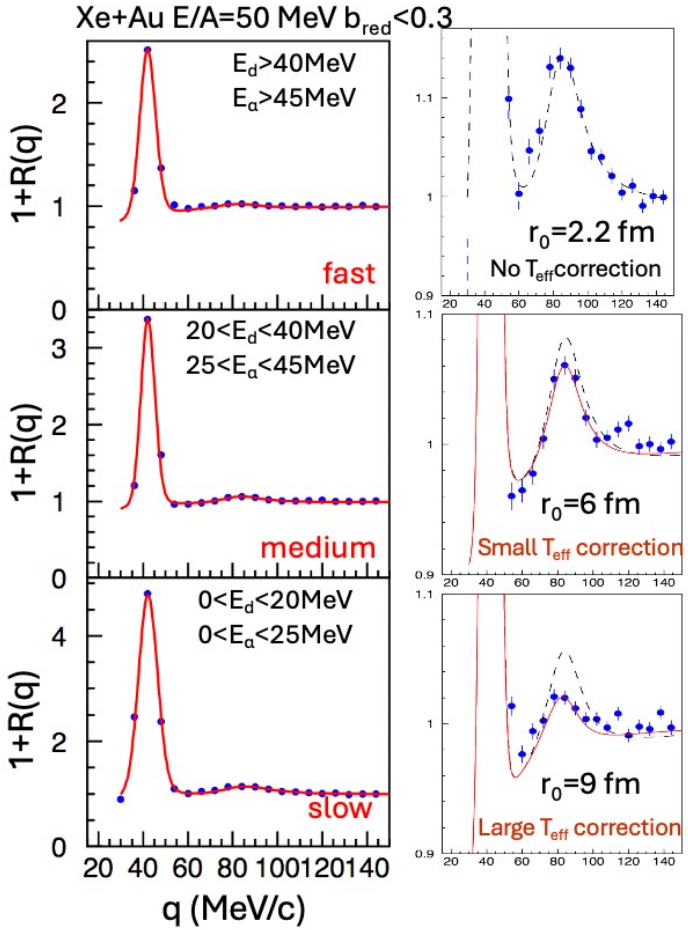
Integral of source → Y_{fast}/Y_{slow} provides contributions from secondary decay emissions

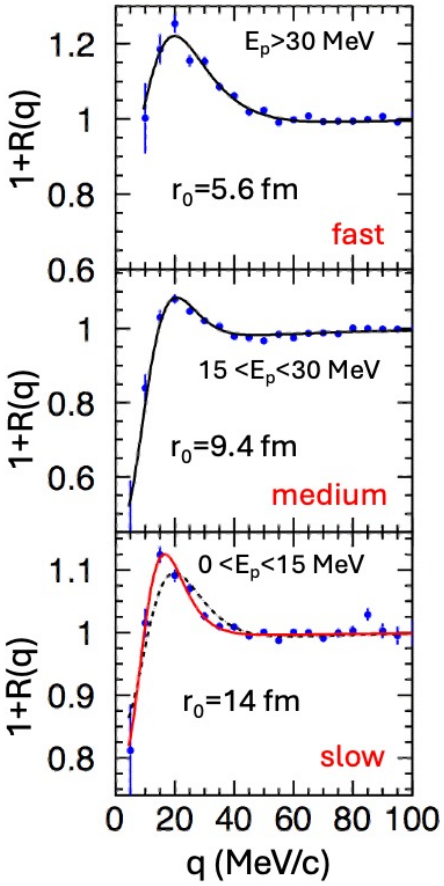
Guess what? The sources are not Gaussian shaped

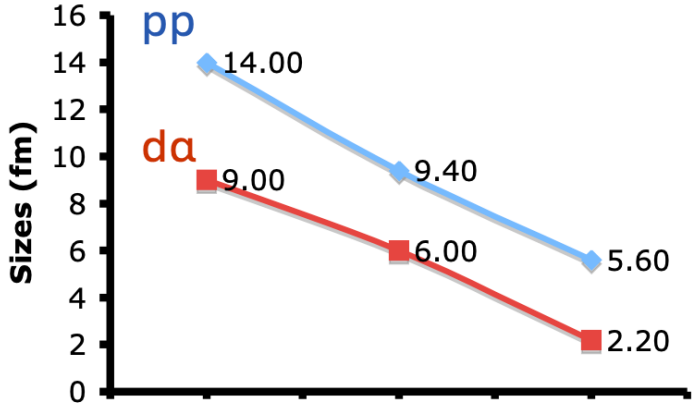


Cannot use simple parameterizations

Guess what? Different sizes for different particle species

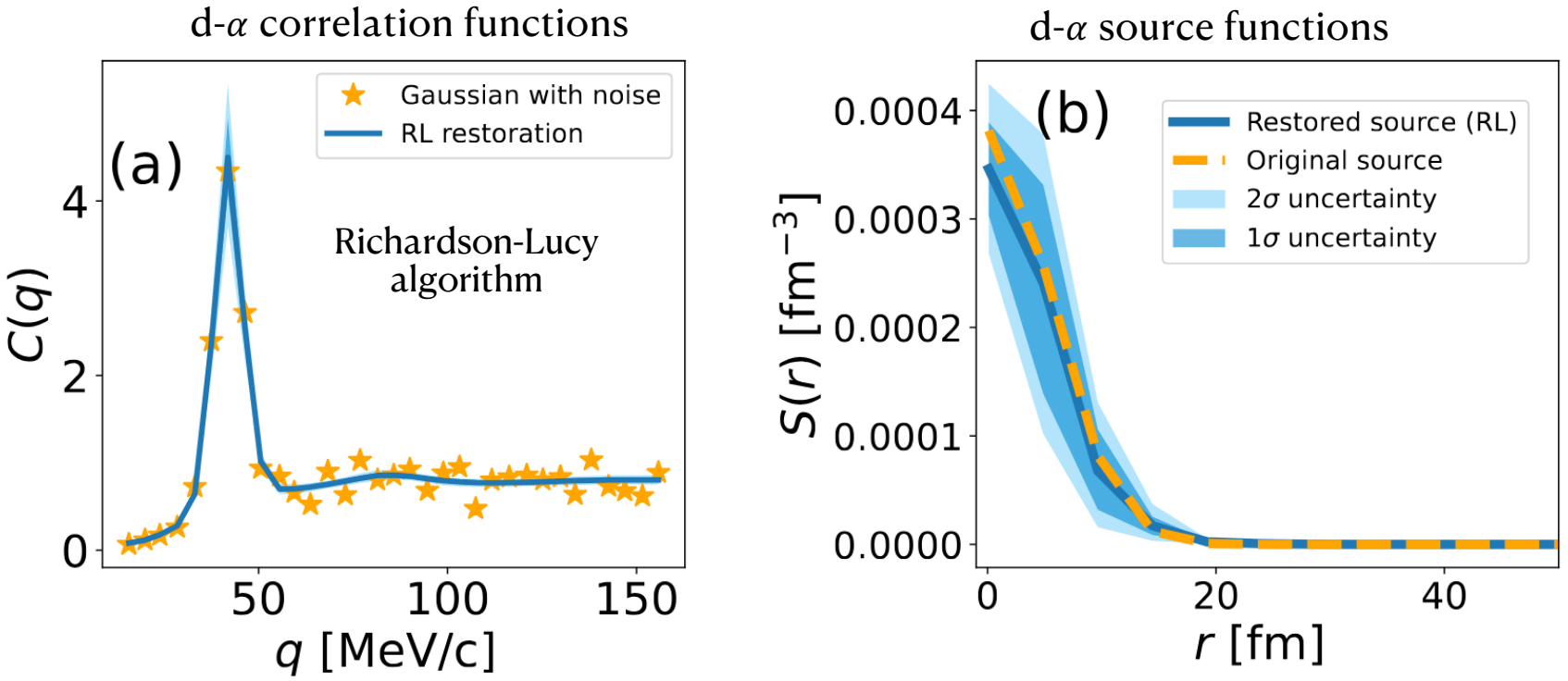






Inverting the KP eqn: Deblurring (RL)

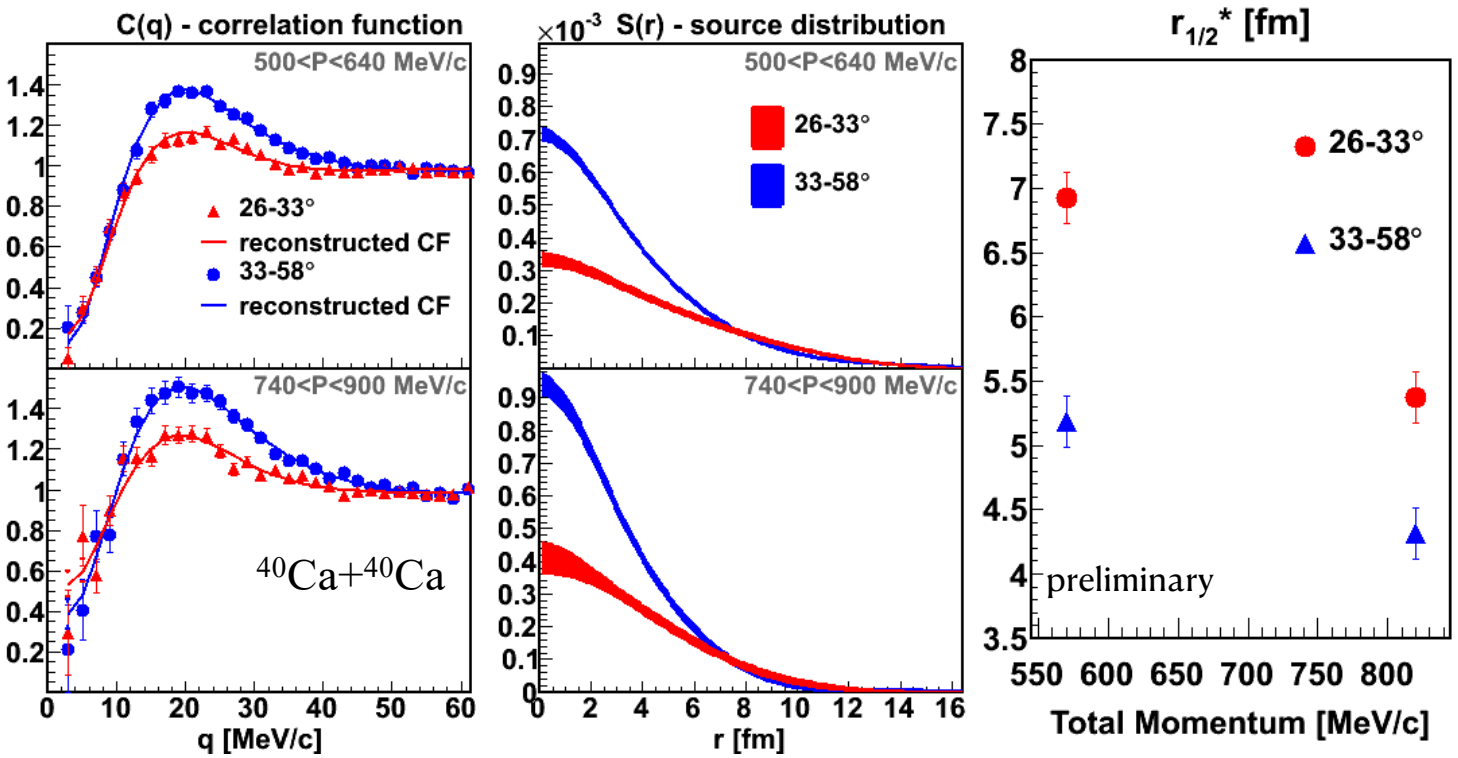
$$R(q) = 4\pi \int dr \cdot r^2 \cdot S(r) \cdot K(r,q)$$



P. Nzabahimana and P. Danielewicz, Physics Letters B 846, 138247 (2023)
P. Nzabahimana, P. Danielewicz, G. Verde, PRC submitted (2025)

Momentum and angular dependence

$^{40}Ca+^{40}Ca$ E/A=80 MeV - Central

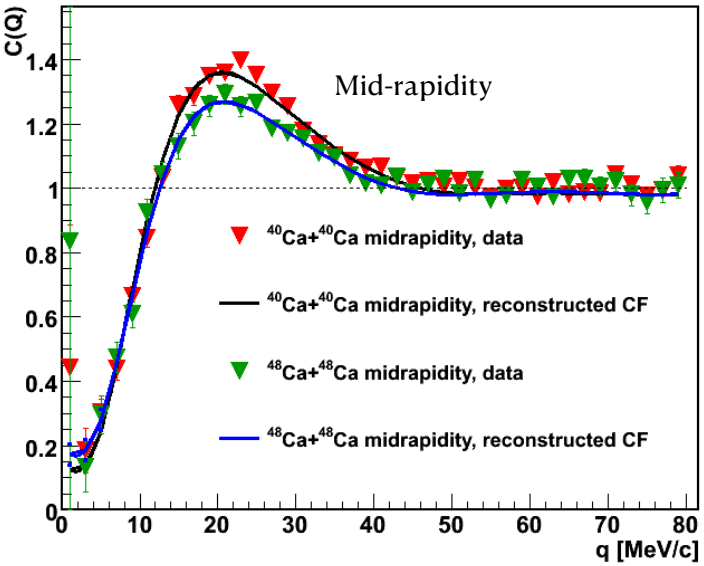


Improving probes of transport model parameters (σ_{NN}, Eos, etc.)

V. Henzl et al., Phys.Rev. C85 (2012) 014606

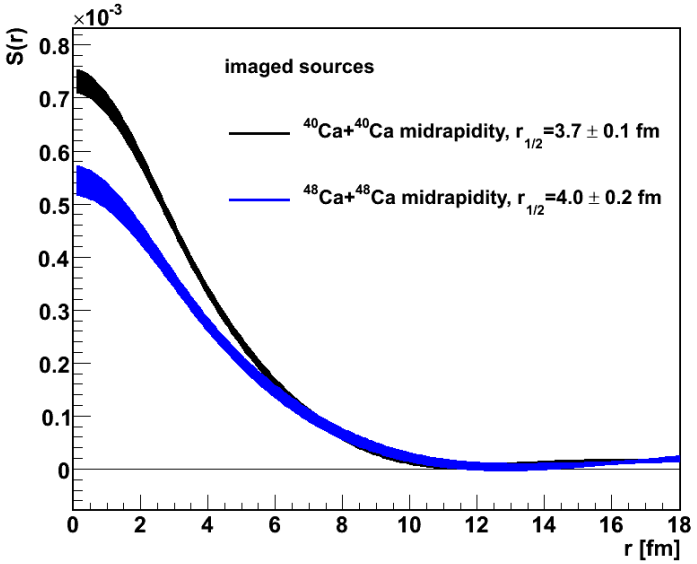
Isospin effects on p-p

$^{48}Ca+^{48}Ca$ vs $^{40}Ca+^{40}Ca$
E/A=80 MeV - Central



Correlations

R 48Ca+ 48Ca > R 40Ca+ 40Ca



Emitting sources

Larger size for more n-rich system → effect of N/Z asymmety? Size? EoS? σ_{NN}

V. Henzl et al., Phys.Rev. C85 (2012) 014606

BUU and source function – KP equation

From the single-particle phase-space density $f(\boldsymbol{r},\boldsymbol{p},t)$

to

→

The two-particle source function $S^{BUU}(\boldsymbol{r})$

$$\left(\frac{\partial}{\partial t}+\mathbf{v_p}.\nabla_r-\nabla_r U(\mathbf{r}).\nabla_p\right)f=I_{col}(\sigma_{in},f)$$

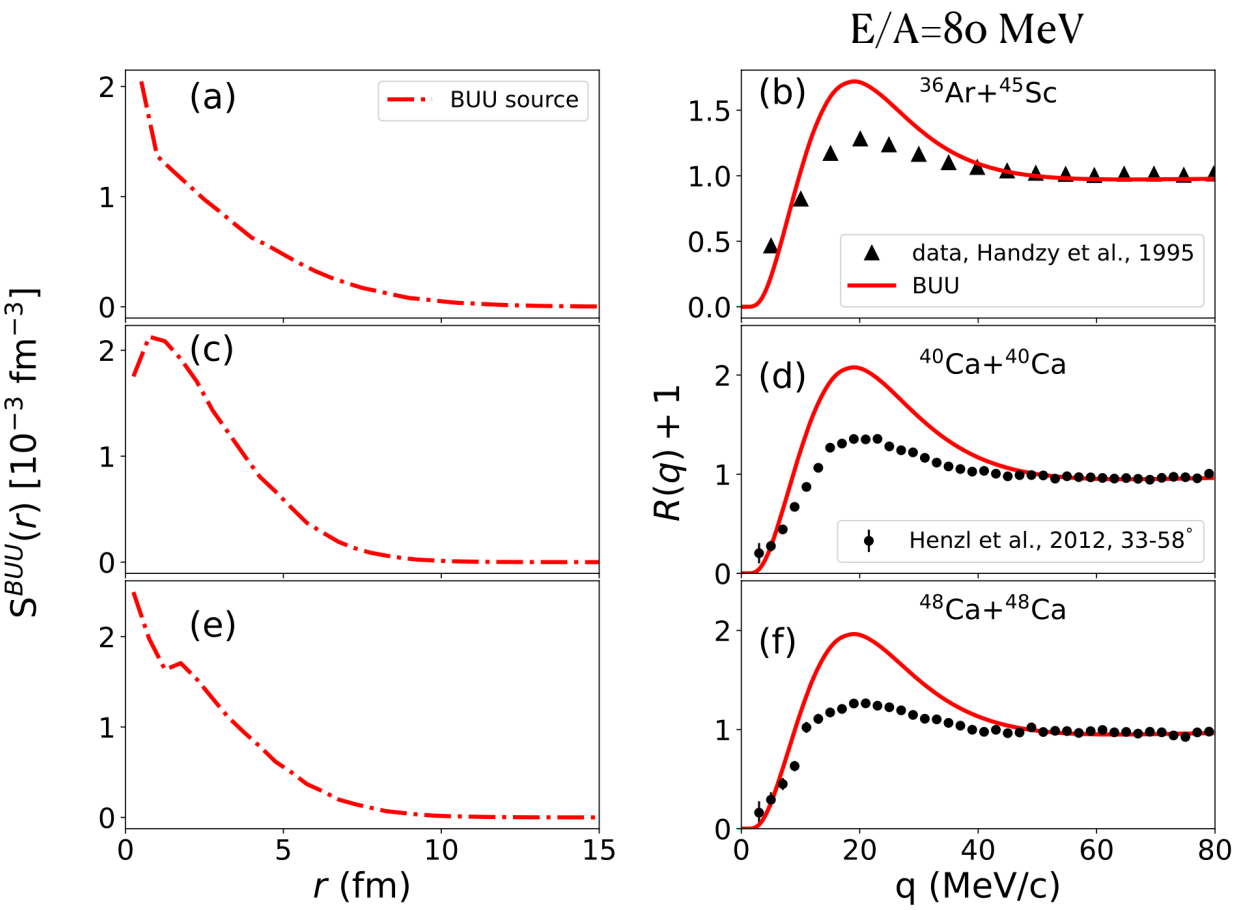
$$I_{col}=\frac{g}{h^3}\int d^3\mathbf{p}_2\int d\Omega\,\frac{d\sigma_{12}}{d\Omega}\,v_{12}$$
$$*\left[\hat{f}_1\hat{f}_2 f_{1'}f_{2'}-f_1 f_2\hat{f}_{1'}\hat{f}_{2'}\right]$$

$$S^{BUU}(\boldsymbol{r})=\frac{\int d^3R\, f(\mathbf{P}/2,\mathbf{R}+\mathbf{r}/2,t')\, f(\mathbf{P}/2,\mathbf{R}-\mathbf{r}/2,t')}{\left(\int d^3r'\, f(\mathbf{P}/2,\mathbf{r}',t')\right)^2}$$

Koonin-Pratt equation: calculate the BUU correlation function and compare data

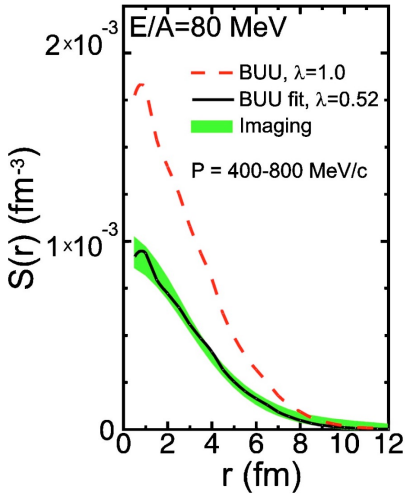
$$C(\mathbf{q})\equiv 1+R(\mathbf{q})=\int d^3r\,|\Psi_{pp}(\mathbf{q},\mathbf{r})|^2\cdot S^{BUU}(\boldsymbol{r})$$

BUU and source function – KP equation



- BUU overpredicts the atio between early emissions and secondary delayed emissions (expected)
- Requires re-normalization procedure

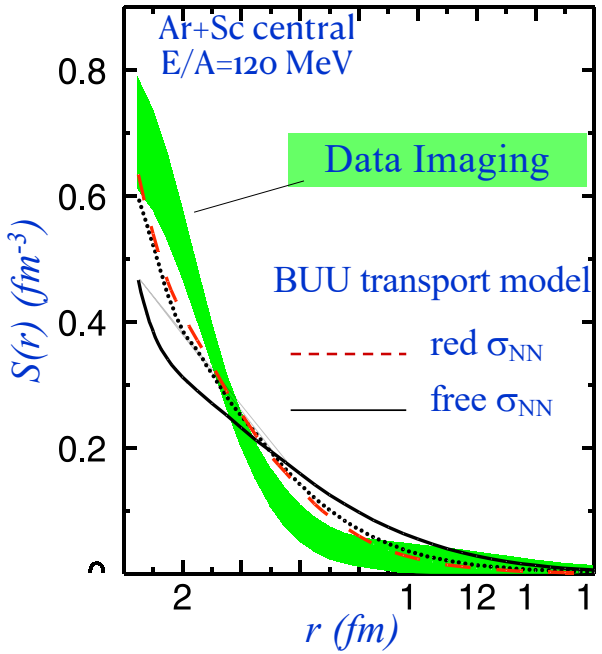
$$S(r) = \lambda \cdot S_{BUU}(r)$$



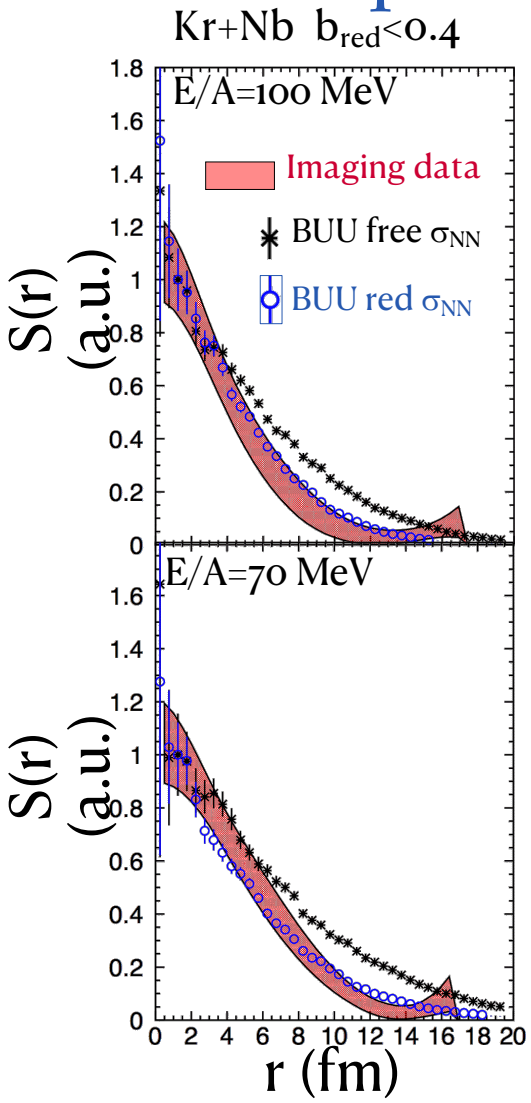
P. Nzabahimana, P. Danielewicz, G Verde (2025) – submitted

pp Femtoscopy and transport models

NN collision cross section σ_{NN}



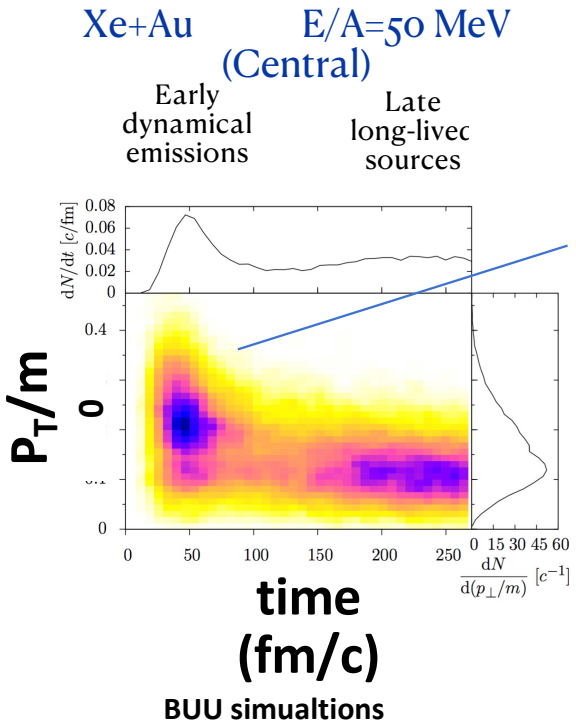
G. Verde, P. Danielewicz et al. PRC67, 034606 (2002)
B. Barker, PhD thesis @ MSU-NSCL (2014)
P. Nzabahimana, P. Danielewicz, G. Verde (2025) TBS



Sensitivity to in-medium σ_{NN}

$$\sigma_{in,\eta}(\rho) = \eta\rho^{-2/3}\tanh\left[\frac{\sigma_{free}}{\eta\rho^{-2/3}}\right]$$

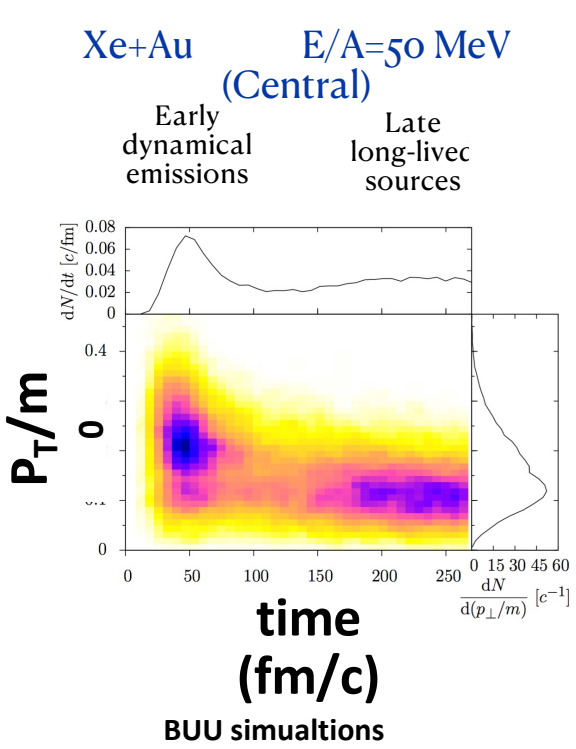
Imaging sources at different emission times/stages

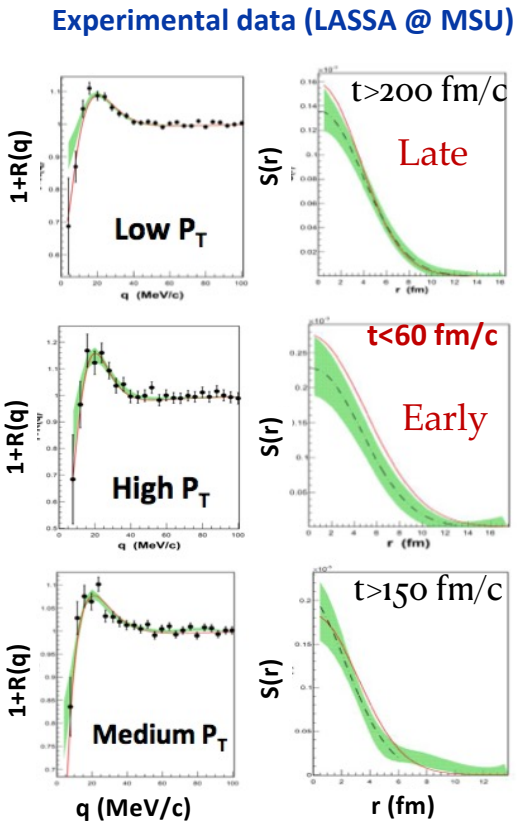


High PT protons emitted at early dynamical stage

G. Verde, B. Barker, P. Danielewicz

Imaging sources at different emission times/stages





T. Minniti (Uni-ME), G. Verde (INFN-CT), B. Barker (MSU), P. Danielwewicz (FRIB/MSU)

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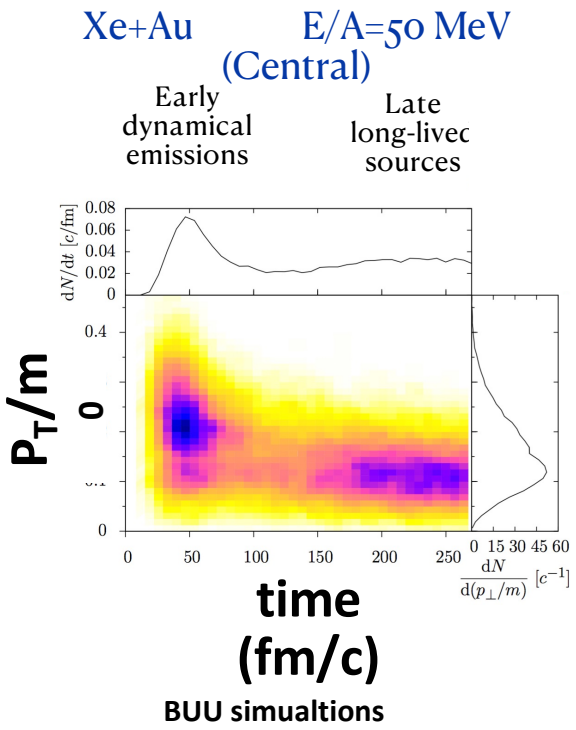

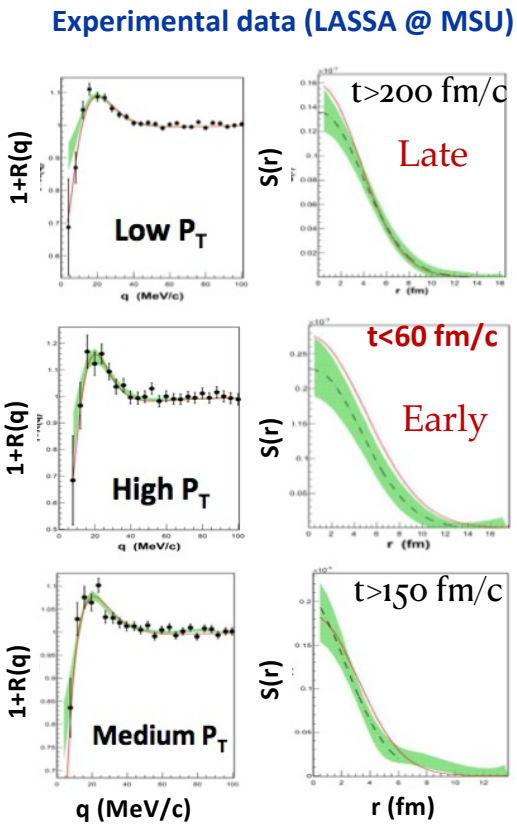

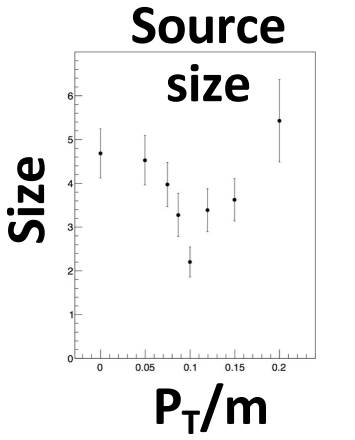

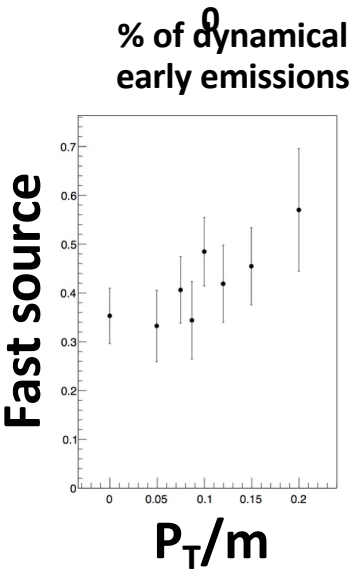

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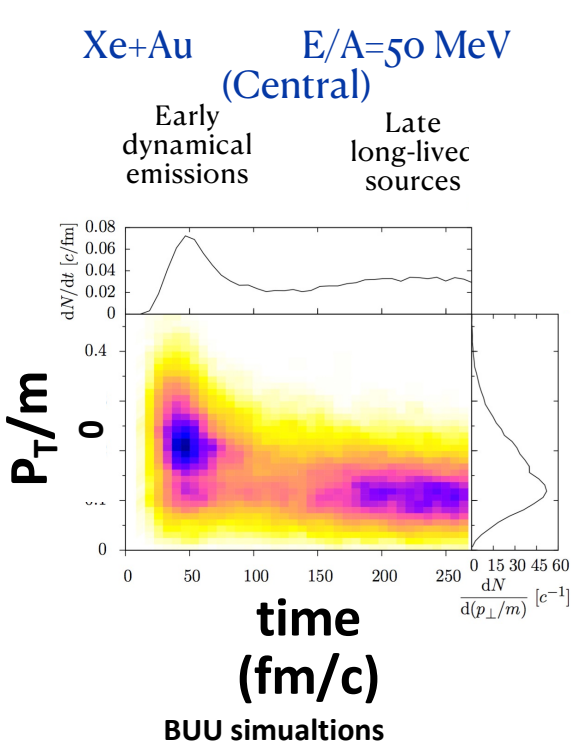
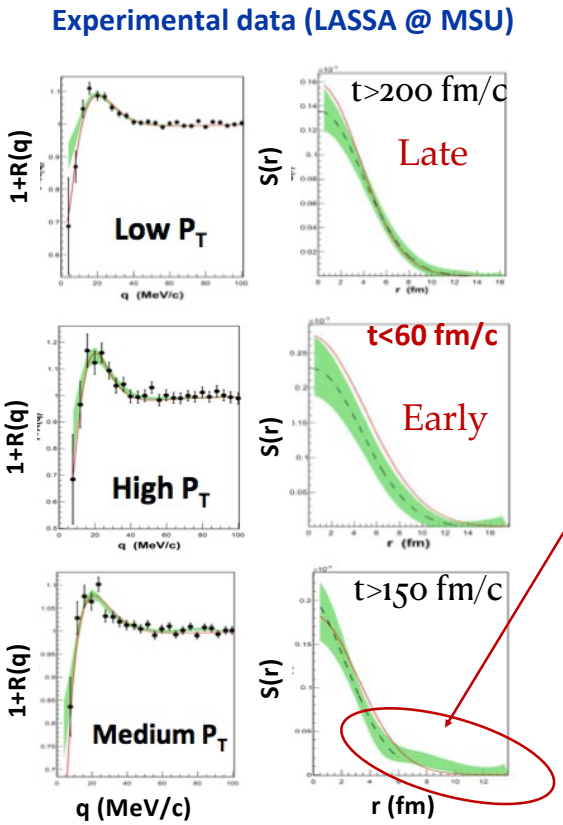




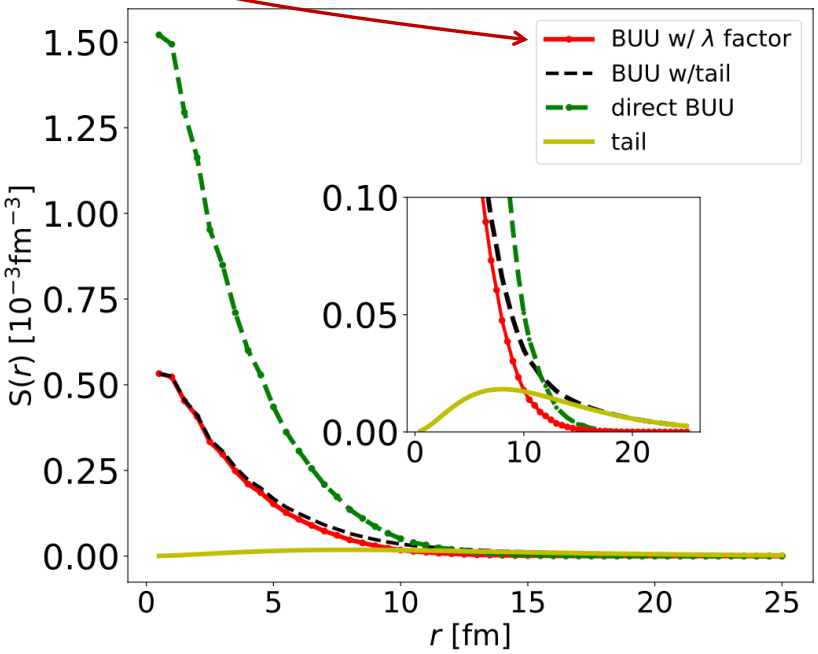
Strong deviations from Gaussian shape

- Interesting for Levy-α analyses?
- How to deal with long-lived sources?
  λ-renormalization cannot fix it

T. Minniti (Uni-ME), G. Verde (INFN-CT), B. Barker (MSU), P. Danielewicz (FRIB/MSU)

Imaging long-lived sources & transport models

$$S(r) = \lambda' \cdot S^{BUU}(r) \qquad \lambda' < 1$$



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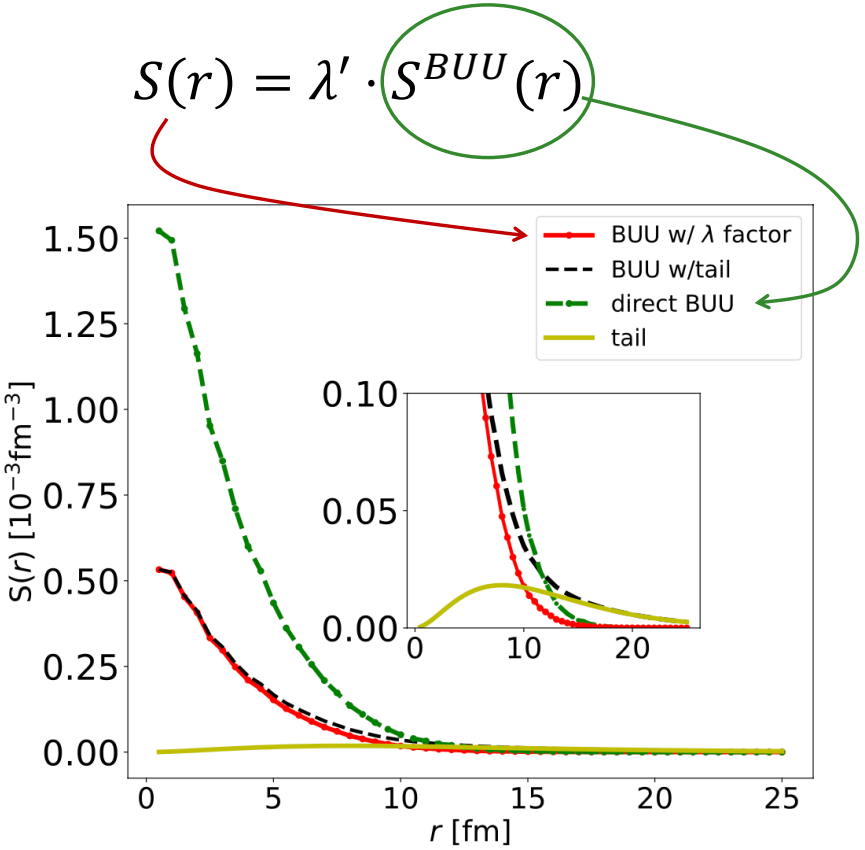
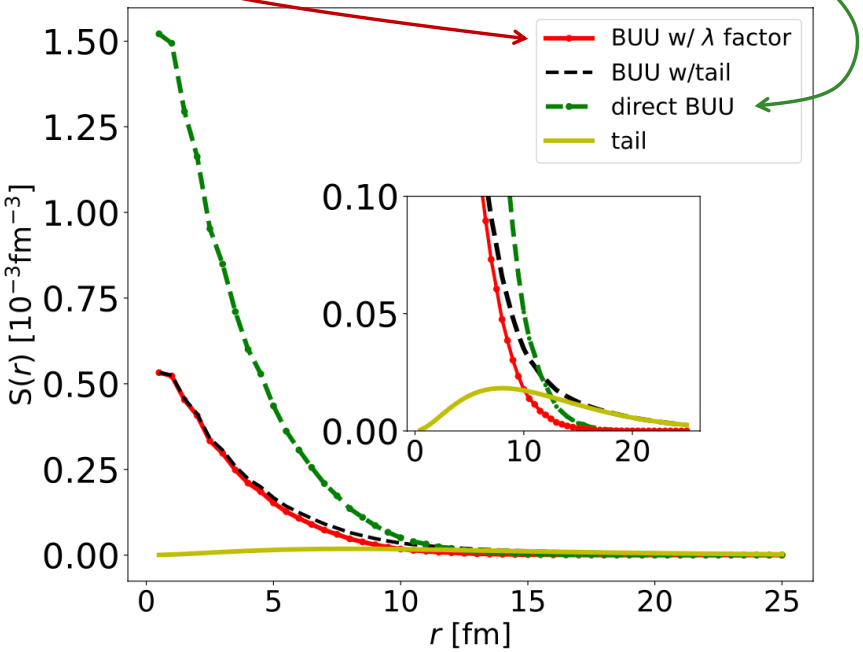


Imaging long-lived sources & transport models

$$S(r) = \lambda' \cdot S^{BUU}(r) \longrightarrow S(r) = \lambda' S^{BUU}(r) + \frac{1-\lambda'}{96\pi B^5} r^2 \exp(-r/B)$$



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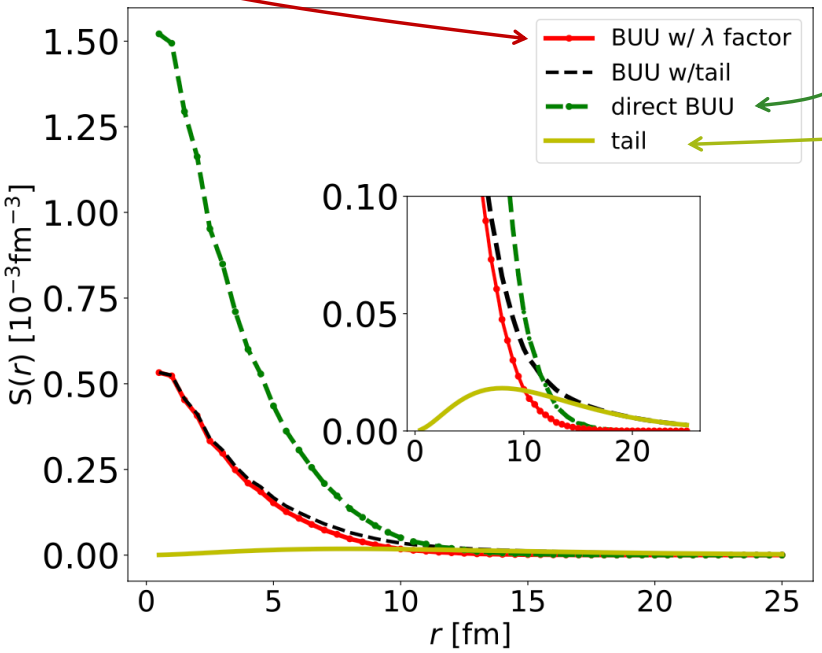

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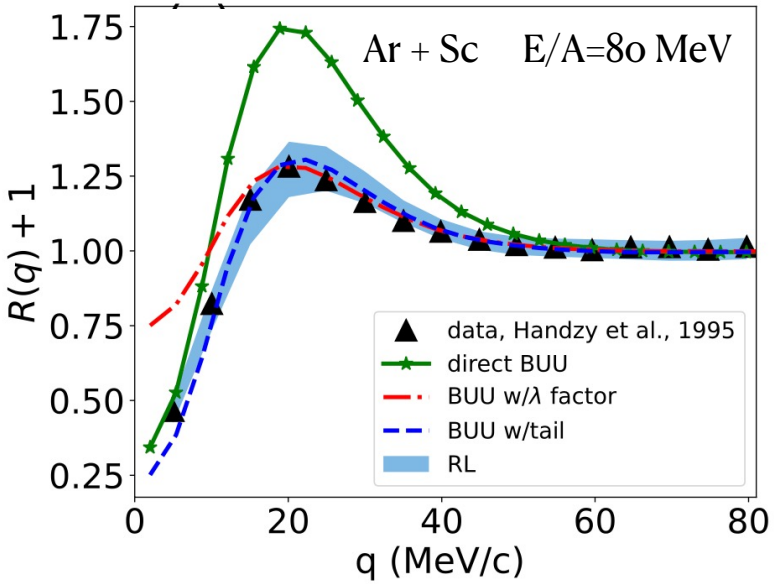
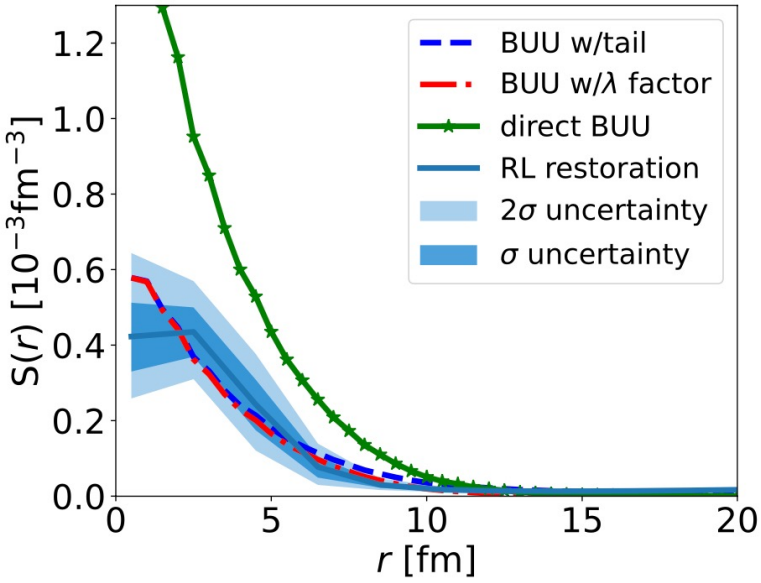
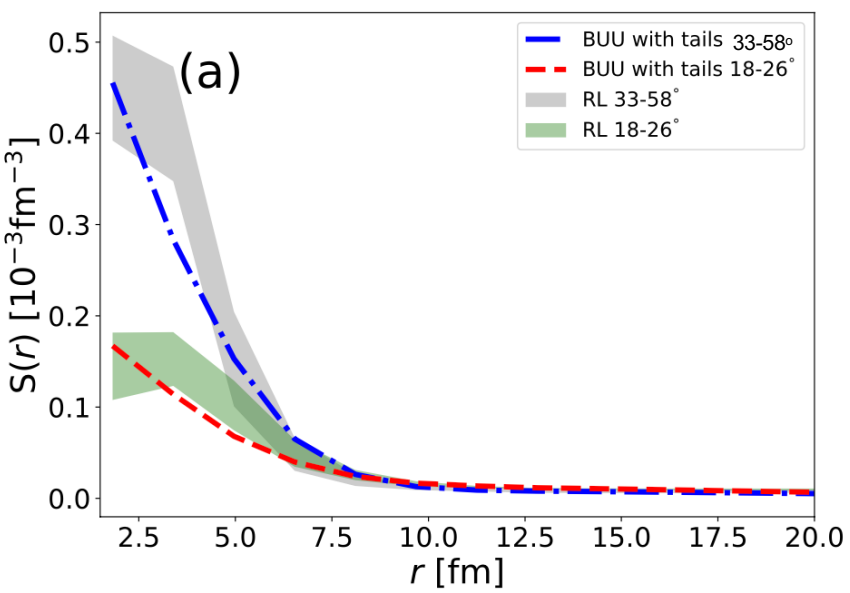


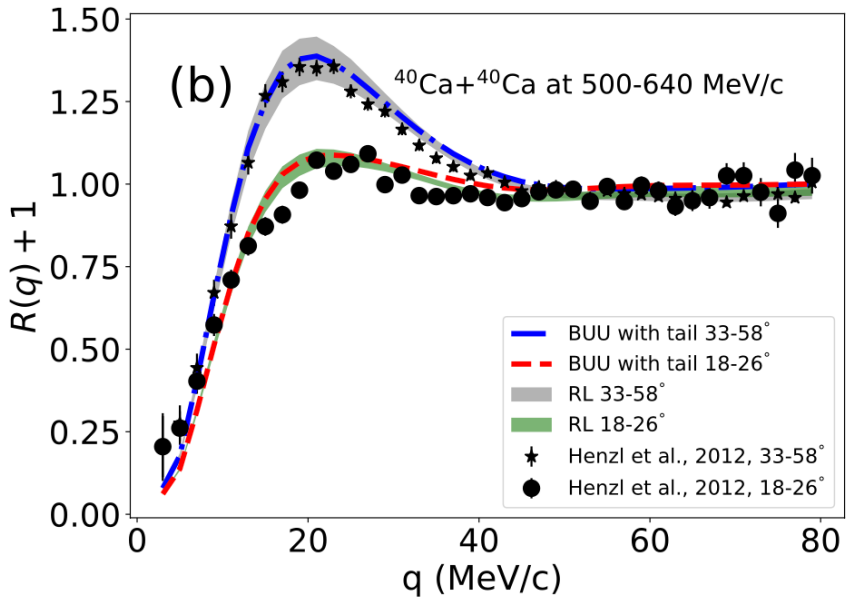


Angular dependence of proton emitting sources

^{40}Ca + ^{40}Ca. E/A = 80 MeV HiRA + μBall @ FRIB
Proton pairs: P = 500-640 MeV/c

Backward emisions: θ=33°-58°
Foward emission: θ=18°-26°





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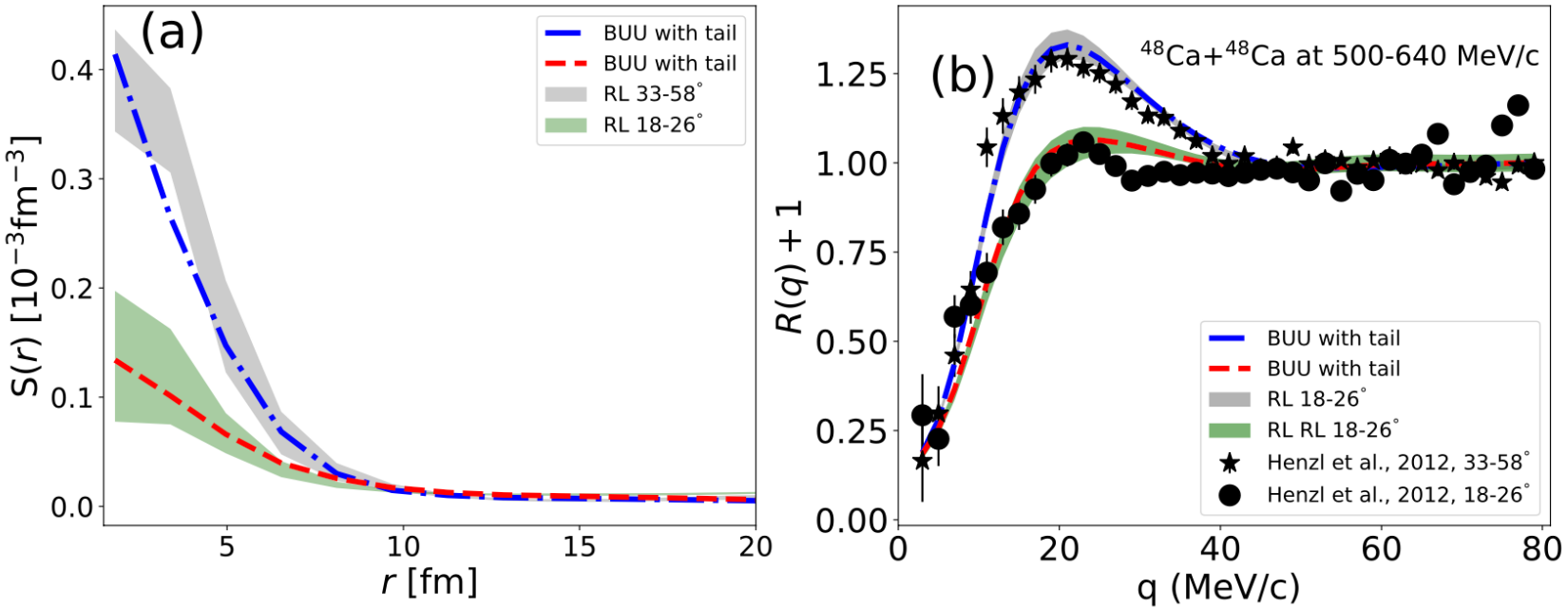

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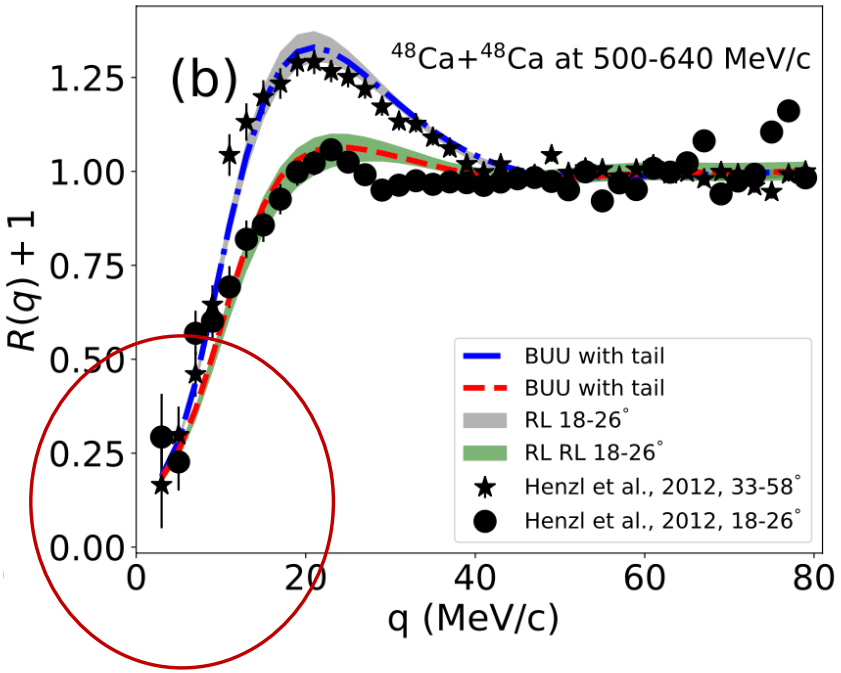
Foward emission: θ=18°-26°

Position resolution of 1 mm not enough...



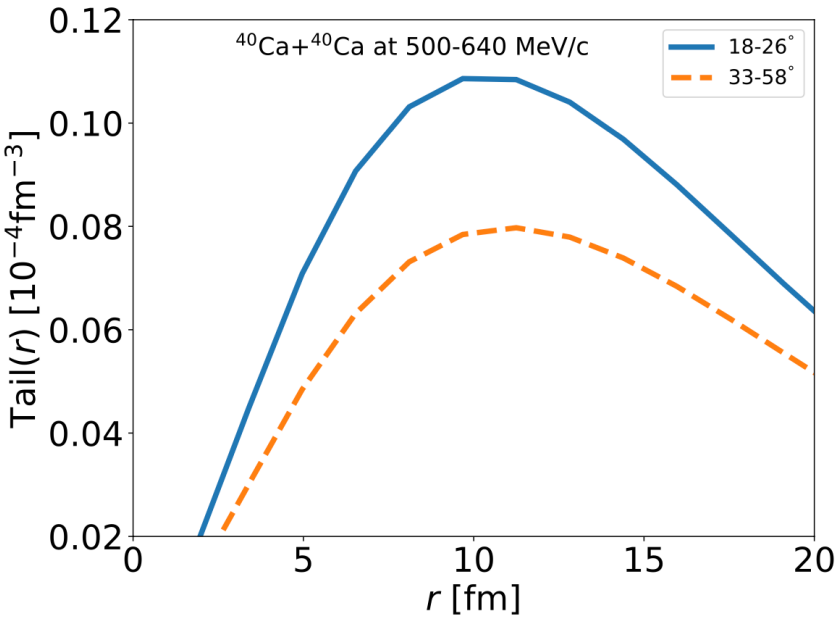
Angular dependence of proton emitting sources

^{40}Ca + ^{40}Ca E/A = 80 MeV HiRA + μBall @ FRIB
Proton pairs: P = 500-640 MeV/c

Backward emisions: θ=33°-58°
Foward emission: θ=18°-26°

Tail function parameters

(lr)1-5 (lr)5-7 Reaction/parameters	Angles	λ	B[fm]
^{40}Ca + ^{40} Ca	$18 < \theta \leq 26°$	0.15	5.19
	$33 < \theta \leq 58°$	0.251	5.52



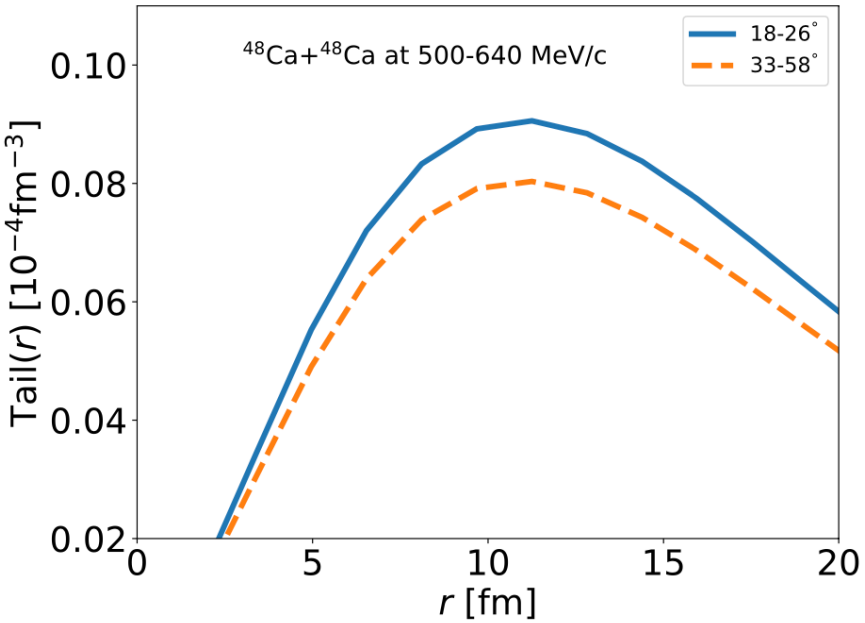
Angular dependence of proton emitting sources

^{48}Ca + ^{48}Ca E/A = 80 MeV HiRA + μBall @ FRIB
Proton pairs: P = 500-640 MeV/c

Backward emisions: θ=33°-58°
Foward emission: θ=18°-26°

Tail function parameters

(lr)1-5 (lr)5-7 Reaction/parameters	Angles	λ	B[fm]
^{40}Ca + ^{40} Ca	$18 < \theta \leq 26°$	0.15	5.19
	$33 < \theta \leq 58°$	0.251	5.52
^{48}Ca + ^{48} Ca	$18 < \theta \leq 26°$	.17	5.5
	$33 < \theta \leq 58°$	.26	5.5

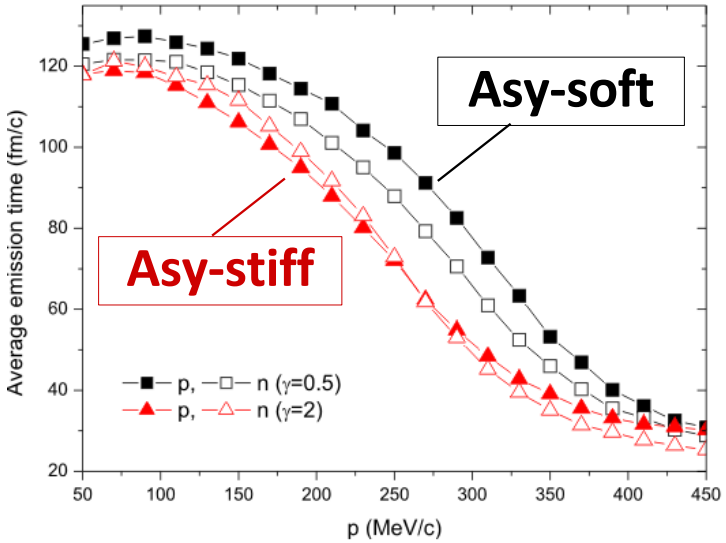


Reduced angular dependence of long-lived sources in neutron-rich system

Sensitivity to $E_{sym}(\rho)$

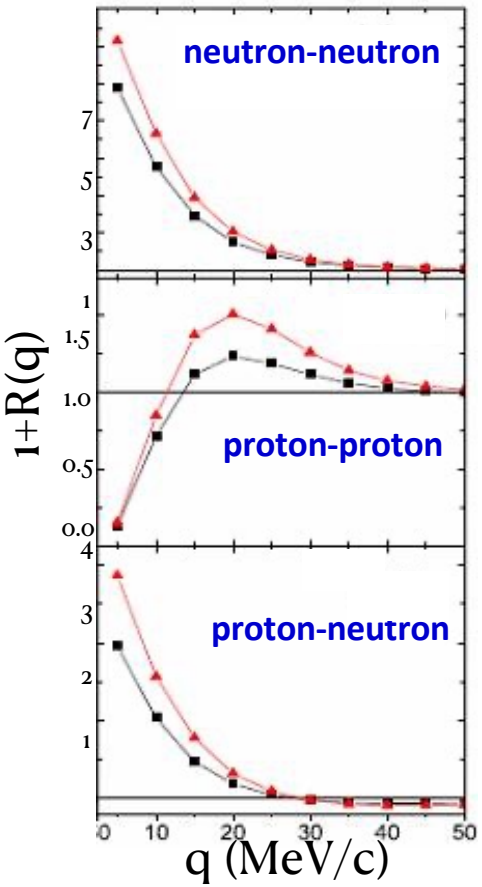
IBUU simulations

$^{52}Ca+^{48}Ca$ E/A=80 MeV Central collisions



Lie-Wen Chen et al., PRL (2003), PRC(2005)

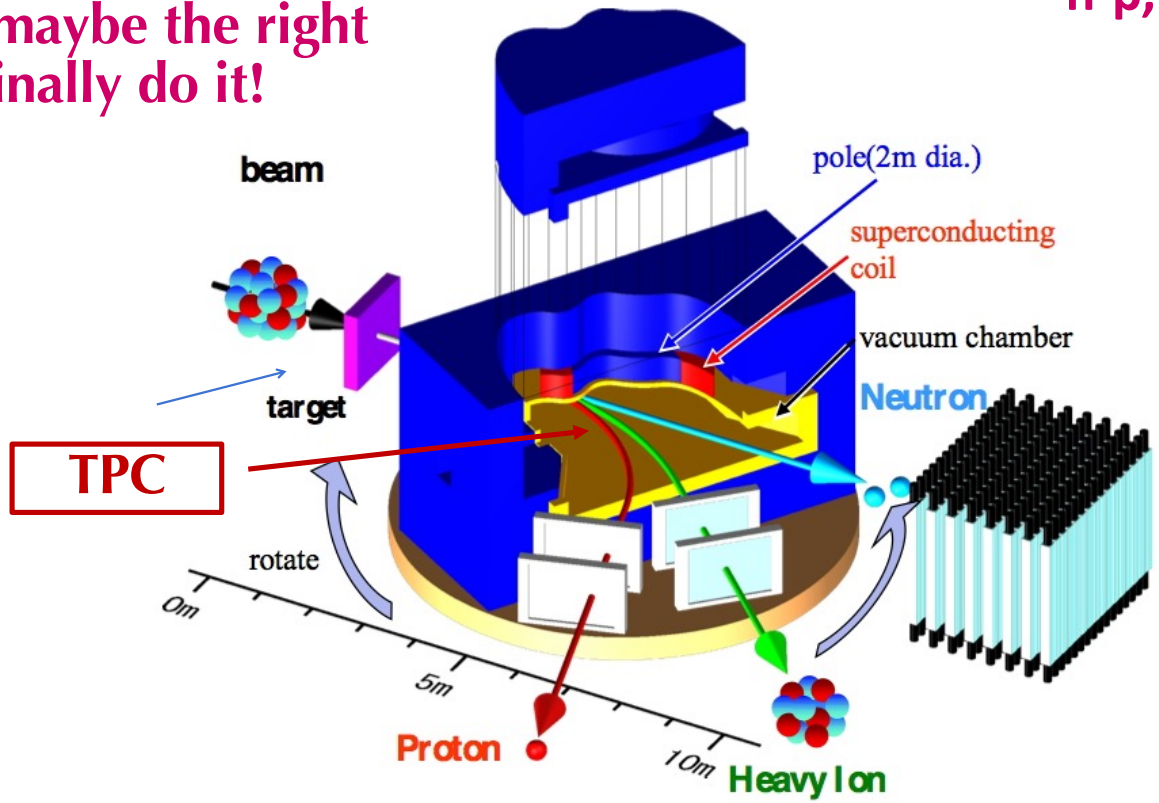
Correlation functions



Probing sources and n-n, n-p and n-cluster interactions

FRIB@MSU: maybe the right time to finally do it!

n-p, n-n and p-p correlations at E/A>100 MeV



Neutron detectors

Correlation detectors

Samurai image (courtesy of T. Nakamura)

Conclusions and outlook

- Two-ways character of Femtoscopy at intermediate energies – more work needed on the profile and size of the source
  - Sources deviate from Gaussian shape
  - Different particles emitted from different sources (in space and time)
- Numerical inversion techniques (imaging, deblurring, …) to compare profiles to transport models – need to take microscopic emission mechanisms in transport
  - Understand long-lived emissions
  - Promising probes of the density dependence of the symmetry energy – understand N/Z dependence data
- Need special effort in position resolution to improve low-q data
- Need for an extended HICA from low to high energies to address common problems in femtoscopy