Unscreened potential and charmonium width from lattice QCD

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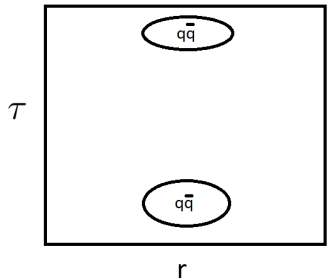
Physical Measurements on the lattice

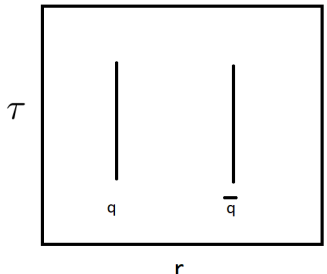
Charmonium at $T > 0$

- Energy of Charmonium states and their interaction rates extracted from the position and width of their spectral functions
- Relativistic QCD (HISQ) configurations with Wilson Clover Fermions operator used for charmonium

The quark-antiquark potential at $T > 0$

- The finite temperature potential, used for modeling the interactions of heavy quark systems in QGP found from the position and width of real-time spectral function, extracted from imaginary time behavior
- The quark-antiquark potential is found from static quarks moving through time





Lattice connection to real-time physics

- All results in this talk use that real-time spectral functions $\rho(\omega)$ can be expressed in an imaginary time formulation on the lattice

$$C(\tau) = \int_{-\infty}^{\infty} \rho(\omega) K(\omega, \tau) d\omega \tag{1}$$

- $C(\tau)$ is the correlation on the lattice
- $\rho(\omega)$ is the real-time spectral function

- $K(\omega, \tau)$ is a kernel that transforms spectral information into correlation information
- For real time $K = \exp(-i\omega t)$
- For imaginary time non-periodic effective-theories measurements $K = \exp(-\omega\tau)$
- For imaginary time periodic measurements $K = \frac{\cosh(\omega\tau - \omega/(2T))}{\sinh(\omega/(2T))}$

The potential picture at finite temperature

- Since real time quantum field theory is not possible to solve, a potential picture for heavy quarks is used instead

$$i\partial_t W(r,t) = V(r,t)W(r,t) \tag{2}$$
$$V(r) = \lim_{t\to\infty} V(r,t) \tag{3}$$

- W is the wilson loop
- $V(t,r)$ is the effective potential as a function of distance between a static quark and an anti quark

$$W(r,t) = \int d\omega \rho_W(r,\omega) e^{-it\omega} \tag{4}$$

- If ρ_W is a Lorentzian the imaginary part of the potential becomes constant in t
- If ρ_W is a gaussian the imaginary part of the potential becomes linear in t

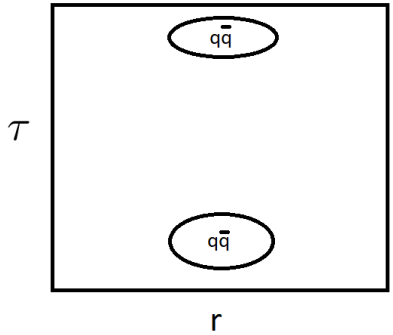
Charmonium at $T > 0$

- We want to learn about charmoniums behavior in high temperature mediums
- We do this by creating a charmonium state at $\tau = 0$ and then propagate it through imaginary time

$$\int d^3x < O(\tau, x) O^\dagger(0,0) > \quad = \quad C(\tau)$$

$$\int_{-\infty}^{\infty} \rho(\omega) \frac{\cosh[\omega(\tau - 1/(2T))]}{\sinh(\omega/(2T))} d\omega \quad = \quad (5)$$

- We are interested in the zero momentum state, so we integrate over space
- Relativistic QCD (HISQ) configurations with Wilson Clover Fermions operator used for charmonium



Lattice setup

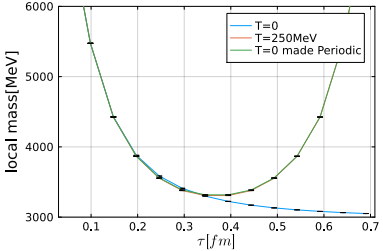
- We use
  - $\beta = 7.825$ with lattice spacing $a = 0.0404fm$ and $\beta = 7.596$ with lattice spacing $a = 0.0493fm$
  - HISQ configurations with $ms/ml = 20$, so almost physical
  - SIMULATeQCD for measuring and generating configurations
  - $N_x = 64$

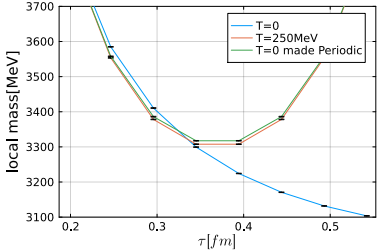
β	Nt	T[MeV]	#conf	#S	S Type	Time/C	T Total[h]	GPU
7.825	16	305	5152	16	S123D	45min	3862	2A100
7.825	18	271	4534	16	S123D	52min	3929	2A100
7.825	20	244	4093	16	S123D	60min	4093	2A100
7.825	22	222	3423	16	S123D	65min	3708	2A100
7.825	24	203	1634	16	S123D	50min	1362	4A100
7.825	28	174	3368	16	S123D	61min	3425	4A100
7.825	32	153	2125	16	S123D	58min	2054	4A100
7.825	64	76	1155	32	S123D	281min	5409	8V100

β	Nt	T [MeV]	configurations	sources	source Type
7.596	16	250	1800	16	S123D
7.596	18	222	3494	16	S123D
7.596	20	200	2805	16	S123D
7.596	22	182	1563	16	S123D
7.596	24	167	3037	16	S123D
7.596	64	63	1126	16	S123D

The Inversion problem

- Extracting $\rho(\omega)$ from $C(\tau)$ is an inversion problem
- $\int d^3x < O(\tau, x)O^\dagger(0,0) > \; = \; C(\tau) = \int_{-\infty}^{\infty} \rho(\omega) \frac{\cosh[\omega(\tau - 1/(2T))]}{\sinh(\omega/(2T))} d\omega$



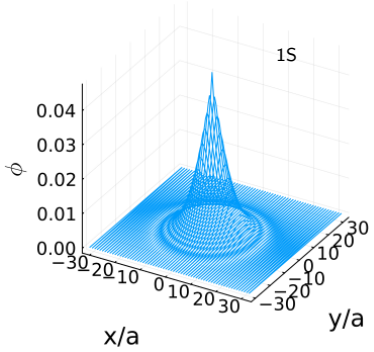


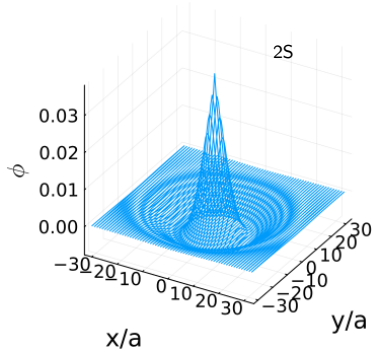
- Correlators shown in terms of effective or local mass, which is the mass explaining the change in the correlator, assuming a single exponential $\partial_\tau \log(C(\tau))$ (effective mass) or assuming a single $\cosh$ function (local mass)
- Zero temperature result approaches plateau of physical mass of $\eta_c(1S)$
- Almost no difference in finite and zero temperature
- Even less difference when making $T = 0$ periodic like finite temperature

Extended Sources

- Sources that approximate the shape of the state, improve signal, less overlap with other states
- Sources are calculated from a discretized schroedinger equation with a confining potential that reproduces zero temperature spectrum

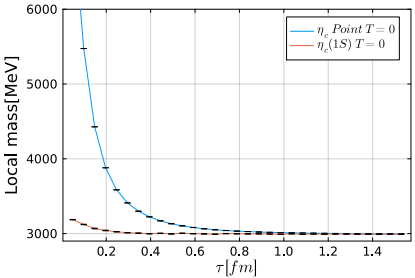
$$O_i(\mathbf{x}, t) = \sum_{\mathbf{r}} \phi_i(\mathbf{r}) \bar{q}(\mathbf{x} + \mathbf{r}, t) \Gamma q(\mathbf{x}, t) \tag{6}$$



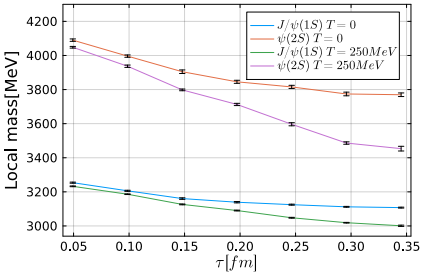


Effect of extended sources

- Extended sources projects into specific states better
- Plateaus of the local mass $M_{eff} ->$ Mass state exists in $\rho(\omega)$



- Slight difference in local mass at zero and finite temperature
- Smearing at zero and finite temperature has to be the same
- 1 Hyp smearing used



Zero temperature subtraction

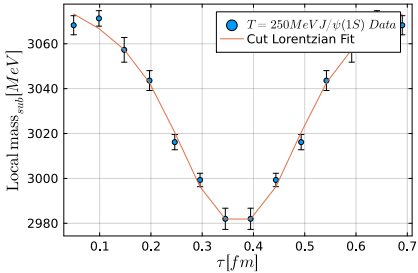
Zero temperature subtraction

- Find continuum at zero temperature and subtract it from finite T correlator

$$C_{cont}(\tau) = C_{T=0}(\tau) - a_{T=0}\exp(-\tau m_{T=0}) \quad (7)$$

$$C_{sub}(\tau) = C_T(\tau) - C_{cont}(\tau) - C_{cont}(N_\tau - \tau) \quad (8)$$

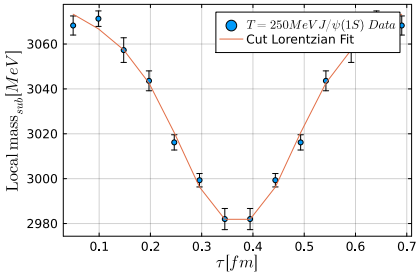
- This is done, since the effective mass at small τ is very similar for all temperatures, with the lowest temperature always having the largest mass
- This is an approximation, useful since data is not too sensitive to small changes in the spectral function, at energies high above the lowest energy state.
- Works best when the overlap with other states is small

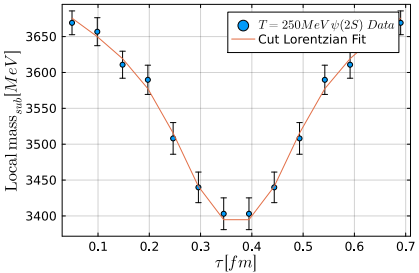


Extracting $\rho(\omega, r)$

- Subtracted zero temperature correlator fitted by Lorentzian cut at 4 times the width
- Delta function at $\omega = 0$ needed for vector channel
- Wilson Clover fermions on HISQ configurations
- Excited states obtained using wave functions

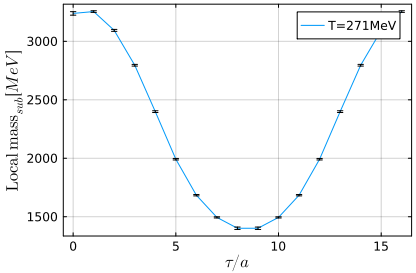
$$\rho(\omega) \quad = \quad \frac{a_1}{(\omega - \omega_0)^2 + \Gamma^2} \Theta(4\Gamma - |\omega - \omega_0|) + a_2 \delta(\omega) \tag{9}$$

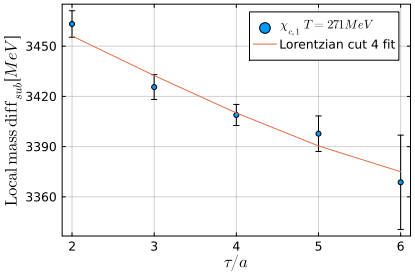




Extracting $\rho(\omega, r)$ P states

- P waves dominated by large zero mode contribution
- Look at derivative of correlator instead, since $\partial_\tau Constant = 0$
- Large fluctuations in the difference
- Local mass definition, but with difference instead

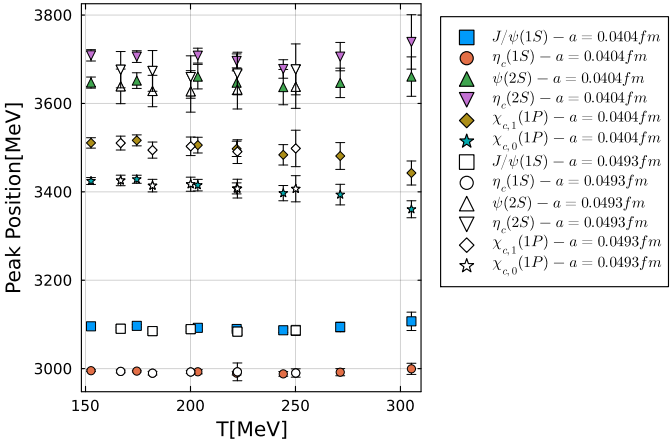




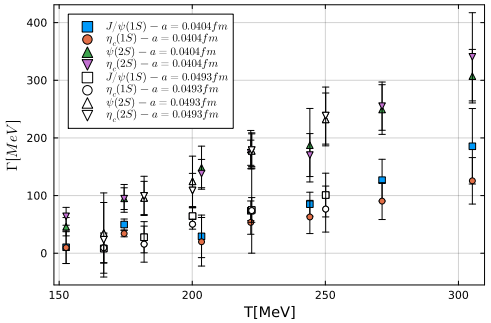
Mass of Bottomonia and Charmonium states at finite temperature

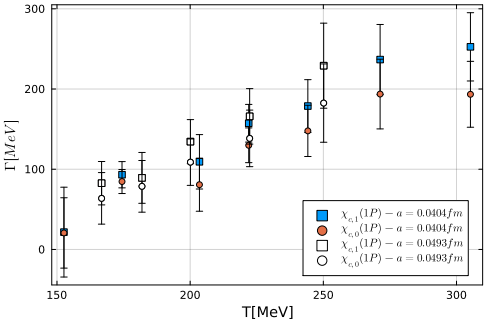
Extended Sources Result

- Results for charmonium with relativistic quarks

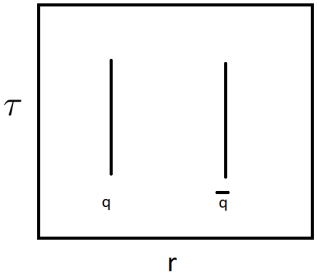


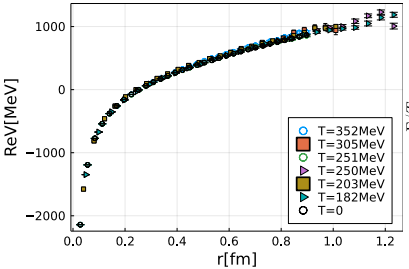
Spectral Width Γ

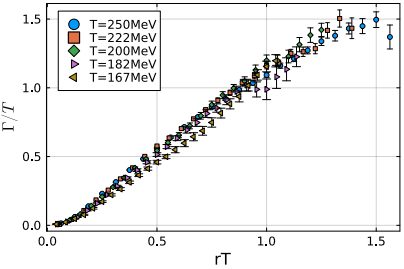




- Position $Re(V(r,T))$ and width $\Gamma(r,T)$ of spectral function for infinitely heavy quark and anti-quark separated by distance r [arXiv:2308.16587]



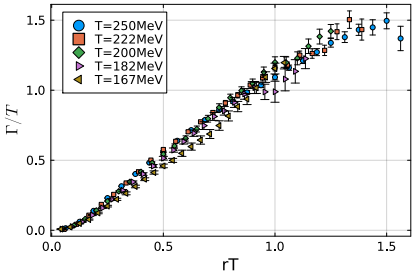


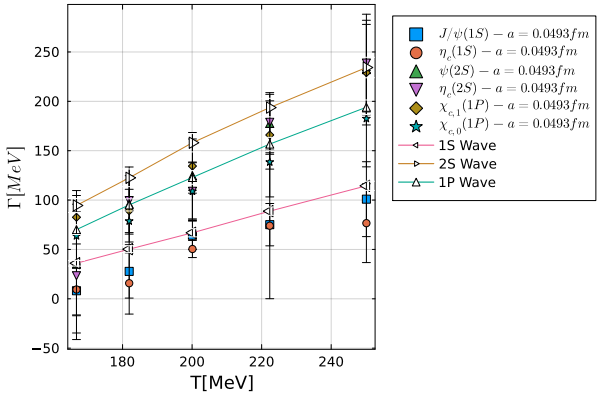


The wave function model and relation to Thermal Width of Quarkonium

- $|\phi_i|^2$ is the probability to find a quark and anti-quark separated by distance r in a potential model
- Estimate Width of Charmonium $\Gamma_i(T)$ from the distribution of a quark and antiquark

$$\Gamma_i(T) \quad = \quad \int d^3x \Gamma(r,T)|\phi_i(x)|^2 \quad (10)$$





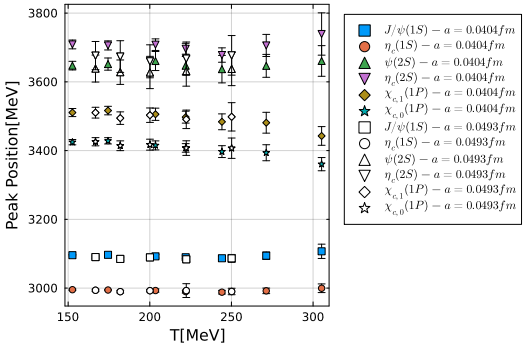
Summary

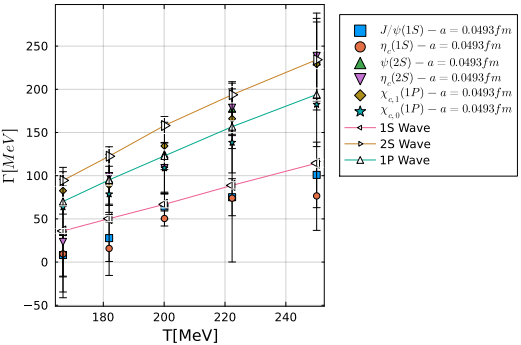
Charmonium at $T > 0$

- No shift in Energy
- Slight drop in mass for P states above 300MeV, however large error bars

Width at $T > 0$

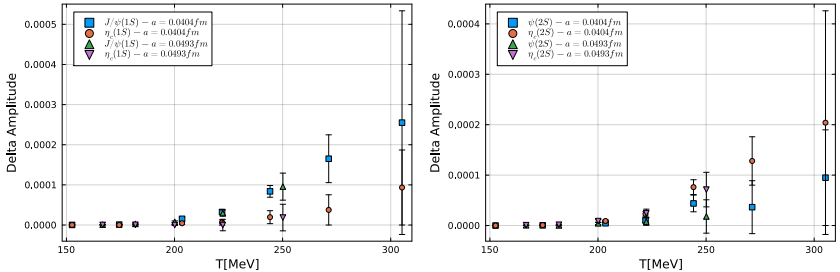
- Width (interaction rates) grows with the physical size of the state
- Width of quark-antiquark potential grows with radius and with temperature





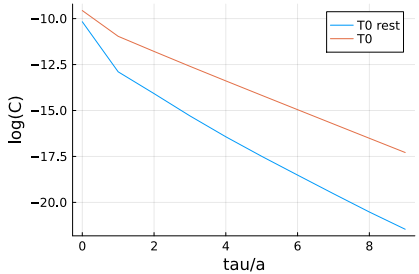
Delta function at zero

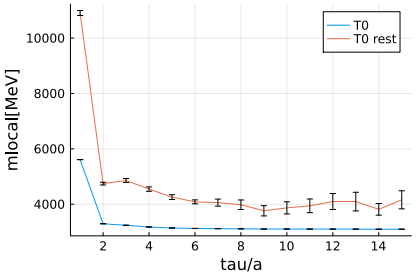
- Ratio of amplitude between cut lorentzian and delta function at zero



Continuum Subtracted Charmonium

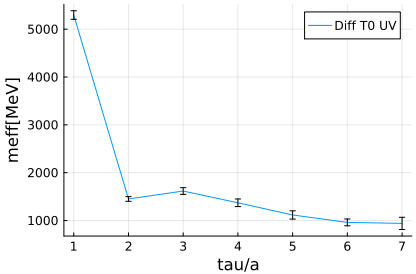
- $T = 0$ fitted with exponential where effective mass has reached plateau
- Fitted exp subtracted from T0 Correlator
- Excited states around 1GeV higher than $T = 0$

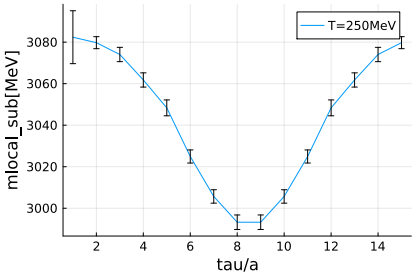


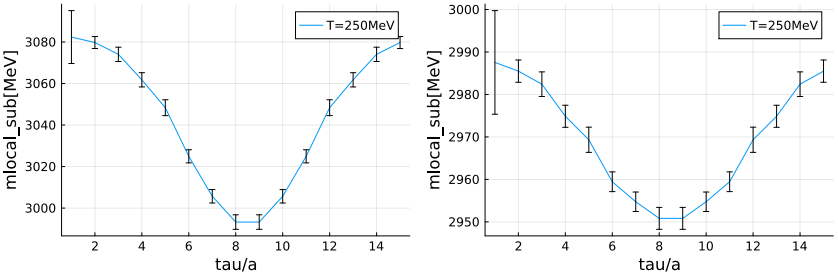


Continuum Subtracted Charmonium 2

- (left) How much the $T = 0$ UV part is higher than $T = 0$
- (right) UV part of $T = 0$ subtracted from finite temperature from forward and backwards, assuming mass large enough that effects from period behavior are well suppressed.





Vector vs Pseudo Scaler

- Vector Left, Pseudo Scaler Right
- Splitting around 95MeV at shown lattice spacing, slightly lower than physical splitting