Challenges in QCD + QED thermodynamics

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Why QED?

- QCD non-perturbative theory of strong interactions

  Relevant for heavy-ion collisions, neutron stars, hadron spectroscopy, ...

- QED interactions can usually be treated *perturbatively*

- Lattice QCD + QED important for $T = 0$ precision physics:
  - Muon g-2
  - $p - n$ mass difference
  - Scattering amplitudes
  - ...

- What about $T \neq 0$ phase transitions? $\rightarrow$ *Non-perturbative* phenomena

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- What about $T \neq 0$ phase transitions? $\rightarrow$ *Non-perturbative* phenomena

- QCD + QED possesses **exact center symmetry**

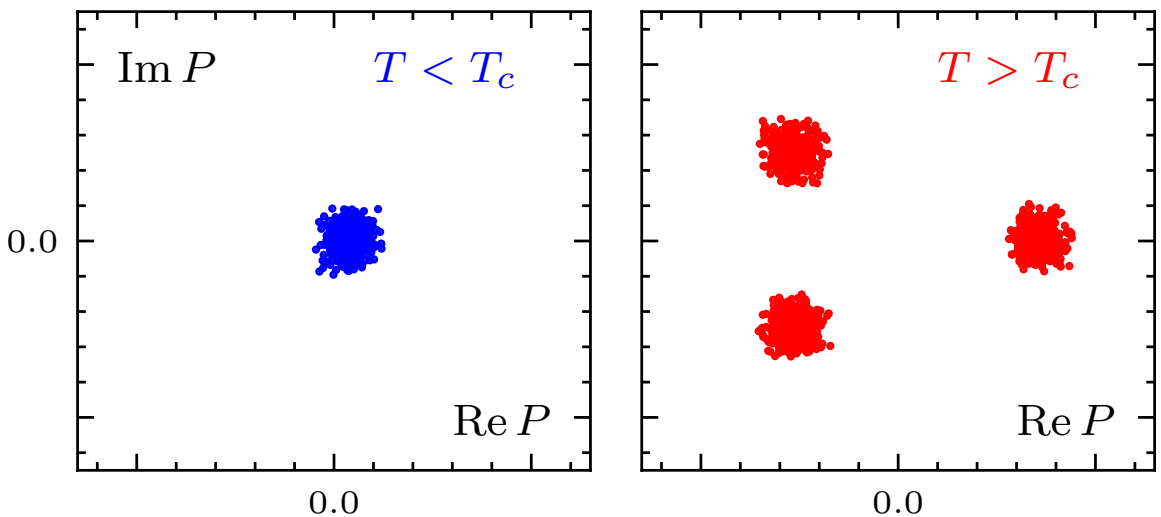
  Preliminary hints of deconfinement phase transition!

  Scior, Edwards, von Smekal, Lattice '10,11,13

Center symmetry

- **Pure SU(3) theory**: symmetric under center group $Z_3 = \{1, e^{\pm i2\pi/3}\}$
- Z_3 spontaneously broken $T > T_c \rightarrow 1^{\text{st}}$-order phase transition

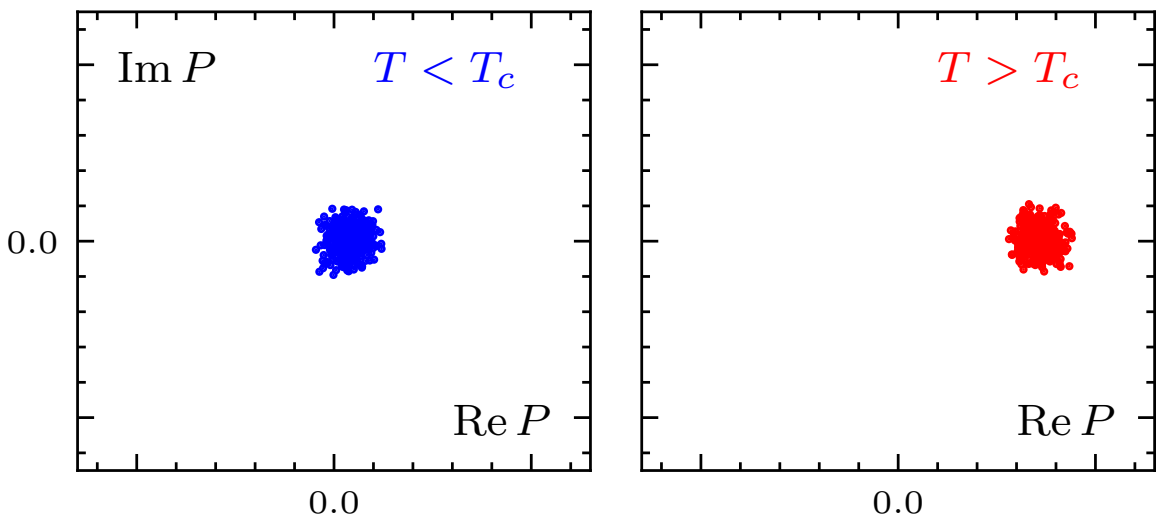
  Boyd et al. '96
- Polyakov loop P order parameter



Center symmetry

- **QCD**: fermion determinant breaks Z_3 *explicitly* $\rightarrow$ P is real
- No phase transition $\rightarrow$ chiral/deconfinement crossover

  Aoki et al. '06
- P is not an exact order parameter



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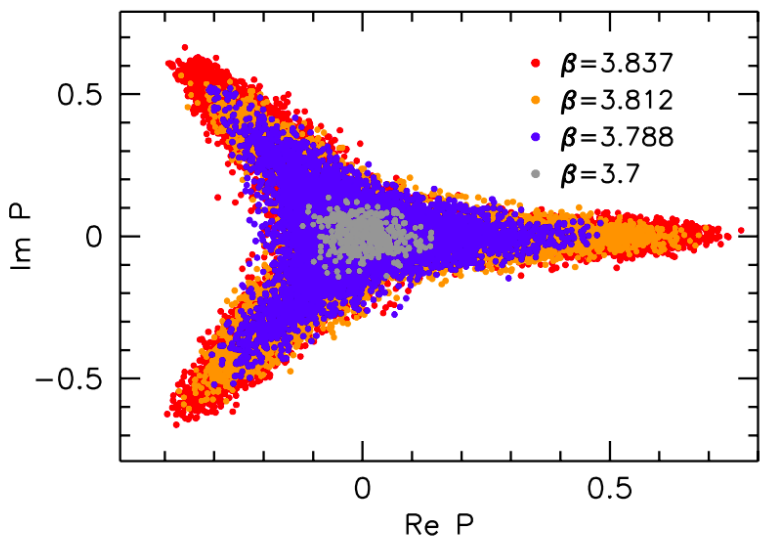
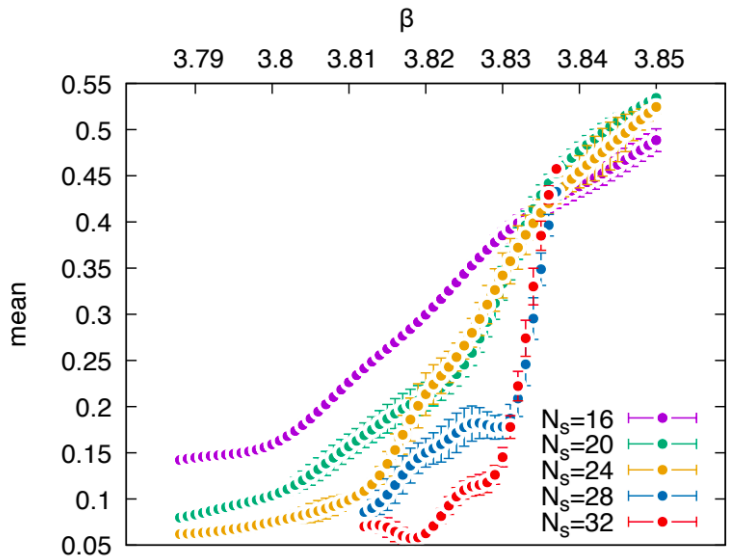
- Flavor permutation can cancel Z_3 transformation of quarks
- 1^{st}-order deconfinement phase transition reappears!

Endrődi, Moore, Sciarra, Lattice '24





Center symmetry

- Is there a more physical example? $\rightarrow$ Yes!
- QCD + QED:

$$\not{D}^f_{nl} = \frac{1}{2}\sum_\mu \gamma_\mu \left[U_\mu(n)\, u^f_\mu(n)\, \delta_{l,n+\hat{\mu}} - U^\dagger_\mu(n-\hat{\mu})\, (u^f_\mu)^\dagger(n-\hat{\mu})\, \delta_{l,n-\hat{\mu}} \right]$$

with $u^f_\mu(n) = e^{ia\phi_\mu(n) q_f/e}, \quad \phi_\mu(n) \in [-\pi, \pi)$

- Different Polyakov loops:

$$P = \frac{1}{V}\sum_{\vec{n}} \prod_{n_t=0}^{N_t-1} U_4(\vec{n}, n_t) \qquad p_f = \frac{1}{V}\sum_{\vec{n}} \prod_{n_t=0}^{N_t-1} u^f_4(\vec{n}, n_t)$$

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- U(1) rotation can cancel center transformation $P \rightarrow z\,P$

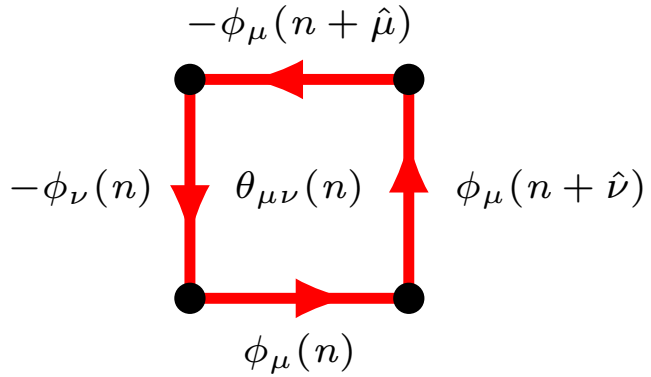
$$\phi_4(n) + 2\pi\, T \Rightarrow \begin{cases} p_u \rightarrow z^2\, p_u = z^{-1}\, p_u \\ p_{d,s} \rightarrow z^{-1}\, p_{d,s} \end{cases}$$

- Determinant is now Z_3 invariant

Lattice setup

- Lattice QCD + QED $N_f = 2 + 1$ improved staggered quarks physical[1] masses
- Compact QED version plaquette action

$$S_{\mathrm{U}(1)} = \beta_{\mathrm{em}} \sum_{\substack{n,\mu,\nu, \\ \mu<\nu}} \Big(1 - \cos\big[\theta_{\mu\nu}(n)\big]\Big)$$



- Remember $\beta \leftrightarrow T, \quad \beta_{\mathrm{em}} = 1/e^2$

[1] QED effects not included in the LCP

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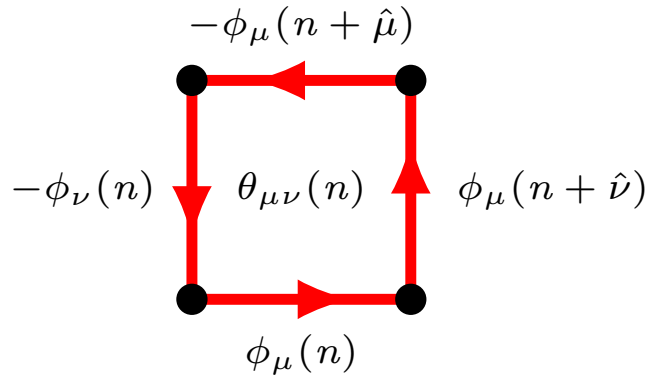
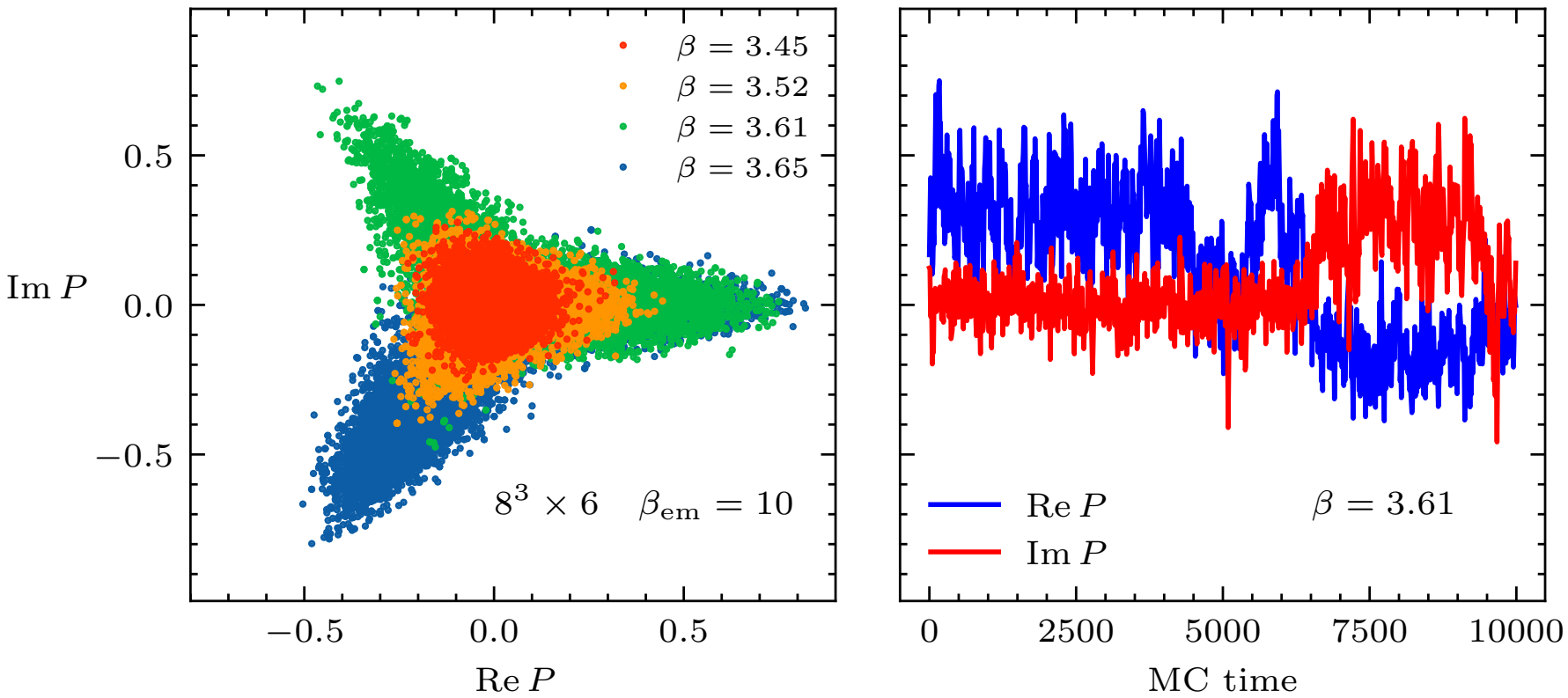


- Remember $\beta \leftrightarrow T, \quad \beta_{\mathrm{em}} = 1/e^2$
- Previous preliminary study: Scior, Edwards, von Smekal, Lattice '10,11,13
  - QCD_2 with strongly coupled compact QED $\beta_{\mathrm{em}} \sim 1$
  - Evidence for pure gauge SU(2) transition
- We mainly explore weak QED coupling regime $\beta_{\mathrm{em}} = 10$

[1] QED effects not included in the LCP

Polyakov loop

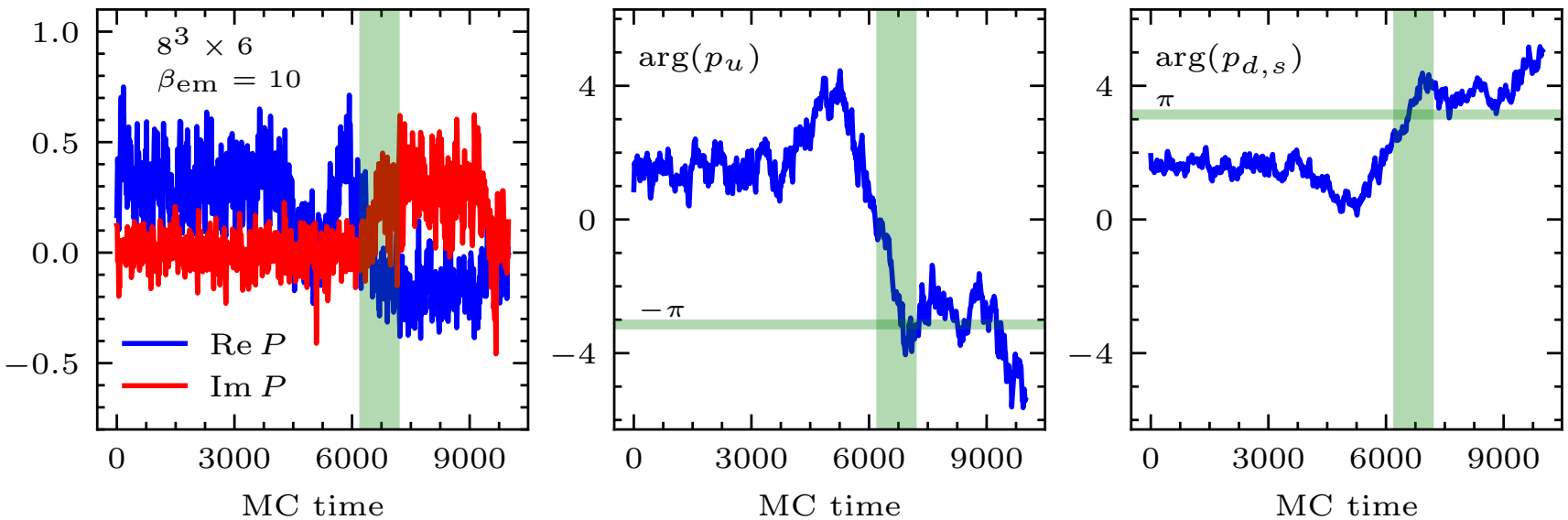
- Consider first small volume $8^3 \times 6$



- Sampling Polyakov sectors is challenging

QED Polyakov loops

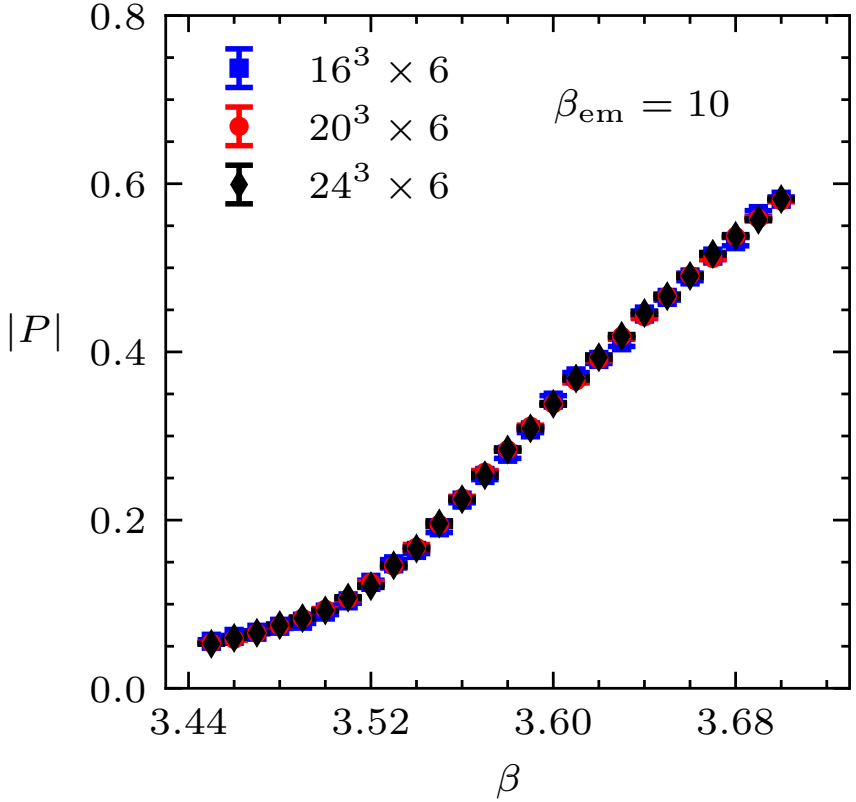
- Why is sampling different SU(3) Polyakov sectors difficult?
- Transition among P sectors achieved when $p_u, p_{d,s}$ rotate *simultaneously*

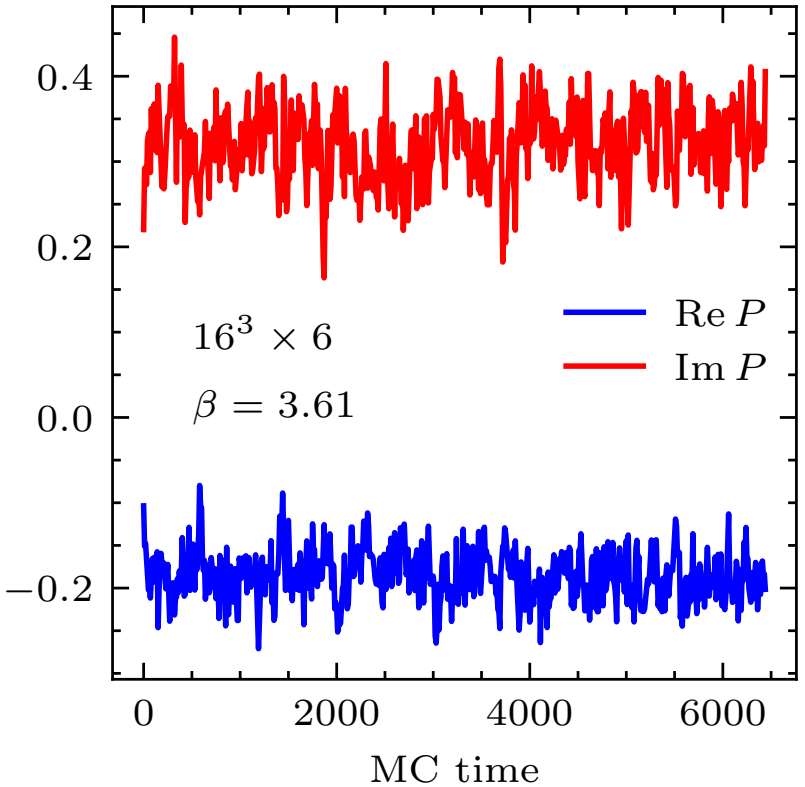


- Tunneling is unlikely with usual sampling algorithms

Larger volumes

- In larger volumes tunneling is further suppressed

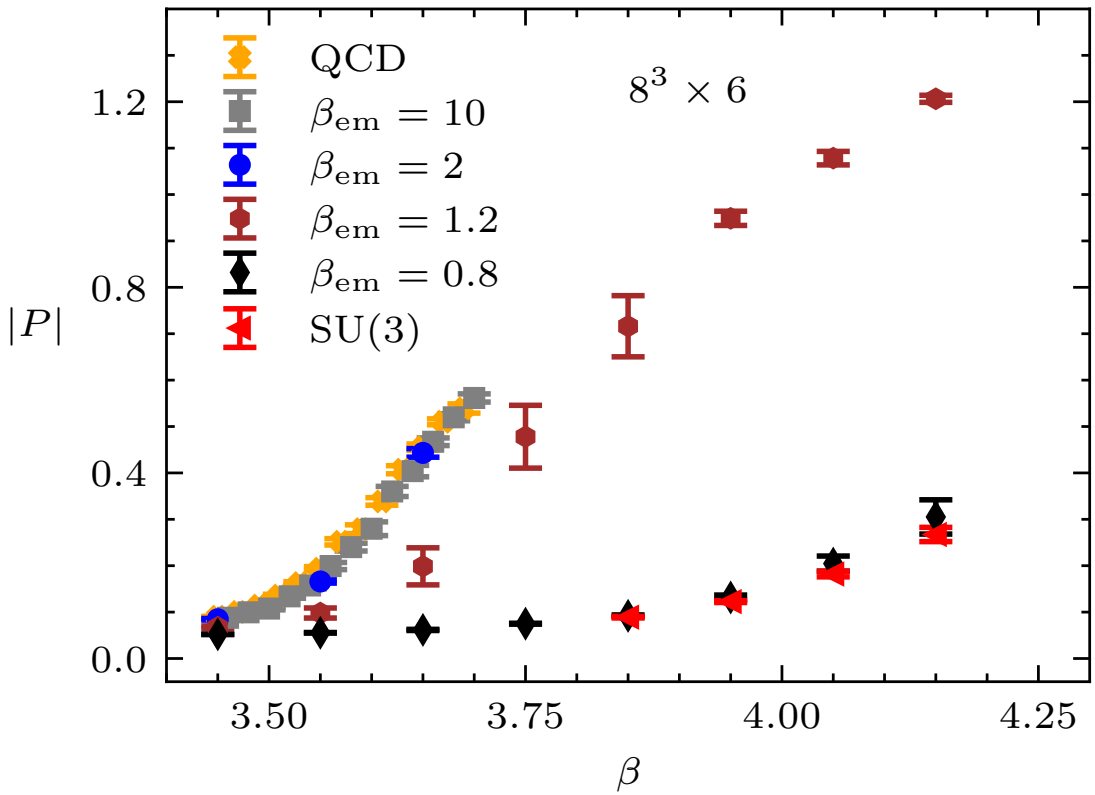




- Polyakov loop shows no apparent volume dependence
- Need tunneling among sectors for definite answer on the phase transition

QED coupling dependence

- We study β_{em} dependence
- Strong QED coupling takes Polyakov loop to SU(3) result
- Qualitatively consistent with Scior, Edwards, von Smekal, Lattice '13



- Precise explanation currently under investigation

Summary

Take-home messages

- QCD + QED has exact center symmetry $\rightarrow$ interesting for thermodynamics
- Sampling Polyakov loop sectors challenging, even in small volumes
- Only strong QED coupling takes P to pure SU(3) values

Outlook

- Improved algorithms: local updates, parallel tempering,...
- Non-compact QED actions
- Continuum limit